T-models of a sphere in general relativity theory (II)

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Spherically symmetric singular solutions of general relativity theory in a comoving T-system, or T-models of the sphere, are considered; the range of solutions is broader than in previous articles [V. A. Ruban, a) Sov. Phys. JETP **39**, 1027 (1969); b) JETP Lett. **8**, 414 (1968); c) LIYF Preprint 412 (1978)]. The solutions are characterized by an infinite gravitational mass defect. The properties and dynamics of the solutions are discussed in relation to the problem of quantum "explosion" of white holes [Ya. B. Zel'dovich, I. D. Novikov, and A. A. Starobinskiĭ, Sov. Phys. JETP **39**, 933 (1974)]. T-models of a sphere are a modification of the T-regions of Schwarzschild-Reisner-Nordstrem (SRN) fields filled with an unlimited quantity of gravitationally totally bound matter. This matter alters substantially the global structure of V_4 and precludes the existence of external R-regions. Generalizations of the SRN vacuum T-metrics in different versions of scalar-tensor theories are found and it is shown that these generalizations lack zero-horizons of the Schwarzschild-sphere type.

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INTRODUCTION

The so-called spherically symmetric T-models of zero-pressure dust matter ($P = 0$) were previously defined and discussed in Refs. 1a and 1b. They are singular solutions of general relativity theory in a synchronously comoving T-system[2–4] with metric of the form [1–8]

$$-ds^2 = -d\tau^2 + e^{\omega(\tau,\chi)} d\chi^2 + r^2(\tau)\,[d\vartheta^2 + \sin^2\vartheta\, d\varphi^2]. \qquad (1)$$

The T-models are distinguished by the fact that the radii $\chi = \text{const}$ of all the Lagrangian distribution spheres of matter are the same if $\tau = \text{const}$ and that the spatial cross sections in the comoving T-system (1) form an unbounded hypercylinder $V_3 = (S_2 \times R_1)$ of infinite volume containing an infinite quantity of matter but lacking a Euclidean "center" ($-\infty < \chi < \infty$). T-models have no classical analog and correspond to a special physically interesting case of T-regions of space-time[2–4] V_4 filled with matter at rest relative to the T-system, this matter turns out to be gravitationally totally bound in such a maximally strong Einstein gravitational field.

The purpose of the present article is to study the most general group of T-models of a sphere for different type sources of matter (chiefly of an ideal liquid with $P \neq 0$, as well as of a free electromagnetic or scalar field) so as to extend the physical interpretation and basic results of Ref. 1 to these T-models.

T-models of a sphere may be interpreted as a modification of vacuum T-regions of a Schwarzschild-Kruskal field[2–4] filled with an infinite quantity of totally bound matter all of whose rest energy is exactly compensated by the gravitational mass defect for each spherical layer. As a result, the active mass of the T-distribution—its total energy equivalent—remains unchanged as the layers with infinite mass are added without limit in the hypercylindrical V_3 ($\tau = \text{const}$). Though cosmological T-models (1) are causally complete V_4 and as if closed in themselves, as in the closed Friedmann model[2,4], it is possible to construct bounded T-spheres and to correctly determine their total mass-energy integral $E = Mc^2 > 0$ by directly splicing their semi-infinite parts ($-\infty < \chi \leqslant 0$) to the external Schwarzschild field in space. The total mass-energy integral is a purely field-dependent geometrodynamic "massless mass" (due to the initial singularity) of the initial vacuum T-regions by means which it is possible to construct T-models of the sphere, which models these regions also resemble to a considerable extent. Consequently, T-spheres yield a method of realizing the maximally possible total gravitational mass defect (for general relativity theory); this model differs qualitatively from the closed Friedmann model. Here the mass defect is equal to the infinite rest mass of the matter.[1]

Such T-metrics are also of interest for relativistic astrophysics, besides serving as an anisotropic axisymmetric "quasiclosed" type of model in cosmological applications[1,6] (including the version with "primary" electromagnetic field based on the Reisner-Nordstrem T–regions[7]). Like the T-sphere,[16] relativistic hypothetical objects which include and always retain beneath the Schwarzschild sphere an arbitrarily unlimited quantity of gravitationally totally bound matter may, in principle, arise in a quantum "explosion" in white holes.[8] In an external R-region, these objects should appear as an ordinary sphere with finite gravitational mass and with a radius that expands only to its own Schwarzschild sphere, with a characteristic pattern of congealing and of formation of a gray hole in anti-collapse; further, its collapse phase is, in general, not observable.[1b,1c,8]

Our general analytic solution for T-models of a sphere, which contain a maximally rigid liquid with $P = \varepsilon$ (as well as a free electromagnetic field),[9] may be interpreted as a generalization of the Schwarzschild-Reisner-Nordstrem vacuum T-metrics in the presence of a scalar-massless field, whether of the canonical ($\Psi = \Psi(\tau)$, $\Box\Psi = 0$) or conformal ($\Box\Phi + 1/6 \cdot R\Phi = 0$) type. The T-models no longer have Schwarzschild-sphere type zero-horizons that are destroyed by the uniform mode of the scalar field. The latter result confirms a

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previous hypothesis, to the effect that a black hole cannot form upon collapse of a massive body in scalar-tensor theories.[10]

LEMAITRE MASS FUNCTION.[11] DESCRIPTION[2] OF T-REGIONS IN V_4

1. Without any limitation on generality, the metric of any spherically symmetric field V_4 may be presented in diagonal form [2–4]:

$$-ds^2=-e^{\sigma(x_0,x_1)}dx_0^2+e^{\omega(x_0,x_1)}dx_1^2+e^{\mu(x_0,x_1)}[d\vartheta^2+\sin^2\vartheta d\varphi^2],\tag{2}$$

where the angular coefficient $e^{\mu/2}=R(x_0,x_1)$ is the radius of curvature of the spheres S_2 ($x_0=$ const, $X_1=$ const) and is a scalar invariant relative to any choice of the coordinates $\tilde{x}_0=\Theta(x_0,x_1)$, $\tilde{x}_1=L(x_1,x_0)$.

For an arbitrary orthogonal metric (2), the Einstein equations reduce to a simple and convenient system in terms of the Lemaitre invariant mass function[11]:

$$\begin{gathered}\frac{\varkappa}{4\pi}m(x_0,x_1)=R(1+e^{-\sigma}\dot{R}^2-e^{-\omega}R'^2),\\ m'=4\pi R^2(T_0^0R'-T_1^0\dot{R}),\quad \dot{m}=4\pi R^2(T_1^1\dot{R}-T_0^1R'),\end{gathered}\tag{3}$$

where the prime and dot denote differentiation with respect to the radial x_1 and time x_0 coordinates. This function $m(x_0,x_1)$ is the active gravitational mass, the total energy equivalent of the distribution of sources of matter. The latter are described by an energy-momentum tensor with components

$$T_0^0,\ T_0^1,\ T_1^1,\ T_2^2=T_3^3=f(x_0,x_1)$$

which do not vanish identically in the case of spherical symmetry and which satisfy the conservation law $T^k_{i;k}=0$.

