Some features of generation of electromagnetic wave packets in atomic coherent ensembles upon deviation from wave synchronism

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(Submitted 28 January 1983)
Zh. Eksp. Teor. Fiz. **85**, 1168–1174 (October 1983)

We investigate the most general case of emission of an electromagnetic field in a medium with phase memory when the medium is excited beforehand. We show that in this case, upon deviation from the wave-synchronism conditions, the medium can emit two pulses; in addition, a special polariton-field cluster can be produced in the medium and move with a velocity v which is not restricted in principle. The physical causes of this effect are elucidated and the main features of the fields are studied. Methods of their experimental realization and verification are proposed.

PACS numbers: 41.10.Hv

A typical situation in coherent atomic ensembles at time intervals $t < T_2$, where T_2 is the phase-relaxation parameter, is one in which a successive action of several optical-pumping pulses induces macroscopic polarization in a medium. In the presence of inhomogeneous resonance-line broadening, the instant of appearance of this polarization does not coincide in general with the instants of action of the pump pulses. It is known that when the wave-synchronization condition is exactly satisfied this macroscopic polarization generates electromagnetic waves that can be detected (photon echo.[1]) We study here a more general case, when the polarization-wave velocity does not coincide with that of light in the given medium, and the macroscopic polarization itself takes the form of a wave packet smaller in size than the sample. The features of such polarization wave packets (PWP) were investigated earlier,[2] but it was left unclear how such a PWP generates an electromagnetic field upon deviation from wave synchronism, and what are the properties of this field.

The distinguishing features of this radiation were ascertained at arbitrary parameters of the PWP. The general form of the packet, after generalizing the results of Ref. 2 under the same assumptions (powerful and strong pump pulses $\delta t < 2\pi\Omega/\Delta_{in}^2$, $\Omega > \Delta_{in}$, where δt is the pump-pulse duration, Ω is the Rabi frequency, and Δ_{in} is the line half-width of the inhomogeneous broadening of the transition) are given by

$$\mathbf{P}_1(r,t)=\mathbf{P}_0(\theta_j,p_{12})\exp\left[-\left(t-\frac{\mathbf{k}_p\mathbf{r}}{\omega_0}\right)^2\Delta_{in}^2\right]\exp[i(\omega_0 t-\mathbf{k}_p\mathbf{r})], \tag{1}$$

where p_{12} is a matrix element of the transition dipole moment; ω_0 is the absorption-line center, $\theta_j = \Omega_j\delta t_j$; j is the index of the pump pulse; the wave vector $\mathbf{k}_p$ for each specific PWP depends on the type of the nonlinear interaction. For example, $\mathbf{k}_p = 2\mathbf{k}2 - \mathbf{k}_1$ for two-pulse and $\mathbf{k}_p = \mathbf{k}_3 + \mathbf{k}_2 - \mathbf{k}_1$ for three-pulse pumping.

The PWP group velocity in (1) coincides with the phase velocity $v = \omega_0/k_p$ of the carrier polarization wave and is in general not equal to the speed of light in the medium ($c_0 = c/n$). The reason is that the modulus of $\mathbf{k}_p$ is a function of the wave vectors of the pump pulses and can be larger as well as smaller than ω_0/c_0.[2]

Consider an extended region bounded in the z direction and described by the expression

$$Q(r)=\eta[z+{}^1/_2L_1(x,y)]-\eta[z-{}^1/_2L_2(x,y)],$$
$$\eta(\chi)=1 \quad\text{at}\quad \chi\geqslant 0,\qquad \eta(\chi)=0\quad\text{at}\quad \chi<0. \tag{2}$$

This expression corresponds to a sample in the form of a plane-parallel plate with slight deviation of the front and rear surface from planar at different points (x, y). The shapes of the front and rear surfaces are described by the functions $L_1(x, y)$ and $L_2(x, y)$, respectively, so that the sample thickness at an arbitrary section point is

$$L(x,y)={}^1/_2[L_1(x,y)+L_2(x,y)]$$

(the origin is at the center of the sample).

