Propagation of ultrashort optical pulses in resonant nonlinear light guides

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The propagation of ultrashort pulses along a nonlinear light guide containing resonant impurities is considered in the single-mode approximation. It is shown that the presence of impurities alters qualitatively the character of the evolution of such pulses, an evolution that can be described by the generalized Maxwell-Bloch equations. The formation of optical solitons as well as self-induced-transparency conditions is possible only if a certain relation exists between the light-wave frequency and the parameters of the nonlinear medium.

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Substantial progress was achieved quite recently in methods of producing extremely short optical pulses (down to several dozen femtoseconds[1]). Such a pulse is considerably shorter than all the characteristic relaxation times in matter (we shall call it an ultrashort pulse—USP). A USP is usually characterized by a high electic field intensity, so that nonlinear optical effects frequently take place, such as the onset of a soliton propagation regime. Under intensive study at present is also the passage of optical pulses through light guides, where for practical purposes the aim is to produce as short pulses as possible. By using USP it is possible to transmit information at very high speed. To offset the broadening due to the dispersion of the group velocities it was proposed[2] to make the light pipes of a material having a Kerr-type dielectric constant, i.e., quadratic in the electric field. The pulses are transformed in this case into solitons and it is estimated[3] that the expected information-transmission rate is of the order 0.01 Tbit/sec over 20 km.

However, practically all materials used for light-guide fabrication contain impurities that contribute to the radiation-absorption spectrum.[4] Owing to the inhomogeneous broadening of the impurity energy levels there is always a group of levels that are at resonance with the radiation transmitted through the light guide. The losses due to the resonant absorption decrease greatly if the pulse duration is made shorter than the characteristic relaxation times of the resonance states,[5] i.e., if the pulse is made ultrashort. This phenomenon, known as self-induced transparency,[6] was considered for light guides in Ref. 7. There, however, the Kerr-type nonlinearity and the linear dispersion spreading were not taken into account. In sufficiently long light guides this approximation may turn out to be invalid.

In the present paper, with a single-mode light guide as the example, we consider the influence of resonant absorption in the self-induced transparency on USP propagation in a Kerr-type nonlinear medium. We show that resonant impurities (simulated, as in Ref. 7, by two-level atoms) alter radically the condition of propagation of a USP as a soliton.

The evolution of a USP moving through a light pipe in the z direction is described by equations that generalize the known Maxwell-Bloch equation[5]:

$$iE_Z-\varepsilon_1 aE_{TT}+\varepsilon_2 g|E|^2E+\langle\sigma\rangle=0, \tag{1a}$$

$$\sigma_T=i\delta\sigma+ifEu, \tag{1b}$$

$$u_T=2if(\sigma E^*-\sigma^*E). \tag{1c}$$

The subscripts Z and T denote derivatives with respect to Z and T: $Z=z/L_a$, $T=t-z/v$, where L_a is the resonant-absorption length[5] and v is the USP propagation velocity in the linear approximation. The term in (1a) with the second derivative with respect to T describes the USP dispersion spreading and the coefficient a is the ratio of the length of the resonant absorption to the dispersion length[8,9]

$$L_g=4\beta t_p^2/|\partial^2k^2(\omega)/\partial\omega^2|.$$

The self-action effect is taken into account by the third term in (1a). The coefficient g is equal to the ratio of the absorption length to the Kerr length

$$L_K=\beta c^2/(2\pi\omega^2R_0^2|\bar{\chi}_K|).$$

Here β is the light-guide propagation constant, ω is the frequency of the carrier wave, t_p and R_0 are the duration and the maximum amplitude of the USP at $z=0$, and $\bar{\chi}_K$ is the effective nonlinear susceptibility responsible for the Kerr effect. In contrast to a uniform infinite medium, where the Kerr susceptibility χ_K is constant, the effective susceptibility $\bar{\chi}_K$ depends on the mode of the wave propagating in the light guide. If the electric field of the wave is written in the form

$$\vec{\mathscr{E}}(x,y,z;t)=\mathbf{e}A(z,t)\Phi(x,y),$$

where $\mathbf{e}$ is the polarization vector, $A(z,t)$ is a slowly varying complex amplitude, and $\Phi(x,y)$ is a function that determines for the given mode the transverse distribution of the electric field in the light pipe, then $\bar{\chi}_K$ is defined as the average over the light-guide cross section:

$$\bar{\chi}_K=\int d\boldsymbol{\rho}\chi_K(\boldsymbol{\rho})|\Phi(\boldsymbol{\rho})|^4\Big/\int d\boldsymbol{\rho}|\Phi|^2,\quad \boldsymbol{\rho}=(x,y).$$

