Degenerate liquid metallic hydrogen—a microscopic approach

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Migdal's semiphenomenological approach (1962) to two-component degenerate Fermi liquids is generalized to include different particle masses in the two components. The formulas are obtained without allowance for the isotopic invariance of the effective interaction between the particles. Only a potential interaction between the particles is considered, so that the spin response of the system is considerably simplified. The screening of the long-range Coulomb forces is introduced by assuming that the zeroth Fourier transform of the interparticle interaction has an extremely large but finite modulus. The limiting transition to infinity is made only in the final stages of the calculation. The results are confirmed of the phenomenological approach of Oliva and Ashcroft, and new results are obtained. A Ward identity hitherto unknown even for one-component systems is derived for the spin current components.

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1. INTRODUCTION. SELECTION RULES FOR TWO-PARTICLE GREEN FUNCTIONS

The transformation of hydrogen into a metal at appropriate high pressures and low temperatures is an obvious consequence of quantum mechanics. The question of the structure of metallic hydrogen is completely open from the experimental viewpoint and is at least partly open theoretically. The main source of the theoretical uncertainty is the inaccuracy of the calculations of the correlation energy of an intermediate-density gas.[1] A quasiliquid character in one direction and a solid-state triangular structure in the perpendicular plane were obtained at zero pressure in the calculations of Yu. Kagan's group.[2] At higher pressures this group goes over into others, including triangular ones.[3] Calculations by Ashcroft's group[4,5] lead to the conclusion that at a Wigner-Seitz parameter $r_0 \approx 1.6$ a liquid metal phase should appear, although at higher densities, $r_0 \approx 0.8$–1.6, the solid-metal phase is favored. It is worth while noticing that the liquid-metal phase is preferred at astrophysically high densities, since the energy of the lattice zero-point vibrations increases with density more rapidly than the potential energy of the system.[6]

In recent work, Oliva and Ashcroft used a model of a degenerate electron-proton liquid to investigate its static properties and to derive stability conditions,[7] as well as to investigate its kinetic and transport properties.[8] They used the Landau phenomenological approach.[9]

A two-component Fermi liquid (FL) as a model of nuclear matter was introduced by Migdal.[10,11] The initial equations of Ref. 10 are valid for any two-component FL, but the final conclusions formulated in Ref. 11 are valid only for systems with isotopic invariance. In potential interaction, which is a very good approximation in our case at astrophysical densities, the spin of each component is also a conserved quantity. This leads to specific selection rule—diagonality with respect to particle species—for the spin-exchange part of the two-particle Green function, and hence also for the spin-exchange part of the quasiparticle interaction. This selection rule for quasiparticle interaction was noted by Oliva and Ashcroft,[7] although its derivation was not wholly consistent.

The restriction imposed by isotopic invariance in the derivation of the stability conditions was lifted by us in Ref. 12. This leads to an important analogy with the theory of a ferromagnetic FL, if the particle species in the ferromagnetic FL is taken to be its spin. In Ref. 12 was used the proof[13] of the conditions of stability of a ferromagnetic FL. In addition, an analog of the Leggett equation[14] was obtained for a two-component system.[12] We add that our stability conditions for a two-component FL coincide with the conditions of Oliva and Ashcroft.

Our purpose is to generalize and supplement Migdal's results.[11] The generalization is due mainly with allowance for the difference between the particle masses of the two components. Our approach is very close to that of Ref. 10, and is also close to the approach of Nozieres and Luttinger.[15]

The method developed in Ref. 11 was adapted to a treatment of long-range Coulomb forces. This was done by a procedure introduced in Ref. 15: all the quantities of interest to us are expressed in terms of a sum of proper diagrams. For a single-component charge FL one must introduce a charge of opposite sign, uniformly spread over the entire volume of the system and canceling completely the charge of the particles. In our case there is no need for such an artifice, since the charges in two-component thermodynamic systems are mutually cancelled. The terms connected with the closed loops of the particle lines cancel one another if it is recognized that $V_{ff}(r) + V_{ff}(r) = 0$, where $V_{fg}(r)$ is the potential of the interaction of particles of species f and g, and the variables f and g run through values ± 1, with $\bar{f} \equiv -f$. To avoid infinities in the intermediate calculations we must assume that $U_{fg}(0)$, which the fourier transform $V_{fg}(r)$ at $k = 0$, has a very large modulus but is finite.

At finite values of $U_{fg}(0)(f,g = \pm 1)$ we can use all the relations that are characteristic of uncharged systems, expressing the improper diagrams in terms of proper ones, only

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for the sake of performing the subsequent limiting transition $U_{fg}(0) \to \pm\infty$ (plus at $f=g$, minus at $f=g$), so that $U_{ff}(0) + U_{\bar{f}\bar{f}}(0) = 0$. Therefore the theory of a charged two-component FL is a variant of the theory of an uncharged two-component FL. This is not the case of one-component FL. We start from the theory of an uncharged FL and obtain the properties of a charged FL via a special transition to the limit. The present paper deals only with normal nonsuperconducting systems, although electronic superconductivity of liquid metallic hydrogen is apparently possible, and furthermore at relatively high temperatures.[7] In addition, at $r_0 \approx 1.6$ there can arise a ferromagnetic state in a proton liquid.[7] For the system to be degenerate at $r_0 \approx 1.6$, temperatures not higher than several kelvin are needed.

We consider the interaction potential

$$\frac{1}{2}\sum_{fg\alpha\beta}\int \Psi_{f\alpha}^{+}(\mathbf{r})\,\Psi_{g\beta}^{+}(\mathbf{r}')\,V_{fg}(|\mathbf{r}-\mathbf{r}'|)\,\Psi_{g\beta}(\mathbf{r}')\,\Psi_{f\alpha}(\mathbf{r})\,d^3\mathbf{r}\,d^3\mathbf{r}',$$

where $\Psi_{f\alpha}(\mathbf{r})$ is the second-quantized operator of the field of the particles of species f at the point $\mathbf{r}$ with spin $\alpha(\hbar/2\equiv 1, \alpha = \pm 1)$; for liquid metallic hydrogen $V_{fg}(r) = fge^2/r$, $f, g = \pm 1$, and e is the elementary charge. In systems with this interaction the spin of each particles species is separately conserved, i.e., we have

$$\sum_{\alpha\beta}\sigma_{\alpha\beta}{}^{a}\int \Psi_{f\alpha}^{+}(\mathbf{r})\,\Psi_{f\beta}(\mathbf{r})\,d^3\mathbf{r},$$

where $\sigma_{\alpha\beta}{}^{a}$ denotes the a-th Pauli matrix, which commutes with the Hamiltonian, while $a = x, y, z$ (or 1, 2, 3). The two-particle Green function is defined as $\langle T\Psi_3\Psi_4\Psi_1{}^{+}\Psi_2{}^{+}\rangle$, where T stands for time ordering and the subscript of Ψ is the set of variables, the arguments of Ψ in the Heisenberg representation: f and α running through two values and the Galilean space-time vector $X_n \equiv t_n, x_n$. The angle bracket $\langle \ldots \rangle$ denotes averaging over the ground state of the system. From the Hamiltonian responsible for the temporal evolution of the operators Ψ we have subtracted the term $\mu_1 N_1 + \mu_{\bar{1}} N \bar{1}$, where μ_f and N_f are respectively the chemical potential and the number of particles of species f. Since the number of particles of each species is preserved, we need consider, without loss of generality, only the quantity

$$K_{\alpha\beta,\gamma\delta}^{fg}(X_3X_4;X_1X_2) \equiv \langle T\Psi_{f\alpha}(X_3)\,\Psi_{g\beta}(X_4)\,\Psi_{f\gamma}^{+}(X_1)\,\Psi_{g\delta}^{+}(X_2)\rangle.$$

For both a normal and a superfluid system, this quantity is invariant to spin rotation. From this we get

$$K_{\alpha\beta,\gamma\delta}^{fg}(X_n) = \delta_{\alpha\gamma}\delta_{\beta\delta}K_s{}^{fg}(X_n) + (\sigma_{\alpha\gamma}\sigma_{\beta\delta})\,K_a{}^{fg}(X_n), \tag{1}$$

where $K_{a,s}^{fg}(X_n)$ are scalar functions of their arguments. The Hamiltonian considered is invariant to the following gauge transformation:

$$\Psi_{f\alpha} = \exp(i\varphi_{f\alpha})\,\Psi_{f\alpha}{}', \qquad \Psi_{f\alpha}{}^{+} = \exp(-i\varphi_{f\alpha})\,\Psi_{f\alpha}{}^{+\prime},$$

where $\varphi_{f\alpha}$ is a real phase that depends on f and α in arbitrary fashion. The function $K_{\alpha\beta,\gamma\delta}^{fg}(X_n)$ is also invariant to this transformation. From this we find that

$$K_{\alpha\beta,\gamma\delta}^{fg}(X_n) = \exp[i(\varphi_{f\alpha}+\varphi_{g\beta}-\varphi_{f\gamma}-\varphi_{g\delta})]\,K_{\alpha\beta,\gamma\delta}^{fg}(X_n).$$

This leads to the relation $K_a{}^{fg} = K_a{}^{f}\delta_{fg}$, i.e., to a specific selection rule.