In polar coordinates for curvature with $r=R(x_1,x_0)$, which are acceptable from the standpoint of physics only for R-regions in V_4 having a canonical metric of the form[2–4]

$$-ds^2=-e^{\nu(r,t)}dt^2+e^{\lambda(r,t)}dr^2+r^2[d\vartheta^2+\sin^2\vartheta d\varphi^2],\tag{4}$$

the field equations (3) are greatly simplified:

$$e^{-\lambda}=1-\frac{\varkappa m(r,t)}{4\pi r},\quad m=(r,t)=4\pi\int_0^r T_0^0(r,t)r^2dr\tag{5}$$

and a exhibit far-reaching analogy with Newtonian gravitational theory for non-static centrally symmetric systems.[12]

In an external empty region of V_4 with Schwarzschild metric

$$e^\nu=e^{-\lambda}=1-(\varkappa M/4\pi r),\qquad M=\text{const},$$

the Lemaitre mass function coincides with the total gravitational energy-mass integral of an isolated sphere in general relativity theory.[11]

In a reference frame comoving with the liquid and having a metric[2]

$$-ds^2=-e^{\sigma(\tau,\chi)}d\tau^2+e^{\omega(\tau,\chi)}d\chi^2+R^2(\tau,\chi)[d\vartheta+\sin^2\vartheta d\varphi^2]\tag{6}$$

we obtain from (3) the system of equations

$$\begin{gathered}m'=4\pi\varepsilon R^2R',\qquad \dot{m}=-4\pi PR^2\dot{R},\\ e^{-\sigma}\dot{R}^2=e^{-\omega}R'^2-1+\varkappa m/4\pi R+\Lambda R^2/3\end{gathered}\tag{7}$$

for the hydrodynamic energy-momentum tensor $T_0^0=\varepsilon(\tau,\chi)$, $T_1^1=T_2^2=T_3^3=-P(\tau,\chi)$. The system is complemented with the conservation law $T^{i,i}_{k;}=0$:

$$\dot{\omega}+4\dot{R}/R=-2\dot{\varepsilon}/(P+\varepsilon),\qquad \sigma'=-2P'/(P+\varepsilon)\tag{8}$$

and with a dynamic equation of the form

$$D_\tau v=-\frac{\varkappa}{4\pi R^2}(m+4\pi PR^3)-\frac{W^2}{P+\varepsilon}\left(\frac{\partial P}{\partial R}\right)_\tau.\tag{9}$$

Here $v=D_\tau R=e^{-\sigma/2}\dot{R}$ is the rate of variation of the circumferential distances on the Lagrangian spheres $\chi=$ const, measured with respect to local proper time. Equation (9) is the relativistic analog of the inverse square law. That is, acceleration of a layer is determined by the gravitational mass $m^*=m+4\pi PR^3$ of the inner sphere and by the pressure gradient ($\sigma'\neq0$). In the static case, this equation turns into the condition of equilibrium of a gravitating liquid with $P=P(\varepsilon)\neq0$ in the R-region (4). Its first integral

$$v^2=W^2-1+\varkappa m/4\pi R+\Lambda R^2/3\tag{10}$$

has the physically lucid meaning of the equivalent of the Newtonian energy equation $f=W^2-1=2\mathcal{E}$ for a spherical layer $\chi=$ const. Therefore, the function

$$W(\chi,\tau)=e^{-\omega/2}R'=\left(1+v^2-\frac{\varkappa m}{4\pi R}+\frac{\Lambda R^2}{3}\right)^{1/2}\tag{11}$$

is interpreted as the relativistic specific energy of the Lagrangian spherical layers, which includes not only the self mass-energy of the liquid, but also the potential gravitational binding energy and the kinetic energy of the radial motion.

The active gravitational mass

$$m(\chi,\tau)=4\pi\int_0^\chi\varepsilon R^2R'd\chi=4\pi\int_0^\chi\varepsilon WR^2e^{\omega/2}d\chi$$

yields the total energy contained within the Lagrangian spheres, and at $W\neq1$ it is different from the total self energy of the liquid in the sphere:

$$\mathcal{M}(\chi,\tau)=4\pi\int_0^\chi\varepsilon R^2e^{\omega/2}d\chi,\quad dV=4\pi R^2e^{\omega/2}d\chi,\tag{12}$$

so that even for dust with $P=0$, it is different from the conserved rest mass of the sphere[1–4]:

$$\mathcal{M}_0(\chi)=4\pi\int_0^\chi\rho R^2e^{\omega/2}d\chi.$$

If a spherical layer $\chi=$ const is added, the ratio of the increments

$$\left(\frac{\partial m}{\partial\mathcal{M}}\right)_\tau=R'e^{-\omega/2}=W=\pm\left(1+v^2-\frac{\varkappa m}{4\pi R}+\frac{\Lambda R^2}{3}\right)^{1/2}\tag{13}$$

differs at $W<1$ by the gravitational binding energy, and at $W>1$ by the excess kinetic energy of the liquid. These general-relativisty nonlinear effects—the gravitational mass defect and the gravitational force created by the kinetic energy of the layer—manifest themselves in a unique way via the non-Euclidean nature of the physical space V_3 of the comoving frame (6), since the active specific energy $W=m'/\mathcal{M}'$ determines also the geometry of the spatial cross sections $\tau=$ const:

$$dl^2 = dR^2/W^2(R) + R^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2),$$
$$e^{\omega} = R'^2/W^2. \qquad (14)$$

In particular, the sign of the function $f = W^2 - 1$, which (at $\Lambda = 0$) characterizes the infinite ($f \geqslant 0$) or finite ($f < 0$) types of motion of the layer, is opposite to the sign of the spatial scalar curvature[13]

$$K = \frac{2}{R^2}\frac{d}{dR}(fR).$$

From the comoving condition in the form

$$D_\tau \ln W = -\frac{v}{P+\varepsilon}\left(\frac{\partial P}{\partial R}\right)_\tau \qquad (15)$$

it follows that the rate of change of the specific active energy of the layer is determined by the pressure gradient. Therefore, at $P = 0$, as well as for distributions of a liquid with $P = P(\tau)$ which are uniform in (6), the integral $W(\chi) = [1 + f(\chi)]^{1/2}$ is conserved. In the case of isotropic Friedmann models, this integral ensures separation of variables in the metric (6):

$$-ds^2 = -d\tau^2 + a^2(\tau)\{d\chi^2 + S^2(\chi)[d\vartheta^2 + \sin^2\vartheta d\varphi^2]\},$$

$$R(\chi,\tau) = a(\tau)S(\chi), \quad S(\chi) = \begin{cases} \sin\chi & (k=+1) \\ \chi & (k=0), \\ \operatorname{sh}\chi & (k=-1) \end{cases} \qquad (16)$$

$$m(\chi,\tau) = \mu(\tau)S^3(\chi), \quad \dot{\mu} = -4\pi P a^2 \dot{a}, \quad \mu(\tau) = (4\pi/3)\varepsilon a^3.$$

The active mass $m(\chi,\tau)$, which is the equivalent of the total energy of the liquid sphere (7), is changed by the work done by the pressure forces $P \neq 0$ on its boundary sphere, so that in the Tolman-Bondi models[1–4,12] the distribution $m(\chi)$ of the active mass for dust with $P = 0$ is [like (11)] a motion integral as well as being the conserved intrinsic rest mass $\mathscr{M}_0(\chi)$ and the specific relativistic energy[12,13]

$$W(\chi) = m'/\mathscr{M}_0' = (dm/d\mathscr{M}_0).$$

At the boundary $\chi = \chi_0$ between the sphere and vacuum, where we must have $P(\chi_0,\tau) = 0$, the Lemaitre mass function is constant, and, because the metric (6) of the interior region of V_4 is a continuation of the external Schwarzschild field, its limiting value will coincide with the total energy integral of the mass of the sphere[12] $E = Mc^2$:

$$M = m(\chi_0, \tau) = \text{const}. \qquad (17)$$

2. As has been pointed out by Novikov,[3] in the case of spherically symmetric fields V_4 of general form (2), there exist singular essentially nonstatic T-regions which are not "contained" in R-frames of the type (4), since the angular metric coefficient $e^{\mu/2} = R(x_0,x_1)$ is time-like

$$e^{-\sigma}\dot{R}^2 = v^2 > W^2 = e^{-\omega}R'^2$$

and cannot be used as a spatial radial coordinate. However, in T-regions of V_4 this radius of curvature of the spheres S_2 ($x_0 = \text{const}, x_1 = \text{const}$) may be used as a time-like monotonic coordinate $T = R(x_0,x_1)$ and can realize a canonical T-system with metric[2]

$$-ds^2 = -e^{\sigma(\chi,T)}dT^2 + e^{\omega(T,\chi)}d\chi^2 + T^2[d\vartheta^2 + \sin^2\vartheta d\varphi^2]. \quad (18)$$

A feature of this metric is that the spatial cross sections $T = \text{const}$ are infinite $(-\infty < \chi < \infty)$ hypercylinders $V_3 = (S_2 \times R_1)$ without a classical center $r = 0$.