The mode function $\Phi(\boldsymbol{\rho})$ is obtained for each specific light guide, being the solution of the corresponding boundary-value problem.[8,9] The interaction of the radiation with the resonant impurities is characterized by the dimensionless constant $f=\bar{d}R_0t_p\hbar^{-1}$, where $\bar{d}$ is the effective element of the dipole transition between the resonant states:

$$\bar{d}=\int d\boldsymbol{\rho}\mathbf{ed}(\boldsymbol{\rho})\Phi(\boldsymbol{\rho})|\Phi(\boldsymbol{\rho})|^2\Big/\int d\boldsymbol{\rho}|\Phi(\boldsymbol{\rho})|^2.$$

The angle brackets in (1a) denote summation over all the normalized frequency detunings $\delta=t_p\Delta\omega$ from the center

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of the inhomogeneously broadened resonant-absorption line, where $\Delta\omega$ is the difference between the pulse carrier frequency and the central frequency of the atomic transition, and $E = A/R_0$.

In the variables that describe the resonant impurities, just as in $\vec{\mathscr{E}}$, the dependence on $\boldsymbol{\rho}$ is separated:

$$\rho_{12}=\sigma(z,t)\Phi(\boldsymbol{\rho}), \quad \rho_{11}-\rho_{22}=u(z,t)\Phi(\boldsymbol{\rho}).$$

Here ρ_{ij} are the density-matrix elements of the two-level atoms ($j,i = 1,2$).

The nonspreading nonlinear pulses correspond to the soliton solutions of the system (1). To find the condition for the existence of such solutions, we must find at what ratio of the parameters in (1) this system admits of the Lax representation,[10] i.e., can be written in the matrix form

$$\hat{L}_Z=\hat{A}_T+[\hat{A},\hat{L}], \tag{2}$$

for a certain pair of matrices $\hat{L}$ and $\hat{A}$. the Lax pair $\hat{L}$ and $\hat{A}$, besides depending on E, σ, and u, must contain an arbitrary constant that assumes the role of the spectral parameter of the method of the inverse problem of scattering theory.[11]

Following Ref. 10, we choose $\hat{L}$ and $\hat{A}$ in the form

$$\hat{L}=\begin{bmatrix} -i\lambda & \alpha_1 E \\ \alpha_2 E^* & i\lambda \end{bmatrix}, \quad \hat{A}=\begin{bmatrix} A & B \\ C & -A \end{bmatrix}. \tag{3}$$

The constants α_1, α_2 and A, B, and C (which are functions of E, σ, u, and E_T) must be chosen such that (2) coincides with (1). Taking (3) into account, we can rewrite (2) in expanded form

$$A_T=\alpha_1 EC-\alpha_2 E^* B, \tag{4a}$$

$$B_T+2i\lambda B=\alpha_1 E_Z-2\alpha_1 EA, \tag{4b}$$

$$C_T-2i\lambda C=\alpha_2 E_Z^*+2\alpha_2 E^* A. \tag{4c}$$

Let B and C have the form of the linear combination

$$B=\langle b_1\sigma\rangle+b_2E+b_3E_T, \quad C=\langle c_1\sigma^*\rangle+c_2E^*+c_3E_T^*, \tag{5}$$

where the unknown coefficients b_j and c_j ($j = 1,2,3$) depend only on λ. Substituting (5) in (4b) and (4c) and equating the coefficients of σ, σ^*, E_T, E_T^*, E_{TT} and E_{TT}^*, we can find with allowance for (1) that

$$\begin{gathered} b_1=\alpha_1/(\delta+2\lambda), \quad b_2=-2\lambda\alpha_1\varepsilon_1 a, \quad b_3=-i\alpha_1\varepsilon_1 a, \\ c_1=\alpha_2/(\delta+2\lambda), \quad c_2=-2\lambda\alpha_2\varepsilon_1 a, \quad c_3=i\alpha_2\varepsilon_1 a \end{gathered} \tag{6}$$

and

$$A=\frac{i}{2}\left\{\varepsilon_2 g|E|^2+4\lambda^2\varepsilon_1 a-\left\langle\frac{fu}{\delta+2\lambda}\right\rangle\right\}. \tag{7}$$

Thus,

$$\begin{gathered} B=\alpha_1\left\{-2\lambda\varepsilon_1 aE-i\varepsilon_1 aE_T+\left\langle\frac{\sigma}{\delta+2\lambda}\right\rangle\right\}, \\ C=\alpha_2\left\{-2\lambda\varepsilon_1 aE^*+i\varepsilon_1 aE_T^*+\left\langle\frac{\sigma^*}{\delta+2\lambda}\right\rangle\right\}. \end{gathered} \tag{8}$$