For a ferromagnetic system or for one strongly polarized in spin, the function $K_{\alpha\beta,\gamma\delta}^{fg}$ is invariant only to spin rotation about the magnetization axis. Expressing this function in terms of the spin invariants, we get (see Ref. 16)

$$\begin{aligned} K_{\alpha\beta,\gamma\delta}^{fg}(X_n) &= \delta_{\alpha\gamma}\delta_{\beta\delta}K_s{}^{fg}(X_n) + \delta_{\alpha\gamma}\delta_{\beta\delta}\beta K_m{}^{fg}(X_n) + \delta_{\alpha\gamma}\alpha\delta_{\beta\delta}\tilde{K}_m{}^{fg}(X_n) \\ &\quad + \delta_{\alpha\gamma}\alpha\delta_{\beta\delta}\beta K_{\parallel}{}^{fg}(X_n) + (\boldsymbol{\sigma}_{\alpha\gamma}{}^{\perp}\boldsymbol{\sigma}_{\beta\delta}{}^{\perp})\,K_{\perp}{}^{fg}(X_n), \end{aligned}$$

if the spin quantization axis coincides with the magnetization vector. Here $K_s{}^{fg}$, $K_m{}^{fg}$, $\tilde{K}_m{}^{fg}$, $K_{\parallel}{}^{fg}$, $K_{\perp}{}^{fg}$ denote scalar functions, and $\boldsymbol{\sigma}^{\perp}$ is the component, perpendicular to the magnetization axis, of the pseudovector $\boldsymbol{\sigma}$ made up of Pauli matrices. Using the foregoing gauge transformation, we find that

$$K_{\perp}{}^{fg} = \delta_{fg}K_{\perp}{}^{f}.$$

We shall use a diagram technique with the rules introduced in Refs. 17 and 18 for the correspondences between the diagrams and the analytic expressions.

2. BASIC RELATIONS FOR AN UNCHARGED TWO-COMPONENT FERMI LIQUID

We generalize Migdal's approach,[10] regarding this section as an introduction to the problem of charged two-component FL. We consider the Fourier transform of the two-particle Green function

$$\begin{aligned} &\int K_{\alpha\beta,\gamma\delta}^{fg}(X_3X_4;X_1X_2)\exp[i(P_3X_3+P_4X_4-P_1X_1-P_2X_2)] \\ &\quad \times d^4X_1\,d^4X_2\,d^4X_3\,d^4X_4 \\ &\quad = (2\pi)^4\,\delta^{(4)}(P_3+P_4-P_1-P_2)\,K_{\alpha\beta,\gamma\delta}^{fg}(P_3P_4;P_1P_2), \end{aligned}$$

where P_n is a four-momentum with zero frequency reckoned from μ_g for even n and from μ_f for odd n. The scalar product XP is specified by the expression $\omega t - \mathbf{p}\cdot\mathbf{x}$. The function $K^{fg}(P_n)$ is meaningful only if $P_1 + P_2 = P_3 + P_4$. We define

$$K^{fg}(K;P_1P_2) = K^{fg}(P_1+{}^1/_2K,\ P_2-{}^1/_2K;\ P_1-{}^1/_2K,\ P_2+{}^1/_2K),$$

where 4-vector describing the transfer of energy and momentum $(\omega, \mathbf{k})$. The expression of the two-particle Green function in the momentum representation $K^{fg}(K;P_1P_2)$ in terms of the two-particle vertex part $\Gamma^{fg}(K;P_1P_2)$ is of the form[17,18]

$$\begin{aligned} K_{\alpha\beta,\gamma\delta}^{fg}(K;P_1P_2) &= (2\pi)^4[\delta^{(4)}(K)\,G_f(P_1)\,G_g(P_2)\,\delta_{\alpha\gamma}\delta_{\beta\gamma} \\ &\quad - \delta^{(4)}(P_1-P_2)\,\delta_{fg}\delta_{\alpha\delta}\delta_{\beta\gamma}G_f(P_1-{}^1/_2K)\,G_f(P_1+{}^1/_2K)] \\ &\quad + iG_f(P_1+{}^1/_2K)\,G_f(P_1-{}^1/_2K)\,\Gamma_{\alpha\beta,\gamma\delta}^{fg} \\ &\quad \times (K;P_1P_2)\,G_g(P_2+{}^1/_2K)\,G_g(P_2-{}^1/_2K), \end{aligned}$$

where $G_f(P)$ is the single-particle Green function of the particles of species f in the momentum representation (cf. Refs. 17, 18). According to this formula the function Γ^{fg} has the same spin structure and satisfies the same selection rules as the function K^{fg}. The function Γ^{fg} is the sum of all the connected diagrams with two incoming and outgoing particle lines. We introduce the two-particle irreducible function Γ—the sum of all the diagrams that cannot be divided into two nonconnected diagrams by dissecting two particle lines. We use the symbol $\bar{\Gamma}$ for this function. We have (cf. Refs. 10, 11, 17, 18)

$$\Gamma^{fg}_{\alpha\beta,\gamma\delta}(K;P_1P_2)$$

$$=\bar{\Gamma}^{fg}_{\alpha\beta,\gamma\delta}(K;P_1P_2)-i\sum_{h=\pm1}\int\frac{d^4Q}{(2\pi)^4}\bar{\Gamma}^{fh}_{\alpha\mu,\gamma\nu}(K;P_1Q)$$
$$\times G_h(Q+{}^1/{}_2K)\,G_h(Q-{}^1/{}_2K)\,\Gamma^{hg}_{\nu\beta,\mu\delta}(K;QP_2); \tag{2}$$

summation over repeated spin indices is implied. We confine ourselves hereafter to small 4-momentum transfers. In this case we neglect the dependence of the function $\bar{\Gamma}$ on K. Expressing the functions Γ^{fg} and $\bar{\Gamma}^{fg}$ in terms of their parts that are direct and exchange in spin, as was done with the function K^{fg}, we find that only those parts of these functions Γ_s^{fg} and $\bar{\Gamma}_s^{fg}$ which are direct in spin have nonvanishing elements with $f=\bar{g}$. From this we get

$$\Gamma^{fg}_{s,a}(K;\,P_1P_2)=\bar{\Gamma}^{fg}_{s,a}(P_1P_2)-2i\sum_{h=\pm1}\int\frac{d^4Q}{(2\pi)^4}$$
$$\times\bar{\Gamma}^{fh}_{s,a}(P_1Q)\,G_h(Q+{}^1/{}_2K)\,G_h(Q-{}^1/{}_2K)\,\Gamma^{hg}_{s,a}(K;QP_2), \tag{3}$$

where $\bar{\Gamma}_a{}^{fg}, \Gamma_a{}^{fg}\propto\delta_{fg}$. It is convenient to rewrite this equation in operator form

$$\Gamma_{s,\,a}(K)=\bar{\Gamma}_{s,\,a}+\bar{\Gamma}_{s,\,a}(GG)\,(K)\,\Gamma_{s,\,a}(K)$$

(cf. Refs. 10, 11, 15). At low energy-momentum transfers the product of single-particle Green functions can be divided into a regular part that is independent of K and a singular K-dependent part:

$$G_f(P+{}^1/{}_2K)\,G_f(P-{}^1/{}_2K)$$
$$=\varphi_f(P)+2\pi iZ_f{}^2\delta(p_0)\,\delta(p-p_F)\,[\omega-v_f\hat{\mathbf{p}}\mathbf{k}+i\delta\omega]^{-1}, \tag{4}$$

