Contribution of magnon-magnon interaction to the thermodynamics of anisotropic ferromagnets

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A diagram technique for Hubbard operators is used to study a Heisenberg ferromagnet with uniaxial anisotropy in the form of an arbitrary function $\varphi(S_f^z)$ and an arbitrary spin. The expressions for the effective interactions and Green functions obtained explicitly in the zeroth approximation of the self-consistent field method make it possible to construct the perturbation theory series both in terms of the reciprocal interaction radius r_0^{-3} and in terms of T/T_C. The temperature correction to the energy and the magnon damping are calculated in the low-temperature region. The corrections free-energy and magnetization corrections necessitated by the magnon interaction are calculated in second order in r_0^{-3}. It is shown that in an anisotropic ferromagnet there appear additional contributions that can exceed by several orders the corrections previously obtained by Dyson for the isotropic case.

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1. INTRODUCTION

It is well known that low-temperature thermodynamics of Heisenberg ferromagnets is determined by the properties of the spin-wave excitations and by the character of the interaction between them.[1,2] The small number of magnons at $T \ll T_C$ (T_C is the Curie temperature) allows us to assume that the temperature corrections necessitated by the interaction of the quasiparticles are determined by the two-particle scattering amplitude.[3] Dyson has shown[3] that for isotropic ferromagnets the temperature correction to the free energy for the magnon-magnon interaction is

$$\delta F^{(2)} \propto T^5 Z_{5/2}^2(\Delta/T);$$

and for $\langle S^z \rangle$ the correction is

$$\delta S^z \propto Z_{3/2}(\Delta/T) Z_{5/2}(\Delta/T) T^4$$

(the notation is defined in the text below).

A characteristic feature of anisotropic ferromagnetics is the non-equidistance of the single-ion energy levels. This circumstance leads, when the anisotropy is included in the zeroth Hamiltonian $\mathscr{H}_0$, to violation of the generalized Wick theorem for spin operators[4–6] and to the need of using special devices to calculate single-cell blocks.[7,8] On the other hand, the use of the Dyson-Maleev formalism[3,9] to investigate the physical properties of anisotropic ferromagnets[10–12] reduces the problem to a nonideal Bose gas only for the simplest anisotropy of the $D(S^z)^2$ and is restricted to low temperatures because of the unwieldy projection operator.[1)]

Zaĭtsev[14–16] proposed a new approach to the theory of Heisenberg magnets, based on representation of the spin operators in terms of Hubbard operators. This made it possible to generalize the Vaks-Larkin-Pikin diagram technique[4,5] to include the arbitrary single-node operators. This approach yielded[6] the spectrum of the spin waves and permitted an analysis of the singularities of the susceptibility for $S=1$ and the anisotropy $D(S_f^z)^2$. For an arbitrary value of S, the representation obtained in Ref. 17 for the spin operators in terms of Hubbard operators was found to be convenient for the investigation of both isotropic and anisotropic ferro- and antiferromagnets.

In this paper the diagram technique for the Hubbard operators[14–16] is used to investigate the spectrum and the damping of spin-wave excitations as well as the thermodynamic corrections, due to magnon-magnon interaction, to the free energy and magnetization of a uniaxial Heisenberg operator with arbitrary spin S. The uniaxial anisotropy in the form of an arbitrary function of the operator S_f^z is taken into account exactly by including it in the zeroth Hamiltonian. It is shown that besides the Dyson term there appear in the anisotropic ferromagnets additional contributions that contain a lower power of the small parameter T/T_C and therefore exceed by one or two orders the corrections obtained by Dyson.