An invariant partitioning of V_4 into R- and T-regions may be carried out with respect to the orientation of the hyperplanes $e^{\mu/2} = R(x_0,x_1) = \text{const}$, which in the T-regions must be space-like with a normal vector inside the local light cones[3,4] $ds^2 = 0$:

$$n_i n^i = \Delta_1 R = e^{-\sigma}\dot{R}^2 - e^{-\omega}R'^2 = v^2 - W^2 \geqslant 0, \qquad (19)$$

whereas in the R-region they are time-like with $n_i n^i = \Delta_1 R = v^2 - W^2 < 0$, and at the boundaries between the T- and R-regions they are zero-isotropic, i.e., $n_i n^i = \Delta_1 R = 0$; $|v| = |W|$. From the definition of an invariant mass function (3), (10), it is obvious that the T-regions of V_4 with $v^2 \geqslant W^2$ are inside the current gravitational radius of distribution of matter at $\Lambda = 0$:

$$R(x_0, x_1) \leqslant (\varkappa/4\pi)m(x_0, x_1). \qquad (20)$$

Their boundaries $v^2 = W^2$, the zero-horizons of Schwarzschild-sphere-type events (5), become semi-permeable causal membranes through which light rays and particles may penetrate in only one direction, from an R- to a T-region or the other way round. Using the equation of propagation of radial light rays $ds^2 = 0$ in the form

$$dx_1/dx_0 = e^{(\sigma-\omega)/2},$$

it can be easily proved that the radius of curvature of the spheres $R = R(x_0,x_1)$ varies along both rays as

$$e^{-\sigma/2}(dR/dx_0) = D_\tau R = v \pm W. \qquad (20')$$

Consequently, for both the outgoing and the incoming rays, in T-regions with $|v| \geqslant |W|$ the radius is always monotonically decreasing of $v < 0$ and is increasing if $v > 0$. Hence, there are two physically distinct types of isometric T-regions with distinct initial conditions under which the regions are produced by the time direction[2–4]: (a) a contracting $T^{(-)}$-region with $v < 0$ in which all causal influences propagate towards the time singularity $R = T = 0$, as in the case of the phase of gravitational collapse of a massive body into a black hole; (b) expanding $T^{(+)}$-region with $v > 0$ in which all light rays and particles travel exclusively outwards from the initial singularity $R = T = 0$, as in the expansion of the metagalaxy, of from hypothetical white holes from under the Schwarzschild sphere in nonuniform cosmology.[14]

Note that the sign of the "velocity" $v = e^{-\sigma/2}\dot{R}$ must be preserved in T-regions of V_4, and that regular "reflection" is impossible, since the turning point $v = 0$ must be in the R-region, while the "throat" with $W = e^{-\omega/2}R' = 0$ is located only in the T-region for arbitrary orthogonal frames (2), including the system (6) comoving with the matter with $T_0^1 = 0$ (cf. Ref. 6).

The Einstein equations assume a very simple form[3] in the canonical T-system (18):

$$e^{-\sigma}(\omega_T/T + 1/T^2) + 1/T^2 = \varkappa T_0^{\,0} + \Lambda,$$
$$e^{-\sigma}(\sigma_T/T - 1/T^2) - 1/T^2 = -\varkappa T_1^{\,1} - \Lambda, \qquad (21)$$
$$e^{-\sigma}\frac{\sigma_\chi}{T} = -\varkappa T_0^{\,1}, \quad \omega_T = \left(\frac{\partial\omega}{\partial T}\right)_\chi, \quad \sigma_\chi = \left(\frac{\partial\sigma}{\partial\chi}\right)_T.$$

By (3), the general relations for T-regions of V_4 (18) that follow from (21) resemble the analogous equations in the quasi-

Newtonian R-system (4) and may be easily obtained by the formally substitution $r \to T$: (cf. Refs. 2–4 and 14):

$$e^{-\sigma}=\varkappa m/4\pi T-1+\Lambda T^2/3,$$
$$m_T=4\pi T^2 T_1^{\ 1}, \qquad m_\chi=4\pi T^2 T_1^{\ 0}, \tag{22}$$
$$\omega_T=-\sigma_T+\varkappa(T_0^{\ 0}-T_1^{\ 1})Te^{-\sigma}=-(\varkappa e^\sigma/4\pi T^2)(m-4\pi T^3 T_0^{\ 0}).$$

The well-known metrics of Schwarzschild[2,4,15a] and deSitter-Kottler (SDK) at ($\Lambda \neq 0$) which have according to (21) the form[3]

$$-ds^2=-U^{-1}(T)dT^2+U(T)d\chi^2+T^2[d\vartheta^2+\sin^2\vartheta d\varphi^2],$$
$$U(T)=(2M/T+\Lambda T^2/3-1)\geqslant 0, \qquad M=\text{const}, \tag{23}$$

as well as the exact solution of the Einstein-Maxwell equations assuming a free electromagnetic field $\mathbf{E}(T)\|\mathbf{H}(T)\|\chi$, the Reisner-Nordstrem (RN) metric[15b] of the form (23) with

$$U(T)=(2M/T-q^2/T^2-1)\geqslant 0,$$
$$q^2=e^2+\mu^2, \quad e,\ \mu=\text{const}$$

and with energy-momentum tensor

$$\mathscr{E}_0^{\ 0}=\mathscr{E}_1^{\ 1}=-\mathscr{E}_2^{\ 2}=-\mathscr{E}_3^{\ 3}=w=\frac{1}{8\pi}(E^2+H^2);$$
$$\varkappa w=\frac{q^2}{T^4}, \quad q=\text{const} \tag{24}$$

(which in the R-region is usually considered as the external field of a charged massive sphere), yield typical examples of uniform Novikov T-regions. Because of the enhanced mobility of $G_4\mathrm{VIII}=(G_3\mathrm{IX}\times G_1)$ on V_3 ($T=$ const), by virtue of the Birkhoff theorem[9b] for these spherically symmetric metrics (23) and (24), the boundaries between the R- and T-regions of V_4 corresponding to the removable pseudo-singularities[15] $U(T_i^*)=0$ are Killing zero-horizons on which the type of symmetry of V_4 changes. That is, the spatial homogeneity in the T-region is replaced by the static character of the R-region, where $\xi_i\xi^i=U(T)<0$, though in the general case both are merely regular zero-hypersurfaces with $n_i n^i=\Delta_i R=0$ and constitute Cauchy-event horizons.

PROPERTIES AND DYNAMICS OF T-MODELS OF THE SPHERE

1. By definition, the above singular spherically symmetric distributions of material sources for which the comoving frame (6) is a T-system with $R^2=r^2(\tau)=T^2$ and a canonical metric [in fact, a uniform metric by (21) if $P\neq 0$] of the form (18) correspond to a special case of T-regions in V_4 with matter at rest, in which there is no radial flow of energy-momentum ($T_0^1=0$) along the hypercylinder $V_3=(S_2\times R_1)$.

One typical feature of T-models of the sphere is that according to (7) and (22) their active mass $m(T)$ is not related to the distribution of sources of matter (and if $P=0$, is even independent of these sources), though the spatial cross-sections in the comoving T-system (18)—the hypercylinders $V_3=(S_2\times R_1)$—contain an unlimited amount of matter with an infinite total mass-energy (12). The latter, however, is entirely gravitationally bound in the initial T-regions of the Schwarzschild-Kruskal field,[2–4] since the negative potential binding energy for each Lagrangian of the spherical layer $W\equiv 0$ (11) cancels exactly its proper mass. In addition, this infinite gravitational mass defect is closely related to the essentially non-Euclidean hypercylindrical geometry of the comoving space V_3 ($T=$ const) in the T-system (18). Though the active mass of the T-models of the sphere does not, in general, contain any mass contribution and must be of field origin, its time variation is caused according to (7) by the work of the pressure forces:

$$m_T=4\pi T^2 T_1^{\ 1}=-4\pi T^2 P(T) \tag{25}$$

(as in the case of Friedmann's isotropic models (16) at $P\neq 0$).

