Circulation effect and quantum interference phenomena in a nonuniformly heated toroidal vessel with superfluid helium

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(Submitted 18 May 1983)
Zh. Eksp. Teor. Fiz. **85**, 1606–1614 (November 1983)

The circulation effect that must take place in a nonuniformly heated toroidal vessel with a superfluid liquid (He-II) under conditions of nonzero complete superfluid-velocity circulation is discussed. Two types of devices for the observation of quantum interference phenomena in He-II are considered.

PACS numbers: 67.40.Hf, 67.40.Pm

1. In a toroidal vessel filled with a superfluid liquid (specifically, helium II) and having two "narrow links," the presence of a temperature gradient should give rise to superfluid flow through the entire vessel. This thermomechanical circulation effect was considered in Refs. 1–3 and its experimental study was started.[4] Not lacking in interest is perhaps also the fact that the possible existence of a thermoelectric effect in superconductors was deduced[5] by analogy with the internal convection that occurs in helium II in the presence of a temperature gradient. On the contrary, the conclusion that a circulation effect is produced in an annular vessel with nonuniformly heated helium II was arrived at[1] on the basis of an analogy with the thermal effect in a superconducting circuit.

The circulation effect in helium II was considered in Refs. 1 and 3 only for the simplest particular case when the circulation of the velocity $\mathbf{v}_s$ of the superfluid motion in the closed (filled) loop is zero for the considered vessel. To be sure, the case of nonzero circulation is mentioned in Ref. 1 and is briefly discussed in Ref. 2. It seems to us, however, that this case deserves more attention, since the accompanying jumps of the circulation-flow velocities, hysteresis phenomena, and residual circulation fluxes can yield, in principle, abundant information on the nature of the critical velocities of the superfluid flow in helium II, let alone the possibility of directly measuring the circulation quantum. Besides, the use of the thermocirculation effect creates favorable conditions for observing in helium II quantum interference phenomena[6] similar to the corresponding interference phenomena in superconductors.[7] This group of problems is precisely the subject of this article. At the same time, it will be useful to repeat the entire analysis in greater detail.

2. Consider the annual helium-II-filled vessel shown schematically in Fig. 1. If the "narrow links" are round capillaries ($i = 1,2$) having a radius r_i (cross section area $S_i = \pi r_i^2$) and a length l_i, a temperature gradient δT produces through the capillaries fluxes $I_{ni} = \rho_{ni} v_{ni} S_i$ of the normal liquid, where the normal velocity averaged over the cross section is[8]

$$v_{ni}=(\rho\sigma S_i/8\pi\eta_{ni}l_i)\,\delta T; \quad (1)$$

where ρ is the helium density, σ the entropy per unit mass, and η_{ni} the corresponding viscosity coefficient [the choice of the signs in (1) and in other expressions will be discussed below].

It is sometimes more convenient in practice[4] to express the velocities v_{ni} not in terms of δT but in terms of the heat output Q of the heater per unit time (this heater should be regarded as located on the left side of the vessel in Fig. 1). Since[8]

$$Q=\rho\sigma T\,(v_{n1}S_1+v_{n2}S_2) \quad (2)$$

we have for capillaries having a cross section of arbitrary shape (but the same for both links)

$$v_{ni}=C\,(\rho\sigma S_i/\eta_{ni}l_i)\,\delta T,$$

we get according to (2) (at $\eta_{n1} = \eta_{n2} = \eta_n$),

$$\delta T=C\,[(\rho\sigma)^2 Ta/\eta_n]\,Q \quad (3)$$

and independently of the constant C

$$v_{ni}=(S_i/l_i\rho\sigma Ta)\,Q; \qquad a=(S_1^2/l_1)+(S_2^2/l_2). \quad (4)$$

For narrow links of another type (say channels filled with powder and others), Eqs. (1)–(4) are generally speaking unsuitable and we shall simply regard the velocities v_{ni} as given.