2. LARKIN'S EQUATION FOR AN ANISOTROPIC FERROMAGNET

We shall describe the uniaxial anisotropy of a Heisenberg ferromagnet with arbitrary spin by the term

$$\mathscr{H}_a = -\sum_f \varphi(S_f^z), \qquad (1)$$

where φ is an arbitrary function of the operator S_f^z, and f is the number of the site. We use the representation of the spin operators in terms of the Hubbard operators[17,18]:

$$S_f^+ = \sum_{M=-S}^{S} \gamma_S(M) X_f^{M+1,M}, \quad S_f^z = \sum_{M=-S}^{S} M X_f^{M,M}, \quad S_f^- = (S_f^+)^+,$$

where the Hubbard operator

$$X_f^{M'M} \equiv |f, M'\rangle\langle f, M|$$

is defined on the eigenfunctions $|f, M\rangle$ of the operator S_f^z, and

$$\gamma_S(M) = [(S-M)(S+M+1)]^{1/2}.$$

This representation is convenient because $\mathscr{H}_a$ assumes a simple operator structure:

$$\mathscr{H}_a = -\sum_f \sum_{M=-S}^{S} X_f^{M,M} \varphi(M),$$

and the entire complicated form of the anisotropy goes over

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into a c-number function $\varphi(M)$. This enables us easily to include $\mathscr{H}_a$ in the zeroth Hamiltonian, and as a result we can write the Hamiltonian of an Heisenberg magnet with anisotropy (1) in the form

$$\mathscr{H}=\mathscr{H}_0+\mathscr{H}_{int}+{}^1/_2NI_0\langle S^z\rangle^2, \quad (2)$$

$$\mathscr{H}_0=\sum_{fM}E_M h_{fM}, \quad E_M=-(g\mu_B H+I_0\langle S^z\rangle)M-\varphi(M), \quad (3)$$

$$\mathscr{H}_{int}=-{}^1/_2\sum_{fm}\sum_{\alpha\beta}I_{fm}^{\alpha\beta}X_f^\alpha X_m^{-\beta}-{}^1/_2\sum_{fm}\sum_{MM'}I_{fm}^{MM'}\tilde{h}_{fM}\tilde{h}_{mM'}. \quad (4)$$

The Hubbard operators X_f^α act in the space of the eigenfunctions of the operator S_f^z. The vector $\boldsymbol{\alpha}$ is related to the transition from the state $|M'\rangle$ into the state $|M\rangle$ in accord with the rule

$$X_f^{MM'}\to X_f^{\alpha(MM')}\equiv X_f^\alpha.$$

It follows from the completeness condition

$$\sum_M X_f^{M,M}=1$$

that in place of $2S+1$ diagonal operators $X_f^{M,M}$ we can use $2S$ zero-trace operators, as was done in fact in Refs. 14–16. We, however will make use of all the operators $X_f^{M,M}\equiv h_{fM}$ (with nonzero traces) that make up the vector $\mathbf{h}_f$. It is then easy to calculate the components of the root vectors:

$$\alpha_M(M_1, M_2)=\delta_{MM_1}-\delta_{MM_2}$$

and the same commutation rules as in Ref. 16 are preserved:

$$[h_{fM}, X_{f'}^\alpha]=\delta_{ff'}\alpha_M X_f^\alpha, \quad [X_f^\alpha, X_{f'}^{-\alpha}]=\delta_{ff'}(\alpha\mathbf{h}),$$
$$[X_f^\alpha, X_{f'}^\beta]=\delta_{ff'}N_{\alpha+\beta}X_f^{\alpha+\beta},$$

where $N_{\alpha+\beta}$ differs from zero and is equal to ± 1 if $\alpha+\beta$ is again a root vector. Diagonal operators with nonzero trace were used earlier in the anisotropic s–f metal problem.[19] in Eq. (2) we separated the self-consistent field[4–6] $\tilde{h}_{fM}=h_{fM}-\langle h_{fM}\rangle$. The dependence of the matrix elements of the transverse interaction is determined by the relations

$$I_{fm}^{\alpha\beta}=I_{fm}\gamma^{(\alpha)}\gamma^{(\beta)},$$
$$\gamma(\alpha)=[(S-M)(S+M+1)]^{1/2}, \quad \alpha=\alpha(M+1, M), \quad (5)$$
$$\gamma(\alpha)=0, \quad \alpha\neq\alpha(M+1, M).$$

The components of the longitudinal interaction are of the form $I_{fm}^{M,M'}=I_{fm}MM'$.