T-models of the sphere do not, in general, have any Newtonial equivalent, and their very existence and unusual properties are due to the specific nonlinearity of general relativity theory, namely: (a) to the existence of Cauchy-Killing zero-horizons (CKH) such as the Schwarzschild sphere and vacuum T-regions (22) and (23), variable maximally strong fields with a "longitudinal" nonwave structure in which matter may be totally bound and is always confined; (b) to the gravitational mass defect[2–4,12] which ensures exact cancellation of the contribution of the matter to the active mass $m(T)$, the equivalent of their total energy, and constancy of this energy under unrestricted accretion of the spherical layers.[1]

Such singular solutions of the Einstein equations (21) in the comoving T-system (6), (18) with $T_0^1=0$ and accordingly $\sigma=0$ and $\sigma=\sigma(T)$, are possible for different types of sources of matter[1,5–8] T_k^i, including an ideal liquid with a specified equation of state $P=P(\varepsilon)$. Further, if $P\neq 0$, the T-models must be homogeneous:

$$\sigma=\sigma(T), \quad \omega=\omega(T), \quad \varepsilon=\varepsilon(T), \quad P=P(T).$$

2. The simplest example, previously investigated in Ref. 1b, of an inhomogeneous T-distributions of dust with $P=0$ yields an analytic solution for the metric (6) at $R=r(\tau)$ and $\sigma=0$ in a synchronously comoving system. This solution supplements the class of Tolman-Bondi models[1–4,13] and assumes the following form in a canonical T-system (18):

$$e^{\sigma(T)}=\left(\frac{2M}{T}-1\right)^{-1}, \; e^{\omega(T,\chi)}=\left(\frac{2M}{T}-1\right)Z^2(T,\chi),$$
$$\rho(T,\chi)=\frac{\mathscr{M}_0'(\chi)}{4\pi T^2 e^{\omega/2}}\geqslant 0, \tag{26}$$
$$Z(T,\chi)=\epsilon+\mathscr{M}_0'(\chi)[(2M/T-1)^{-1/2}-\arcsin(T/2M)^{1/2}],$$
$$\epsilon=0,\ \pm 1.$$

These T-models of a sphere possess a constant active mass $M>0$ for an arbitrary unlimited amount of contained dust in $V_3=(S_2\times R_1)$ with an arbitrary distribution of the proper rest mass $\mathscr{M}_0(\chi)$ whose contribution is exactly offset by the gravitational binding energy ($W\equiv 0$) in the initial T-regions of the Schwarzschild-Kruskal field ($\rho=0$, $\mathscr{M}_0=0$). Since the rest mass is fully neutralized, such passive matter with $W\equiv 0$ is simply inscribed in the vacuum T-regions, preserves their local properties, and replaces, as it were, the trial particles of the synchronous T-frame (6) with $R=r(\tau)$. However, the presence of dust radically alters the global causal structure of V_4 and, in particular, transforms the Schwarzschild zero-sphere $T^*=2M$ into a regular space-like boundary of V_4 for (26), where $e^{\omega/2}\approx 2\mathscr{M}_0'\neq 0$.

It can be shown that such limiting Cauchy-Killing zero-horizons $\xi_i \xi^i = U(T_i^*) = 0$ between causally bound vacuum uniform T- and static Schwarzschild-Kruskal R-regions, which are membranes $T^* = 2M$ semipermeable to light rays and particles, become an absolutely impervious barrier to causal effects in T-models of the sphere (26). This is clear from an analysis of the basic equation for the geodesics of the T-metric (26):

$$\left(\frac{dT}{d\lambda}\right)^2 = \frac{w^2}{Z^2} + U(T)\left(\epsilon + \frac{l^2}{T^2}\right), \qquad U(T) = \left(\frac{2M}{T} - 1\right) > 0, \tag{27}$$

an equation obtained using the integrals of the angular momentum l and of the radial momentum $w(\chi)$ of the particles ($\epsilon = +1$) and light rays ($\epsilon = 0$),

$$\vartheta = \frac{\pi}{2}, \quad l = T^2 \frac{d\varphi}{d\lambda}; \qquad \frac{w(\chi)}{w(\chi_0)} = \frac{\mathscr{M}_0'(\chi)}{\mathscr{M}_0'(\chi_0)} = \text{const}, \tag{28}$$

together with the normalization condition (18):

$$2L = g_{ik} \frac{dx^i}{d\lambda} \frac{dx^k}{d\lambda} = \epsilon = 0, \quad \pm 1. \tag{29}$$

The behavior of the geodesics for the T-models (26) differs from the vacuum case[15a] only by the factor $Z(T,\chi) \neq 1$, which diverges at the boundaries of the T-regions when $U(T_i^*) = 0$:

$$Z(T, \chi) \approx 2\mathscr{M}_0'(2M/T - 1)^{-1/2} \to \infty.$$

Along the geodesics that extend to the boundaries of the T-models, $T(\lambda) \to T^* = 2M$, the radial momentum is finite, whereas $Z(T,\chi) \to \infty$, which as a result leads to a halt $[-(dT/d\lambda) = 0]$ of all the particle and light ray paths on this limiting barrier $T^* = 2M$ and to their reflection in the other T-region, which is now absolutely impermeable to causal effects. Thus, the pair of locally equivalent $T^{(+)}$- and $T^{(-)}$-regions of the Schwarzschild-Kruskal vacuum metric[2-4,15a] forms a cosmological T-model (26) when they are filled with gravitationally totally bound matter with $W \equiv 0$; this T-model is geodesically complete and does not contain any external R-regions (see Fig. 1a).

Cosmological T-models of a sphere can be interpreted analogously in the case[1a] $\Lambda \neq 0$ and when there is a free electromagnetic field present.[16,7] These T-models may be constructed on the basis of the corresponding T-regions of the Schwarzschild-deSitter-Kottler (23) and Reisner-Nordstrem (24) metrics, which they generalize. They may also be considered as a singular branch, with $W = 0$, $R = r(\tau)$, of the general solution (6) of the Einstein-Maxwell equations ($\Lambda \neq 0$) for the spherically symmetric problem of collapse of neutral dust ($P = 0$) in the field of a charged central mass[16] in a synchronous comoving T-system (cf. Ref. 1a):

$$-ds^2 = -d\tau^2 + e^{\omega(\tau,\chi)} d\chi^2 + r^2(\tau)\,[d\vartheta^2 + \sin^2\vartheta\, d\varphi^2],$$

$$\dot{r}^2 = \frac{2M}{r} - \frac{q^2}{r^2} + \frac{\Lambda r^2}{3} - 1,$$

$$\rho = \frac{\mathscr{M}_0'(\chi)}{4\pi r^2 e^{\omega/2}} \geqslant 0, \quad \varkappa w = \frac{q^2}{r^4}; \quad q = \text{const},$$

$$e^{\omega/2} = \dot{r}\left\{\epsilon + \mathscr{M}_0' \int^{r(\tau)} \frac{du}{u}\left(\frac{2M}{u} - \frac{q^2}{u^2} + \frac{\Lambda u^2}{3} - 1\right)^{-3/2}\right\}, \tag{30}$$

$$\epsilon = 0, \pm 1.$$

This constitutes an anisotropic axisymmetric cosmological quasi-closed model[6] that contains, besides dust with $W \equiv 0$, an additional uniform variable electromagnetic field $[\mathbf{E}(\tau) \| \mathbf{H}(\tau)]$. Its lines of force are directed along the generatrices of the hypercylinder $V_3 = (S_2 \times R_1)$ and are, as it were, "frozen" as a consequence of the law of conservation of "chargeless charges" ($e = \text{const}, \mu = \text{const}$) for the initial T-regions (24).

