Behavior of surface (two-dimensional) superconductors and of a very thin superconducting film in a magnetic field

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(Submitted 6 June 1983)
Zh. Eksp. Teor. Fiz. **85**, 1707–1715 (November 1983)

The magnetic properties of a very thin superconducting cylindrical film and of a two-dimensional superconductor of cylindrical form are studied. Below the Kosterlitz-Thouless transition temperature the magnetic moment of such systems oscillates as a function of the magnetic field. In a metastable state the magnetic moment of the cylinder may exceed its equilibrium value considerably. This effect permits, in principle, observation of surface superconductivity on Tamm levels in dielectrics.

PACS numbers: 73.60.Ka, 74.30.Gn

1. INTRODUCTION

The investigation of ordering and phase transitions in two-dimensional systems (in particular, on the surfaces of three-dimensional objects) is now attracting very much attention. Among the problems involved is two-dimensional (surface) superconductivity, more specifically the superconductivity of a "surface metal," i.e., the superconductivity in the case of a degenerate partly filled band of electron surface (Tamm) levels.[1] Such a band can exist in principle both on the surface of a three-dimensional metal and on the surface of a three-dimensional dielectric. It is hardly necessary to prove how interesting and possibly how important for technical applications it is to investigate such superconducting systems.

To be sure, the feasibility of two-dimensional superconductivity was considered in Ref. 1 only within the framework of a self-consistent approximation and specifically the BCS approximation. Yet, as became clear subsequently (see, e.g., Ref. 2), when account is taken of fluctuations, no long-range order can exist in two-dimensional systems, let alone one-dimensional ones. This, however, does not prevent an actual appearance of superconductivity in very thin wires and films, as well of quite remote metal layers in layered compounds (see Ref. 3, Chap. 6 and the cited literature). The resultant problem is the subject of many studies, and it was made clear in the upshot that two-dimensional ordering and such phenomena as superconductivity, superfluidity, and ferromagnetism[1)] can be observed also in two-dimensional systems, the influence of the fluctuations notwithstanding (see Ref. 3, Chaps. 1 and 6, and Ref. 5, as well as the literature cited there). When the mechanism of Kosterlitz and Thouless is considered, the foregoing pertains to a temperature below a certain value T_c corresponding to the dissociation of vortex pairs. We note that for systems with finite dimensions (for a cylinder of finite length, etc.) even a self-consistent description is practically always well applicable at $T<T_c$, owing to the weak (logarithmic) divergence of fluctuations of spin-wave type in a two-dimensional system.

The discussion of the properties, particularly electrodynamic, of two-dimensional (surface) superconductors is thus of real physical significance. The possible applications are to layered compounds, very thin (atomic) metallic films on various surfaces, and others (it is not excluded that the anomalous diamagnetic effects observed in CuCl and CdS are also somehow connected with surface superconductivity[6]).

We point out that we shall consider below the magnetic properties of surface superconductors only in the case when the "superconducting electrons" are localized near the surface, as is the case for filled Tamm levels in dielectric samples or for metallic layers of atomic thickness on dielectric substrates. A system of Tamm levels on the surface of a metal in the presence of electron scattering from the surface into the interior of a metal, and a superconducting film of atomic thickness on the surface of a normal metal were considered in Ref. 7.

It seems quite obvious from physical considerations that a two-dimensional superconducting "film" in which the superconducting electrons are localized in a layer of thickness a (generally speaking, atomic in scale) will always behave below T_c as a thin macroscopic superconducting film having a thickness d and satisfying the conditions

$$d\ll\xi_0,\quad d\ll\lambda_L, \tag{1}$$

where ξ_0 is the superconducting correlation length at $T=0$, while λ_L is the London penetration depth of the magnetic field in a bulky sample of the same material.

