Critical dynamics of the homogeneous magnetization in CdCr₂S₄

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The paper reports the results of an experimental investigation of the critical dynamics of the homogeneous magnetization in $CdCr_2S_4$. It is shown that the behavior of the homogeneous relaxation in the reduced-temperature region $\tau \sim 1 \times 10^{-2}$ is in accord with the current ideas. But near T_c , as follows from the data obtained, the critical dynamics is not only determined by the dipole interaction, but is also characterized by a low-frequency process, the nature of which is not yet clear. It is possible that this process is responsible for the anomalous behavior of the susceptibility at low frequencies and the non-power-law dependence of the critical-damping factor on the reduced temperature. It is concluded that the dynamic phenomena occurring in the immediate neighborhood of the Curie point are more complex than currently expected on the basis of the dynamic scaling hypothesis.

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Homogeneous magnetization dynamics is determined by interactions that do not conserve the total spin of the system. In the case of cubic ferromagnets the main such interaction is, as a rule, the dipole interaction, which is usually the only interaction that is taken into account in the description of the dynamics of the critical fluctuations of the homogeneous magnetization in such ferromagnets. In this case, according to current ideas, the decay of the fluctuations occurs in a relaxational manner, and the main quantity characterizing this process is the critical damping factor Γ_0 , or the homogeneous-magnetization relaxation time t_0 ($t_0 = \Gamma_0^{-1}$). The dynamic scaling hypothesis predicts a power-law dependence of the damping factor on the reduced temperature $\tau = (T - T_c)/T_c$, the sign and magnitude of the corresponding critical exponent for the cubic ferromagnets being determined by the strength of the effect of the dipole forces, in other words, by the magnitude of the static susceptibility χ_0 . According to the conclusions of the theory, in the exchange region of temperatures above T_c (where $4\pi\chi_0 \ll 1$), $\Gamma_0 \sim \tau^{-1}$ (Ref. 1), whereas in the dipole region (where $4\pi\chi_0 \ge 1$) $\Gamma_0 \rightarrow 0$ as $T \rightarrow T_c$. (It is customary in the theory to consider two versions of the dipole dynamics: hard and soft versions. In the first case² $\Gamma_0 \sim \tau^{4/3}$, while in the second case³ $\Gamma_0 \sim \tau^{2/3}$.)

One of the basic problems of the experimental study of the homogeneous relaxation is the determination of the behavior of the damping factor in the entire critical region. The choice of the experimental method for the investigations depends largely on the degree of proximity to T_c (essentially on the magnitude of the static susceptibility). Thus, for the study of the dynamic phenomena occurring in the exchange region, the EPR method is effective, whereas for the purpose of understanding the behavior of Γ_0 in the dipole region the most informative are the frequency and temperature dependences of the dynamic susceptibility determined by radiofrequency methods in zero external constant magnetic field.

It should be noted that the results of the experimental investigations of the critical behavior of the homogeneous relaxation in various ferromagnets essentially agree with the conclusions of the theory about the role of the dipole forces in the exchange region, where these forces can be taken into account with the aid of perturbation theory. But for a number of reasons the character of the behavior of the relaxation in the dipole region is still not clear. Besides methodological difficulties connected with the proximity to T_c , the absence of reliable experimental data on the behavior of Γ_0 in the region where $4\pi\chi_0 \ge 1$ is due to the following fundamental circumstance. As has already been noted, the results of radio-frequency measurements of the dynamic susceptibility $\chi(\omega)$ are used to study the critical behavior of the homogeneous relaxation. In this case the quantities Γ_0 and χ_0 are usually determined with the aid of the Lorentz formula, which relates $\chi(\omega)$ and Γ_0 :

$$\chi(\omega) = \chi_0 \Gamma_0 (\Gamma_0 - i\omega)^{-1}. \tag{1}$$

But for $4\pi \chi_0 \gg 1$ the characteristic critical-fluctuation energy (which in the dipole region is equal in magnitude to the damping factor) tends to zero as $T \rightarrow T_c$, and, as shown in Refs. 4-6, the relation (1) can no longer be used to determine the values of the damping factor when ω is of the order of Γ_0 . At the same time most published radio-frequency investigations of the critical dynamics were performed at frequencies of 10⁶ to 10⁸ Hz, which are comparable to Γ_0 , and, as a rule, the inapplicability of the Lorentz formula in this case was ignored in the analyses of the obtained data. On the other hand, the very scanty data pertaining to the frequency range from 10 to 10⁵ Hz point to disagreement with the current ideas about the character of the behavior of the susceptibility in this frequency region. For example, the study of the critical dynamics in yttrium iron garnet⁷ has revealed quite a strong dispersion of the real part of the susceptibility even in the frequency range from 20 to 80 Hz. Therefore, of indubitable interest would be investigations covering a broad frequency range, and performed on single-crystal samples of toroidal shape (with a demagnetization factor N = 0), which would allow us to carry out in a "purer" form the comparison of the experimental and theoretical results. In the present work we attempted such a study of the critical behavior of the homogeneous relaxation in a cubic ferromagnet. And,