The dynamic characteristics of the ferromagnet in question are connected with the properties of the Matusbara Green function

$$D_{\alpha\beta}(f\tau;\ m\tau')=-\langle T\tilde{X}_m^\alpha(\tau)\tilde{X}_m^{-\beta}(\tau')\rangle. \quad (6)$$

We denote by $\Sigma^{\alpha\beta}(\mathbf{k},\omega_n)$ the complete aggregate of the diagrams that are irreducible in one transverse interaction line, with incoming and outgoing vectors α and β, respectively. From an examination of the diagram series for the Fourier transform $D_{\alpha\beta}(\mathbf{k},\omega_n)$ of the Green function (6) we get the Larkin equation:

$$D_{\alpha\beta}(\mathbf{k},\omega_n)=\Sigma^{\alpha\beta}(\mathbf{k},\omega_n)-\frac{1}{2}\frac{(\Sigma_\perp I_\perp\Sigma_\perp)_{\alpha\beta}}{1+{}^1/_2\,\mathrm{Sp}(I_\perp\Sigma_\perp)}, \quad (7)$$

where $\Sigma_\perp$ and $I_\perp$ are matrices with respective components $\Sigma^{\alpha\beta}(\mathbf{k},\omega_n)$ and $I_\mathbf{k}^{\alpha\beta}$. A distinguishing feature of Eq. (7) is the explicit connection between $D_{\alpha\beta}$ and the components of the matrices $\Sigma_\perp$ and $I_\perp$, whereas in Refs. 14–16 $D_{\alpha\beta}$ was expressed in terms of the reciprocal matrix $\Sigma_\perp^{-1}$, whose determination at arbitrary S calls for unwieldy transformations. It is easy to obtain from (7) the Larkin equation for the transverse Green function $K^{+-}(\mathbf{k},\omega_n)$ in the anisotropic case

$$K^{+-}(\mathbf{k},\omega_n)=\sum_{\alpha\beta}\gamma(\alpha)\gamma(\beta)D_{\alpha\beta}(\mathbf{k},\omega_n)=\frac{\Sigma^{+-}(\mathbf{k},\omega_n)}{1+{}^1/_2I_\mathbf{k}\Sigma^{+-}(\mathbf{k},\omega_n)} \quad (8)$$

$$\Sigma^{+-}(\mathbf{k},\omega_n)=\sum_{\alpha\beta}\gamma(\alpha)\gamma(\beta)\Sigma^{\alpha\beta}(\mathbf{k},\omega_n). \quad (9)$$

As $\mathscr{H}_a\to 0$ Eq. 8 goes over into the corresponding equation of Refs. 4–6.

For the Fourier transform of the longitudinal function

$$D_{MM'}(f\tau;\ m\tau')=-\langle T\tilde{h}_{fM}(\tau)\tilde{h}_{mM'}(\tau')\rangle \quad (10)$$

we obtain similarly

$$D_{MM'}(\mathbf{k},\omega_n)=\Sigma^{MM'}(\mathbf{k},\omega_n)+\frac{(\Sigma_\parallel I_\parallel\Sigma_\parallel)_{MM'}}{1-\mathrm{Sp}(I_\parallel\Sigma_\parallel)}, \quad (11)$$

where $\Sigma^{MM'}(\mathbf{k},\omega_n)$ is the corresponding irreducible part for the longitudinal function (10), and $I_\parallel$ is a matrix whose components are the integrals of the longitudinal interactions $I_\mathbf{k}^{MM'}$.