The behavior of such films in a magnetic field is known, and it seems at first glance that the magnetic-field component parallel to the field can be regarded as continuous in the absence of transport (total, flowing over the film) current. Indeed, at $a\sim 3\times10^{-8}$ cm we have $(a/\lambda_L)^2\sim10^{-6}$ even at $\lambda_L\sim3\times10^{-5}$ cm and the "film" diamagnetic moment due to its superconductivity is not larger or is of the order of the diamagnetic contribution to the magnetic moment in the normal state (for some more details see Ref. 8). In accord with the foregoing, in the theory of superconducting layered compounds (see Refs. 9 and 10, as well as Ref. 3, Chap 6) the magnetic field parallel to the layers is regarded as continuous. If it were always possible to proceed in this manner, a very thin superconducting film and two-dimensional superconductor would in fact not interact with an external magnetic field parallel to their surface. As a consequence, an experimental study of the magnetic properties of the film (of its magnetic moment, susceptibility, etc.) would be difficult.

As noted in Ref. 8, however, for multiply connected

 0038-5646/83/110994-05$04.00

two-dimensional superconductors (e.g., in the case of superconductivity on the surface of a cylinder), the magnetic moment produced in a parallel magnetic field is considerably stronger than in the normal state. Yet this effect was not analyzed in detail in Ref. 8, and its investigation is precisely the purpose of the present article.

Specifically, we consider below the behavior of a two-dimensional superconductor in the form of a round cylinder of radius R in an uniform external magnetic field parallel to the cylinder axis and having an intensity H_0. We assume in Sec. 2 that the temperature T is close to T_c and to the critical temperature T_{c0} of the self-consistent-field approximation (it is useful to single out the latter case since it is especially simple and known results can be employed). In Sec. 3 the temperature is already assumed arbitrary ($0 < T < T_c$).

2. TEMPERATURE REGION CLOSE TO T_{c0}

In the temperature region close to T_{c0} we can use the well known Ginzburg-Landau (GL) approximation, which was used in a number of detailed studies of the behavior of macroscopic cylindrical films in an external field (see, e.g., Ref. 11 and the bibliography therein). In particular, an expression was obtained in Ref. 11 for the thermodynamic potential $\mathscr{F}$ of a thin superconducting cylindrical film of radius R in an external field that can be expressed in the case $d/R \ll 1$ of interest to us in the form

$$\mathscr{F} = -2\psi^2 + \psi^4 + \frac{2\xi^2(T)\psi^2(n-\phi_a)^2}{R^2(1+\mu\psi^2/2)}, \qquad (2)$$

where $\mu = Rd/\lambda_L^2(T)$, $\phi_a = \pi R^2 H_0/\Phi_0$, $\Phi_0 = hc/2e$ is the flux quantum, ψ is the modulus of the order parameter (of the wave function) of the superconductor in relative units,[12] $\xi(T)$ is the temperature-dependent coherence length, and $n = 0, 1, 2, \ldots$ is the (integer) number of flux quanta trapped inside the cylindrical cavity.

In units of the flux Φ_0, the total magnetic flux inside the cavity, with account taken of the leakage of the field through the cylinder wall, is equal to[11]

$$\phi_i = \frac{\pi R^2 H_i}{\Phi_0} = n\frac{\mu\psi^2/2}{1+\mu\psi^2/2} + \frac{\phi_a}{1+\mu\psi^2/2}, \qquad (3)$$

where H_i is the magnetic field inside the cavity.

For a magnetic moment M per unit length of the cylinder we have

$$\frac{4\pi M}{\Phi_0} = \phi_i - \phi_a = (n-\phi_a)\frac{\mu\psi^2/2}{1+\mu\psi^2/2}. \qquad (4)$$

A characteristic feature of expresions (2)–(4) is that the denominators contain the screening factor $\mu = Rd/\lambda_L^2(T)$, which can be large at $R \gg d$ even for a very thin film $d \ll \lambda_L$ (thus, at $d \sim 3\times 10^{-8}$ cm, $\lambda_L \sim 3\times 10^{-5}$ cm, and $R \sim 0.3$ cm we have $\mu \sim 10$). As a result, the thin cylindrical film screens the internal cavity against the external field. Indeed, as follows from (3) at $\mu \gg 1$ (and $\psi \approx 1$)[2)] the total flux inside the cavity is equal to the captured number of flux quanta $\phi_i \approx n$ and is independent of the external-field flux ϕ_a. In the immediate vicinity of T_{c0}, when the length $\lambda_L(T)$ becomes large, the factor μ decreases and the cylinder becomes transparent to the external field (in this case, naturally $\phi_i \approx \phi_a$).