3. Since V_4 is closed and causally complete in cosmological infinitely extended ($-\infty < \chi < \infty$) T-models of a sphere, the concept of a total-energy integral for these T-models is without physical meaning, as in the case of Friedmann's topologically closed model.[2,12] But by analogy with the "semi-closed" models, [3,4] we may consider bounded T-spheres[1b] surrounded by an external empty region with a Schwarzschild field. The total energy-mass integral $E = Mc^2 > 0$ of such singular T-distributions of matter may be properly determined by means of a regular matching of the intrinsic T-metric (26) in the bounded region ($-\infty < \chi \leqslant \chi_0 = \text{const}$) and in the Schwarzschild-Kruskal vacuum field,[2-4,15a] while satisfying the boundary conditions that the invariant characteristics of the geometry of the complete V_4 must be continuous (this is obviously possible for $P = 0$) through the vacuum T-region, and emerge to the asymptotically planar R-region (see Fig. 1b). The bounded T-sphere may contain an arbitrary infinite quantity of gravitationally totally bound matter with $W \equiv 0$, always enclosed under its Schwarzschild sphere, though from the standpoint of gravitation it appears as an object with finite mass-energy. This mass-energy is a purely field-like geometrodynamic massless mass of the vacuum Schwarzschild-Kruskal T-regions and is always positive.[1)]

Consequently, T-spheres provide a method of realizing the maximally possible (in general relativity theory) gravitational mass defect, which is precisely equal to the infinite proper mass of matter. The method is different in principle from Friedmann's closed model.

The boundary of the T-sphere $\chi = \chi_0 = \text{const}$ consists of particles of dust ($P = 0$) that travel along the radial geodesics (27)–(29) in an external Schwarzschild field with $Z = 1$ like a trial "reference liquid" of the T-system (26). Therefore, even using the Kruskal diagram (see Fig. 1, where the limiting sphere $\chi = 0$ is depicted by a segment of the time axis $u = 0$), it is easy to elucidate the theoretical possibility of unilateral exchange of information between its internal T-regions and the external R-region of V_4. From the standpoint of an observer in the R-region, the pattern of evolution of the T-sphere does not differ from the case of a gray hole—which is a semi-closed world with equatorial sphere $\chi_0 = \pi/2$ as its boundary,[3,4] and is characterized by an unusual asymmetry between the unobservable stage of collapse $T^{(-)}$ and the visible anti-collapse phase $T^{(+)}$ in which the expansion of

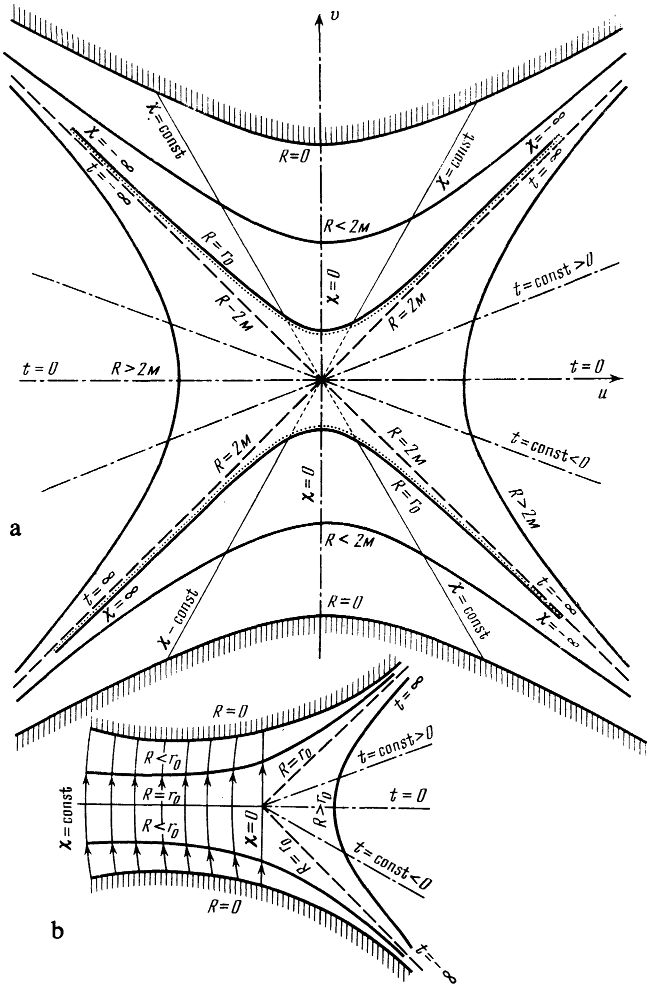

Fig. 1 (a) Kruskal diagram for uniform T-models of the sphere (26) at $\mathscr{M}' > 1/\pi$, and $\epsilon = +1$ in isothermal coordinates of the form[15]

$$u\pm v=\exp\left\{\frac{\mathcal{T}\pm\chi}{2r_0}\right\},\qquad \mathcal{T}=\mathcal{T}(T),$$

$$ds^2=F^2(T)[dv^2-du^2]-R^2(u,v)\,d\Omega^2,$$

$$F^2=4r_0^2e^{\omega(T)}\exp\{\mathcal{T}(T)/2r_0\},\qquad v^2-u^2=\exp\{-\mathcal{T}(T)/r_0\},$$

$$\mathcal{T}(T)=\int_0^T(r_0/T-1)^{-1}Z^{-1}(T)\,dT,\qquad r_0=2M=\text{const}.$$

The world lines $\chi = 2r_0\,\operatorname{artanh}(v/u) = \text{const}$ of the dust particles are straight lines, and the diametrically opposite ends at the spatial absolutely reflecting boundary $R(u,v) = r_0 = 2M$ are the same. (b) Global structure of causally complete V_4 for a bounded $(-\infty < \chi \leqslant \chi_0 = 0)$ T-sphere at $P = 0$ with an external Schwarzschild vacuum field.

$V_3 = (S_2 \times R_1)$ begins from the "central" singularity $R = T = 0$, and concludes with asymptotic congealing on the Schwarzschild sphere $R = r(\tau) = 2M$. The anti-collapse pattern of the T-sphere at first resembles parabolic expansion of a glowing spherical shell from a central point,[17] which then decelerates and approaches asymptotically (relative to the time of an external observer in the R-region, i.e., $t \to \infty$) the gravitational radius $R(T) \to 2M$ with a typical pattern of congealing of the process on the Schwarzschild sphere (as in the formation of a black hole),[2,4] while its collapse stage is not amenable at all to observation.[1c,8]

The existence of T-spheres in general relativity theory leads to an important physical conclusion: Within the Schwarzschild sphere (which may arise, for example, in the collapse of massive bodies) it is possible, at least in principle, for the entire proper rest mass of an arbitrary amount of matter to become bound once its energy equivalent has been released in the form of radiation, though it is still unclear what might be the mechanism by which such an ideal gravitational "engine" operates (cf. Ref. 4).

