Surface waves in smectics *A*

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The propagation of surface waves in smectic *A* is considered in the limit in which permeation effects are not important. The form of the possible modes and the region of their existence are found. It is shown that under ordinary experimental conditions propagating modes are absent far from the smectic-nematic transition point.

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The propagation of surface waves in liquid crystals of the smectic *A* type has been considered repeatedly in recent years. Interest in this phenomenon is connected with the fact that smectic *A* represents a layered system, which possesses solid elastic properties in one direction (perpendicular to the layers) and liquid properties in the remaining two directions. The viscous properties of such liquid crystals are strongly anisotropic. It is evident that the study of the dynamics of such systems is of interest by itself and, in addition, furnishes additional possibilities for the experimental determination of a number of characteristic parameters.

The propagation of surface waves in incompressible smectic *A* was considered in Ref. 1 without account of viscosity and permeation effects. Two propagating modes were found. One represents a capillary wave, similar to the capillary waves in a liquid, while the other is an elastic wave, corresponding to the bulk mode of second sound of smectics. In the present work we shall show that account of the viscosity imposes additional limitations on the region of existence of these modes and in practice leads to their disappearance in any region that is not very close to the smectic-nematic transition point.

References 2–4 were devoted to the study of surface waves under conditions in which the permeation effects are important. This case corresponds to very low frequencies and wave vectors, or to the same geometry in which the pinning of the layers to the walls of the vessel in which the sample is located is important. In addition, boundary conditions on the free surface of the liquid crystal were chosen in Refs. 2 and 3 in a form which corresponds to a fixed, immovable structure of the layers near the surface, in which case the upper layer does not coincide with the surface itself. Thus, the boundary conditions are written down in principle for a liquid and do not take into account the elastic properties of smectic *A*. Such an assumption does not appear to be well substantiated, the more so in the case in which the permeation effects are not important.

In connection with what has been said above, it would be useful to consider surface waves with account of viscosity in the case in which the permeation efects do not make the principal contribution, i.e., the geometry of the sample is such that the effects of pinning of the layers to the wall are not important (a sufficiently long sample), while the frequencies and the wave vectors are sufficiently large.

We consider smectic *A* which occupies the half-space $z<0$ in a geometry in which the z axis is perpendicular to the layers. Its free energy as a function of the derivatives of the displacement of the layers $u(\mathbf{r})$ is the following:

$$F=F_0+\frac{1}{2}B\left(\frac{\partial u}{\partial z}\right)^2+\frac{1}{2}K\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)^2, \qquad (1)$$

where F_0 is the free energy of the undistorted state, the second term describes the elastic energy of compression of the layers, and the third, the energy of the lateral flexure. The equations of hydrodynamics for an incompressible smectic are written as follows[5]:

$$\begin{gathered}\operatorname{div}\mathbf{v}=0,\\ \rho\dot v_i=-p_{,i}+\eta_{ij,\,j}+g\delta_{iz},\\ \dot u-v_z=\nu g.\end{gathered} \qquad (2)$$

Here ν is the permeation coefficient, η_{ij} the viscous stress tensor, which has the form

$$\begin{gathered}\eta_{ij}=2\mu_2A_{ij}+2(\mu_1-\mu_2)(A_{iz}\delta_{jz}+A_{zj}\delta_{iz})+\mu_0A_{zz}\delta_{iz}\delta_{jz},\\ A_{ij}=\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right),\end{gathered} \qquad (3)$$

where μ_α is the coefficient of viscosity. The restoring force g actng on the layer along a direction normal to it is obtained from (1) in the form

$$g=B\frac{\partial^2 u}{\partial z^2}-K\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2u. \qquad (4)$$

We shall seek a solution of Eqs. (2) in the form of a surface wave propagating in the x direction and damped in the interior of the sample. As usual, we represent the variables entering into the hydrodynamic equations in the following form;

$$\begin{pmatrix}\mathbf{v}\\ \mathbf{p}\\ u\end{pmatrix}=\sum_q\begin{pmatrix}\mathbf{v}_q\\ \mathbf{p}_q\\ u_q\end{pmatrix}e^{lqz}e^{iqx+i\omega t}. \qquad (5)$$

The symbol l with positive real part corresponds to a wave that is damped out as $z\to-\infty$.

It is easy to see from (2) that the effects of permeation will be unimportant at $\omega\gg\nu Bq^2(l^2-\lambda^2q^2)$, where $\lambda=(k/B)^{1/2}$ is a characteristic length. In this case, to the study of which we limit ourselves, the third equation of the system (2) has the form $\dot u=v_z$ and means that the molecules of the smectic move together with the layers.