If the z axis is chosen in the PWP motion direction ($z\|k_p$), the total polarization takes the form

$$P(\mathbf{r},t)=P_1(z,t)Q(\mathbf{r}). \tag{3}$$

The electromagnetic field radiated by this polarization was obtained under conditions when this field was weak and its action on the polarization could be neglected (the given-polarization approximation). This approximation is valid when weak fields are generated and for dilute gases and crystals.

The sought field was obtained, assuming the polarization wave to be transverse ($k_p \perp P_1$), by solving the wave equation

$$\left(\frac{\partial^2}{\partial z^2}-\frac{n^2}{c^2}\frac{\partial^2}{\partial t^2}\right)E(r,t)=\frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}P(r,t)=\Phi(r,t),$$

$$\Phi(r,t)=-\frac{4\pi(\omega_0^2+2\Delta_{in}^2)}{c^2}P_0\exp\left[-\left(t-\frac{z}{v}\right)^2\Delta_{in}^2\right]$$

$$\times Q(r)\exp\left\{i\left[\omega_0 t-k_p z+\gamma\Delta_{in}\left(t-\frac{z}{v}\right)\right]\right\},\qquad \gamma=\frac{4\Delta_{in}\omega_0}{\omega_0^2+2\Delta_{in}^2}. \tag{4}$$

In the expression for Φ (r, t) we used the substitution

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$$1+i\gamma\Delta_{in}\left(t-\frac{z}{v}\right)-\gamma\frac{\Delta_{in}}{\omega_0}\left[\Delta_{in}\left(t-\frac{z}{v}\right)\right]^2$$

$$\approx \exp\left[i\gamma\Delta_{in}\left(t-\frac{z}{v}\right)\right], \qquad (5)$$

which is valid for the region of optical frequencies ($\Delta_{in} < \omega_0$). This approximation is not essential in principle for the calculation, but simplifies it significantly without loss of generality.

Equation (4) differs from the known cases in that the oscillations of the polarization waves and the electromagnetic wave generated by the polarization are close in frequency, and the velocities of these waves are unequal (k_p is arbitrary at fixed ω_0). This holds also for the envelopes of these waves.

To solve this equation we change to a coordinate frame (r', t') that moves with velocity v and is connected with the laboratory frame (r, t) by the Lorentz transformation

$$z'=(z-vt)\beta,\quad t'=(t-vz/c_0^2)\beta,\quad y'=y,\quad x'=x.$$

The inverse transformation is of the form

$$z=\alpha(z'+vt')\beta,\quad t=\alpha(t'+vz'/c_0^2)\beta,\quad y=y',\quad x=x',$$

where $\alpha = 1, \beta = (1-v^2/c_0^2)^{-1/2}$, and $\alpha = -1, \beta = (v^2/c_0^2-1)^{-1/2}$ at $v < c_0$ and $v > c_0$, respectively.

To find the field we use next the Fourier transformation for $E(r', t')$ and $\Phi\ (r', t')$:

$$E(r',t')=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{E}(\omega,k)\exp[i(\omega t'-kz')]d\omega\, dk,$$

$$\Phi(r',t')=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{\Phi}(\omega,k)\exp[i(\omega t'-kz')]d\omega\, dk, \qquad (6)$$

where the function $\Phi\ (\omega, k)$ is obtained by taking the inverse Fourier transform

$$\tilde{\Phi}(\omega,k)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\Phi(r',t')\exp[-i(\omega t'-kz')]dt'\, dz'$$

$$=-\frac{P_0(\omega_0^2+2\Delta_{in}^2)}{\pi c^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp[-(z'\Delta_{in}')^2]$$

$$\times\left\{\eta\left[\alpha(z'+vt')\beta+\frac{1}{2}L_1(x',y')\right]\right.$$

$$\left.-\eta\left[\alpha(z'+vt')\beta-\frac{1}{2}L_2(x',y')\right]\right\}\exp\{-i(\omega_0'+\gamma\Delta_{in}')z'$$

$$-i(\omega t'-kz')\}dt'\, dz'$$

$$=-\frac{P_0(\omega_0^2+2\Delta_{in}^2)}{\pi c^2}\frac{1}{(i\omega)}\left\{\exp\left[\frac{i\omega L_{(1,2)}(x',y')}{2v\beta}\right]\right.$$