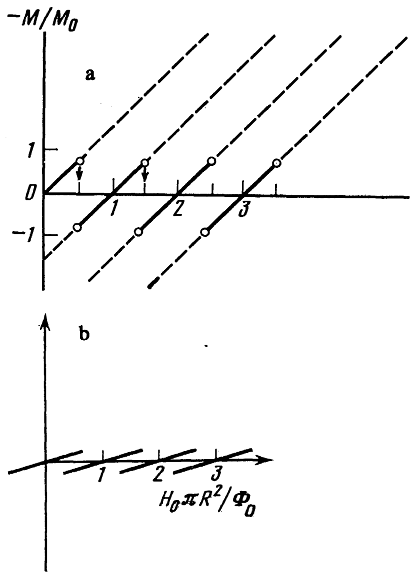

FIG. 1. Dependence of the magnetic moment of a thin superconducting cylinder on an external magnetic field H_0 parallel to the cylinder axis: a) the parameter $\mu \gg 1$, b) $\mu \ll 1$.

It can be seen from (2) that when ϕ_a increases the minimum of the potential $\mathscr{F}$ corresponds to a state inside the cavity such that the number n of flux quanta is as close as possible to ϕ_a. As a result, when ϕ_a increases transitions from one quantum level to another $n+1$ can take place in the system, and in the absence of hysteresis the transitions occur at values $\phi_a = n + 1/2$. Such transitions are observed in experiment and are manifest, in particular, by oscillatory dependences of the critical temperature of a very thin cylindrical film on the magnetic field (the Little-Parks effect[13]). These oscillations are easiest to see at $T \approx T_{c0}$ in mirocylinders of small radius.[14] With increasing R and also at lower temperatures, when the factor μ increases, hysteresis phenomena are strongly pronounced.[11] The quantum number n no longer adjusts itself to the external field ϕ_a and states with $n \neq \phi_a$ may occur, although such states are thermodynamically unfavorable, i.e., are metastable.

The foregoing is illustrated in Fig. 1, which shows schematically the dependence of the magnetic moment of a cylindrical film on an external magnetic field in accord with (4) at $\mu \gg 1$ and $\psi \approx 1$ (Fig. 1a) and at $\mu \ll 1$ and $\psi \approx 1$ (Fig. 1b). The arrows mark the values of $\phi_a = n + 1/2$ corresponding to the points of equilibrium transition from the level n to the level $n+1$. The dashed line continues the $M(\phi_a)$ dependence into the metastable region. It can be seen that the magnetic moment of the film increases rapidly with increasing ϕ_a at $\mu \gg 1$, and a transition of the system to another quantum level should be accompanied by a magnetic-moment jump that is in principle observable in experiment.

As already mentioned, an analogy can be drawn between a very thin superconducting cylindrical film and the two-dimensional superconducting state, if the latter is produced on the surface of a cylindrical sample (the connection between the phenomenological parameters of the functional

(2) and the microscopic parameters of a "surface" superconductor will be established in Sec. 3 of the article). Therefore the magnetic moment of a two-dimensional superconducting film should also have the anomalies noted above.

3. EQUILIBRIUM STATES OF SYSTEM AT ARBITRARY TEMPERATURES

The semiphenomenological GL theory used above is valid, generally speaking, only near the critical temperature T_{c0} of the self-consistent-field approximation. For two-dimensional supercondcutors, the GL approximation can be used when the temperature T is close to the Kosterlitz-Thouless transition temperature T_c, and the latter is close to T_{c0}. Far from T_{c0} the quantitative treatment is by other methods, although it can be assumed beforehand that the results of the GL approximation will remain qualitatively correct also far from T_{c0}. We consider therefore a thin superconducting "dirty" cylindrical film in a magnetic field parallel to the axis, at arbitrary temperatures, within the framework of the BCS theory. To this end we use the Eilenberger equations[15]