as follows from the data obtained, the dynamic phenomena that occur in the paramagnetic phase near the Curie point have quite a complex character, which is not in accord with the current theoretical ideas.

EXPERIMENT

As the experimental material we chose the ferromagnetic spinel CdCr₂S₄ ($T_c \approx 84$ K), which has a cubic magnetic symmetry, and is an example of the Heisenberg ferromagnet.⁸

The investigated single-crystal sample had the form of a ring with dimensions $\phi 2.7 \times \phi 1.5 \times 0.3$ mm, cut in such a way that the plane of the ring coincided with the (111) plane. The sample with a copper-wire pickup loop uniformly wound on it was placed in a thermostat that allowed us to perform the investigations in the temperature range from 77.5 to 300 K. The thermostat was a hermetic cell placed in a Dewar vessel with liquid nitrogen. Inside the cell was a closed spherical heater that was used to vary the sample temperature. The supporting structural elements were made from thin-walled stainless-steel tubes. The cell and the heater were filled with gaseous helium at atmospheric pressure. A system of heat screens was used to reduce the convection currents and the temperature gradients in the thermostat. The heating element itself consisted of tightly joined copper hemispheres (of thickness 1 mm and diameter 27 mm), on which the heating coil, which occupied 90% of the area of each hemisphere, was bifilarly wound. The sample and a measuring thermometer were mounted close to each other at the center of the heater with the aid of a holder made of thin mica. To reduce the heat supply the leads of the pickup loop and the thermometer from the heater were made from thin ($\phi 0.05 \text{ mm}$) wire. The measures taken ensured the uniformity of the temperature field in the sample. As the measuring thermometer we used a KD102-type high-sensitivity ($\sim 2.5 \text{ mW/K}$) silicon diode. The temperature was determined, using the linear temperature dependence of the diode resistance for a stable constant current flowing in the forward direction. The diode was graduated with the aid of a calibrated KG-type germanium resistance thermometer. The voltage drop across the diode was measured by a Shch-31 digital voltmeter, which allowed the determination of the changes in the temperature with the 0.001-K resolution. The power consumed by the thermometer did not exceed 3×10^{-6} W. The power generated in the pickup loop during the dynamic susceptibility measurements also did not exceed 3×10^{-6} W; at this rate of heat release the sample temperature was constant within the limits of sensitivity of the thermometer. Let us note that the power supplied to the heater in the region of temperatures close to T_c was roughly 0.3 W. The temperature-stabilization system, which was similar to the one described in Ref. 9, allowed us to maintain the sample temperature constant to within $(1-3) \times 10^{-3}$ K. The geomagnetic field in the sample unit was reduced with the aid of Permalloy screens to 10-15 mOe.

The investigations were performed in the high-frequency range from 1 to 12 MHz by the resonance method with the use of a series oscillatory circuit⁹ and in the low-frequency

the components of the impedance of the pickup loop were determined from the modulus of the total resistance and the phase shift between the current flowing through the coil and the voltage across it. In determining the susceptibility of the investigated sample we took into account the so-called residual parameters of the measuring circuit (the resistance and inductance of the leads, the self-capacitance and inductance of the active resistors of the circuit, etc.), as well as the influence of the skin effect on the intrinsic parameters of the pickup loop.¹⁰ The high- and low-frequency circuit parameters were measured to within 0.2% and 0.5% respectively. The errors made in the determination of the quantities γ' and χ " for $\chi_0 > 0.1$ did not exceed 1% at high, and 2% at low, frequencies. For $\chi_0 < 0.1$ the errors increased in proportion to the decrease in the magnitude of the susceptibility, and were as high as 100% for $\chi_0 \approx 10^{-4}$. The values of the critical-damping factor were deter-