If the vessel as a whole is open ("cut" along the line AB in Fig. 1), the flux density in each of the links is

$$\mathbf{j}_i=\mathbf{j}_{si}+\mathbf{j}_{ni}=0, \qquad \mathbf{j}_{si}=\rho_{si}\mathbf{v}_{si}$$

and consequently the corresponding velocities of the superfluid part of the liquid are

$$\mathbf{v}_{si}^{(0)} = -(\rho_{ni}/\rho_{si})\,\mathbf{v}_{ni}. \quad (5)$$

The following condition should hold for a closed vessel (m is the helium-atom mass)

$$\oint \mathbf{v}_s d\mathbf{l}=2\pi(\hbar/m)\,n, \quad n=0, \pm 1, \pm 2, \ldots, \quad (6)$$

where the integration is along an arbitrary contour, such as indicated dashed in Fig. 1. In addition, to use vectors, it is more convenient in this case to use the projections of the velocities on contour length elements $d\,\mathbf{l}$. Doing this and assuming $\delta T > 0$, we have

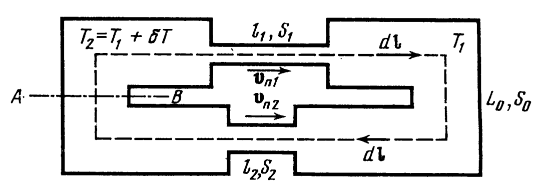

FIG. 1. Toroidal vessel with superfluid helium.

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$$v_{n1}>0, \quad v_{n2}<0, \quad v_{s1}^{(0)} = -\frac{\rho_{n1}}{\rho_{s1}}|v_{n1}|, \tag{7}$$
$$v_{s2}^{(0)} = \frac{\rho_{n2}}{\rho_{s2}}|v_{n2}|.$$

Denoting by v_{s_0}, ρ_{s_0}, S_0, and l_0 the corresponding quantities for the "main" part of the vessel and by $v_{si} = v_{si}^{(0)} + v_{si}^{(\prime)}$ the superfluid velocities in the links $i = 1, 2$ we write the condition (3) in the form

$$(v_{s1}^{(0)} + v_{s1}^{(\prime)})l_1 + (v_{s2}^{(0)} + v_{s2}^{(\prime)})l_2 + v_{s0}l_0 = (2\pi\hbar/m)n. \tag{8}$$

By virtue of the flux conservation we have ($j_n = 0$ in the main part of the vessel; the constraints (5) are also taken into account)

$$I_{s0} = \rho_{s0}v_{s0}S_0 = \rho_{s1}v_{s1}^{(\prime)}S_1 = \rho_{s2}v_{s2}^{(\prime)}S_2. \tag{9}$$

It is assumed here for simplicity that the velocities v_{s0}, v_{s1}, and v_{s2} are constant along the corresponding cross section, an incorrect assumption in a certain vicinity of the "junctions." The corresponding error can apparently always be made small enough.

The density ρ_{si} of the superfluid part of the liquid in the narrow links can differ from ρ_{s0} (the density of the superfluid component in the volume) on account of the proximity effect in the presence of narrow pores etc.[3]

From (8) and (9) we obtain, taking (5) into account

$$v_{s0} = \left[\frac{2\pi\hbar}{m}n + \frac{\rho_{n1}}{\rho_{s1}}l_1|v_{n1}| - \frac{\rho_{n2}}{\rho_{s2}}l_2|v_{n2}|\right] \times \left[l_0 + \rho_{s0}S_0\left(\frac{l_1}{\rho_{s1}S_1} + \frac{l_2}{\rho_{s2}S_2}\right)\right]^{-1}. \tag{10}$$

In the case of narrow links—round capillaries far from the λ point we can put $\rho_{s1} = \rho_{s2} = \rho_{s0} = \rho_s$, $\eta_{n1} = \eta_{n2} = \eta_n$ and use Eq. 1. Then

$$v_{s0} = \left[\frac{2\pi\hbar}{m}n + \frac{\rho\sigma\rho_n}{8\pi\eta_n\rho_s}(S_1 - S_2)\delta T\right]\left[l_0 + S_0\left(\frac{l_1}{S_1} + \frac{l_2}{S_2}\right)\right]^{-1}. \tag{11}$$

At $n = 0$ this equation goes over in fact into the equations used in Refs. 1–4, and specifically into Eq. (1) of Ref. 3. Near the point ρ_{s1} and ρ_{s2} can differ from ρ_{s0} even at not too thin (macroscopic capillaries); account must also be taken in general of the temperature dependences of ρ_{si} and of other quantities. The corresponding analysis is perfectly feasible on the basis of the Ψ theory of superfluidity.[9]