(cf., e.g., Refs. 10, 11, 15, 17, 18). Here p_f and v_f denote respectively the Fermi momentum and the Fermi velocity of the particles of species f; Z_f is the jump of the density of these particles on their Fermi surfaces; the 4-momentum $P\equiv(p_0, \mathbf{p})$, the unit vector $\mathbf{p}=\mathbf{p}/p$, and p_0 are reckoned from the quantity μ_f, and $\delta=0^+$. The diagonal operators that correspond to multiplication by the first and second terms of the right-hand side of (4) will be designated respectively by φ and $\Delta\,(K)$. It will also be convenient to write

$$G_f(P+{}^1/{}_2K)\,G_f(P-{}^1/{}_2K)$$
$$=\bar{\varphi}_f(P)+2\pi iZ_f{}^2\delta(p_0)\,\delta(p-p_f)\,\omega v_f{}^{-1}[\omega-v_f\hat{\mathbf{p}}\mathbf{k}+i\delta\omega]^{-1}, \tag{5}$$

where

$$\bar{\varphi}_f(P)\equiv\varphi_f(P)-2\pi iZ_f{}^2\delta(p_0)\,\delta(p-p_f)/v_f.$$

In matrix notation these terms will be designated $\bar{\varphi}$ and $\bar{\Delta}\,(K)$. According to (4) and (5) we have

$$\varphi_f(P)=\lim_{\omega\to0}\,\lim_{k\to0}G_f(P+{}^1/{}_2K)\,G_f(P-{}^1/{}_2K),$$
$$\bar{\varphi}_f(P)=\lim_{k\to0}\,\lim_{\omega\to0}G_f(P+{}^1/{}_2K)\,G_f(P-{}^1/{}_2K); \tag{6}$$

these limits will hereafter be defined as ω- and k-limits. As applied to the function $\Gamma\,(K)$ these limits determine the functions ${}^\omega\Gamma$ and ${}^k\Gamma$. In operator form we have

$${}^\omega\Gamma_{s,\,a}=\bar{\Gamma}_{s,\,a}+\bar{\Gamma}_{s,\,a}\varphi\,{}^\omega\Gamma_{s,\,a},\qquad {}^k\Gamma_{s,\,a}=\bar{\Gamma}_{s,\,a}+\bar{\Gamma}_{s,\,a}\varphi\,{}^k\Gamma_{s,\,a}. \tag{7}$$

With the aid of algebraic transformations we obtain from (2) and (7)

$$\Gamma_{s,\,a}(K)={}^\omega\Gamma_{s,\,a}+{}^\omega\Gamma_{s,\,a}\Delta\,(K)\,\Gamma_{s,\,a}(K)$$
$$={}^k\Gamma_{s,\,a}+{}^k\Gamma_{s,\,a}\bar{\Delta}(K)\,\Gamma_{s,\,a}(K). \tag{8}$$

From this we get the relation

$${}^\omega\Gamma_{s,\,a}={}^k\Gamma_{s,\,a}+{}^k\Gamma_{s,\,a}\Delta\,{}^\omega\Gamma_{s,\,a}. \tag{9}$$

Here Δ is $\Delta\,(K)$ in the k-limit or $-\bar{\Delta}\,(K)$ in the ω-limit. The nondimensionalized function $\Gamma\,(K)$ will be designated $f(K)$ and defined in accord with Ref. 10 (cf. also Ref. 13) in the following manner:

$$f^{fg}_{s,a}(K;P_1P_2)=(\nu_f\nu_g)^{1/2}\,\Gamma^{fg}_{s,a}(K;P_1P_2)\,Z_fZ_g, \tag{10}$$

where ν_f is the density of states on the Fermi surface of the particles of species f, and $\nu_f=p_f^2/\nu_f\pi^2$ ($\hbar\equiv1$); a unity-volume system is assumed. Relations (9), rewritten for the dimensionless quantities (10), take the form

$${}^\omega f^{fg}_{a,s}(P_1P_2)={}^kf^{fg}_{a,s}(P_1P_2)+\sum_{h=\pm1}\langle{}^\omega f^{fh}_{a,s}(P_1Q)\,{}^kf^{hg}_{a,s}(QP_2)\rangle_Q, \tag{11}$$

where $Q\equiv(0,p_h\hat{\mathbf{q}})$, is the 4-momentum and the bracket $\langle\ldots\rangle_Q$ denotes averaging over the spherical angles that define the unit vector $\hat{\mathbf{q}}$. At $P_1=(0,p_f\hat{\mathbf{p}})$ and $P_2=(0,p_g\hat{\mathbf{p}}')$ the matrix functions ${}^\omega f$ and ${}^kf$ should depend only on p_f, p_g, and the scalar product $\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}'$. The Landau parameters in our case are the quantities ${}^\omega f^{fg}_{a,s;l}$, $l=0, 1, \ldots$, defined by the relation

$${}^{\omega,k}f^{fg}_{a,s}(\hat{\mathbf{p}}\hat{\mathbf{p}}')=\sum_{l=0}^{\infty}{}^{\omega,k}f^{fg}_{a,s;l}(2l+1)P_l(\hat{\mathbf{p}}\hat{\mathbf{p}}'), \tag{12}$$

where P_l is a Legendre polynomial. In contrast to the usual definition of the Legendre amplitudes, we have introduced the factor $(2l+1)$ in the sum (12). Substituting (12) in (11) we get

$${}^kf^{f}_{a;l}={}^\omega f^{f}_{a;l}(1+{}^\omega f^{f}_{a;l})^{-1},$$
$${}^kf^{gf}_{s;l}=\delta_{gf}-(\delta_{gf}+gf\,{}^\omega f^{\bar{g}\bar{f}}_{s;l})\,[(1+{}^\omega f^{11}_{s,l})\,(1+{}^\omega f^{\bar{1}\bar{1}}_{s,f})-({}^\omega f^{1\bar{1}}_{s,l})^2]^{-1}, \tag{13}$$

where $g,f=\pm1$ and $\bar{g}\equiv-g$; the Landau-parameter matrices are diagonal in the indices f and g:

$${}^{k,\omega}f^{f}_{a;l}\equiv{}^{k,\omega}f^{ff}_{a;l},\quad {}^{k,\omega}f_{s;l}={}^{k,\omega}f_{s;l}.$$

The function

$$\delta_{\alpha\gamma}\delta_{\beta\delta}\,{}^k\Gamma_a{}^{fg}(\hat{\mathbf{p}}\hat{\mathbf{p}}')+(\boldsymbol{\sigma}_{\alpha\gamma}\boldsymbol{\sigma}_{\beta\delta})\,{}^k\,\Gamma_s{}^{fg}(\hat{\mathbf{p}}\hat{\mathbf{p}}')$$

has for two-component systems the meaning of the forward-scattering amplitude of the quasiparticles of species f and g, with corresponding spins and moment $p_f\hat{\mathbf{p}}$ and $p_g\hat{\mathbf{p}}'$ on the Fermi surface (cf. Refs. 17 and 18). Accordingly, this quantity should vanish at $f=g$, $\alpha=\beta=\gamma=\delta$ and $\hat{\mathbf{p}}=\hat{\mathbf{p}}'$ on account of the Fermi statistics (see Refs. 17, 18, as well as 19). Expressing this fact with the aid of the quantities (12) we obtain two sum rules for our system:

$$\sum_{l=0}^{\infty}(2l+1)\,({}^kf^{f}_{a;l}+{}^kf^{ff}_{s;l})=0. \tag{14}$$

To express relations (14) in terms of ω-quantities, a substitution using (13) must be made.