3. ZEROTH APPROXIMATION OF THE SELF-CONSISTENT-FIELD (SCF) METHOD AND EFFECTIVE INTERACTIONS

In the approximation considered, the irreducible parts in (7) and (11) are equal to

$$\Sigma^{\alpha\beta}(\mathbf{k},\omega_n)=\delta_{\alpha\beta}b(\alpha)D_\alpha(\omega_n), \quad D_\alpha(\omega_n)=\frac{1}{i\omega_n+\alpha E}, \quad (12)$$

$$\Sigma^{MM'}(\mathbf{k},\omega_n)=\delta_{MM'}n_M-n_Mn_{M'}, \quad (13)$$

where, as in Refs. 14–16, we use the concept of the end factor $b(\alpha)=\langle\alpha h_f\rangle_0$; the occupation numbers n_M are defined by the relation

$$n_M=\langle h_{fM}\rangle_0. \quad (14)$$

From (12) and (7) we easily obtain $D_{\alpha\beta}(\mathbf{k},\omega_n)$ in the zeroth approximation of the SCF method:

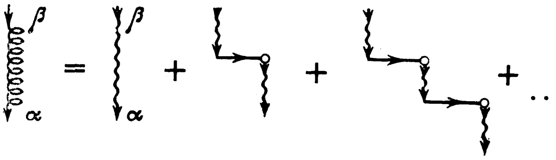
$$D_{\alpha\beta}^{(0)}(\mathbf{k},\omega_n)=\delta_{\alpha\beta}D_\alpha(\omega_n)b(\alpha)-{}^1/_2I_\mathbf{k}\Gamma(\alpha,\omega_n)\Gamma(\beta,\omega_n)/[1+{}^1/_2I_\mathbf{k}\sum_{\alpha_1}\gamma(\alpha_1)\Gamma(\alpha_1,\omega_n)]^{-1}, \quad (15)$$

$$\Gamma(\alpha,\omega_n)=\gamma(\alpha)b(\alpha)D_\alpha(\omega_n). \quad (16)$$

The Green function (8) takes consequently, in the same approximation, the form

$$K^{+-}(\mathbf{k},\omega_n)=\sum_\alpha\gamma(\alpha)\Gamma(\alpha,\omega_n)\left[1+{}^1/_2I_\mathbf{k}\sum_\alpha\gamma(\alpha)\Gamma(\alpha,\omega_n)\right]^{-1}. \quad (17)$$

We introduce the effective interaction $\tilde{I}^{\alpha\beta}(\mathbf{q},\omega_n)$ defined by the following diagram series[2)]:

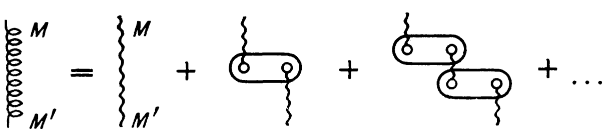


(18)

After summing this series we have

$$\Gamma^{\alpha\beta}(\mathbf{q},\omega_n)=I^{\alpha\beta}(\mathbf{q})\left[1+{}^1/_2 I_{\mathbf{q}}\sum_{\alpha_1}\gamma(\alpha_1)\Gamma(\alpha_1,\omega_n)\right]^{-1}. \quad (19)$$

For a longitudinal effective interaction



(20)

we have after summation

$$\Gamma^{MM'}(\mathbf{q},\omega_n)=I^{MM'}(\mathbf{q})\left[1-T^{-1}\sum_{M_1M_2}I^{M_1M_2}(\mathbf{q})\,b(M_1,M_2)\,\delta_{n,0}\right]^{-1}. \quad (21)$$

The explicit expressions obtained here for $D^{(0)}_{\alpha\beta}(\mathbf{k},\omega_n)$ and for the effective interactions permits construction of a series of successive approximations in powers of the reciprocal interaction radius, as was done for the isotropic case by Vaks, Larkin, and Pikin.[4,5] On the other hand, in the region $T\ll T_C$ we can go over consistently to a spin-wave description of a ferromagnet with arbitrary form of uniaxial anisotropy.