For narrow links made up of powder with microscopic pores the density ρ_{si} can differ substantially from ρ_{s0} also far from the λ point. For the particular case when (see Ref. 3)

$$\rho_{s2} \gg \rho_{s1}, \quad \rho_{n2} \ll \rho_{n1} \approx \rho, \quad S_1 = S_2 = S, \quad l_1 = l_2 = l,$$

we obtain from (20)

$$v_{s0} \approx \frac{(2\pi\hbar/m)n + (\rho/\rho_{s1})l|v_{n1}|}{l_0 + \rho_{s0}S_0 l/\rho_{s1}S} \tag{12}$$

(strictly speaking, it is assumed here also that

$$(\rho_{n2}/\rho_{s2})|v_{n2}| \ll (\rho_{n1}/\rho_{s1})|v_{n1}|,$$

since this inequality is not automatically guaranteed by the inequalities above). If $l_0 \ll \rho_{s0}S_0 l/\rho_{s1}S$, Eq. (12) yields at $n = 0$ the velocity $v_{s0} \approx \rho S v_{ni}/\rho_{s0}S_0$ and the total flux

$$I_{s0} \approx \rho_{s0}v_{s0}S_0 \approx \rho v_{n1}S \approx \rho_{n1}v_{n1}S = I_{n1},$$

given in Ref. 3.

3. One of the important results to emphasize is that the minimum of the energy (or of some other thermodynamic potential) does not necessarily always correspond to $n = 0$, i.e., to absence of circulation. If $l_0 \gg S_0(l_1/S_1 + l_2/S_2)$, the kinetic energy of the superfluid flow is

$$\mathcal{H} \approx {}^1/_2\rho_{s0}v_{s0}^2 l_0 S_0.$$

It is then clear from (10)–(12) that if initially $n = 0$, the energy $\mathcal{H}$ increases like $(\delta T)^2$. Subsequently, however, a state with $n = 1$ or $n = -1$, will be energetically more favorable, next a state with $n = 2$ or $n = -2$, etc. We use for the sake of argument expression (11) with sufficiently large l_0 and $S_2 \gg S_1$, when

$$v_{s0} = \frac{1}{l_0}\left(\frac{2\pi\hbar}{m}n - \frac{\rho\sigma\rho_n S_2}{8\pi\eta_n\rho_s}\delta T\right) = \frac{2\pi\hbar}{ml_0}(n - A\delta T), \tag{13}$$
$$A = \rho\sigma\rho_n S_2/16\pi^2\hbar\eta_n\rho_s.$$

Then

$$\mathcal{H} = (2\pi^2 h^2\rho_s S_0/m^2 l_0)(n - A\delta T)^2. \tag{14}$$

The behavior of the function

$$\mathcal{N} = (m^2 l_0/2\pi^2\hbar^2\rho_s S_0)\mathcal{H}$$

as a function of $A\delta T$ at $n = 0$, 1, and 2 is also clear from Fig. 2. Obviously, the state with $n = 2$ is energetically favored at $A\delta T > 1/2$, that with $n = 2$ at $A\delta T > 3/2$, etc. The situation is reminiscent here of the case of a hollow superconducting cylinder (we have in mind the dependence of the free energy on the external magnetic field and on the number n of the quanta in the flux, see Ref. 10). Of course, near the critical temperature T_c of the superconductor or near the λ point in helium II the picture is more complicated than in the specially simple case considered above.

To get a clear picture of the velocity scale, we recall that the circulation quantum is

$$2\pi\hbar/m = 2\pi\hbar/m_{He} \approx 10^{-3}\ \text{cm}^2\cdot\text{sec}^{-1},$$

and at the λ point we have $\rho = 0.146$ g/cm^{-3}, $\sigma = 1.6\cdot 10^7$

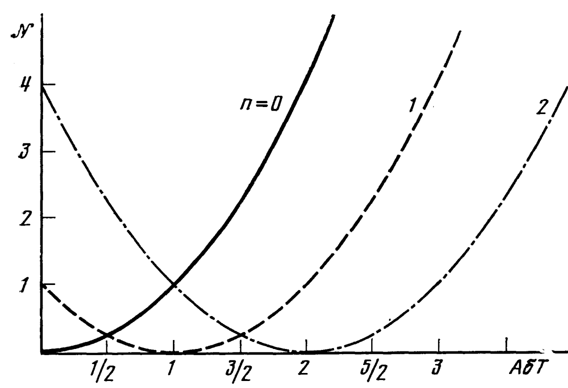

FIG. 2. Dependence of the kinetic energy of superfluid flow on the temperature difference δT at $n = 0$, 1, and 2.