$$\left.-\exp\left[-\frac{i\omega L_{(2,1)}(x',y')}{2v\beta}\right]\right\}\int_{-\infty}^{\infty}\exp[-(z'\Delta_{in}')^2]$$

$$\times\exp\left\{-i(\omega_0'+\gamma\Delta_{in}')z'\right.$$

$$\left.+ikz'+i\frac{\omega}{v}z'\right\}dz'=-\frac{i}{\omega c_0^2}\mu\exp\left\{-\left(\frac{\omega_1-\beta\omega-\beta vk}{2\Delta_{in}}\right)^2\right\}$$

$$\times\left\{\exp\left[\frac{i\omega L_{(1,2)}(x',y')}{2v\beta}\right]-\exp\left[-\frac{i\omega L_{(2,1)}(x',y')}{2v\beta}\right]\right\}, \qquad (7)$$

where

$$\mu=4v\beta\omega_0P_0/\sqrt{\pi}n^2\gamma,\ \omega_1=\omega_0+\gamma\Delta_{in},\ \omega_0'=\omega_0/v\beta,\ \Delta_{in}'=\Delta_{in}/v\beta.$$

The first and second subscripts of $L_{(i,m)}(x', y')$ correspond to $v < c_0$ and $v > c_0$, respectively.

Substituting $E(r', t')$ and $\Phi\ (r', t')$ in terms of their Fourier transforms in the wave equation (4), we obtain for $\tilde{E}(\omega, k)$ the expression

$$\tilde{E}(\omega,k)=[c_0^2/(\omega^2-k^2c_0^2)]\tilde{\Phi}(\omega,k). \qquad (8)$$

Finally, substituting (7) in (6), we obtain the following expression for the field in a moving coordinate system

$$E(r',t')=-\frac{2i\mu}{c_0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{k\omega}\exp(-ikz')\left(\frac{1}{\omega+c_0k}-\frac{1}{\omega-c_0k}\right)$$

$$\times\exp\left[-\left(\frac{\omega_1-\beta\omega-\beta vk}{2\Delta_{in}}\right)^2\right]\left\{\exp\left[i\omega\left(\frac{L_{(1,2)}(x',y')}{2v\beta}+t'\right)\right]\right.$$

$$\left.-\exp\left[-i\omega\left(\frac{L_{(2,1)}(x',y')}{2v\beta}-t'\right)\right]\right\}d\omega\, dk. \qquad (9)$$

The integral can be analytically obtained in two cases, when the region of integration with respect to (ω, k) can be divided into two nonoverlapping regions.

1. In the first interval of (ω, k) we integrate using the fact that $e^{i\omega x}(\omega \pm c_0k)^{-1}$ is a δ function at sufficiently large $x = (L_{(l,m)}/2v\beta \pm t')$. This holds true when the integration region $\Delta\omega$ satisfies near $\omega = \mp c_0k$ the conditions

$$\frac{\Delta\omega}{4\Delta_{in}^2}\left[2\omega_1\mp2\beta kc_0\left(1\pm\frac{v}{c_0}\right)+\beta^2\Delta\omega\right]\ll1, \qquad (10)$$

$$\Delta\omega\approx\pi\left|\frac{L_{(l,m)}(x',y')}{2v\beta}\pm t'\right|^{-1}. \qquad (11)$$

Thus, when conditions (10) and (11) are satisfied we have after integrating with respect to ω the following expression for the field:

$$E(r',t')$$

$$=-\frac{2\pi}{c_0}\mu\int_{-\infty}^{\infty}\frac{1}{k^2}e^{-ikz}\left[\exp\left[-\left(\frac{\omega_1-\beta k(c_0+v)}{2\Delta_{in}}\right)^2\right]\right.$$

$$\times\{\exp[iD(1,2,+)]-\exp[-iD(2,1,-)]\}$$

$$+\exp\left[-\left(\frac{\omega_1-\beta k(c_0-v)}{2\Delta_{in}}\right)^2\right]$$