$$\begin{gathered}\left[\tilde{\omega}+i\frac{e}{c}\mathbf{vA}(\mathbf{r})\pm{}^1/_2\mathbf{v}\nabla\right]f^{\mp}(\mathbf{v},\mathbf{r})=\tilde{\Delta}^{\mp}(\mathbf{r})g(\mathbf{v},\mathbf{r}),\\ g^2(\mathbf{v},\mathbf{r})+f^-(\mathbf{v},\mathbf{r})f^+(\mathbf{v},\mathbf{r})=1,\qquad \omega=\pi T(2n+1),\\ \tilde{\omega}=\omega+\bar{g}(\mathbf{r})/2\tau,\qquad \tilde{\Delta}^{\pm}=\Delta^{\pm}(\mathbf{r})+\bar{f}^{\pm}(\mathbf{r})/2\tau,\\ \Delta^{\pm}(\mathbf{r})=\lambda\pi T\sum_{\omega}\bar{f}^{\pm}(\mathbf{r}),\quad \bar{\Delta}(\mathbf{r})=\Delta(\mathbf{r}),\quad \Delta^+(\mathbf{r})=\Delta^*(\mathbf{r}),\end{gathered}\tag{5}$$

where λ is the parameter of the electron-phonon interaction, τ is the electron mean free path time, the functions $\bar{f}^{\pm}(\mathbf{r})$ and $\bar{g}(\mathbf{r})$ are the values of the functions $f^{\mp}(\mathbf{v},\mathbf{r})$ and $g(\mathbf{v},\mathbf{r})$ averaged over the direetion $\mathbf{v}$ of the electron motion, and $\mathbf{A}(\mathbf{r})$ is the vector potential.

Equations (5) are obtained from the condition that the functional[15]

$$\mathscr{F}=\int d\mathbf{r}\left[\frac{|\Delta^2(\mathbf{r})|}{\lambda}-2\pi N(0)\sum_{\omega}\int\frac{do}{2\pi}I\right],$$

$$I=\frac{1}{2}(f^-\Delta^++f^+\Delta^-)+g\left(\omega+i\frac{e}{c}\mathbf{vA}-\frac{1}{2}i\mathbf{v}\nabla\ln\frac{f^+}{f^-}\right)\tag{6}$$

$$-|\omega|+\frac{1}{4\tau}\bar{f}^-f^+$$

be a minimum when varied with respect to $f^{\pm}$, g, and $\Delta^{\pm}$. The minimum value of the functional (6) yields the difference between the equilibrium values of the free energy in the superconducting and normal states.

We transform to a cylindrical coordinate frame (z, r, φ). The vector potential has one nonzero component A_φ, which is independent of φ in the gauge $\operatorname{div}\mathbf{A}=0$. The functions $f^{\pm}$ and g depend only on φ and this dependence is given by

$$f^{\mp}(\mathbf{v},\mathbf{r})=f^{\mp}(\mathbf{v})e^{\pm in\varphi},\qquad g(\mathbf{v},\mathbf{r})=g(\mathbf{v}),\qquad \Delta^{\mp}(\mathbf{r})=\Delta e^{\pm in\varphi},\tag{7}$$

where Δ is independent of the coordinates.

Substituting (7) in (5) we obtain equations for $f^{\pm}(\mathbf{v})$, $g(\mathbf{v})$, and Δ:

$$\begin{gathered}\left[\tilde{\omega}+iv_\varphi\left(\frac{e}{c}A_\varphi(R)-\frac{n}{2R}\right)\right]f^{\mp}(\mathbf{v})=\tilde{\Delta}^{\mp}g(\mathbf{v}),\\ g^2(\mathbf{v})+f^+(\mathbf{v})f^-(\mathbf{v})=1,\quad \tilde{\omega}=\omega+\bar{g}/2\tau,\\ \tilde{\Delta}^{\pm}=\Delta^{\pm}+\bar{f}^{\pm}/2\tau,\quad \Delta=\lambda\pi T\sum_{\omega}\bar{f}^-,\\ A_\varphi(R)={}^1/_2H_iR,\end{gathered}\tag{8}$$

where H_i is the field inside the cylinder.