4. FIRST APPROXIMATION OF THE SCF METHOD

At $T\ll T_C$, in first order in $1/r_0^3$, the irreducible part of the function (7) is equal to

$$\Sigma^{\alpha\beta}(\mathbf{k},\omega_n)=\Sigma^{\alpha\beta}_{(0)}(\mathbf{k},\omega_n)+\Sigma^{\alpha\beta}_{(1)}(\mathbf{k},\omega_n), \quad (22)$$

where the first term is described by Eq. (12), and the second is given by the sum of diagrams of Fig. 1.

The first three diagrams are valid also in the isotropic case, since diagrams d) and e) make a nonzero contribution only in the presence of anisotropy. As a result we find that near their poles the transverse function (17) can be written in the form

$$K^{+-}(\mathbf{k},\omega_n)=2\langle S^z\rangle/[i\omega_n-\varepsilon_{\mathbf{k}}^0+P(\mathbf{k})], \quad (23)$$

where

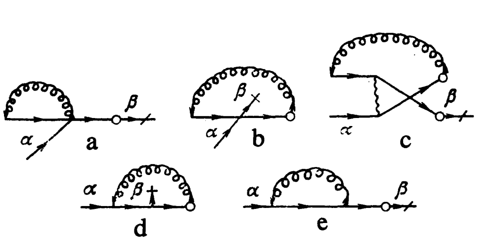


FIG. 1. Diagrams of first order in $1/r_0^3$ at $T\ll T_C$ for $\Sigma^{\alpha\beta}(\mathbf{k},\omega_n)$.

$$\varepsilon_{\mathbf{k}}^0=\Delta+S[I_0-I_{\mathbf{k}}],\quad \Delta=g\mu_B H+\varphi(S)-\varphi(S-1),$$

$$P(\mathbf{k})=\frac{1}{N}\sum_{\mathbf{q}}[I_0+I_{\mathbf{k}-\mathbf{q}}-I_{\mathbf{k}}-I_{\mathbf{q}}]n_{\mathbf{q}}-\left(\frac{1}{N}\right)(2S-1)R[\varphi]$$

$$\times\sum_{\mathbf{q}}(I_{\mathbf{k}}+I_{\mathbf{q}})\quad n_{\mathbf{q}}[\varepsilon_{\mathbf{k}}^0+\varepsilon_{\mathbf{q}}^0+E_S-E_{S-2}]^{-1}, \quad (24)$$

$$R[\varphi]=2\varphi(S-1)-\varphi(S)-\varphi(S-2),\quad n_{\mathbf{q}}=[\exp(\varepsilon_{\mathbf{q}}^0/T)-1]^{-1}.$$

It follows from (23) that the spectrum of the spin-wave excitations is of the form

$$E_{\mathbf{k}}=\varepsilon_{\mathbf{k}}^0-\frac{I_0}{6}\frac{\pi v}{r_0^3}K_{5/2}(T)\sum_{i=1}^{3}R_{0i}^2k_i^2+\frac{2(2S-1)}{r_0^3}\frac{I_0R[\varphi]}{2SI_0+R[\varphi]}A(T,\mathbf{k}), \quad (25)$$

$$K_\alpha(T)=\left(\frac{3T}{2\pi SI_0}\right)^\alpha Z_\alpha\left(\frac{\Delta}{T}\right),\quad Z_\alpha(x)=\sum_{n=1}^{\infty}\frac{e^{-nx}}{n^\alpha}.$$

The function A is written in the form

$$A(T,\mathbf{k})=\left[1-\frac{R[\varphi]}{2SI_0+R[\varphi]}\frac{1}{4}\sum_{i=1}^{3}x_{0i}^2k_i^2\right]K_{3/2}(T)-\frac{\pi}{2}\frac{R[\varphi]}{2SI_0+R[\varphi]}K_{5/2}(T).$$