region (0.3-100 kHz) by the phase method. In the latter case

mined from the results of the susceptibility measurements performed at frequencies of 1-12 MHz. For this purpose we used two different methods, depending on whether the quantity ω was large or small compared to Γ_0 . In the reducedtemperature region $\tau > 7 \times 10^{-3}$, where the condition $\omega \ll \Gamma_0$ was fulfilled, the χ_0 and Γ_0 values were computed with the aid of the expression (1) from the quantities χ' and χ'' obtained. At lower temperatures, because of the violation of the condition $\omega \ll \Gamma_0$ at high frequencies, this formula is no longer applicable, and therefore in this case we used the method proposed in Ref. 4. This method which is based on the general principle of the dynamic scaling hypothesis, consists in the search at a given temperature of that frequency f_c at which the real and imaginary parts of the susceptibility are equal in magnitude. Since, according to this hypothesis, $\chi(\omega)$ is a homogeneous function of ω/Γ_0 , f_c is in order of magnitude equal to Γ_0 . It is natural that the applicability of such an approach to the determination of the Γ_0 values is confined within the limits of validity of the dynamic scaling hypothesis. Notice that in both cases Γ_0 is actually determined in terms of χ' and χ'' ; therefore, the error in the Γ_0 values is the sum of the errors made in the determination of the quantities χ' and χ'' in the corresponding temperature region.

THE CHOICE OF THE CURIE POINT

The most general arguments were used to choose the Curie point. One of them is connected with the phenomenon of critical slowing down, according to which $\Gamma_0 \rightarrow 0$ as $T \rightarrow T_c$. In the case when the dynamic scaling hypothesis is applicable we can take as the T_c the temperature at which f_c vanishes. But under real experimental conditions the instability of the temperature will lead to a situation in which f_c will assume a finite nonzero value at the Curie point. Therefore, experimentally, this method of determining the Curie point was realized in the following manner. From the functions $\chi'(T)$ and $\chi''(T)$ obtained at different frequencies we found that frequency $f_{c,\min}$ for which the ratio χ''/χ' is equal to unity only at one temperature point. Notice that the

ratio χ''/χ' for $f < f_{c,\min}$ is always smaller than unity at any temperature, while for $f > f_{c,\min} \chi''/\chi'$ is equal to unity at two temperature points corresponding to the paramagnetic and ferromagnetic phases. The accuracy achieved in the determination of T_c by this method evidently depends on the degree of instability of the temperature and the accuracy achieved in the determination of χ' and χ'' (in our experiments the error made when we determined the T_c by finding the temperature corresponding to $f_{c,\min}$ was $\pm (15 20) \times 10^{-3}$ K (see Fig. 4b below)). Although such an error could not explain the deviation of $\Gamma_0(\tau)$ from a power law near T_c (see below), it was desirable to determine the position of the Curie point by a different method.

For this purpose we used the nonlinear properties of the susceptibility in the critical region. In the absence of an external constant magnetic field, and under the condition that $\omega \ll \Gamma_0$, the susceptibility can, as is well known (see, for example, Ref. 11), be represented in the form

$$\chi(h) = \chi_0 + 3\chi_2 h^2 + \dots \qquad (2)$$

This series expansion in the applied-field amplitude h is valid if¹²

$$g\mu h \ll (kT_c) \tau^{5\nu/2}.$$
 (3)

Here g is the g factor, μ is the Bohr magneton, and ν is the critical exponent of the correlation length. The second term in the expansion (2) is directly connected with the third harmonic of the voltage drop induced across the pickup loop upon the application of a variable magnetic field to the sample, while χ_2 is none other than the four-particle spin correlator, ¹² which is singular at the Curie point. Consequently,



the temperature behavior of the third harmonic should also have a singular character at the transition point. Therefore, as T_c we can take the temperature corresponding to the singularity in the temperature dependence of the third-harmonic signal intensity obtained for a variable-field amplitude satisfying the condition (3). In the experimental determination by this method of the Curie point the thirdharmonic signal measurement error did not exceed 0.5%, which allowed us to determine the temperature corresponding to the third-harmonic signal peak to within $\pm (5 10 \times 10^{-3}$ K (see Fig. 6b below). The choice of the Curie point as the temperature corresponding to the third-harmonic signal peak is apparently more general than the method based on the dynamical scaling. Nevertheless, it should be noted that the two methods gave practically the same T_c value.