We introduce the vertex functions for the scalar, vector, spinor, and spinor-vector fields. They will be defined, in the spirit of Ref. 16, somewhat differently than in Refs. 10 and 11. We begin with a definition of the free vertices $v^{cf}, 0 \leqslant c \leqslant 7$, $f = \pm 1$. We have

$$v_g^{0f}(\mathbf{p}) = \delta_{fg}, \quad v_g^{af}(\mathbf{p}) = \delta_{fg} p_a / m_f, \quad a = 1, 2, 3;$$
$$v_g^{4f}(\mathbf{p}) = \sigma^z \delta_{fg}, \quad v_g^{a+4,4}(\mathbf{p}) = \sigma^z \delta_{fg} p_a / m_f, \tag{15}$$

where the vertices v^{cf} are proportional at $c < 4$ to a unit spin matrix. The vertex particles dressed by the interparticle interactions are defined in operator form by the relation

$$\mathcal{T}^i(K) = v^i + \Gamma_{s,a}(K)(GG)(K) v^i,$$

or by the equation

$$\mathcal{T}^i(K) = v^i + \bar{\Gamma}_{s,a}(K)(GG)(K) v^i.$$

We have used here one variable i in place of the two c and f, with $0 \leqslant c \leqslant 7$, $f = \pm 1$. More expanded, these relations take the form

$$\mathcal{T}_g^i(K;P) = v_g^i(P) - 2i \sum_{h=\pm 1} \int \frac{d^4Q}{(2\pi)^4} \Gamma_{s,a}^{gh}(K;PQ) G_h(Q - {}^1/_2K)$$
$$\times G_h(Q + {}^1/_2K) v_h^i(Q) = v_g^i(P) - 2i \sum_{h=\pm 1} \int \frac{d^4Q}{(2\pi)^4} \bar{\Gamma}_{s,a}^{gh}(PQ)$$
$$\times G_h(Q - {}^1/_2K) G_h(Q + {}^1/_2K) \mathcal{T}_h^i(K;Q). \tag{16}$$

It is necessary to substitute here Γ_a or $\bar{\Gamma}_a$ if $c > 3$, and Γ_s or $\bar{\Gamma}_s$ if $c \leqslant 3$ $(i = c, f)$; there is no summation over the spin variables in these equations.

Taking the ω- and k-limits in (16), we find with the aid of (4) and (5)

$${}^\omega\mathcal{T}^i = v^i + {}^\omega\Gamma_{s,a} \varphi v^i = v^i + \bar{\Gamma}_{s,a} \varphi {}^\omega\mathcal{T}^i,$$
$${}^k\mathcal{T}^i = v^i + {}^k\Gamma_{s,a} \bar{\varphi} v^i = v^i + \bar{\Gamma}_{s,a} \bar{\varphi} {}^k\mathcal{T}^i. \tag{17}$$

Using the algebraic methods developed in Refs. 10 and 15, we obtain from (16), (17) and (7), (8), with account taken of (4) and (5),

$$\mathcal{T}^i(K) = {}^\omega\mathcal{T}^i + \Gamma_{s,a}(K) \Delta(K) {}^\omega\mathcal{T}^i = {}^\omega\mathcal{T}^i + {}^\omega\Gamma_{s,a} \Delta(K) \mathcal{T}^i(K),$$
$$\mathcal{T}^i(K) = {}^k\mathcal{T}^i + \Gamma_{s,a}(K) \bar{\Delta}(K) {}^k\mathcal{T}^i = {}^k\mathcal{T}^i + {}^k\Gamma_{s,a} \bar{\Delta}(K) \mathcal{T}^i(K). \tag{18}$$

We introduce correlation functions developed by

$$S^{i,j}(K) = -i \sum_{g=\pm 1} \mathrm{Tr} \int \frac{d^4P}{(2\pi)^4} [v_g^i(P) G_g(P + {}^1/_2K) G_g$$
$$\times (P - {}^1/_2K) \mathcal{T}_g^j(K;P)], \tag{19}$$

where Tr stands for the trace over the spin indices (cf. Ref. 15). It can be easily seen that $S^{ij} = S^{ji}$ by virtue of Eq. (16) and of the symmetry of the function Γ. The quantity $S^{ij}(k)$ can differ from zero at $i = c, f$ and $j = d, g$ only when we have simultaneously $c < 4$ and $d < 4$ or $c \geqslant 4$ and $d \geqslant 4$, and in the last case furthermore if $f = g$. If the correlation of the orbital vertices $(c, d < 4)$ or of the spin vertices $(c, d \geqslant 4)$ is considered, the Tr symbol can be replaced by the factor 2. We shall consider hereafter only such cases. From (4), (16), (18), and (19) we obtain

$$S^{ij}(K) = {}^\omega S^{ij} - 2i \sum_{g=\pm 1} \int \frac{d^4P}{(2\pi)^4} {}^\omega\mathcal{T}_g^i(P) \Delta_g(K;P) {}^\omega\mathcal{T}_g^i(P)$$
$$- 4 \sum_{g,h\pm 1} \int \frac{d^4P\, d^4Q}{(2\pi)^8} {}^\omega\mathcal{T}_g^i(P) \Delta_g(K;P) \Gamma_{s,a}^{gh}$$
$$\times (K;PQ) \Delta_h(K;Q)\, {}^\omega\mathcal{T}_h^j(K;Q). \tag{20}$$

Applying in the same manner Eq. (16) and the third equation of (18) to Eq. (19) we obtain

$$S^{ij}(K) = {}^k S^{ij} - 2i \sum_{g=\pm 1} \int \frac{d^4P}{(2\pi)^4} {}^k\mathcal{T}_g^i(P) \bar{\Delta}_g(K;P)\, {}^k\mathcal{T}_g^i(P)$$
$$-4 \sum_{g,h=\pm 1} \int \frac{d^4P\, d^4Q}{(2\pi)^8} {}^k\mathcal{T}_g^i(P) \bar{\Delta}_g(K;P) \Gamma_{s,a}^{gh}$$
$$\times (K;PQ)\, \bar{\Delta}_h(K;P) {}^k\mathcal{T}_h^j(P). \tag{21}$$

We made use of Eq. (5) (cf. Ref. 15). Relations (20) and (21) complete the algebraic part of the relations of the theory.

3. WARD IDENTITIES AND THEIR COROLLARIES FOR THE VERTEX AND CORRELATION FUNCTIONS OF AN UNCHARGED TWO-COMPONENT FL

The Ward identities for two-component FL were established by Migdal.[10,11] We shall express them in terms of our system of vertex functions (15), taking into account the mass difference of the two components, which was neglected in Refs. 10 and 11. In addition, we establish the Ward identity for the vertex of the spin current, i.e., $\mathcal{T}_g^{cf}(P)$ at $c > 4$, which was hitherto apparently unknown. We begin with the identities that follow from gauge invariance. For our system, the x-dependent phase function can also depend arbitrarily on the spin and on the particle species. If the phase function is independent of spin, the response to such a field can be expressed by a combination of vertices with $i = 0, f$ and $i = a, f$, with $a = 1, 2, 3$. Using a procedure similar to that used in Ref. 18, we obtain

$$\delta_{fg} \left[\omega \frac{\partial}{\partial p_0} + (\mathbf{k} \nabla_\mathbf{p}) \right] G_g^{-1}(P)$$
$$= \omega \mathcal{T}_g^{0f}(K;P) - \sum_{a=1} k_a \mathcal{T}_g^{af}(K;P). \tag{22}$$

If the phase function is odd in the spin variable, then

$$\sigma^z \delta_{fg} \left[\omega \frac{\partial}{\partial \eta} + (\mathbf{k} \nabla_\mathbf{p}) \right] G_g^{-1}(P)$$
$$= \omega \mathcal{T}_g^{4f}(K;P) - \sum_{a=1}^{3} k_a \mathcal{T}_g^{4+a,f}(K;P). \tag{23}$$

In the ω- and k-limits we obtain from (22)

$${}^\omega\mathcal{T}_g^{0f}(P) = \delta_{fg} (\partial G_g^{-1}(P) / \partial p_0), \tag{24}$$
$${}^k\mathcal{T}_g^{af}(P) = -\delta_{fg} (\partial G_g^{-1}(P) / \partial p_a); \quad a = 1, 2, 3,$$

and similarly from (23)

$$\begin{aligned} {}^{\omega}\mathcal{T}_g^{4f}(P) &= \delta_{fg}\sigma^z(\partial G_g^{-1}(P)/\partial p_0), \\ {}^{k}\mathcal{T}_g^{4+a,f}(P) &= -\delta_{fg}\sigma^z(\partial G_g^{-1}(P)/\partial p_a), \quad a=1,2,3. \end{aligned} \tag{25}$$