$$\left.\times\{\exp[-iD(1,2,+)]-\exp(iD(2,1,-))\}\right]dk, \qquad (12)$$

$$D(l,m,\pm)=c_0k\left(\frac{L_{(l,m)}(x',y')}{2v\beta}\pm t'\right).$$

We calculate this integral by expanding $1/k^2$ in powers of $(k-k_0)$ near $1/k_0^2$, where $k_0=\omega_1[\beta(c_0\pm v)]^{-1}$. Integrating term by term we find that each term of the result is of the form

$$E_j(r',t')\infty e^{i\xi_j\omega}\sum_{n=0}^{\infty}\left(-\frac{i\Delta_{\text{in}}}{k_0c}\frac{\partial}{\partial\xi_j}\right)^n\exp[-\xi_j^2]$$

$$=A(b,\xi_j)\exp[-\xi_j^2], \tag{13}$$

$$A(b,\xi_j)=\sum_{n=0}^{\infty}a_n(\xi_j)b^n,\quad b=\frac{\Delta_{\text{in}}}{k_0c},\quad \xi_j=\xi_{j\pm}(t,r)\Delta_{\text{in}}.$$

At $b\ll1$ and $a_{n+1}/a_n\ll b^{-1}$ we can obtain the desired accuracy by choosing the required number of terms of the series $\Sigma a_n(\xi_j)b^n$. For example, when only the first term is taken into account the accuracy of the expression in the case $\Delta_{\text{in}}=10^{11}\ \text{sec}^{-1}$ and $\omega_0=10^{14}\ \text{sec}^{-1}$ is determined by the third significant figure of $\varepsilon\sim b\approx10^{-3}$.

The final integration result obtained in this approximation takes after reverting to the laboratory frame the form

$$E_1(r,t)=-P_0(\theta_jp_{12})\frac{\pi}{n^2}\frac{\omega_0^2+2\Delta_{\text{in}}^2}{\omega_1^2}\frac{v}{c_0}\alpha\frac{1}{(1-v/c_0)}\Big[\exp[-(\xi_+ +f_{(1,2)}(x,y))^2\Delta_{\text{in}}^2]\exp\{i\omega_1[\xi_++f_{(1,2)}(x,y)]\}$$

$$-\exp[-(\xi_++f_{(2,1)}(x,y))^2\Delta_{\text{in}}^2]\times\exp\{i\omega_1[\xi_+-f_{(2,1)}(x,y)]\}\Big]$$

$$+P_0(\theta_jp_{12})\frac{\pi}{n^2}\frac{\omega_0^2+2\Delta_{\text{in}}^2}{\omega_1^2}\frac{v}{c_0}\frac{1}{(1+v/c_0)}$$

$$\times[\exp[-(\xi_-+\alpha\varphi_{(1,2)}(x,y))^2\Delta_{\text{in}}^2]\exp\{i\omega_1[\xi_-+\alpha\varphi_{(1,2)}(x,y)]\}$$

$$-\exp[-(\xi_--\alpha\varphi_{(2,1)}(x,y))^2\Delta_{\text{in}}^2]\exp\{i\omega_1[\xi_--\alpha\varphi_{(2,1)}(x,y)]\}], \tag{14}$$

where the first and second subscripts of $f_{(l,m)}(\ldots)$ and $\varphi_{(l,m)}(\ldots)$ correspond to $v<c_0$ and $v>c_0$, respectively; $\xi_+=t-z/c_0$, $\xi_-=t+z/c_0$, with $\pm$ corresponding to $v\lessgtr c_0$,

$$f_{(l,m)}(x,y)=\pm\frac{1}{2}L_{(l,m)}(x,y)\left(\frac{1}{v}-\frac{1}{c_0}\right),$$

$$\varphi_{(l,m)}(x,y)=\frac{1}{2}L_{(l,m)}(x,y)\left(\frac{1}{v}+\frac{1}{c_0}\right).$$

It can be seen from this expression that the resultant field is a combination of four terms, two in the form of wave packets propagating in the same direction as the PWP, and the other two propagating in the opposite direction.