As we have shown, T-spheres have unexpected applications in relativistic astrophysics, for example, in the discussion of the quantum effects of generation of particle pairs (explosion of a vacuum in primordial white holes[4,8]). White holes were first considered in a classical treatment[14] as nuclei against the Friedmann background that had lagged behind in cosmological expansion due to an initial stage of strong inhomogeneity of a singular state of the universe in which super-dense matter may escape from under the Schwarzschild sphere in an anti-collapse stage at any moment of time, for example, at once for an external observer, i.e., as in the time-dependent transformation of the pattern of collapse in the formation of black holes.[2,4] But in a neighborhood of the Schwarzschild central singularity $R = T = 0$ with $M > 0$, corresponding to the anisotropic vacuum state of the initial expansion of the lagging nucleus with an asymptotically uniform Kasner-Taub metric[2] of the form (6) ($\sigma = 0$, $R \propto \tau^{2/3} \to 0$, $e^{\omega/2} \propto \tau^{-1/3} \to \infty$), the quantum vacuum is unstable relative to spontaneous generation of particle pairs of this strong variable gravitational field if $\tau \lesssim \tau_P \sim (G\hbar/c^5)^{1/2} \sim 10^{-43}$ sec.[4] Therefore, the T-region must be filled with super-dense hot matter consisting of created and rapidly thermalizing (except, probably, the gravitons) particles with $\varepsilon_P \sim 1/\varkappa\tau_P^2 \sim 10^{93}$ g/cm^3 (Ref. 8). The product matter is initially at rest relative to the T-system[18] (where the Schwarzschild singularity $R = T = 0$ is simultaneous and the initial metric uniform) and is simply a bounded T-distribution with vacuum surrounded by the external isotropic background of the Friedmann model. Consequently, the particles of product matter in the initial T-region are gravitationally totally bound ($W \equiv 0$) and always contained within their own Schwarzschild sphere. These particles do not contribute at all to the total mass-energy $m(T)$ (which is due to the initial Schwarzschild singularity), but influence markedly the dynamics and properties of a white hole.[1c,8]

Thus, as a result of intense quantum production of particle pairs near the Schwarzschild anisotropic singularity $R = T = 0$, the classical white hole will turn into a bounded T-sphere and may contain an arbitrary proper mass of generated matter totally bound gravitationally to the initial T-region in V_4. To a first approximation, the intrinsic T-model may be regarded as uniform in the form (18) with $\sigma = \sigma(T)$ and $\omega = \omega(T)$, ignoring outflow (and accretion) of matter: $T_l^0 \neq 0$ due to the discontinuity of the pressure $P \neq 0$ at the boundary $\chi = \chi_0$ with the outer Friedmann (or even empty) region, since even at the speed of light a shock wave can travel within finite proper time $\Delta\tau \sim R_g/c \sim \varkappa M/c$ of expansion of the T-sphere only a finite radial distance along the hypercylinder $V_3 = (S_2 \times R_1)$ to its gravitational radius $\Delta\chi \sim \Delta\tau \approx (\varkappa/4\pi) m_{\max}$; in addition, the shock wave will draw in only a small fraction of the matter in the T-sphere.[8]

4. For uniform T-models of the form

$$-ds^2 = -e^{\sigma(T)} dT^2 + e^{\omega(T)} d\chi^2 + T^2 [d\vartheta^2 + \sin^2\vartheta d\varphi^2], \qquad (18')$$

filled with an ideal liquid with equations of state $P = n\varepsilon$ ($0 \leqslant n \leqslant 1$), the Einstein equations (21) and (22) admit of a number of integrals[1c]

$$e^{-\sigma(T)} = \frac{\varkappa m(T)}{4\pi T} - 1, \quad P = -\frac{m_T}{4\pi T^2} > 0,$$
$$e^{\omega} = \frac{A}{T^4 P^{2/(1+n)}}, \quad A = \text{const} > 0. \qquad (31)$$

Due to the work of pressure forces $P \neq 0$, the active mass $m(T)$ of the T-models and, correspondingly, the gravitational radius $F(T) = (\varkappa/4\pi) m$, must vary, so that by using (31) the problem can be reduced to that of solving a single *nonlinear equation of the form*

$$(F-T) F_{TT} + \frac{(3n-1)}{2} \frac{F F_T}{T} - \frac{(1+n)}{2n} F_T^2 + 2n F_T = 0. \qquad (32)$$

In the case $P = \varepsilon/3$, it is an easy matter to indicate its particular solution with

$$F_{TT} = 0, \quad F = b - {}^1/_3 T > 0, \quad e^{-\sigma} = \dot r^2 = b/r - {}^4/_3,$$
$$T = r(\tau), \quad b = 2M = \text{const} > 0,$$

namely:

$$-ds^2 = -\left(\frac{b}{T} - \frac{4}{3}\right)^{-1} dT^2 + \frac{B^2}{T} d\chi^2 + T^2 [d\vartheta^2 + \sin^2\vartheta d\varphi^2]. \qquad (33)$$

In this special T-model of the sphere, overall expansion of the hypercylinder $V_3 = (S_2 \times R_1)$ begins and ends (after passing through the regular extremum: $r_{\max} = T_{\max} = F_{\min} = (3/2)M$, $(e^{\omega/2})_{\min} \neq 0$) with Kasner linear singularities:

$$T = r(\tau) \propto \tau^{2/3} \to 0, \quad e^{\omega/2} \propto \tau^{-1/3} \to \infty,$$
$$F(\tau) \to b = 2M = \text{const},$$

just as in the case of the T-regions of the Schwarzschild-Kruskal field.

As before, the T-models of a sphere exhibit a considerable degree of similarity with the initial T-regions, though the existence of gravitationally totally bound liquid with $W \equiv 0$ not only radically alters the global causal structure of V_4 (eliminating the vacuum zero Cauchy-Killing horizons $U(T_l^*) = 0$ such as of the Schwarzschild sphere $T_l^* = 2M$, as well as external R-regions), but also markedly influences the local properties of a white hole in view of the variability of the active mass $m(T)$ if $P \neq 0$.

The characteristic features of the dynamics of T-models of a sphere, (31) and (32) for $P < \varepsilon$, are reflected by an exact solution more general than (33) with $P = \varepsilon/3$ when $F_{TT} \neq 0$, in the Shikin parametric form[18,7] with $\xi = -F_T > 0$:

$$\frac{F(\xi)}{b} = \alpha + \frac{1}{\xi} + \frac{\xi}{3} > 0, \quad \frac{T(\xi)}{b} = \alpha + \frac{\xi}{3} - \frac{\xi^3}{3} \geqslant 0,$$
$$T(\xi_{1,2}) = 0, \quad e^{\omega} = \frac{B^2 |\xi|^3}{T(\xi)}, \qquad (34)$$
$$\varkappa P = \frac{\varkappa\varepsilon}{3} = \frac{1}{\xi^2 T^2(\xi)}, \quad -\frac{2}{\sqrt{3}} \leqslant \alpha < \infty, \quad B = \text{const}.$$

They must have two singularities, initial and final ($\varepsilon, P \to \infty$), with one of them ($\alpha > 0$) or even both together ($-2/\sqrt{3} < \alpha < 0$) being vacuum singularities of the Schwarzschild field $R = T = 0$, and with a form independent of the presence of matter. The two singularities correspond to anisotropic Kasner collapse of $V_3 = (S_2 \times R_1)$ into a line

$$T = r(\tau) \propto \tau^{2/3} \to 0, \quad e^{\omega/2} \infty \tau^{-1/3} \to \infty,$$
$$\varepsilon \propto V^{-(1+n)} \infty \tau^{-(1+n)} \to \infty, \qquad (35)$$

with the active mass remaining finite:

$$m(T) = m(\xi_{1,2}) = M_{1,2} = \text{const}, \quad M_1 \neq M_2. \qquad (36)$$

The second possible type of singularity in the *T-models* ($\alpha > 0$) is described by the kinematic Kasner asymptote of the collapse of V_3 into the disk S_2:

$$e^{\omega/2} \propto (\tau - \tau^*) \to 0, \quad r(\tau^*) \to T^* = \text{const} \neq 0,$$
$$\varepsilon \propto V^{-(1+n)} \propto (\tau - \tau^*)^{-(1+n)} \to \infty. \qquad (37)$$

The latter apparently replaces the vacuum Schwarzschild zero sphere $T^* = 2M$, which in T-models of a sphere turns into a physical singularity ($\varepsilon, P \to \infty$) without altering its zero-orientation if $P < \varepsilon$. There is therefore no causal radial horizon[18] in this type of singularity (unlike the Schwarzschild linear singularity), and the light rays traverse an infinite distance along the generatrices of $V_3 = (S_2 \times R_1)$ as $\tau \to \tau^*$, $r(\tau^*) \to T^*$. In such a longitudinal contraction of the hypercylinder V_3 into a spherical δ-layer S_2, the active mass becomes infinite:

$$m(\tau) \propto V^{-(1+n)} \propto (\tau - \tau^*)^{-(1+n)} \to \infty \tag{38}$$

as a consequence of the effect of the pressure forces $P(\tau) \propto (\tau - \tau^*)^{-(1+n)} \to \infty$ on a Lagrangian sphere of finite radius $r(\tau^*) = T^* = \text{const}$.