Using (18) we can verify that Eq. (22) is satisfied identically by virtue of the relation

$$\omega\Delta_g(K;\ Q)=v_g(\mathbf{k}\hat{\mathbf{q}})\,\bar{\Delta}_g(K;\ Q).$$

In the proof we use the form of the pole part of the Green function, i.e.,

$$G_f(P)\approx Z_f/[p_0-v_j(p-p_f)+ip_0\delta], \qquad \delta=0^+.$$

By performing similar transformations with identities (5) we can prove that relation (23) is also identically satisfied. Relations (22) can be rewritten with the aid of (24) in the form

$$\begin{aligned} &\omega\mathcal{T}_g^{0f}(K;P)-\sum_{a=1}^{3}k_a\mathcal{T}_g^{af}(K;P) \\ &\quad=\omega\,{}^{\omega}\mathcal{T}_g^{0f}(P)-\sum_{a=1}^{3}k_a\,{}^{k}\mathcal{T}_g^{af}(P). \end{aligned} \tag{26}$$

From (23) and (25) we obtain similarly

$$\begin{aligned} &\omega\mathcal{T}_g^{4f}(K;P)-\sum_{a=1}^{3}k_a\mathcal{T}_g^{4+a,f}(K;P) \\ &\quad=\omega\,{}^{\omega}\mathcal{T}_g^{4f}(P)-\sum_{a=1}^{3}k_a\,{}^{k}\mathcal{T}_g^{4+a,f}(P). \end{aligned} \tag{27}$$

Introducing a static scalar field $\delta\varphi_j$ that varies slowly in space, and recognizing that in such a case $\mu_f+\delta\varphi_j=\text{const}$, we get

$${}^{k}\mathcal{T}_g^{0f}(P)=\partial G_g^{-1}(P)/\partial\mu_f. \tag{28}$$

We now establish the Ward identity that follows from the conservation of the total momentum. A transformation to a reference frame that moves with infinitesimal velocity $\delta\mathbf{w}$ and varies slowly with time leads to perturbation of the system Hamiltonian by a term $i(\delta\mathbf{w}\nabla)$ that acts equally on both components. This leads in turn to a change $p_0\to p_0+\mathbf{p}\cdot\delta\mathbf{w}$ of the argument of the single-particle Green function. The change of the single-particle Green function is therefore

$$\delta G_f(P)=\mathbf{p}\delta\mathbf{w}(\partial G_f(P)/\partial p_0). \tag{29}$$

Expressing this change in terms of the vertices and comparing the results, we get

$$p_a[\partial G_g^{-1}(P)/\partial p_0]=\sum_{f=\pm1}m_f\,{}^{\omega}\mathcal{T}_g^{af}(P). \tag{30}$$

Let us, following Ref. 15, investigate the Ward identities for the correlation functions. By virtue of (24), (25), and (21) we have

$$\begin{gathered} {}^{k}S^{0f,0g}=-\partial N_f/\partial\mu_g=-\partial N_g/\partial\mu_f, \\ {}^{k}S^{4f,4g}=-\delta_{fg}\chi_f/\beta_f^2, \\ {}^{k}S^{af,bg}={}^{k}S^{4+a,f;\ 4+b,g}=-\delta_{gf}\delta_{ab}N_f/m_f; \qquad a,\ b=1,\ 2,\ 3, \end{gathered} \tag{31}$$

where N_f and χ_f are respectively the density of the particles of species f and their specific static spin susceptibility. The remaining quantities ${}^{k}S^{ij}$, $i, j=c, g$; $0\leqslant c\leqslant 7$, $g=\pm1$ are equal to zero. In the ω-limit only the correlation functions ${}^{\omega}S^{af,ag}$ and ${}^{\omega}S^{4+a,f;4+a,g}$, $a=1, 2, 3$ differ from zero. On the other hand, by virtue of (28) we have from (19)

$$\sum_{f=\pm1}m_f\,{}^{\omega}S^{af,ag}=0. \tag{32}$$

The spectral representation of the autocorrelation functions makes them negative at $\omega=0$.[14] Hence ${}^{\omega}S^{4+a,f;\,4+a,f}<0$, and the symmetric matrix ${}^{\omega}S^{af,ag}$, which does not depend on a for isotropic systems, is negatively semi-definite. That this matrix is not definite follows from (32). These inequalities lead to inequalities for the amplitudes (12). From (22) lead to the following equations for the correlation functions

$${}^{\omega}S^{0f,cg}(K)-\sum_{a=1}^{3}k_aS^{af,cg}(K)=\delta_{fg}k_cN_f/m_f, \quad c=0,1,2,3, \quad k_0\equiv0, \tag{33}$$

as can be proved by using (4), (5), (19), (24), and (26), as well as the form of the pole part of the fuanction $G_g(P)$. A similar application of Eqs. (27) and (31) leads to the relations

$${}^{\omega}S^{4f,cf}(K)-\sum_{a=1}^{3}k_aS^{4+a,f;cf}(K)=k_{c-4}N_f/m_f; \quad c=4,5,6,7. \tag{34}$$

It follows from these relations that the continuity equation is automatically satisfied for the induced density and for the induced current of the particles of any component of (33), and also for the induced spin density and spin current of any component of (34). On the other hand it follows from these relations that a field that depends arbitrarily on the spin-species variable and on the spin, but is a 4-gradient, induces no densities or currents. Expressing the response functions of the phenomenological approach[7,8] in the form used in Ref. 15, and comparing them with ours, we can prove that both approaches are identical provided that the effective interaction of the phenomenological approach is taken equal to $Z_fZ_g\,{}^{\omega}\Gamma^{fg}(\mathbf{p}_1\mathbf{p}_2)$.

We introduce, following Ref. 15, the effective vertices ${}^{\omega,k}\tau^i$ on the Fermi surface. They are defined as

$${}^{\omega,k}\tau_g^i(P)=Z_g\,{}^{\omega,k}\mathcal{T}_g^i(P)\big|_{\mathrm{FS}}. \tag{35}$$

Using (24), (25), and the form of the pole part of the Green function, we obtain

$$\begin{gathered} {}^{\omega}\tau_g^{0f}=\delta_{fg}, \quad {}^{\omega}\tau_g^{4f}=\sigma^z\delta_{fg}, \quad {}^{k}\tau_g^{0f}=v_g(\partial p_g/\partial\mu_f), \quad {}^{k}\tau_g^{af}=\delta_{fg}v_f\hat{p}_a, \\ {}^{k}\tau_g^{4+a,f}=\delta_{fg}\sigma^zv_f\hat{p}_a, \quad a=1,2,3. \end{gathered} \tag{36}$$

The vertices ${}^{k}\tau_g^{0f}$, ${}^{k}\tau_f^{4f}$, ${}^{\omega}\tau_g^{af}$ and ${}^{\omega}\tau_f^{4+a,f}$ can be expressed in terms of the quantities in (36) with the aid of Eq. (18) and the Landau amplitudes. Comparing the dual form of the vertices involved in this procedure and using (30) we can obtain equations for the observable quantities of the two-component FL. We shall do this by direct application of (20) and (21) together with (31), (32), and (36). In addition, we write down the formula

$${}^{\omega}\tau_g^{af}=v_f\hat{p}_a[\delta_{fg}+{}^{\omega}f_{s;1}^{fg}(\nu_f/\nu_g)^{1/2}]. \tag{37}$$

Choosing $i=0$ and $f, j=0, g$ in (30), substituting there the corresponding τ from (36), taking the k-limit in (20), and making use of the fact that ${}^{\omega}S^{0f,0g}=0$, we obtain with the aid of (31)

$$\partial N_f/\partial\mu_g=\delta_{fg}\nu_f-(\nu_f\nu_g)^{1/2}\,{}^{k}f^{fg}_{s;0}\,. \tag{38}$$