Equations (8) contain the effective vector potential $\tilde{A}=A_\varphi-\Phi_0 n/2\pi R$, and the equations (8) themselves are obtained from the equations for a superconductor in the absence of an external field by replacing ω with $\omega+i\mathbf{q}\cdot\mathbf{v}$, where $q_\varphi=e\tilde{A}/c$ and $q_r=q_z=0$. They are equivalent to the equations for a superconductor in the presence of a uniform current, when the momentum of the mass center of the Cooper pairs is equal to $2\mathbf{q}$ (Ref. 16).

Our task in the first stage is to calculate the superconductor free-energy functional that depends on the parameters Δ and $\tilde{A}$. Adding next the free energy of the magnetic field, we obtain the total functional, whose minimization with respect to Δ, n, and A_φ determines the thermodynamic parameters of the system. We confine ourselves to the most realistic case, when the superconductor is "dirty" and the inequality $\tau\, T_{c0}\ll 1$ is satisfied. Equation (8) can then be solved by using the smallness of the parameter $\tau\, T_{c0}$. As a result, the functions $f^{\pm}(\mathbf{v})$ and $g(\mathbf{v})$ are determined accurate to terms of order $(\tau\, T_{c0})^2$. We can next substitute $f^{\pm}$ and g in the functional (6) and obtain the sought functional $\mathscr{F}(\Delta,\tilde{A})$. An alternate procedure is to obtain an algebraic self-consistency equation for Δ and find a function $\mathscr{F}(\Delta,\tilde{A})$ that would yield this equation when varied with respect to Δ. In addition, this functional must define the known free energy of the superconductor in the absence of the field, i.e., at $A=0$. These conditions yield uniquely the functional $\mathscr{F}(\Delta,\tilde{A})$. We obtain as a result

$$\begin{gathered}\mathscr{F}(\Delta,\tilde{A})=\mathscr{F}_s(\Delta)+\mathscr{F}_{int}(\Delta,\tilde{A}),\\ \mathscr{F}_s(\Delta)=N(0)\left\{\frac{\Delta^2}{\lambda}-2\pi T\sum_{\omega}|\omega|\left[\left(1+\frac{\Delta^2}{\omega^2}\right)^{1/2}-1\right]\right\},\\ \mathscr{F}_{int}(\Delta,\tilde{A})=\frac{\pi}{2}N(0)\frac{\Delta}{\tau_m}\operatorname{th}\frac{\Delta}{2T},\qquad \tau_m^{-1}=\frac{1}{2}v_F^2\left(\frac{e}{c}\tilde{A}\right)^2\tau,\end{gathered}\tag{9}$$

where $\mathscr{F}_s(\Delta)$ is the functional of the superconductor (per unit area) relative to the homogeneous order parameter Δ in the absence of external fields, the term $\mathscr{F}_{\text{int}}$ describes the suppression of the superconductivity by the magnetic field, and τ_m is the effective time of breaking of the Cooper pair by the magnetic field. The quantity $N(0)$ is the two-dimensional denisty of states.

With account taken of the magnetic-field energy, the total functional of the system (per unit cylinder length) is of the form

$$\begin{gathered}\mathscr{F}(\Delta,n,A_\varphi)=2\pi R\left[\mathscr{F}_s(\Delta)+\frac{1}{8\pi}Q(\Delta)\tilde{A}^2\right]+\frac{1}{8}R^2(H_i-H_0)^2,\\ Q(\Delta)=\frac{\Delta}{\delta\Delta_0}\operatorname{th}\frac{\Delta}{2T},\quad \delta^{-1}=\frac{2\pi N(0)v_F^2e^2l}{\xi_0c^2}=\frac{4\pi^2\Delta_0}{c^2\rho},\\ \xi_0=\hbar v_F/\pi\Delta,\quad l=v_F\tau,\end{gathered}\tag{10}$$

where the quantity $(\delta a)^{1/2}$ plays in this problem the role of the London penetration depth, $Q(\Delta)$ is the electromagnetic kernel for a thin superconductor with a homogeneous order parameter Δ, and ρ is the resistivity of the two-dimensional superconductor in the normal state. In order of magnitude, $\delta\approx 10^5 a(\xi_0/l)$. In the case of a pure two-dimensional superconductor we have $\delta = mc^2/4\pi l^2 n_s$, where n_s is the two-dimensional density of the superconducting electrons.