RESULTS

1. Low-frequency relaxation

Apparently, the most interesting results were obtained in the low-frequency measurements. As follows from the data on the behavior of Γ_0 in the dipole region (see below), Γ_0 is, in order of magnitude, greater than 1 MHz in the entire accessible temperature region, right down to the point closest to T_c ($\tau_{\rm lim} \approx 1 \times 10^{-4}$). Therefore, it might have been expected that the condition $\omega \ll \Gamma_0$ would be fulfilled in the investigation of the behavior of the susceptibility in the lowfrequency region (0.3–100 kHz). Then, according to the relation (1), at a given temperature the real part of the susceptibility should practically not be frequency dependent, and the magnetic losses should be proportional to the frequency. But

FIG. 1. Dependence of χ' on f for different τ : 1) + 1×10⁻³, 2) + 8×10⁻⁴, 3) + 6×10⁻⁴, 4) + 4×10⁻⁴, 5) + 2×10⁻⁴, and 6) + 1×10⁻⁴. The χ' values were obtained for $h \approx 10$ mOe.



FIG 2. Dependence of χ'' on f for different τ . The τ values are the same as in Fig. 1.

in actual fact it turned out that in the dipole region the behavior of the susceptibility at low frequencies does not correspond to these expectations. In the first place the real part does not depend on the frequency in the entire investigated region, and the function $\chi'(\omega)$ itself has a roughly logarithmic character at low frequencies (Fig. 1). In the second place, in this same temperature region the imaginary part varies slowly with frequency, and $\chi''(\omega)$ has a broad peak at frequencies of the order of 10^3-10^4 Hz (Fig. 2). This behavior of the dynamic susceptibility is in accord with the Kramers-Krönig relations:

$$\chi'(\omega) = \frac{2}{\pi} \int_{0}^{\infty} \frac{\omega' \chi''(\omega') d\omega'}{\omega'^2 - \omega^2}.$$
 (4)

Indeed, if we taken account of the fact that χ " varies very slowly with ω in a fairly broad frequency range, covering almost three orders of magnitude, then it seems we can average the values of the imaginary part over the whole of this range, and consider χ " to be independent of ω . Then, as follows from the relation (4), the ω dependence of χ ' in this frequency range will have a logarithmic form:

$$\chi'(\omega) = \frac{2}{\pi} \chi'' \ln \frac{\Omega_0}{\omega}, \qquad (5)$$

where Ω_0 is some characteristic frequency that depends only on the temperature. We found upon verification that the quantities χ' and χ'' obtained in the low-frequency region are indeed connected by the relation (5) to within 20–30%.

Let us note that the experimental data obtained for the interval $3 \times 10^2 < f < 1 \times 10^4$ Hz can be described by a power



$$\chi(\omega) = \chi_0 \left[1 - \left(\frac{\omega}{i\Omega_0}\right)^{\rho} \right], \quad \chi'(\omega) = \chi_0 \left[1 - \left(\frac{\omega}{\Omega_0}\right)^{\rho} \cos\frac{\pi\rho}{2} \right],$$
$$\chi''(\omega) = \chi_0 \left(\frac{\omega}{\Omega_0}\right)^{\rho} \sin\frac{\pi\rho}{2}.$$
(6)

Such a description has the advantage that χ " vanishes at $\omega = 0$.

The ρ value determined from the formulas (6) turned out to be roughly equal to 0.1. So small a value for this exponent did not, unfortunately, allow us to estimate the quantity Ω_0 with sufficient reliability.

Thus, it follows from the behavior of the susceptibility at low frequencies that, at least for $4\pi\chi_0 \ge 1$, the critical dynamics of the homogenous magnetization is apparently determined by not only the dipole interaction, but also another interaction whose effect on the behavior of the susceptibility manifests itself in the low-frequency region. This conclusion is corroborated by the shape of the dependences $\chi''(\chi')$ obtained for different distances from T_c (Fig. 3), which clearly exhibit two regions: a high-frequency and a low-frequency region. The dipole forces are apparently responsible for the high-frequency part of the relaxation. As for the nature of the low-frequency part, it is still obscure.

It should be noted that a fairly strong low-frequency dispersion of the susceptibility has also been observed in the critical region in a number of experiments,^{7,13,14} but no systematic investigations of this phenomena has thus far been carried out.



FIG. 3. Plots of the dependence $\chi''(\chi')$ for different τ . The τ values are the same as in Fig. 1.