Similar calculations with $i=j=4$ and f yield

$$\chi_f=\nu_f\beta_f^2(1+{}^{\omega}f^{f}_{a;0})^{-1} \tag{39}$$

in accord with Ref. 7. Choosing $i=a,f;j=a,g$ in (21), and making there the appropriate substitution from Eqs. (31) and (36), we obtain in the limit

$$-{}^{\omega}S^{af,ag}=N_f(1/m_f-1/m_f^{*})\delta_{fg}-{}^{\omega}f^{fg}_{s,1}(N_fN_g)^{1/2}(m_g^{*}m_f^{*})^{-1/2}, \tag{40}$$

where the effective mass $m_f{}^{*}$ of the particles of species f is defined as p_f/v_f. In the proof of (4) we used the relation $N_f=p_f^3/3\pi^2$ ($\hbar\equiv1$). For two-component systems this relation can also be obtained by direct application of the formalism of the single-particle Green function without using the formalism developed in this paper (cf. Refs. 20 and 21). Substitution of (40) in (32) leads to an equation for the effective mass

$$1=(1+{}^{\omega}f^{ff}_{s,1})m_f/m_f^{*}+{}^{\omega}f^{1\bar{1}}_{s,1}(m_{\bar{f}}N_{\bar{f}}/m_fN_f)^{1/2}(m_1m_{\bar{1}}/m_1^{*}m_{\bar{1}}^{*})^{1/2} \tag{41}$$

in accord with Ref. 7. Taking (41) into consideration, we find that a positive-semidefinite character of the matrix (40) is equivalent to the inequality

$${}^{\omega}f^{1\bar{1}}_{s;1}\geqslant0. \tag{42}$$

This inequality coincides with the Leggett inequality for ferromagnetic systems,[14] if the spin projection is regarded as a particle "species." Choosing $i=j=4+a,f$ in Eq. (21) taken in the ω-limit, and making in it the substitution from (31) and (36), we get

$$-{}^{\omega}S^{4+a,f;4+a,f}=N_f\left(\frac{1}{m_f}-\frac{1+{}^{\omega}f^{f}_{a;1}}{m_f^{*}}\right)>0, \tag{43}$$

from which follow two Leggett inequalities,[14] a separate one for each component. For the sake of completeness, we add the inequalities for the Landau amplitudes, which we obtained[12] in 1970 by the Leggett microscopic methods,[14] and the analogous Pomeranchuk inequalities[22] for a two-component FL. We have

$$1+{}^{\omega}f^{f}_{a;l}>0,\quad 1+{}^{\omega}f^{ff}_{s;l}>0,\quad \mathrm{Det}(\delta_{fg}+{}^{\omega}f^{fg}_{s;l})>0 \tag{44}$$

in accord with Ref. 7.

When using the analogy beteen particle species and spin it is useful to mention that some other inequalities that mix terms with different l and do not reduce to inequalities (42)–(44) should be satisfied for ${}^{\omega}f^{fg}_{s;l}$, although even the simplest of these inequalities is quite unwieldy (see Refs. 23 and 24).

Taking the matrix inverse to (38), we get with the aid of (13)

$$(\partial\mu_f/\partial N_g)=(\nu_f\nu_g)^{-1/2}(\delta_{fg}+{}^{\omega}f^{fg}_{s;0}), \tag{45}$$

in agreement with Ref. 7. Substituting these quantities in the thermodynamic formulas for the compressibility

$$\varkappa=\left[\sum_{f,g=\pm1}N_fN_g(\partial\mu_f/\partial N_g)\right]^{-1}, \tag{46}$$

we can obtain it for our system. The heat capacity of a two-component FL can also be obtained by the method of Luttinger and Ward.[20,25] It can be proved with the aid of the Ward identity (28) that the principal term of the low-temperature entropy can be written as

$$S=-2\frac{\partial}{\partial T}\left[T\sum_{\varepsilon}\int\frac{d^3p}{(2\pi)^3}\sum_{g=\pm1}\ln G_g(0;\varepsilon\mathbf{p})\right], \tag{47}$$

where $G_g(0;\varepsilon\mathbf{p})$ denotes the Green function at zero temperature, with p_0 replaced by the Matsubara frequency ε. For single-component systems there is summation over g in (47). From this we obtain for the low-temperature heat capacity a formula that coincides with that obtained phenomenologically.[7]

4. CHARGED FERMI LIQUIDS

It was assumed in the two preceding sections that the only source of singularities at small K is connected with the product (4) of the Green functions. For a system with long-range forces it is easy to find diagrams for the function Γ with an interaction line connected with K; a typical diagram of this kind consists of two contributions at the vertex of the function with small transfer of energy-momentum K, connected by a single (wavy) interaction line (Fig. 1). The diagrammatic contribution to the function Γ may not contain such lines at all, i.e., it can be a proper diagram in the sense of Nozières and Luttinger[15] or be an improper diagram and contain such a line only once, etc. We introduce an effective interaction line, shown in Fig. 2 by a thick wavy line, where the circle denote the sum of all the diagrams that terminate in interaction lines with quasimomentum K. The analytic form of this equation is

$$V^{fg}(K)=fgu(k)+\sum_{h,h'=\pm1}fhu(k)\tilde{S}^{0h,0h'}(K)V^{h'g}(K), \tag{48}$$

where $f,\ g=\pm1$, $u(k)=4\pi e^2/k^2$ and e is the electron charge, $\tilde{S}^{0h,0h'}$ is the proper correlation function of densities of the particles of species h and h'. Solving (48) for the quantities $V^{fg}(K)$ we obtain

$$V^{fg}(K)=fgu(k)[1-u(k)\tilde{S}^{qq}(K)]^{-1}, \tag{49}$$

where

$$\tilde{S}^{qq}(K)=\sum_{f,g=\pm1}fg\tilde{S}^{0f,0g}(K).$$

This quantity has the meaning of the autocorrelation function of the charge in units of the elementary charge, and the term in the square brackets in (49) is the dielectric function of the system. The improper vertex function Γ^{fg} of (2) can be expressed in term of the proper $\tilde{\Gamma}^{fg}$ with the aid of Fig. 3 and the following equation:

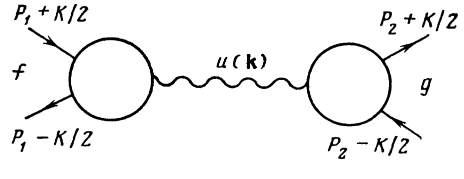

FIG. 1.

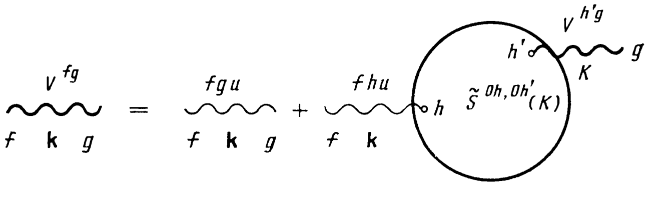

FIG. 2.

$$\Gamma^{fg}(K;P_1P_2)$$
$$=\tilde{\Gamma}^{fg}(K;P_1P_2)+\sum_{h;h'}\tilde{\mathcal{T}}^{0h}(K;P_1)\tilde{\mathcal{T}}^{0h'}(K;P_2)V^{hh'}(K),$$

where a tilde over a symbol denotes its proper part. Substituting in this formula $V_{hh'}(K)$ in the form (49) we obtain

$$\Gamma^{fg}(K;\ P_1P_2)=\tilde{\Gamma}^{fg}(K;\ P_1P_2)$$
$$+u(k)\tilde{\mathcal{T}}_f{}^q(K;\ P_1)\tilde{\mathcal{T}}_g{}^q(K;\ P_2)[1-u(k)\tilde{S}^{qq}(K)]^{-1}, \quad (50)$$

where the proper vertex functions are

$$\tilde{\mathcal{T}}_f{}^q(K;P)=\sum_{g=\pm1}g\tilde{\mathcal{T}}_f{}^{0g}(K;P).$$

Taking into account the character of the vertices $\tilde{\mathcal{T}}^q$, we can show that the last term in (50) modifies only Γ_s^{fg}. Substituting Γ in such a form in (16) and expressing the vertices in terms of this function we obtain with the aid of (19)

$$\mathcal{T}_g{}^{cf}(K;\ P)=\tilde{\mathcal{T}}_g{}^{cf}(K;\ P)+\tilde{\mathcal{T}}_g{}^q(K;\ P)\tilde{S}^{q,cf}(K)u(k)/\varepsilon(K).$$
$$c=0,\ 1,\ 2,\ 3, \quad (51)$$

where

$$\varepsilon(K)\equiv[1-u(k)\tilde{S}^{qq}(K)],\quad \tilde{S}^{q,cf}(K)=\sum_{h=\pm1}h\tilde{S}^{0h,cf}(K);$$

at $c\geqslant4$ we get

$$\mathcal{T}_g{}^{cf}(K;\ P)=\tilde{\mathcal{T}}_g{}^{cf}(K;\ P).$$

Substituting (51) in (19) we can express the improper correlation functions in terms of the proper ones:

$$S^{cf,dg}(K)=\tilde{S}^{cf,dg}(K)+\tilde{S}^{q,cf}(K)\tilde{S}^{q,dg}(K)/\varepsilon(K). \quad (52)$$