$\mathrm{erg \cdot g^{-1} \cdot deg^{-1}}$ and $\eta_n \approx 2\times 10^{-5} \mathrm{g\cdot cm^{-1}\cdot sec^{-1}}$. Therefore at a temperature several tenths of a degree below the λ point we have $A \sim 10^{13} S_2$ and, e.g., at $S_2 \sim 10^{-10}$ cm^2 ($r \sim 10^{-5}$ cm) we have $A\delta T \sim 1$ already at $\delta T \sim 10^{-3}$ K.

In general, as is clear from estimates and experiment,[4] the conditions under which $A\delta T \gtrsim 1$–10 are not only easily attained, but are in fact also usually realized.

If no circulation had existed at $\delta T = 0$, when a difference δT appears the velocity v_{s0} increases in accord with (10) with $n = 0$. In the particular case (13) at $A\delta T = 1/2$, a transition to the state with $n = 1$ is already possible, but generally speaking hysteresis should be observed. How long the state with $n = 0$ is preserved also at $A\delta T > 1/2$ depends on many factors: the shape of the vessel, the presence of roughnesses, jolts, etc. Actually, under ordinary conditions (i.e., sufficiently smooth channels and no mechanical vibrations) the state with $n = 0$ can remain metastable until the velocity v_s reaches in some section of the torus (vessel) its critical value v_{sc1} corresponding to the start of vortex formation. The appearance of vortices will cause transitions into states with nonzero values of n. In this case, however, the velocity of the circulating flux need not necessarily decrease; it can even increase in some cases. In fact, if the velocity $v_{sc\,1}$ is first reached in the weakest link (link 2 in Fig. 1), the decrease of the total velocity $v_{s1} = v_{s1}^{(0)} + v_{s1}^{(\prime)}$ on this section of the torus gives rise to an increase of the velocity of the circulating flow $v_{si}^{(\prime)}$ (we recall that the velocities $v_{s1}^{(0)}$ and $v_{s1}^{(\prime)}$ are oppositely directed here). We shall return to this question in the discussion of the experimental situation, and note at present that as a result of one transition or of a cascade of transitions between states with equal values of n the circulation of the superfluid velocity in the ring changes by an exactly integer number of quanta. This makes it possible in principle to measure in experiment the ratio $\hbar/m$ [Eq. (10)]. The ratio $\hbar/m$ can also be assessed from the residual superfluid flows remaining in the torus after removal of the temperature gradient, if previously (during the heating) spontaneous transitions took place into states with nonzero n. The solution of this problem is not only of methodological interest, in view of the known doubts concerning the character of the quantization in helium II (we refer here to the assumed possible existence in helium II of a condensate of pairs with mass $2m$ besides the usual single-particle condensate).

4. Turning to the experimental situation, we note that in the only experiments performed to date[4] the characteristic scale of the controllable temperature changes was 10^{-2}–10^{-3} K, which leads to temperature gradients that are too large to reveal circulating-flow velocity jumps corresponding to one or a few circulation quanta. At the same time, a number of interesting peculiarities of the thermocirculation effect were revealed in Ref. 4, on which we would like to dwell.