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FIG. 4 a) Plot of $\Gamma_0(\tau)$. The symbol \bullet indicates the Γ_0 values obtained from the formula (1); O, the values obtained from the frequencies f_c . Plotted along the upper axis are the $4\pi\chi_0$ values determined from the formula (1). b) $(T - T_c)$ dependences of the ratio χ''/χ' obtained at frequencies close to $f_{c \min}$.

2. The nature of the behavior of the damping factor

The behavior of the critical-damping factor was determined from the data obtained in the measurements performed in the frequency range from 1 to 12 MHz. And, as has already been noted, the Γ_0 values were, depending on the distance from T_c , determined either from the formula (1) (for $\omega \ll \Gamma_0$, or from the frequencies corresponding to the equality of χ' and χ'' . It is clear from the dependence $\Gamma_0(\tau)$ obtained (Fig. 4a) that, in accord with the theoretical ideas about the role of the dipole forces in the critical dynamics of the cubic ferromagnets, the homogeneous relaxation changes the character of its behavior, at the temperature corresponding to $4\pi\chi_0 \approx 1$, specifically, in the exchange region Γ_0 increases in magnitude with decreasing temperature, while in the dipole region it decreases. In the region of very low susceptibility values the errors made in the determination of χ' and χ'' (and, consequently, of Γ_0) are quite high, and therefore we can speak only of the general character of the behavior of the damping factor in the exchange

region. Nevertheless, the agreement between the Γ_0 values obtained for the transition region by both the radio-frequency method (the present work) and the EPR method (Refs. 15 and 16) should be noted.

For $4\pi\chi_0 > 1$ the errors made in the determination of Γ_0 are not high, but, as can be seen from Fig. 4, the character of the dependence $\Gamma_0(\tau)$ in the dipole region is quite unexpected. Whereas in the $1 \times 10^{-2} < \tau < 1 \times 10^{-1}$ region Γ_0 varies roughly like $\tau^{0.8}$ (which is not at variance with the results obtained in Ref. 3, where roughly the same exponent is predicted), at lower temperatures the $\Gamma_0(\tau)$ dependence does not have a power-law form.

It should be noted that the character of the temperature dependence of the real part of the susceptibility (Fig. 5), as found at high frequencies, clearly demonstrates the phenomenon of critical slowing down. Indeed, it can be seen from this dependence that, in the $(T - T_c) < 0.1$ K region, $\chi'(T)$ behaves differently at different frequencies: For $T \rightarrow T_c$, χ' either decreases slightly in magnitude (7 and 6 MHz), or it



FIG. 5. $(T - T_c)$ dependence of χ' obtained at different frequencies: 1)–7) pertain to f = 7, 6, 5, 4, 3, 2, and 1 MHz; 8)–11) were obtained at the frequencies 0.3, 1.6, 12.8, and 100 kHz for $h \approx 10$ mOe.

practically does not vary with temperature (3–5 MHz), or it continues to increase (at lower frequencies). A similar behavior of $\chi'(T)$ at different frequencies near T_c has been observed in an investigation of the homogeneous relaxation in EuS by Shino and Hashimoto,¹³ who, apparently, were the first to suggest a connection between this behavior of the real part of the susceptibility and the phenomenon of critical slowing down.

It should be noted from the results of the investigations that, for the temperature region where the values of the static susceptibility could still be determined with the aid of (1), the reduced-temperature dependence of the susceptibility complies with the prediction of the static scaling theory, and the obtained χ_0 values correspond with the data reported in Ref. 15.

3. Temperature dependence of the third-harmonic signal

As has already been indicated, to determine the Curie point, we measured the temperature dependence of the third-harmonic in the low-frequency region. In these experiments we took special measures to eliminate signals with frequency $3\omega_0$ that were not connected with the nonlinear properties of the susceptibility. Thus, the relative content of the third harmonic in the initial voltage of the fundamentalfrequency (ω_0) generator did not exceed 10⁻⁵, and to remove the overload on the input amplifying devices rejection filters tuned on the fundamental frequency were placed at their points of entry. In determining the Curie point from the temperature anomaly of the third-harmonic signal, we paid particular attention to the fulfillment of the condition for a small variable-field amplitude, a condition which is quite rigid. Indeed, as can be seen from (3), in the case of, for example, the cubic ferromagnet (v = 2/3) the applied-field strength satisfying this condition should decrease like $\tau^{5/3}$ as T_c is approached. Estimates show that, for $\tau = 1 \times 10^{-4}$, the field amplitude should not exceed 0.1 Oe. Therefore, in determining T_c , we carried out the third-harmonic signal measurements with a variable field with $h \approx 0.01$ Oe (as was established, an increase by a factor of 2-3 of the variable-field amplitude did not lead to a change in the temperature corresponding to the third-harmonic peak). As an example, we show in Fig. 6 the $(T - T_c)$ dependence of the third-harmonic signal amplitude $A_{3\omega}$ obtained at 12.8 kHz. The lowest-temperature peak on this curve pertains to the ferromagnetic phase, and is possibly connected with the processes of domain formation.