Hence

$$S^{qq}(K)=\tilde{S}^{qq}(K)/\varepsilon(K). \quad (53)$$

We note that at c, $d\geqslant4$ we have

$$S^{cf,dg}=\delta_{fg}\tilde{S}^{cf,df}.$$

If $u(0)$ is very large but finite (see the Introduction), all the relations obtained in the preceding section, including the Ward identities, are valid in the present case. Equations (50)–(53) enable us then to express all the quantities in terms of proper ones. We note that two-particle irreducibility can be introduced also for proper diagrams. This enables us to repeat for the proper quantities the previous algebraic transformations. However, proper vertex functions in the ω- or k-limits can be obtained only with the aid of the relations for the improper quantities (24), (25), and (28), which are connected with the corresponding proper ones through Eqs. (51). This question does not arise for the spin vertices (at $c\geqslant4$), which consist of only proper parts.

The Ward identities yield ${}^{\omega}S^{0f,cg}=0$ at all c. Going to the ω-limit in (52) and (53) at finite $u(0)$ we find that also ${}^{\omega}\tilde{S}^{0f,cg}=0$ at $0\leqslant c\leqslant3$. Using this result in (51) we get

$${}^{\omega}\tilde{\mathcal{T}}_g{}^{cf}(P)={}^{\omega}\tilde{\mathcal{T}}_g{}^{cf}(P),\qquad 0\leqslant c\leqslant3. \quad (54)$$

In addition, recognizing that ${}^{k}S^{0f,ag}=0$ at $a=1$, 2, 3, we can find from (52) at $c=0$, $d=a$, and finite $u(0)$ that ${}^{k}S^{0f,ag}=0$, too. Hence and from (51) we get

$${}^{k}\tilde{\mathcal{T}}_g{}^{af}(P)={}^{k}\mathcal{T}_g{}^{af}(P),\qquad a=1,\ 2,\ 3. \quad (55)$$

We can prove similarly that

$${}^{k}\tilde{S}^{af,bg}=-N_f\delta_{fg}\delta_{ab}/m_f,\qquad a,\ b=1,\ 2,\ 3.$$

Relations (54) at $c=0$ and (55) are sufficient to establish the form of the proper correlation functions. They are expressed in terms of the proper Landau parameters in the same manner as the improper functions are expressed in terms of the improper parameters. The equations for the effective masses (41) are therefore valid also for charged systems. We add that relations (54) and (55) lead jointly with the equations for the vertex functions to relations (22) and (26), which are satisfied by the proper vertex functions. Relations (33) are therefore valid also for the proper correlation functions. Relations (54) and (55) yield the effective interaction for charged systems if Eqs. (35) and (36) are taken into account:

$$Z_fZ_g{}^{\omega}\Gamma^{fg}(P_1P_2)|_{\mathrm{FS}}=Z_fZ_g{}^{\omega}\tilde{\Gamma}^{fg}(P_1P_2)|_{\mathrm{FS}}+4\pi e^2fg/k^2, \quad (56)$$

where the ω-limit must be understood in the sense of $0<kv_F\ll\omega\ll\mu_j$. Equation (56) can be interpreted as a verification of the Landau-Silin term for two-component charged FL. It can be seen from (56) that the exchange part of the effective interaction, just as the components of the nonexchange part with $l<0$, are not altered by the long-range action.

It remains to consider only the vertex ${}^{k}\tau_g^{0f}(P)$ and the correlation functions ${}^{k}S^{0f,0g}$, $f,g=\pm1$. We can let $u(0)$ tend here to infinity right away, since this done not result in ambiguity. In the calculations we shall need to know the values of ${}^{k}\tilde{\mathcal{T}}_g^{0f}|_{\mathrm{FS}}$ and ${}^{k}\tilde{S}^{0f,0g}$. The quantities ${}^{k}\mathcal{T}_f^{q}|_{\mathrm{FS}}$, ${}^{k}\tilde{S}^{q,0f}$, or ${}^{k}\bar{S}^{qq}$ that enter in (51)–(53) and are taken in the k-limit can be obtained from the preceding ones by direct summation. Expressing ${}^{k}\tilde{\mathcal{T}}_g^{0f}|_{\mathrm{FS}}$ in terms of ${}^{\omega}\tilde{\mathcal{T}}_g^{0f}|_{\mathrm{FS}}=\delta_{fg}/Z_g$ (cf. (35), (36), and (54)) with the aid of the first equation of (18), taken in the the k-limit, we obtain

$${}^{k}\tilde{\mathcal{T}}_g{}^{0f}|_{\mathrm{FS}}=[\delta_{fg}-{}^{k}\tilde{f}_{s;0}^{fg}(\nu_f/\nu_g)^{1/2}]Z_g{}^{-1}$$
$$=fg[\delta_{fg}+{}^{\omega}\tilde{f}_{s;0}^{fg}(\nu_f/\nu_g)^{1/2}]W_0{}^{-1}Z_g{}^{-1}, \quad (57)$$

where $W_l\equiv\mathrm{Det}(\delta^{fg}+{}^{\omega}\tilde{f}_{s;l}^{fg})$ (cf. (13)). Substituting ${}^{\omega}\tilde{\mathcal{T}}_g^{0f}|_{\mathrm{FS}}$ in (20) we get (cf. Eq. 38))

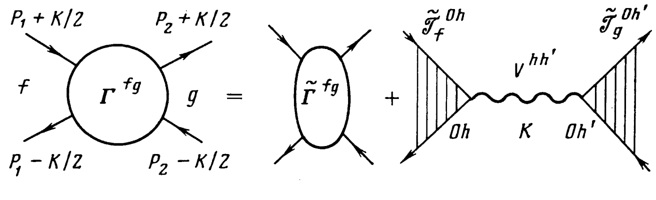

FIG. 3.

$$ {}^{k}\tilde{S}^{0f,0g} = -(\nu_f\nu_g)^{1/2}(\delta_{fg} - {}^{k}\tilde{f}^{fg}_{s;0}) = -fg(\nu_f\nu_g)^{1/2}(\delta_{fg} + {}^{\omega}\tilde{f}^{\bar{f}\bar{g}}_{s;0})\,W_0^{-1}. \tag{58} $$

Going to the k-limit in (52) and (53) at $c = d = 0$, we obtain

$$ {}^{k}S^{qq} = 0, \quad {}^{k}S^{0f,0g} = {}^{k}\tilde{S}^{0f,0g} - {}^{k}\tilde{S}^{0f,q}\,{}^{k}\tilde{S}^{0g,q}/{}^{k}\tilde{S}^{qq}, \tag{59} $$

whence ${}^{k}S^{0f,q} = 0$. That part of the thermodynamic-potential differential which is connected with the change of the number of electrons and protons takes the form $\mu_{\text{I}}dN_{\text{I}} + \mu_{1}dN_{1} = \mu_{+}dN + \mu_{-}dQ$, where $N = (N_1 + N_{\text{I}})/2$, and $Q = N_1 - N_{\text{I}}$ are respectively the number of particle pairs and the charge of the system in units of e. The quantities $\mu_{+} = \mu_1 + \mu_{\text{I}}$, $\mu_{-} = (\mu_1 - \mu_{\text{I}})/2$ are interpreted as chemical potentials connected respectively with the variables N and Q. The identities obtained, expressed in terms of the introduced parameters, take the form $\partial N/\partial\mu_{-} = \partial Q/\partial\mu_{-} = \partial Q/\partial\mu_{+} = 0$ [cf. (31)], i.e., the charge and its chemical potential are no longer thermodynamic variables. Indeed, the charge density of a thermodynaic system should vanish and, hence, all the thermodynamic variables should be independent of the chemical potential connected with the charge. Changing to the k-limit in (51) we find, after multiplying by f and summing over $f = \pm 1$, that ${}^{k}\mathscr{T}^{q}_{f}(P)$ [or $\partial G_f^{-1}(P)/\partial\mu_{-}$ if (28) is taken into account] is equal to zero.