Following Refs. 4 and 5, we have introduced in (25) the quantities

$$vR_{0i}^2=\sum_{\mathbf{r}}r_i^2\sum_{j=1}^{3}\frac{r_j^2}{x_{0i}^2}\frac{I(\mathbf{r})}{I_0},\quad i=1,2,3.$$

It can be seen that for a uniaxial ferromagnet, besides the usual temperature correction $\propto T^{5/2}Z_{5/2}(\Delta/T)$ to the energy there is also a term that contains a lower power of the ratio T/T_C. This circumstance leads in ferromagnets with finite anisotropy to a stronger temperature dependence of the magnon energy than in the isotropic case.

The magnon damping can be represented in the form

$$\Gamma_{\mathbf{k}}=(2S-1)R^2[\varphi]\frac{2\sqrt{\pi}}{Sr_0^3}\left(\frac{3}{2\pi SI_0}\right)^{3/2}u_{\mathbf{k}}^{1/2}n(u_{\mathbf{k}})\theta(u_{\mathbf{k}}), \quad (26)$$

where $u_{\mathbf{k}}=E_{s-2}-E_s-\Delta-\varepsilon_{\mathbf{k}}^0$ and $\theta(x)$ is the Heaviside unit step function. It follows from (26) that the damping of magnons with momentum **k** will differ from zero only in the case when the system contains a magnon having an energy ε_q^0 such that the sum of the energies of these two magnons coincides with the energy difference of the levels E_{S-2} and E_S. Since the minimum magnon energy is Δ, the condition noted above cannot be satisfied in the case when $\varepsilon_{\mathbf{k}}+\Delta>E_{s-2}-E_s$, and the damping vanishes. We note here that the result (26) was obtained for $\varphi(S_f^z)=D(S_f^z)^2$ in Ref. 8.

5. FREE ENERGY AND MAGNETIZATION

In the low temperature region ($T \ll T_C$) the expression for the free energy can be represented in the form

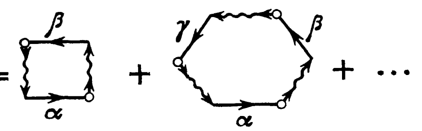
$$F = E_0 + F_1 + \delta F^{(2)}, \tag{27}$$

Where E_0 is the ground-state energy per site

$$E_0 = -{}^1/_2 I_0 S^2 - g\mu_B H S - \varphi(S), \tag{28}$$

and F_1 is the free energy of a gas of noninteracting magnons. The graphic expression for F_1 takes the form

$$-\frac{F_1}{T} =$$



(29)

A distinguishing feature of (29) compared with the corresponding isotropic-case series is the additional summation over the vectors α. In the considered temperature region, owing to the end factors $b(\alpha)$, there remain in the sums only terms with $\alpha = \alpha_0(S, S-1)$. At $\alpha \neq \alpha_0(S, S-1)$ the factors $b(\alpha)$ yield exponential terms of the type $\exp(-T_C/T)$, which we shall neglect. From (29) we find that

$$F_1 = \frac{T}{N} \sum_{\mathbf{q}} \ln\left[1 - \exp\left(-\frac{\varepsilon_{\mathbf{q}}^0}{T}\right)\right]. \tag{30}$$

The quantity $\delta F^{(2)}$ in (27) is due to magnon-magnon interaction. In second order in $1/r_0^3$ of the SCF method, the contribution to $\delta F^{(2)}$ is determined by the diagrams shown in Fig. 2. The last two of them are features of only the anisotropic case. Comparing the diagrams with the analytic expressions, we obtain

$$\delta F^{(2)} = -\frac{1}{2N} \sum_{\mathbf{k}} P(\mathbf{k}) n_{\mathbf{k}} = \delta F_D^{(2)} + \delta F_a^{(2)}, \tag{31}$$