Compatibility of (8), (7), and (4a) imposes constraints on the parameters of the problem, i.e., the following conditions must be satisfied:

$$\varepsilon_2 g=2\alpha_1\alpha_2\varepsilon_1 a, \quad f^2=-\alpha_1\alpha_2.$$

Hence

$$-\varepsilon_1\varepsilon_2 g=2af^2. \tag{9}$$

Since a and g are positive by definition, Eq. (9) leads to a relation for the signs of the Kerr susceptibility and of the dispersion constants: $\varepsilon_1\varepsilon_2 = -1$, meaning simply that self-action and dispersion of the group velocities should lead to opposing effects. In dimensional variables, Eq. (9) takes the form $L_g/L_K = 2f^2$ or

$$4\pi(\hbar\omega/c\bar{d})^2|\bar{\chi}_K|=|\partial^2k^2(\omega)/\partial\omega^2|. \tag{9a}$$

The only condition for the constants α_1 and α_2 is the requirement that the spectral problem of the inverse scattering problem be anti-Hermitian.[10] Hence $\alpha_1 = \alpha_2 = if$.

Were there no resonant impurities in the light guide, soliton existence would be ensured by the following equalities:

$$\alpha_1=\alpha_2=i, \quad -\varepsilon_1\varepsilon_2=2a.$$

By suitable choice of nondimensionalizing parameters, the system (1) would reduce to a nonlinear Schrödinger equation, as in Refs. 9 and 8. The presence of resonant impurities, on the other hand, changes the situation radically: a 2π pulse of self-induced transparency should simultaneously be also a soliton of the nonlinear Schrödinger equation, and the amplitude and duration of the 2π pulse should be precisely such that the corresponding self-action (due to the Kerr effect) would lead to total cancellation of the dispersion spreading of the USP.

The condition for the existence of a soliton solution of Eq. (1) is quite stringent. However, by varying the frequency of the carrier light wave within the limits of the inhomogeneously broadened resonance line it is possible to attain satisfaction of the condition (9) [or 9(a)].

The result is valid also in a homogeneous medium. In this case it suffices to replace $\bar{d}$ and $\bar{\chi}_K$ by the true d and χ_K. Let the light guide be uniform in the sense that the dipole moment d and the Kerr susceptibility χ_K are independent of the transverse coordinates x an y. Then

$$|\bar{\chi}_K|/\bar{d}^2=F|\chi_K|/d^2,$$

where the explicitly separated geometric factor $F = I_4I_2/I_3^2$ reflects clearly the difference in the condition (9) between an unbounded homogeneous medium and the light guide. Here

$$I_n=\int d\boldsymbol{\rho}\,\Phi^n(\boldsymbol{\rho}), \quad n=1-4,$$

where the modulus has been left out, since for a regular dielectric light pipe without a metallic shield we usually have $\Phi^*(\boldsymbol{\rho}) = \Phi(\boldsymbol{\rho})$, as is indeed assumed in the present paper. This restriction is not essential and its generalization is trivial. The appearance of a geometric factor is typical of problem of integrated and fiber optics. It is useful to estimate its influence on the condition for the existence of solutions of (1). For the fundamental mode a good approximation of $\Phi(\boldsymbol{\rho})$ is a Gaussian function of $|\boldsymbol{\rho}|$, having at half-maximum a width equal to the effective thickness of the light pipe. For a fiber with parabolic transverse distribution of the refractive index this is at any rate the exact result.[8,9] It turns out that for this choice of $\Phi(\boldsymbol{\rho})$ the geometric factor $F = 8/9$. Of course for each particular light guide and particular mode the value of F will differ but it is not likely to deviate greatly from unity.

To conclude, it is useful to point out two examples of nonlinear propagation of USP in a light guide; their analysis is also based on the model considered here [on the system (1)]. In Ref. 12 a pulse broadened as a result of group-velocity dispersion was narrowed down by using an intermediate cell with resonant atoms (sodium vapor). The same can be accomplished by introducing resonant impurities in some segment of the light pipe. The absence of a cell with alkali-metal vapor makes such a light-pipe line more convenient to use.

As repeaters for signals transmitted over light pipes it is natural to use an amplifier similar to a laser amplifier. A light-guide segment containing resonant impurities whose population is inverted by optical pumping is precisely such an amplifier. The evolution of the USP is described in this case by the system (1), but with reversed sign of the resonant-level population difference. A very similar situation arises also in the problem of self-induced transparency when light beams are scanned over the surface of a nonlinear medium.[13,14]

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