Depending on the ratio of v and c_0, as well as on the sample thickness $L(x,y)$, the resultant field is of the form of either one or two pulses. Fields propagating in the directions $\xi_\pm$ comprise sequences of two light pulses separated by an interval $L(x,y)(1/v\mp1/c_0)$ if the condition

$$F_\pm=\frac{v}{c_0}\frac{1}{|1\mp v/c_0|}\leqslant\frac{L(x,y)\Delta_{\text{in}}}{c_0}=\Pi \tag{15}$$

which follows from an analysis of (14), is satisfied.

It follows from (15) that at $\Pi<1$ the radiation in the ξ_+ direction consists of two pulses only for a PWP whose velocity is much less than c_0. If $\Pi>1$ a two-pulse sequence becomes possible already for a deviation from phase locking in either direction. The physical cause of generation of two light pulses by one PWP is that the latter is created on the front face of the sample and is annihilated on the rear face.

The employed approximations (10) and (11) do not enable us to track exactly the dynamics of radiation-pulse formation in the entire region (in the boundary regions upon deviation from phase locking, and in the entire sample under phase-locking conditions), since the solution obtained refers only to a field already shaped by the entire radiation process. Using (11), (10), and (13) in the laboratory frame, we can write these approximations in the form

$$\left|\frac{1}{2}L_1(x,y)+z\right|,\quad\left|\frac{1}{2}L_2(x,y)-z\right|>R,$$

$$R=\alpha\left(\frac{\pi v}{2\Delta_{\text{in}}}\right)\left(1-\frac{v^2}{c_0^2}\right)^{-1} \tag{16}$$

Here $\pi v/2\Delta_{\text{in}}$ is a measure of the PWP size, and the last factor in R determines the degree of the phasing of the oscillations at various points of space, a degree governed by the difference between the PWP and field velocities. The accuracy, governed by the approximations (10) and (11), is then

$$\varepsilon\infty R\left|\frac{1}{2}L_{(1,2)}(x,y)\pm z\right|^{-1}.$$

The desired accuracy ϵ can thus be achieved at a sufficient distance z from the sample (this follows from the fact that the remaining terms of the integral (9) are proportional to z^{-1} if z is large enough).

To clarify on the features of the generation of fields in these cases we consider the possible velocity-mismatch situations.

a) Slow PWP ($v\ll c_0$). The parameter R is determined only by the PWP dimensions, i.e., the forward and backward electromagnetic-field packets assume their final form as a result of complete creation of the PWP on the front face and its complete annihilation on the rear face of the sample.

b) Case near wave synchronism ($v\approx c_0$). The field packet is formed here in the entire sample volume ($R\to\infty$). In this case, if the PWP dimension is less than the sample thickness $L(x,y)$, the field radiated backwards retains the two-pulse character. At $v\approx c_0$, in turn, as follows from (14), the forward light pulses merge into a single pulse whose amplitude takes the form of the known diffraction function:

$$|E_{\text{out}}|^2\infty L^2\left(\frac{\sin(\Delta kL/2)}{\Delta kL/2}\right)^2. \tag{17}$$

At PWP dimensions larger than $L(x,y)$ there is only one pulse in either propagation direction, since the creation and annihilation of the PWP on the front and rear faces coincide both in time and in space.

c) Fast PWP ($v > c_0$). The parameter R again decreases here and as $v \to \infty$ the size of the coherence region in which the electromagnetic-field pulses are generated tends to zero.

Far from the phase-locking point, when the condition (15) for two-pulse radiation is satisfied, the expression (14) for the radiated field does not coincide with the diffraction function (17). The radiated fields begin to vary mainly as

$$E_{j\,\mathrm{out}} \infty \frac{1}{(\Delta k)} \left[-\left(t-\frac{z}{c_0}\right)^2 \Delta_{\mathrm{in}}^2 \right] \exp[i(\omega t - k_j z)]. \quad (18)$$

It can be seen from (18) that although the decrease of the intensity as a function of (Δk) is the same as for the usual case of samples with abrupt boundary $[|E_{\mathrm{out}}|^2 \sim 1/(\Delta k)^2]$, the field no longer depends on the oscillating function $\sin(\Delta kL/2)$, i.e, it is entirely independent of the sample size $L(x, y)$. This indicates that the radiation is due to the surface region of the medium.