After minimizing (9) with respect to A_φ (or H_i) we obtain per unit area of the cylinder

$$\mathcal{F}(n,\Delta)=\mathcal{F}_s(\Delta)+\frac{Q(\Delta)\Phi_0^2(\phi_a-n)^2}{32\pi^3R^2[1+Q(\Delta)R/2]},\qquad(11)$$

$$\frac{4\pi M}{\Phi_0}=\phi_i-\phi_a=\frac{Q(\Delta)R(n-\phi_a)}{2[1+Q(\Delta)R/2]}.$$

Relations (10) go over into (2) and (4) if $T\to T_{c0}$, with

$$\Delta^2=7{,}32\psi^2T_{c0}^2\ln(T_{c0}/T),\quad \mu=R/\delta,\qquad(12)$$

$$\xi^2(T)=0.545\xi_0 l/\ln(T_{c0}/T).$$

It can be seen from (10) that independently of the temperature the minimum with respect to n is equal to the nearest integer part of the number ϕ_a, and in the equilibrium state the magnetic moment of the system oscillates between $-M_0$ and M_0, where $M_0=\Phi_0/8\pi[1+2Q^{-1}R^{-1}]$ (see Fig. 1), with $M_0\ll\Phi_0/8\pi$. The deviations of the field inside the cylinder from the external field do not exceed $\Phi_0/2\pi R^2$ in absolute value.

When minimizing with respect to Δ we recognize that in the equilibrium state the ratio of the magnetic contribution of the magnetic functional $\mathcal{F}$ in (10) to the superconducting contribution $\mathcal{F}_s$ is of the order of $[\xi(T)/R]^2$, i.e., the effect of the magnetic field on the value of Δ is small everywhere except in a narrow vicinity of the point T_{c0}, of the order of $(T_{c0}-T)/T_{c0}\approx\xi_0 l/R^2$. Outside this vicinity the equilibrium parameter is practically equal to the parameter of the superconductor without a field.

4. METASABLE STATES AND THEIR LIFETIMES

If no equilibrium with respect to the number n is reached, the magnetic moment of the cylinder can substantially exceed M_0. We obtain now that range of values of $\tilde\phi=\phi_a-n$ at which superconductivity continues to exist in the form of a metastable state. The critical value of $\tilde\phi$ is determined from the condition that the minimum of $\mathcal{F}(\Delta,n)$ with respect to Δ vanish at a specified value of n, i.e., from the conditions $\partial\mathcal{F}/\partial\Delta=\partial^2\mathcal{F}/\partial\Delta^2=0$. They lead to the following expressions for the critical "superheating" field $\tilde H_c=\Phi_0\tilde\varphi/\pi R^2$:

$$\tilde H_c^2=\frac{70{,}5N(0)T_{c0}^2t}{\delta\Phi_0^2}\left(1+0.5\frac{\delta}{Rt}\right),\quad t=\frac{T_{c0}-T}{T_{c0}}\ll 1,$$

$$\tilde H_c^2=4{,}54N(0)\Delta_0^2\delta^{-1},\quad R\gg\delta,\quad T\ll T_{c0}.$$

At low temperatures $\tilde H_c$ is smaller by a factor $a/\lambda_L(0)$ than the thermodynamic magnetic field of a bulky superconductor with the same values of T_{c0}, v_F, and τ. In the case $l\approx a$ this factor is about 10^{-3}. Near T_{c0} the quantity $\tilde H_c$ contains one additional small factor t.

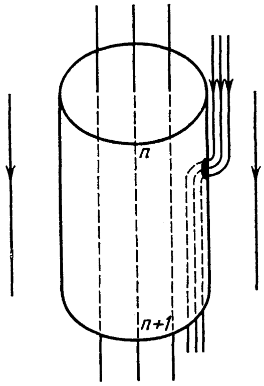

FIG. 2. Transition from a state with the number n (in the upper part of the cylinder). The normal "core" of the vortex (small black circle) moves along the cylinder whose radius is much larger than δ.