The consideration of the effect of the external constant magnetic field leads to a situation in which we should take into account those terms in the expansion (2) which are responsible for the appearance of the even harmonics. But in the paramagnetic phase the second-harmonic signal was practically not observed, which indicates that the external field was sufficiently well screened off. With the appearance of the second-harmonic signal became observable, and increased with distance from T_c even in the absence of a constant field.

A detailed study of the nonlinear properties of the susceptibility was not carried out in the present work, since the main purpose of this part of the investigation was to determine the Curie point. But it is clear that investigations of that sort will furnish useful information about the physics of the dynamic phenomena occurring in the critical region.

CONCLUSION

It follows from the results of the investigations performed that the picture of the dynamic phenomena occurring in the immediate neighborhood of T_c is more complex than the one provided at present by the dynamic scaling hypothesis. We can distinguish three characteristic regions in the paramagnetic phase. In the first two, namely in the exchange region $(\tau > 0.1)$ and in the interval $1 \times 10^{-2} < \tau < 1 \times 10^{-1}$ (the dipole region), the behavior of the dynamic susceptibility is consistent with the predictions of the dynamic scaling theory. But the behavior of the homogeneous relaxation in the region of temperatures in close proximity to the Curie point (i.e., in the region $\tau < 1 \times 10^{-2}$) clearly contradicts the current ideas (the anomalous behavior of the susceptibility at low frequencies, the non-powerlaw form of the dependence $\Gamma_0(\tau)$). Therefore, at this time the explanation of the data obtained may be only a conjecture. Our primary hypothesis is that at large values of the static susceptibility the critical dynamics is apparently determined by the interaction of two subsystems, one of which is evident-



FIG. 6. Dependence of $A_{3\omega}$ on $(T - T_c)$: a) the general form; b) the shape of the third-harmonic signal peak near T_c . The quantity $A_{3\omega}$ is in arbitrary units, $f_0 = 12.8$ kHz, and $h \approx 10$ mOe.

ly the spin subsystem. The nature of the other subsystem, which is responsible for the low-frequency part of the relaxation, is for the present obscure. Arguments of a different sort do not allow us to relate it to the phonons or to the free charge carriers. If we assume that the interaction between the subsystems has a nonlinear character, then it can serve as a condition for the appearance of a spatially-inhomogeneous fluctuation distribution in the sample, a prototype of its kind being a domain structure. This assumption is apparently confirmed by certain results obtained in an investigation of the critical dynamics in YIG,¹⁷ among which results we should note the effects of absorption and evolution of heat by sample that were observed under quasi-adiabatic thermal conditions, as well as some other characteristics, found in the same experiments, of the temperature behavior of the physical quantities. And it should be pointed out that the temperatures at which anomalies of this kind occurred in the paramagnetic phase were determined by the shape of the sample (in the form of a torus, a cylinder, and a sphere were used). And in the case of the spinel $CdCr_2S_4$, as can be seen from Fig. 6, there exist additional anomalies in the temperature dependence of the third-harmonic, besides the peak due to the existence of the Curie point. A similar behavior of the third-harmonic signal has been observed in another ferromagnetic spinel, CdCr₂Se₄, the preliminary investigations of which were also performed on a single-crystal sample having the form of a ring. The above-indicated anomalous phenomena observed both in the case of $Y_3Fe_5O_{12}$ and for $CdCr_2S_4$ and CdCr₂Se₄ can, in our opinion, serve as a basis for the assumption that there exists a spatially-inhomogeneous critical-fluctuation distribution at large values of the static susceptibility.

Generally speaking, the dipole region still remains the least studied. As far as we know, the majority of the published investigations were confined to temperature regions which in terms of τ were not closer to T_c than 10^{-3} , and the available data on the critical behavior of the damping factor in the dipole region are, on account of the inapplicability of the relation (1) in the region $\omega > \Gamma_0$, not reliable enough, in our opinion. Therefore, to understand the dynamic phenom-

ena that occur near T_c , we need further investigations performed by different methods.

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