where

$$\delta F_D^{(2)} = -\frac{\pi^2 I_0 \mu}{18 r_0^6} \left(\frac{3T}{2\pi S I_0}\right)^5 Z_{5/2}^2\left(\frac{\Delta}{T}\right),$$

$$\mu = \sum_{\mathbf{r}} \left\{ \sum_{i,j=1}^{3} \left(\frac{r_i^2 r_j^2}{x_{0i}^2 x_{0j}^2}\right) \frac{I(\mathbf{r})}{I_0} \right\}, \tag{32}$$

$$\delta F_a^{(2)} = \frac{(2S-1) R[\varphi] I_0}{2SI_0 + R[\varphi]} \left(\frac{1}{r_0^3}\right)^2 \left(\frac{3T}{2\pi S I_0}\right)^3 Z_{3/2}\left(\frac{\Delta}{T}\right)$$
$$\times \left[Z_{3/2}\left(\frac{\Delta}{T}\right) - \pi \frac{R[\varphi]}{2SI_0 + R[\varphi]} \left(\frac{3T}{2\pi S I_0}\right) Z_{5/2}\left(\frac{\Delta}{T}\right) \right]. \tag{33}$$

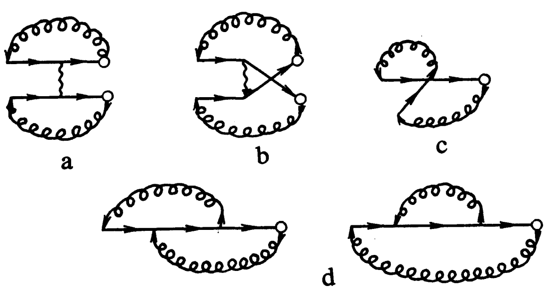


FIG. 2. Plots of second order in $1/r_0^3$ at $T \ll T_C$ for the free energy.

The term $\delta F_D^{(2)}$ in (31) describes the decrease of the free energy on account of the interaction of the spin waves and corresponds to the usual Dyson correction previously obtained for the isotropic case.[3,4–6] The second term in (31) is connected with the additional spin-wave interaction that occurs in anisotropic ferromagnets because of the nonequidistance of the single-ion energy levels.

It can be seen from (32) and (33) that at a finite value of the anisotropy $\delta F_a^{(2)}$ can exceed $\delta F_D^{(2)}$ by several orders, since it contains a lower power of the small parameter T/T_C. We note here that to obtain the analytic expressions we have used the condition

$$\exp\{-(g\mu_B H + I_0 S + R[\varphi])/T\} \ll 1.$$

This means that the results are valid also at anisotropy values that are comparable in order of magnitude with the exchange energy.

Knowing the free energy, we easily obtain for the magnetization, with allowance for the spin-wave interaction, the expression

$$\langle S^z \rangle = S - \frac{1}{N} \sum_{\mathbf{q}} n_{\mathbf{q}} + \delta S^z, \tag{34}$$

where

$$\frac{1}{N} \sum_{\mathbf{q}} n_{\mathbf{q}} = \frac{1}{r_0^3} \left(\frac{3T}{2\pi S I_0}\right)^{3/2} Z_{3/2}\left(\frac{\Delta}{T}\right) + O(T^{5/2}). \tag{35}$$

We represent δS^z in the form of two terms:

$$\delta S^z = \delta_D S^z + \delta_a S^z, \tag{36}$$

$$\delta_D S^z = -\frac{\pi\mu}{6 S r_0^6} \left(\frac{3T}{2\pi S I_0}\right)^4 Z_{5/2}\left(\frac{\Delta}{T}\right) Z_{3/2}\left(\frac{\Delta}{T}\right), \tag{37}$$

$$\delta_a S^z = \frac{3(2S-1)}{\pi S r_0^6} \frac{R[\varphi]}{2SI_0 + R[\varphi]} \left(\frac{3T}{2\pi S I_0}\right)^2 \left\{ Z_{3/2}\left(\frac{\Delta}{T}\right) Z_{1/2}\left(\frac{\Delta}{T}\right) \right.$$
$$- \frac{\pi}{2} \frac{R[\varphi]}{2SI_0 + R[\varphi]} \left(\frac{3T}{2\pi S I_0}\right)$$
$$\left. \times \left[Z_{3/2}^2\left(\frac{\Delta}{T}\right) + Z_{1/2}\left(\frac{\Delta}{T}\right) Z_{5/2}\left(\frac{\Delta}{T}\right) \right] \right\}. \tag{38}$$