One can estimate the maximum amplitude $E_{\max}(v \neq c_0)$, of the electromagnetic field generated on the wings of the dispersion curve and described by Eq. (18). In units relative to the field radiated under phase-locking conditions ($v = c_0$), using (14) and (15), one can obtain

$$E_{\max}(v \neq c_0) \approx SE(c_0),$$

where

$$E(c_0) \infty L(x, y) P_0 \frac{\omega}{c_0}, \quad S = \frac{\Delta_{\mathrm{in}}}{\omega_0}$$

(for example, for $\Delta_{\mathrm{in}} = 10^{11}\ \mathrm{sec}^{-1}$ and $\omega_0 = 10^{14}\ \mathrm{sec}^{-1}$ we have $S = 10^{-3}$, which can be measured in practice).

It is noteworthy that at $v > c_0$ the pulses generated on the front and rear faces of the sample are inverted in time relative to each other. The explanation is that the PWP generates a light pulse on the front face of the medium, then overtakes this light pulse and generates a second one, but now on the other face of the sample. The light pulse generated on the latter surface thus emerges earlier than the first light pulse. Such a situation, of course, is impossible if $v < c_0$. It is interesting that allowance for the reaction of the resultant field does not affect the presence of two-pulse radiation at $v > c_0$, since the PWP "runs away" from the field it generates. This time reversal can be recorded in experiment, since the form functions of the surfaces $L_1(x, y)$ and $L_2(x, y)$ are transmitted in field pulses generated by different faces [see (14)]. In the case considered by us the pump waves have plane wave fronts that undergo no phase perturbations on going through the sample boundary (e.g., in the case when the sample is placed in an immersion medium). Nonetheless, the fields generated at $v \neq c_0$ will contain information on the form of the sample surfaces. The features of the retrieval of information on the shapes of the surfaces (of the sequence in which the pulses and their wave fronts appear) in the generated-field signals are given in Table I.

TABLE I.

v	Radiation forward		Radiation backward	
	Sequence of pulses (1, 2)		Sequence of pulses (1, 2)	
	First	Second	First	Second
$v<c_0$	$f_1(x, y)$	$-f_2(x, y)$	$\varphi_1(x, y)$	$-\varphi_2(x, y)$
$v>c_0$	$f_2(x, y)$	$-f_1(x, y)$	$\varphi_1(x, y)$	$-\varphi_2(x, y)$

It can be seen that signals propagating forward (ξ_+) can be either forward or backward relative to the wave front, depending on whether $v > c_0$ or $v < c_0$, and also on which sample face is the source of the signal. In addition, by varying the PWP velocity it is possible to change the scale of the phase distortions of the wave front. For example, the total phase shift for the first sample surface is

$$\Psi_1(x, y) = \omega_1 f_1(x, y) = \frac{1}{2} \omega_1 L_1(x, y) \left(\frac{1}{v} - \frac{1}{c_0}\right). \quad (19)$$

It can be seen from (19) that as $v \to 0$ the phase perturbation of the field wave front increases without limit. The other terms behave similarly. This property can be used in the study of weak surface distortions.

If the radiation consists of two pulses, the information on the shapes of the sample faces is contained only in the form of field phase perturbations. The situation is entirely different in the single-pulse regime [$v \approx c_0$ or small $L(x, y)$], for here the two pulses begin to interfere with each other and the information from the front and rear faces becomes mixed. Then, as follows from (14), the field takes the form

$$E_1(r, t) \infty \left(\frac{1}{v} - \frac{1}{c_0}\right)^{-1} \sin\left[\frac{1}{2} \omega_1 L(x, y) \left(\frac{1}{v} - \frac{1}{c_0}\right)\right]$$

$$\times \exp\left\{ i\omega_1 \left[\xi_+ + \frac{1}{2} f_1(x, y) - \frac{1}{2} f_2(x, y) \right] \right\}. \quad (20)$$

It can be seen that the field contains both amplitude and phase information. The latter may not be obtainable in the case $v = c_0$ [see (19)] as well as when v differs from c_0 in the case $L_1(x, y) = L_2(x, y) + \mathrm{const}$, when the functions $L_1(x, y)$ and $L_2(x, y)$ are symmetric about the plane $(x, y, 0)$. The entire information on the faces is contained then in the amplitude part. On the other hand, if the front and rear faces are chosen to have identical shapes, the entire information is now contained only in the phase part ($L_1(x, y) = -L_2(x, y) + \mathrm{const}$).