In some respects our system behaves as a one-component system consisting of electron-proton pairs. Summing the second equation of (59) over f and g and using the first equation of (31) we get

$$ \partial N/\partial\mu_{+} = -{}^{1}/{}_{4}\,{}^{k}S^{nn} \equiv -{}^{1}/{}_{4}\sum_{fg}{}^{k}S^{0f,0g} = -{}^{1}/{}_{4}\,{}^{k}\tilde{S}^{nn} + {}^{1}/{}_{4}({}^{k}\tilde{S}^{qn})^{2}/{}^{k}\tilde{S}^{qq}, \tag{60} $$

where

$$ \tilde{S}^{qn} = \sum_{f,g} f\tilde{S}^{0f,0g}. $$

Substituting (58) and (60), we find after some calculations

$$ \partial N/\partial\mu_{+} = \Big[\sum_{f,g=\pm1}(\nu_f\nu_g)^{-1/2}(\delta_{fg} + {}^{\omega}\tilde{f}^{fg}_{s;0})\Big]^{-1}. \tag{61} $$

The thermodynamic formula for the compressibility of the system per unit volume takes the form $\varkappa = (\partial N/\partial\mu_{+})N^{-2}$, from which it follows that Eqs. (45) and (46) lead to the same result if $N_1 = N_{\text{I}} = N$. We have thus confirmed the result of Oliva and Ashcroft,[7] which was obtained by method more appropriate to systems with short-range action. Transforming to the k-limit in (51), summing over f, and using Eq. (28) taken on the Fermi surface, we obtain

$$ 2v_g dp_F/d\mu_{+} = [{}^{k}\tilde{\mathscr{T}}_{g}{}^{n}(P) - {}^{k}\tilde{\mathscr{T}}_{g}{}^{q}(P)\,{}^{k}\tilde{S}^{qn}/{}^{k}\tilde{S}^{qq}]\,|_{\text{FS}}, \tag{62} $$

where p_F denotes the Fermi-momentum value that is common to both components. It can be verified that (62) is satisfied identically at $g = \pm 1$.

For charged particles, the amplitude for forward scattering of particles on the Fermi surface is ${}^{k}\Gamma(P_1P_2)|_{\text{FS}}$ [cf. (19)]. Therefore the sum rules for charged liquids cannot be obtained from (14) by direct replacement of the improper amplitudes by proper ones. Changing to the k-limit in (50) we should get zero at $f = g$ and $P_1 = P_2$ on the Fermi surface and at equal values of the spin indices. Using (57) and (58) to obtain ${}^{k}\tilde{\mathscr{T}}^{q}_{f}|_{\text{FS}}$ and ${}^{k}\tilde{S}^{qq}$, and changing to dimensionless variables, we get

$$ \sum_{l=0}(2l+1)({}^{k}\tilde{f}^{gg}_{s;l} + {}^{k}\tilde{f}^{g}_{a;l}) + \Big[\sum_{f=\pm1} f(\nu_f)^{1/2}(\delta_{fg} - {}^{k}\tilde{f}^{fg}_{s;0})\Big]^{2} \times\Big[\sum_{f,h=\pm1} fh(\nu_f\nu_h)^{1/2}(\delta_{fh} - {}^{k}\tilde{f}^{fh}_{s;0})\Big]^{-1} = 0. \tag{63} $$

It is easily seen that the stability conditions will be satisfied by all the proper Landau exchange parameters and by the proper non-exchange ones at $l > 0$. In addition, all the inequalities (42) and (43) should also be satisfied by proper amplitudes. For non-exchange Landau parameters at $l = 0$, starting from the fact that the static autocorrelation functions are not positive, we can prove with the aid of (60) and (61) that $\partial N/\partial\mu_{+} > 0$, and find from (53) that ${}^{k}S^{qq} < 0$.

5. CONCLUDING REMARKS

For two-component FL, just as for ferromagnetic FL (cf., e.g., Refs. 25 and 13), the number of possible static measurements is smaller than the number of Landau parameters that enter in the equations for the corresponding quantities. For example, for two-component neutral FL we can measure the three quantities $\partial N_f/\partial\mu_g$, the heat capacity, and the spin susceptibility, while the equations for these quantities contain five Landau parameters (${}^{\omega}f^{gh}_{c,0}$, $f^{gh}_{a,0}$) and two effective masses. For charged systems the situation deteriorates, inasmuch as in place of the three quantities $\partial N_f/\partial\mu_g$ we can now measure only $\partial N/\partial\mu_{+}$, although the determination of the static screening length via $\varepsilon(K)$ makes it also possible to obtain the value of ${}^{k}\tilde{S}^{qq}$. For neutral systems there remain therefore two free parameters, and for charged systems even three. These numbers can be reduced by unity, since it is possible to measure separately the static electron susceptibility and the proton spin susceptibility. These possibilities are the consequence of the separate conservations of the electron and proton spins, as a result of which the spin vertices and the correlation functions are diagonal in the particle species.

The proton and electron components of the dynamic susceptibility have the analytic properties of the susceptibility. Their static values are therefore expressed in a known manner in terms of an integral of their imaginary parts with respect to the frequencies. If a static magnetic field h is turned on, such that $|\beta_{\text{I}}|h \ll \mu_1$, the regions of ω in which the electron and proton imaginary parts of χ differ substantially from zero become separated. This yields the static values χ_g and $g = \pm 1$ separately rather than only their sum.

We have considered in this paper only normal FL, with a slight exception made (see the Introduction) for ferromagnetic FL. We note once more that the proton ferromagnetism is more readily feasible at $r_s \approx 1.6$ and a nonzero electron magnetic moment appears in that case as a result of the action of the magnetization field. Generalization of the theory to include this case is quite simple—the single-particle Green functions become dependent on the spin projection α on the magnetization axis. In addition, the two-particle vertex part will take the form considered by us at the end of the

Introduction. We take an analogous expression for $\Gamma^{fg}_{\alpha\beta,\gamma\delta}$. It is easy to note that our approach can be used only for the quantity $\Gamma^{fg}_{\alpha\beta,\alpha\beta}$ if the particle spin in units of $\hbar/2$ is comparable with the number of particles. In this case the quantity Γ^{fg}_{α} is inessential, and $Z_{f\alpha}Z_{g\beta}{}^{\omega}\Gamma^{fg}_{\alpha\beta,\alpha\beta}$ (where $Z_{f\alpha}$ denotes the discontinuity, on the Fermi surface, of the density of the particles of species f with spin α) plays the role of the effective quasiparticle interaction. The aggregate of the vertices determined by the free vertices (15) is also now sufficient to obtain all the results of the theory. Equations (16) for the total vertices will contain summation over the longitudinal spin α, the vertex Γ will be a spin matrix, and the solutions $\mathcal{T}^{i}_{g\alpha}(K,P)$ of Eq. (16) will depend on spin. It is easy to show that the Ward identities and their corollaries can be written for two-component ferromagnetic FL in the form (22)–(30) if the subscript g is replaced everywhere by the double subscript $g\alpha$. It is likewise easily seen that the expressions (50)–(53) for the improper values in terms of the proper ones remain in force (in some cases following the substitution $g \rightarrow g\alpha$), although the form of the proper quantities changes. The number of the free parameters of the theory, i.e., those that cannot be determined from static measurements, increases compared with the normal systems. Indeed, we have here ten different Landau parameters at $l = 0$ and four effective masses. On the other hand, it is possible to measure seven quantities for neutral systems (three values of $\partial N_f/\partial\mu_g$, three values of $\partial M/\partial h$, $\partial M/\partial\mu_f$, and the heat capacity; here M is the magnetization). For charged systems we can measure only four quantities including the screening length. It is therefore interesting to ascertain whether an analog of (62) will be satisfied identically for ferromagnetic FL or not. If not, ${}^{\omega}\tilde{f}^{f\alpha,g\beta}_{0}$ and the effective masses become subject to the restrictions that are valid for charged systems.

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