The first term in (36) corresponds to the Dyson correction, which in our case differs only by the anisotropy-induced renormalization of the gap Δ in the spin-wave spectrum. The second term vanishes when the anisotropy tends to zero, and also at $S = 1/2$. At a finite anisotropy the contribution made to δS^z by the second term can substantially exceed the contribution of the Dyson term. We can conclude from this that in anisotropic feromagnets the magnon-magnon interaction plays a larger role than in the isotropic case. In fact, in the case $\varphi(S_f^z) = D(S_f^z)^2$ at $T \ll \Delta < I_0 S$ we easily obtain the asymptotic expression

$$\delta_D S^z = -\frac{\pi\mu}{6 S r_0^6} \left(\frac{3T}{2\pi S I_0}\right)^4 \exp\left(-\frac{2\Delta}{T}\right),$$

$$\delta_a S^z = -\frac{3(2S-1)}{\pi S r_0^6} \frac{D}{I_0 S - D} \left(\frac{3T}{2\pi S I_0}\right)^2 \exp\left(-\frac{2\Delta}{T}\right).$$

In the case $\Delta \ll T \ll I_0 S$ it follows from (37) and (38) that

$$\delta_D S^z \sim -\frac{1}{r_0^6 S}\left(\frac{T}{I_0 S}\right)^4,$$

$$\delta_a S^z \sim -\frac{(2S-1)}{S r_0^6}\left(\frac{D}{I_0 S}\right)\left(\frac{T}{I_0 S}\right)^2\left(\frac{T}{\Delta}\right)^{1/2}$$

in which case the gap plays an insignificant role in the magnon spectrum.

In conclusion, we discuss the possibility of experimentally observing the contribution $\delta_a S^z$ to the total decrease of the magnetization with rising temperature. To this end we compare the value of the Bloch term (35) with the value of $\delta_a S^z$. We assume for simplicity that $\varphi(S_f^z) = D(S_f^z)^2$. Then $R[\varphi] = -2D$. We consider separately the two cases $D > 0$ and $D < 0$.

a) $D > 0$, anisotropy of the easy axis type. Choosing $D/I_0 S \approx 0{,}2, 3T/2\pi S I_0 \approx 0{,}2$, $1/r_0^3 \approx \frac{1}{2}$, we find that the decrease of the magnetization on account of the magnon interaction is on the order of one percent compared with the decrease of the magnetization due to the Bloch term.

b) $D < 0$, anisotropy of the easy plane type. At $H = 0$ the magnetization of the ferromagnet lies in the easy plane. In an external magnetic field $g\mu_B H > |D|(2S-1)$ perpendicular to this plane, the spins are aligned along the z axis and we obtain an experimental geometry in which our analysis is valid. Choosing a ferromagnet with relatively strong anisotropy, we can choose a magnetic field such that the quantity $Z_{1/2}(\Delta/T)$ in (38) ceases to introduce additional smallness. Therefore the decrease of the magnetization on account of the interaction is much higher in this case and can amount to several times ten percent compared with the magnetization decrease due to the Bloch term (35).

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[1]A new representation of the spin operator in terms of Bose and Fermi operator, with a projection operator of simple form, was proposed in a recent paper.[13]

[2]An arrow headed by a circle denotes the function $\Sigma_0^{\alpha\beta}$. Two circles in an oval denote the function $\Sigma_0^{MM'}$..

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