We point out, first, that in Ref. 4 the thermocirculation effect was revealed by the reaction of the walls of a torus on an elastic suspension to which was applied (within one of its "large" volumes) an alternating heat flux having a frequency $\omega \sim 10^5$ Hz and a power $Q \sim 1$–10 mW. The size and shape of the torus were chosen such as "expand to a maximum the region of the investigated subcritical flow." To this, in particular, the weak links in Ref. 4 were systems of thin capillaries having in the links individual-pore cross sections $S_1 = 2.1\times 10^{-7}$ cm^2 and $S_2 = 4.3\times 10^{-6}$ cm^2 and with total cross section $\Sigma S_1 = \Sigma S_2 = 0.043$ cm^2. Equation (10) is thereby modified to (we assume below, in accord with the experimental conditions, $\rho_{s1} = \rho_{s2} = \rho_{s0} \equiv \rho_s$, $\eta_{n1} = \eta_{n2} = \eta_n$)

$$v_{s0}=\frac{1}{bl_0}\left[\frac{2\pi\hbar}{m}n-\frac{\rho_n(S_2-S_1)}{\sigma\rho T\rho_s a}Q\right],\quad a=\frac{S_1\Sigma S_1}{l_1}+\frac{S_2\Sigma S}{l_2},\quad b=1+\frac{l_1S_0}{l_0\Sigma S_1}+\frac{l_2S_0}{l_0\Sigma S_2}. \tag{15}$$

Besides v_{s0}, interest attaches also to the total velocities of the superfluid component in the pores of the first and second link:

$$v_{s1}=v_{s1}^{(0)}+v_{s1}^{(\prime)}=\frac{1}{bl_0}\left[\frac{2\pi\hbar}{m}n+\frac{\rho_nQ}{\sigma\rho T\rho_s a}\left(\frac{l_0S_1}{l_1}+\frac{S_2S_0}{\Sigma S_1}+\frac{l_2S_1S_0}{l_0\Sigma S_2}\right)\right], \tag{16}$$

$$v_{s2}=v_{s2}^{(0)}+v_{s2}^{(\prime)}=\frac{1}{bl_0}\left[\frac{2\pi\hbar}{m}n-\frac{\rho_nQ}{\sigma\rho T\rho_s a}\left(\frac{l_0S_2}{l_2}+\frac{S_1S_0}{\Sigma S_2}+\frac{l_1S_2S_0}{l_0\Sigma S_1}\right)\right]. \tag{17}$$

Substituting in (15)–(17) the concrete values of the geometric parameters of the torus[4] and the typical value $\sigma\rho T\rho_s/\rho_n = 0.3$ J/cm^3 (at $T \sim 1.5$ K), we obtain

$$v_{s0}\approx 4\cdot 10^{-5}(n-3.2\cdot 10^5 Q), \tag{18}$$

$$v_{s1}\approx 4\cdot 10^{-5}(2.3n-8.2\cdot 10^5 Q), \tag{19}$$

$$v_{s2}\approx 4\cdot 10^{-5}(2.3n+10.4\cdot 10^5 Q), \tag{20}$$

where the power Q should be expressed in W/sec and all the velocities in cm/sec.

It can be seen from (18)–(20) that to observe individual quanta the velocity v_s should be measured accurate to 10^{-5} cm/sec, whereas it was actually[4] monitored only with accuracy of the order of 10^{-3} cm/sec. Nonetheless, the velocity v_{s0} in the experiments[4] was in good agreement with the value calculated from (15) in the entire investigated temperature interval from 1.40 to 2.17 K also in the subcritical flow regime: $Q < Q_c(T) \sim 1 - 10$ mW.

One of the important features of the experimental results is that in the transcritical regime ($Q > Q_c$) the velocity v_{s0} varied more steeply with increasing Q than at $Q < Q_c$. In our opinion this can be attributed to the fact that the critical value of the velocity v_{sc1} was reached earlier in the second (leading) link.

Actually, as seen from (20), at $n = 0$ the total velocity v_s in this section of the ring is a maximum. At the same time, the decrease of the velocity v_{s2} as a result of the onset of the vortices leads to transitions to states with negative values of n, in which case the velocity v_{s0} increases, as follows from (18). With further increase of Q, when the vortices appear also on other sections of the ring, the growth of the velocity v_{s0} should first slow down and then stop completely. Thus, in apparatus of the type used in Ref. 4 one can observe in succession the critical velocities in different sections of the ring.