The transition between states with different quantum numbers n is of first order in the magnetic field. Let us estimate the activation energy of the critical seed of the new phase (of the phase with a different number n) for such a transition. We assume that the intermediate state between two phases with different numbers n and $n+1$ should contain a normal region with area of the order $\xi^2(T)$ on the surface of the cylinder, and near this region the state with value n on one end of the cylinder changes into a state with number $n+1$ on the other end. The change of the phase on circling around the normal region is 2π. The magnetic-flux quantum by which the magnetic fluxes inside the cylinder in the states n and $n+1$ differ passes through the normal region from the outer region of the cylinder into the inner. Near the cylinder, the force lines of the field pass through the normal region in a direction perpendicular to the cylinder surface (Fig. 2). In other words, a vortex passes through the cylinder, with an energy[17]

$$\mathcal{E}=\frac{\Phi_0^2Q(\Delta)}{(4\pi)^2}\ln\frac{1}{\xi(T)Q(\Delta)}.\qquad(13)$$

The vortex is produced on one end of the cylinder, is separated from it, and the further displacement, which requires no activation energy, of the vortex from one edge of the cylinder from the other transfers an additional flux Φ_0 from the outer space into the interior of the cylinder over its entire length. The activation energy of the state with new n is approximately equal to the vortex-formation energy, and according to (10) and (13) it is of the order of $\Delta_{c0}t(l/a)$. At $l\sim a$ the lifetime of the metastable state with nonequilibrium value of n can be large enough only at low temperatures $T\ll T_{c0}$, and near T_{c0} the equilibrium with respect to n is established rapidly. At $l\gg a$ the metastable state can exist for a long time everywhere except in a narrow vicinity of the point T_{c0}. The Kosterlitz-Thouless temperature T_c is also determined essentially by the parameter l/a. The quantity k_BT_c is the energy preceding the logarithmic factor in the right-hand side of (13), i.e.,

$$k_BT_c=\Phi_0^2Q(\Delta)/(4\pi)^2$$

(see Ref. 18). At $l\gg a$ we obtain

$$(T_{c0}-T_c)/T_{c0}=0{,}16\rho e^{-2}\sim a/l.$$

In the dirty superconductor limit $l \gg a$ the value of T_c may be noticeably lower than T_{c0}.

A relatively large macroscopic nonequilibrium moment, of the order of $\widetilde{H}_c R^2$ (per unit length), can be observed for the cylinder in sufficiently clean two-dimensional superconductors with $l \gg a$ in a wide temperature range $((T_{c0} - T)/T_{c0} \gg a/l)$. In the limit of dirty superconductors with $l \sim a$ this is possible only at low temperatures $T < T_c$ and $T \ll T_{c0}$.

We have considered the behavior of a superconducting cylinder in a field parallel to its axis. The presence of a field component perpendicular to the cylinder surface will, naturally, distort the results. To neglect the effect of this perpendicular component $H_\perp$ it is necessary that it cause no penetration of vortices into the cylinder. Such a penetration sets in when $H_\perp$ exceeds a critical value $2\pi\mathscr{E}/\Phi_0 R \approx \Phi_0/\delta R$. Therefore all the results are valid so long as $H_\perp < \Phi_0/\delta R$. This is a rather stringent restriction. Thus, at $l/a \sim 10^2$ and $R \sim 0.1$ the component $H_\perp$ must not exceed 10^{-4} Oe (see expression (10) for δ).

5. CONCLUSION

We list the main conclusions of the present paper.

1. With increasing external magnetic field, the system considered goes over from a state with an azimuthal quantum number n into a state with a number $n + 1$, where n is the integer closest to $H_0 \pi R^2/\Phi_0$. With change of n, the flux inside the cylinder changes by Φ. As n is varied, all the thermodynamic parameters of the system oscillate as functions of the external field H_0, with a period $\Phi_0/\pi R^2$. The magnetic moment of the cylinder also oscillates and reverses sign on going through those values of H_0 at which the quantum number n changes.

2. We note that in the equilibrium state a parallel magnetic field does not destroy the superconductivity in a thin cylinder at temperatures $t \gg \xi_0^2/R^2$, all the way to fields of the order of the paramagnetic limit. As applied to an unclosed film, this conclusion is valid without restriction on the temperature.