Sustitution of (5) in (2) leads to a biquadratic equation for l, the solution of which is

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$$l_{1,2}^2=\left\{\frac{i\omega\rho}{\mu_1 q^2}+\frac{B}{i\omega\mu_1}+2+\frac{\mu_0}{\mu_1}\pm\left[\left(\frac{i\omega\rho}{\mu_1 q^2}+\frac{B}{i\omega\mu_1}+2+\frac{\mu_0}{\mu_1}\right)^2-4\left(1+\frac{i\omega\rho}{\mu_1 q^2}+\frac{Kq^2}{i\omega\mu_1}\right)\right]^{1/2}\right\}. \quad (6)$$

The boundary conditions on the free surface, which the solution must satisfy, have the following form in our case: at $z=\xi$,

$$t_{zx}=K\frac{\partial^3 u}{\partial x^3}-\mu_1\left(\frac{\partial v_x}{\partial z}+\frac{\partial v_z}{\partial x}\right)=0,$$
$$t_{zz}=p-B\frac{\partial u}{\partial z}+K\frac{\partial^3 u}{\partial z\,\partial x^2}-(4\mu_1-2\mu_2+\mu_0)\frac{\partial v_z}{\partial z}=-\sigma\frac{\partial^2\xi}{\partial x^2} \quad (7)$$

while at $z=0$,

$$\frac{\partial\xi}{\partial t}=\dot{u}=v_z.$$

Here t_{ij} are the components of the stress tensor, ξ is the vertical displacement of the surface, σ is the coefficient of surface tension. We represent the velocity, pressure and displacement in the following form

$$v_z=\sum_q(C_1e^{l_1qz}+C_2e^{l_2qz})e^{iqx+i\omega t},$$
$$v_x=\frac{i}{q}\frac{\partial v_z}{\partial z},\quad u=\frac{v_z}{i\omega},$$
$$p=\frac{\mu_1}{q^2}\frac{\partial^3 v_z}{\partial z^3}-(i\omega\rho+2\mu_2q^2-\mu_1q^2)\frac{\partial v_z}{q^2\partial z}, \quad (8)$$

where the coefficients C_1 and C_2 must satisfy the boundary conditions (7). We substitute (8) in (7) and obtain

$$C_1\left(1+l_2^2+\frac{Kq^2}{i\omega\mu_1}\right)l_1+C_2\left(1+l_1^2+\frac{Kq^2}{i\omega\mu_1}\right)l_2=-\frac{\sigma q}{i\omega\mu_1}(C_1+C_2),$$
$$C_1\left(1+l_1^2+\frac{Kq^2}{i\omega\mu_1}\right)+C_2\left(1+l_2^2+\frac{Kq^2}{i\omega\mu_1}\right)=0. \quad (9)$$

The condition of solvability of this system gives the dispersion equation

$$D(\omega)=(l_1-l_2)\left[\left(1+\frac{Kq^2}{i\omega\mu_1}-l_1l_2\right)^2-l_1l_2(l_1+l_2)^2-\frac{\sigma q}{i\omega\mu_1}(l_1+l_2)\right]=0, \quad (10)$$

where $l_{1,2}$ are determined by Eq. (6) (we recall that we should keep only those values of l with positive real part). The solution (10) corresponding to the case $q<\sigma\rho/\mu_1^2$ is not of interest, since here $C_1=-C_2$ and, consequently, all the quantities in (8) are identically equal to zero.

We now investigate the expression in the square brackets in (10). Its analysis (which, although not complicated, is rather tedious) allows us to obtain the form of the possible modes in a number of regions of values of the wave vectors. The complete solution of the dispersion equation can only be found numerically, as a function of the parameters of the problem.

We shall give the final results only for the extreme cases, in which the propagating modes were predicted by the authors of Ref. 1.

a) At $q>B/\sigma$ two solutions satisfy the dispersion equation (10). One, under the additional condition $q>\sigma\rho/\mu_1^2$ represents a propagating capillary wave $\omega^2=\sigma q^3/\rho$, similar to that found in Ref. 1, while the other, at $q>\sigma\rho/\mu_1^2$, is a damped wave and has the forn $\omega=i\sigma q/\mu_1$.

b) At $q<B/\sigma$ the solution of the dispersion equation satisfying the boundary conditions is the propagating mode $\omega^2=Bq^2$, which is similar to that found in Ref. 1. However, even in this case, there is the additional condition $q<(B\rho)^{1/2}/\mu_1$.

Thus, account of the viscosity imposes additional limitations on the region of wave vectors that are characteristic for the existence of the propagating modes. These limitations, as will be seen below, can be very important. In the case in which the smectic A is located far away from the transition point to a nematic liquid crystal, the typical values of the parameters are the following:

$B\sim10^6$ dyn/cm^2, $\sigma\sim10^2$ dyn/cm, $\mu_1\sim10$ dyn.s/cm^2, $\rho\sim1$ g/cm^3.

Then

$B/\sigma\sim10^4$ cm^{-1}, $\sigma\rho/\mu_1^2\sim1$ cm^{-1}, $(\beta\rho)^{1/2}/\mu_1\sim10^2$ cm^{-1}.

It is seen from the above expressions that the capillary waves corresponding to case a) do not exist under such conditions. So far as the surface mode corresponding to case b) is concerned, its wavelength will be too long (of the order of 0.1 cm), and thus it cannot be observed experimentally by light scattering, which is customarily used for the observation of surface waves.

The situation can be changed in the region of the smectic A —nematic transition, since the elastic modulus B decreases in this case and vanishes at the transition point. Depending on the temperature behavior of σ and μ_1, regions of existence of propagating surface waves can arise. Therefore, the experimental investigation of surface waves near the transition point is of interest, since it will give additional information on the temperature dependences of the characteristic parameters.

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