Another interesting fact noted in Ref. 4 is the increase of the critical thermal velocity $Q_c(T)$ as the helium approaches the λ-point temperature. This $Q_c(T)$ dependence seems difficult to explain at first glance, since usually critical velocities only decrease with rising T. We recall, however, that in the experiments[4] the heat flux oscillated rapidly, and vortex formation took a certain time $t_{\rm vort}$. Obviously, at a high oscillation frequency, when $\omega > \tau_{\rm vort}^{-1}$, the time is too short for the vortices to appear and there is no momentum transfer from the superfluid flux to the wall. Although as T_λ is approached the height of the energy barrier that separates the states with n and $n-1$ circulation quanta decreases gradually, the time $\tau_{\rm vort}$ can still increase,[11,12] so that Q_c will also increase.

It is clear even from the foregoing that apparatus of the type used in Ref. 4 can serve as a very sensitive tool for the investigation of the kinetics of vortex formation in superfluid helium II.

5. One of the most highly promising applications of the thermocirculation effect is its use to observe in helium II quantum interference phenomena.[6] To this end, it suffices to place a Josephson junction in the path of a superfluid flux produced in the presence of a temperature gradient. The role of this junction can be assumed, e.g., by a diaphragm with a narrow opening, by a capillary with a seal produced by an electric field,[13] and others. We shall not stop here to describe and calculate the parameters of the corresponding Josephson junction, although the latter is generally speaking difficult to perform near the λ point using the Ψ theory of superfluidity[9] (an example of a calculation of this kind is contained in Ref. 13). We note only that since the coherence length for the spatial changes of the order parameter increases rapidly near T_λ:

$$\xi \approx 10^{-8}[(T_\lambda - T)/T_\lambda]^{-2/3}\ \text{cm},$$

the corresponding opening or capillary can be of macroscopic size ($r \sim 10^{-4}$ cm at $T_\lambda - T \sim 10^{-6}$ K).

In the presence of a Josephson junction D (located, e.g., in section AB, see Fig. 1), Eqs. (8) and (9) are written in the form

$$(v_{s1}^{(0)} + v_{s1}^{(I)})l_1 + (v_{s2}^{(0)} + v_{s2}^{(I)})l_2 + v_{s0}l_0 + (\hbar/m)\Delta\varphi = (2\pi\hbar/m)n, \quad (21)$$

$$I_{s0} = \rho_{s0}v_{s0}S_0 = \rho_{s1}v_{s1}^{(I)}S_1 = \rho_{s2}v_{s2}^{(I)}S_2 = I_c \sin\Delta\varphi. \quad (22)$$

Here $\Delta\varphi$ is the difference between the phases of the macroscopic wave function $\Psi = \eta \exp(i\varphi)$ at the edges of the junction and I_c is the maximum (critical) current passed by this junction.

Expressing with the aid of (22) the quantities v_{s0}, $v_{s1}^{(I)}$ and $v_{s2}^{(I)}$ in terms of $I_c \sin\Delta\varphi$, substituting in (21), and using also Eq. (4), we obtain for the phase difference the equation

$$I_c\left(\frac{l_0}{\rho_{s0}S_0} + \frac{l_1}{\rho_{s1}S_1} + \frac{l_2}{\rho_{s2}S_2}\right)\sin\Delta\varphi = \frac{\hbar}{m}(2\pi n - \Delta\varphi) + \frac{Q}{a\sigma\rho T}\left(\frac{S_1\rho_{n1}}{\rho_{s1}} - \frac{S_2\rho_{n2}}{\rho_{s2}}\right). \quad (23)$$

Equation (23) is similar in form to the expression that describes the dependence of the superconducting current on the external magnetic flux in a superconducting torus with the weak coupling of Zimmermann and Silver.[7] The role of the external magnetic flux is played in this case by the heat flux Q.

It can be seen from (23) (see also Ref. 7) that at

$$\frac{I_c m}{\hbar}\left(\frac{l_0}{\rho_{s0}S_0} + \frac{l_1}{\rho_{s1}S_1} + \frac{l_2}{\rho_{s2}S_2}\right) < 1$$

the superfluid flux $I_s = I_c \sin\Delta\varphi$ circulating in the torus oscillates as a function of Q between 0 and I_c, with a period

$$L_Q = \frac{\pi\hbar}{m} a\sigma\rho T \left|\frac{S_1\rho_{n1}}{\rho_{s1}} - \frac{S_2\rho_{n2}}{\rho_{s2}}\right|^{-1}. \quad (24)$$

In the particular case when $S_2\rho_{n2}/\rho_{s2} \gg S_1\rho_{n1}/\rho_{s1}$ and $S_2^2/l_2 \gg S_1^2/l_1$, the period of the oscillations amounts to

$$L_Q = \frac{\pi\hbar}{m}\frac{\sigma\rho T\rho_{s2}S_2}{\rho_{n2}l_2} \quad (25)$$

so that $L_Q \approx 10^{-3}$ W at $T \approx 1.5$ K and $S_2/l_2 \approx 200$ cm. Thus, observation of the oscillations of I_s calls for an appropriate choice of the geometric parameters of the torus and very low heat power.