In the special case ($\alpha = 0$), the singularity is caused exclusively by the presence of gravitating matter and corresponds to Friedmann quasi-isotropic collapse of V_3 into a point:

$$e^{\omega/2} \propto r(\tau) = T \propto \tau^{2/[3(1+n)]} \to 0,$$

$$\varkappa\varepsilon \approx \frac{4}{3(1+n)^2\tau^2} \to \infty, \tag{39}$$

when $m(\tau) \propto V^{-n} \propto T^{-3n} \propto \tau^{-2/(1+n)} \to \infty$ because the pressure increase is more rapid ($P \propto T^{-3(1+n)} \propto \tau^{-2} \to \infty$), than the decrease in the area of the "liquid" sphere $S = 4\pi T^2 \to 0$.

In Kasner collapse of $V_3 = (S_2 \times R_1)$ into the line (35), conversely, the pressure grows more slowly:

$$P(T) \propto T^{-3(1+n)/2} \propto \tau^{-(1+n)} \to \infty,$$

and as $S = 4\pi T^2 \to 0$, the active mass therefore remains finite: $m(T) \to M = \text{const}$. Since such a linear singularity of the T-models of a sphere is vacuum-like and is independent of the presence of matter with $P < \varepsilon$, the initial value $M > 0$ must be thought of as the bare massless mass of the initial T-regions of the Schwarzschild field in a white hole.

Uniform T-models of the sphere, which are defined as the solutions of the Einstein equations (21) and (22) relative to the T-metric (18) with $T_0^1 = 0$, may also be constructed using a more realistic description of product matter in a white hole, as in anisotropic cosmology.[4,19] Here matter is treated as a mixture of an ideal liquid with $P = n\varepsilon$ ($0 \leqslant n \leqslant 1$) and collision-free nonthermalizing radiation (gravitons, neutrinos) in the form of mutually canceling counter flows of ultrarelativistic particles along the generatrices of the hypercylinder $V_3 = (S_2 \times R_1)$ with an energy-momentum tensor in the uniform T-system (18′) of the form[1c,8]

$$T_0{}^0 = -T_1{}^1 = \Pi, \quad T_2{}^2 = T_3{}^3 = 0,$$

$$\varkappa\Pi = \mathcal{F}/T^2 e^{\omega/2} \geqslant 0, \quad \mathcal{F} = \text{const}. \tag{40}$$

The time variation of the active mass of the T-models of a sphere is now caused by the combined work of the isotropic pressure forces $P \neq 0$ of the liquid and the anisotropic tension of the flows of ultra-relativistic particles to the Lagrangian spheres $\chi = \text{const}$ in accordance with (21) and (22):

$$m_T = -4\pi T^2 (P + \Pi), \tag{41}$$

so that in the course of the general expansion of $V_3 = (S_2 \times R_1)$, the latter must always decrease. The initial active bare mass is therefore a maximum in the Schwarzschild linear singularity (35) (and for the point (39) and disk (37) singularities $m(T)$ is even infinite), and its minimum coincides with the maximum transverse expansion of the hypercylinder V_3:

$$F(T) = T = (\varkappa/4\pi)\, m(T).$$

A detailed analysis of the dynamics of uniform T-models of a sphere (18′) filled with liquid with different equations of state $P = n\varepsilon$ ($0 \leqslant n \leqslant 1$) and different flows of ultra-relativistic particles (40) may be found in Ref. 1c.

Thus, if white holes did actually exist as lagging nuclei in the singular initial state of the universe,[14] then, because of the instability of the quantum vacuum relative to production of particle pairs,[4,8,20] they turned into bounded T-spheres and became gray holes. But their pattern of evolution cannot be fully explained if outflow of matter ($P \neq 0$) or possibly its accretion in the "hot" version is taken into account, and requires further study.[4,8]

APPENDIX

UNIFORM SCALAR-TENSOR GENERALIZATIONS OF SCHWARZSCHILD AND REISNER-NORDSTREM *T*-METRICS

1. For the special case of a liquid with a maximally rigid equation of state $P = \varepsilon$, the T-solutions of general relativity theory may be obtained in the simple analytic form[1c,9]

$$-ds^2 = -d\tau^2 + e^{\omega(\tau)} d\chi^2 + r^2(\tau)\,[d\vartheta^2 + \sin^2\vartheta\, d\varphi^2],$$

$$e^{\omega/2} = X_0 (\mathrm{tg}\,(u/2))^{\pm(1-\nu^2)^{1/2}},$$

$$r = r_0 (\sin\,(u/2))^{1 \mp (1-\nu^2)^{1/2}} (\cos u/2))^{1 \pm (1-\nu^2)^{1/2}},$$

$$\varkappa\varepsilon = \varkappa P = \nu^2/e^{\omega} r^4, \quad \tau = \int_0^u r(u)\,du, \tag{A1}$$

$$0 \leqslant \nu^2 \leqslant 1, \quad r_0 = 2M, \quad X_0 = \text{const}.$$

These T-models of a sphere are distinguished by a number of specific behavioral features, since the maximally rigid liquid with $P = \varepsilon$ alters qualitatively the expansion dynamics of the hypercylinder $V_3 = (S_2 \times R_1)$ compared with vacuum T-regions of the Schwarzschild field ($\nu^2 = 0$, $\varepsilon = 0$). Its effect is always substantial even at the two central singularities $r(\tau) = 0$, one of which becomes an anisotropic point collapse, and the other a linear collapse of $V_3 = (S_2 \times R_1)$ with modified Kasner asymptote:

$$r(\tau) \propto \tau^{\tilde{p}} \to 0, \quad e^{\omega/2} \propto \tau^{1-2\tilde{p}},$$

$$\varkappa\varepsilon = \varkappa P \approx \mu^2/\tau^2 \to \infty,$$

$$\tilde{p} = \frac{1 \pm (1-\nu^2)^{1/2}}{2 \pm (1-\nu^2)^{1/2}} = \frac{1}{3}[1 + (1-3\mu^2)^{1/2}], \quad \mu^2 = \text{const} \leqslant 1/3. \tag{A2}$$

Here the active mass is $m(\tau) \propto \tau^{-2+3\tilde{p}} \to \infty, \tilde{p} < 2/3$. If a liquid with $P = \varepsilon (\mu^2 > 1/4)$ is dominant, only point collapse of V_3, approaching the Friedmann quasi-isotropic regime, is possible if $\mu^2 \to 1/3$, $\nu^2 \to 3/4$, $\tilde{p} \to 1/3$.[9] Note the degenerate case $\nu^2 = 1$ in which the hypercylinder V_3 is static in the longitudinal direction ($e^{\omega/2} = X_0 = \text{const}$) and pulsates only in the transverse direction, whereas the two central singularities $r(\tau)=0$ correspond to the collapse of V_3 into a "keg": $\tilde{p}_1=0, \tilde{p}_2=\tilde{p}_3=\tilde{p}=1/2, \mu^2=1/4$.