2. We consider now the second (ω, k) region in which it is still possible to obtain an analytic solution that describes, in conjunction with (14), the general behavior of the field. In this case the solution can be obtained under the condition

$$|\omega_1 - \omega\beta(1 \pm v/c_0)| \gg 2\Delta_{\mathrm{in}}, \quad (21)$$

that ensues when conditions (9) and (10) are not satisfied.

Using the rapid convergence of the integral of the Gaussian function when (8) is integrated with respect to k, we get an expression for the field in the form

$$E_2(r', t') = B \exp\left(-i\omega_1 \frac{z}{v\beta}\right)$$

$$\times \exp[-(\Delta_{\mathrm{in}}' z')^2] \int_{-\infty}^{\infty} \sin\left[\omega \frac{L(xy)}{2v\beta}\right] e^{i\omega\eta} \omega^{-1}$$

$$\times [\omega_1 - \beta(1 + v/c_0)\omega]^{-1} [\omega_1 - \beta(1 - v/c_0)\omega]^{-1} d\omega, \quad (22)$$

where b is a constant, $\eta = t' + z'/v$, and the crossed integral is evaluated with account taken of (21). It is easy to show that when condition (21) is satisfied the integral converges rapidly near $\omega = 0$ in the region

$$\Delta\omega \leqslant \pi \left| \frac{L(xy)}{2v\beta} \pm \eta \right|^{-1}. \qquad (23)$$

The latter takes in the laboratory frame the form

$$\frac{\alpha}{(1 \mp v/c_0)}, \frac{\pi v}{(^1/_2 L_{(1,2)}(x, y) \pm \alpha z)} \ll \omega_1. \qquad (24)$$

Using (24) and retaining only the symmetric part in the integral (22), we obtain after transforming to the laboratory frame the following result for the field:

$$E_2(r, t) = \frac{4\pi}{n^2} \frac{(\omega_0^2 + 2\Delta_{in}^2)}{\omega_1^2} Q(r) \left(\frac{v}{c_0}\right)^2 \frac{1}{(1 - v^2/c_0^2)} \times \exp\left[-\left(t - \frac{z}{v}\right)^2 \Delta_{in}^2\right] \times \exp[i\omega_1 t - \mathbf{k}_p \mathbf{r})]. \qquad (25)$$

This equation in conjunction with (24) states that the field exists only inside the sample, with the volume of the subsurface regions excluded.

An interesting feature of this expression is that it describes a electromagnetic-field wave packet that coincides in form with the PWP and moves together with it at the same velocity ($v \neq c_0$). This combination of field and polarization can be regarded as a polariton field that differs from the known polaritons in that there are no restrictions in principle on the velocity $v (v < c_0, v > c_0)$. The features of such polariton formations in systems with phase memory must be studied more accurately with account taken of the reaction of the electromagnetic component (24) on the state of the medium, but this is outside the scope of the present article.

The overall picture of electromagnetic-field generation in systems with phase memory under conditions of deviations from wave synchronism ($v \neq c_0$) can thus be represented as follows.

At the start of the PWP formation on the left face of the sample, two fields are produced in the subsurface region. One field [the first and third terms of (14)] is generated so long as the PWP is "covered" by the second field (25); next, breaking away from the PWP (either lagging it or leading it), this field leaves the sample. The field (25), in turn, exists in the medium together with the PWP until the latter reaches the opposite face of the sample. In the course of vanishing of the PWP and of this field, an electromagnetic field is formed [second and fourth terms of (14)] and likewise leaves the sample.

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Translated by J. G. Adashko