3. In the equilibrium state, the field inside the superconducting cylinder changes (compared with the external field) by an amount not exceeding $\Phi_0/\pi R^2$. In the metastable state this change is much larger, and total screening of the field is possible in parallel fields of the order of 0.1 Oe. The lifetimes of the metastable states can be large in sufficiently pure superconductors with $l \gg a$. The same condition is practically essential for the elimination of the influence of field components perpendicular to the cylinder surface. This component must not exceed $10^{-7} R^{-1}(l/a)$ Oe, where R is the cylinder radius in centimeters.

In the case of observation of two-dimensional superconductors, the magnetic measurements that are clear from foregoing exposition will also be found useful.

The authors thank Yu. E. Lozovik and V. V. Shmidt for a helpful discussion.

[1)]The question of surface magnets was posed also in Ref. 4 without allowance for fluctuations. For magnets this approach is all the more suitable since the fluctuations destroy the two-dimensional long-range order, generally speaking, only in the case of a degenerate order parameter. For models of the Ising type, however, and for a number of others, the fluctuations do not destroy the two-dimensional long-range order.

[2)]It can be seen from (2) that the minimum of $\mathscr{F}$ corresponds to values $\psi \approx 1$ in a wide range of ϕ_a, if $\xi(T)/R \ll 1$ (for details see Ref. 11).

[1]V. L. Ginzburg and D. A. Kirzhnits, Zh. Eksp. Teor. Fiz. **46**, 397 (1964) [Sov. Phys. JETP **19**, 269 (1964)]; V. L. Ginzburg, Phys. Lett. **13**, 101 (1964).

[2]P. C. Hohenberg, Phys. Rev. **158**, 383 (1967).

[3]In: Problema vysokotemperaturnoi sverkhprovodimosti, (Problem of High-Temperature Superconductivity) V. L. Ginzburg and D. A. Kirzhnits, eds., Nauka, 1977.

[4]L. N. Bulaevskii and V. L. Ginzburg, Fiz. met. metalloved. **17**, 631 (1964).

[5]M. A. Eddington and A. J. Leggett, Collective Phenomena **2**, 81 (1975).

[6]V. L. Ginzburg, Solid State Commun. **39**, 991 (1981).

[7]A. I. Buzdin and L. N. Bulaevskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **34**, 118 (1981) [JETP Lett. **34**, 112 (1981)]; V. V. Averin, A. I. Buzdin and L. N. Bulaevskii, Zh. Eksp. Teor. Fiz. **84**, 737 (1983) [Sov. Phys. JETP **57**, 426 (1983)].

[8]V. L. Ginzburg, Essays in honor of D. ter Haar. Pergamon Press (in preparation).

[9]W. Lawrence and S. Doniach, Proc. LT-12, Kyoto, p. 361 (1970).

[10]L. N. Bulaevskiĭ, Zh. Eksp. Teor. Fiz. **64**, 2241 (1973) [Sov. Phys. JETP **37**, 1133 (1973)]; Usp. Fiz. Nauk **116**, 449 (1975) [Sov. Phys. Usp. **18**, 514 (1975)].

[11]R. M. Arutyunyan and G. F. Zharkov, Zh. Eksp. Teor. Fiz. **78**, 1530, **79**, 245 (1980) [Sov. Phys. JETP **52**, 124 (1980)]; Preprint FIAN #251, (1982); J. Low Temp. Phys. **52**, 409 (1983).

[12]V. L. Ginzburg and L. D. Landau, Zh. Eksp. Teor. Fiz. **20**, 1064 (1950).

[13]W. A. Little and R. D. Parks, Phys. Rev. Lett. **9**, 9 (1962); R. D. Parks and W. A. Little, Phys. Rev. **A133**, 97 (1964).

[14]R. P. Groff and R. D. Parks, Phys. Rev. **176**, 567 (1968).

[15]G. Eilenberger, Z. Phys. **214**, 195 1968).

[16]K. Maki, In: Superconductivity, ed. by R. D. Parks, New York, 1969.

[17]P. G. de Gennes, Superconductivity of Metals and Alloys, Benjamin, 1966.

[18]J. M. Kosterlitz and D. J. Thouless, J. Phys. **C6**, 1181 (1973).

Translated by J. G. Adashko