Whether the Zel'dovich limiting asymptote $P=\varepsilon$ is realistic for the description of super-dense product matter in white holes is quite open to question, as in cosmology (see the bibliography in Ref. 19). But a maximally rigid liquid with $P=\varepsilon$ simulates a massless free scalar canonical-type field that satisfies the d'Alemberg equation $\Box\Psi=0$, when it assumes in general relativity theory the role of only a source of matter as, e.g., for the Dicke scalar-tensor theory of gravitation (see Ref. 21 for details). Therefore, the T-models of a sphere (A1) in general relativity theory may be considered as a modification of the Schwarzschild vacuum T-region in the Dicke scalar-tensor theory in the presence of a uniform mode of the free scalar field

$$\Box\Psi=0, \quad \Psi(u)=\Psi_0 \ln(\mathrm{tg}\,(u/2))^\sigma, \quad \sigma=\text{const},$$

which is equivalent to a maximally rigid liquid at rest with

$$P_s=\varepsilon_s=\frac{\zeta}{4}\left(\frac{d\Psi}{d\tau}\right)^2, \quad \zeta=\frac{(2\omega+3)}{\varkappa}=\text{const},$$

if we substitute $\nu^2=\sigma^2\varkappa\zeta/4$.

2. T-models of a sphere that contain a maximally rigid liquid with $P=\varepsilon$ admit of an analytic solution also where there is a uniform free Reisner-Nordstrem electromagnetic field (24) of the form[1c,9]

$$-ds^2=-r^4e^\omega d\eta^2+e^\omega d\chi^2+r^2(\eta)\,[d\vartheta^2+\sin^2\vartheta d\varphi^2], \tag{A3}$$

$$e^\omega=\frac{2\alpha}{[e^{-\alpha\eta}+2\alpha/q^2]^2}, \quad r=\frac{\gamma e^{-\omega/2}}{2\,\mathrm{ch}\,(\gamma/2)\,(\eta+\eta_0)},$$

$$\varkappa\varepsilon=\varkappa P=\frac{\beta^2}{4r^4e^\omega}, \quad \varkappa w=\frac{q^2}{r^4},$$

where $\gamma^2=\alpha^2+\beta^2$ and $\eta_0=\text{const}$ (cf. Ref. 5c). These T-models may be interpreted as solutions of the Einstein equations for gravitationally bound free uniform electromagnetic fields and a canonical scalar massless field as sources of matter: $\Box\Psi=0$, $\Psi(\eta)=\Psi_0+S\eta$ if we set $\beta^2=\varkappa\zeta S^2=(2\omega+3)S^2$. These vacuum analogs of the Schwarzschild-Reisner-Nordstrem T-metrics (A1) and (A3) are easily transformed in canonical representation of the Dicke scalar-tensor theory into the initial Jordan-Brans-Dicke form with variable gravitational constant $G\propto\varphi^{-1}(\tau)$, where $d\tilde{s}^2=\varphi^{-1}ds^2$ and $\varphi=e^\Psi$.[1c] They may also be easily obtained for the variant of the scalar-tensor theory with a conformal scalar field $\Box\Phi+1/6R\Phi=0$ as a source of matter in general relativity theory[22] by a dimensional transformation of canonical Schwarzschild-Reisner-Nordstrem T-metrics (A1) and (A3) to the form[23]

$$g_{ik}^*=(\mathrm{ch}\,\lambda\Psi)^{-2}g_{ik}, \quad \Phi=\lambda^{-1}\,\mathrm{th}\,\lambda\Psi,$$
$$F_{ik}^*=F_{ik}, \quad \lambda=(\varkappa\zeta/6)^{1/2}. \tag{A4}$$

Scalar-tensor generalizations of the Schwarzschild-Reisner-Nordstrem T-regions in the canonical representation (A1) and (A3) possess time singularities with asymptote (A2), where

$$\Psi(\tau)\approx 2b\ln\tau\to\pm\infty, \quad \mu^2=\varkappa\zeta b^2=(2\omega+3)\,b^2\leqslant 1/3,$$

and the electromagnetic field in (A3) precludes the possibility of linear collapse and there is left only point collapse of $V_3=(S_2\times R_1)$, when its influence becomes negligible:

$$\frac{w}{\varepsilon_s}\propto\tau^{2(1-\tilde{p})}\to 0, \quad \tilde{p}=\frac{\gamma-\alpha}{2\gamma-\alpha}<1/2,$$
$$\mu^2=\frac{\beta^2}{(2\gamma-\alpha)^2}\leqslant 1/3, \quad \alpha>0 \quad (q\neq 0).$$

For the conformal variant with $\Box\Phi+1/6R\Phi=0$ and the Jordan-Brans-Dicke scalar-tensor theory with variable $G\propto\varphi^{-1}(\tau)\propto\tau^{-2b}\to\infty$ or 0, owing to the singular nature of the gauge transformations of the metric similar to (A4)

$$-ds^{2*}=\Lambda(\tau)\,ds^2, \quad \Lambda\propto\exp\{-|\lambda\Psi|\}\propto\tau^{2\delta}\to 0,$$
$$\delta=|b\lambda|, \quad \Phi(\tau)\to\pm\lambda^{-1},$$

the asymptotic behavior of (A2) is altered and, in terms of the new proper time $\tilde{\tau}^*$ assumes the common form (cf. Ref. 21):

$$r(\tilde{\tau}^*)\propto\tau^{*n}\propto\tilde{\tau}^{p/(1+C)},$$
$$e^{\omega/2}\propto\tau^{*l}\propto\tilde{\tau}^{(1-2p)/(1+C)}, \quad \varphi\propto\tilde{\tau}^{C/(1+C)},$$
$$\Phi(\tau^*)\to\pm\lambda^{-1}, \quad n=\frac{\tilde{p}+\delta}{1+\delta}=\frac{\gamma-\alpha-S}{2\gamma-\alpha-S}, \tag{A5}$$

$$l=\frac{1-2\tilde{p}+\delta}{1+\delta}, \quad C=\frac{S}{2\gamma-\alpha-2S}=-\frac{2B^*}{1+3B^*}, \quad B^*=\frac{b}{2\lambda}>0.$$

The asymptote (A5) may include not only point and linear collapse, but also collapse of $\tilde{V}_3^*=(S_2\times R_1)$ into a plane (but only for the Jordan-Brans-Dicke variant in which $\Lambda(\tau)\propto\varphi^{-1}(\tau)\propto\tau^{-2\delta}\to\infty, \delta>0$. However, like the canonical form (A2), these asymptotes do not contain the Kasner kinematic set of exponents ($l=1, n=0$) of the type of collapse of V_3 into a "pancake" corresponding to the removable pseudo-singularities $U(\tau_i^*)=0$ on the zero-isotropic boundaries $T=T_i^*$ between the T- and R-regions of V_4 in the Schwarzschild-Reisner-Nordstrem metrics (23). Thus, the uniform mode of the free scalar field $\Psi=\Psi(\tau)$ is equivalent to a maximally rigid liquid with

$$P_s=\varepsilon_s=\frac{\zeta}{4}\left(\frac{d\Psi}{d\tau}\right)^2$$

when the T-regions of the Schwarzschild-Reisner-Nordstrem metrics (23) are filled, eliminates the limiting zero-horizons such as the Schwarzschild sphere and turns them into physical singularities of the point-collapse type (A2) or (A5) for the conformal variant and the Jordan-Brans-Dicke scalar-tensor theory. This agrees with the results wherein the Schwarzschild sphere is strongly influenced by a scalar vacuum field in the static R-region of V_4, when the limiting zero-horizons in the generalized Schwarzschild-Reisner-Nordstrem metrics also vanish and are replaced by the bare singularity $R=0$.[10,24] This confirms a previous conclusion according to which it is impossible for a black hole to form in the gravitational collapse of massive bodies in different variants of scalar-tensor theory.[10,25]

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[1)]Similarly, bounded T-spheres with electromagnetic field (30) possess additional geometrodynamic chargeless charges ($q\neq 0$), while $M>0$ if $\Lambda=0$.

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