Another superfluid interferometer[6] similar to the superconducting quantum interference device of Mercereau *et al.*[7] is illustrated in Fig. 3. The lower wide half of the torus contains two identical Josephson junctions $D\,1$ and $D\,2$; in addition, the upper and lower halves of the torus are provided with entrance and exit tubes for the flow of helium through the interferometer. The experiment consists of measuring the maximum transmission of the torus (the maximum flux I_{sm} flowing through the interferometer) as a function of the temperature difference $\delta T = T_2 - T_1$ produced between the upper and lower halves of the ring, or as a function of the heat power Q released in the upper half. In particular, one can measure the dependence on δT (or Q) of the rate at which the level of liquid helium poured into the entrance tube drops under the action of the weight of the helium column in this tube. The temperature difference δT pro-

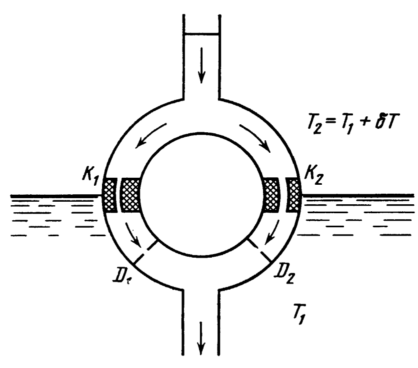

FIG. 3. Schematic form of apparatus for the observation quantum interference effects in helium II. K_1, K_2—capillaries; D_1, D_2—Josephson junctions.

duces in the ring a circulating current that "controls" the carrying capacity of the junctions D_1 and D_2. Here δT plays the same role as the external magnetic flux in the case of superconducting interferometers. An analysis similar to that presented above shows that I_{sm} is an oscillating function of δT and Q and its period is given by (24) or by the equation[6]

$$L_{\delta T}=\frac{\hbar}{m}\frac{\rho_s}{\rho_n}\frac{4\pi\eta_n}{\sigma\rho|S_1-S_2|}, \qquad (26)$$

if the dependence of I_{sm} on δT is sought. The last equation was derived using Eqs. (1) and (2) for round capillaries under the assumptions that the values of ρ_s, ρ_n, and η_n are the same in both capillaries.

The maxima of I_{sm} as a function of δT are located at $\delta T=(\delta T)_0+nL_{\delta T}$, where n is an integer. The presence of the term $(\delta T)_0$ is connected with the difference (which is obligatory in the setup considered) between the areas S_1 and S_2 (the thermocirculation effect is impossible if the cross sections are equal; accordingly $L_{\delta T}\rightarrow\infty$ as $S_1\rightarrow S_2$). For the same reason, simultaneous "blocking" of both arms of the interferometer is impossible, i.e., the minimum flux passing through the interferometer in this setup is always different from zero.

We note that similar phenomena (oscillations of a thermoelectric circulating superconducting current as a function of the applied temperature difference δT) should obviously take place also in inhomogeneous and nonuniformly heated closed superconducting circuits that contain Josephson junctions.[6]

6. It is clear from the foregoing that the theory can and must be generalized to cover also the region near the λ point, allowance for edge effects (the "juncture" regions of the links), the kinetics of the transitions between states between different n, and the approach to the critical velocity. Mention must also be made of more complicated multiply connected vessels, different manners of their "connection" in the circuit with the superfluid flow, and the ensuing interference phenomena. Special notice should be taken of solutions of ^3He in ^4He and of superfluid ^3He. On the whole, in our opinion, study of circulation and interference effect in nonuniformly heated vessels containing superfluid liquids is worthy of attention both theoretically and experimentally, and can lead to many important results.

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Translated by J. G. Adashko