Infrared absorption in small metallic particles

A. G. Mal'shukov

Institute of Spectroscopy of the USSR Academy of Sciences

(Submitted 11 November 1983)
Zh. Eksp. Teor. Fiz. **85**, 700–707 (August 1983)

The coupling between the field and the current acquires a nonlocal form in small metallic particles ($a \ll l_e$, $a \ll v_F/\omega$, where l_e is the electron mean free path in the bulk specimen and ω is the frequency of the external low-frequency field). Under these conditions, the classical theory of electromagnetic absorption based on local coupling is no longer valid. A nonlocal theory of infrared absorption is developed, and it is shown that the form of the absorption coefficient is of the same form as in the classical theory, except that the effective electron-collision times, which govern both the electric-dipole and magnetic-diople parts of the absorption, greatly differ from each other. For magnetic-dipole absorption, the effective collision time has the meaning of the relaxation time of the electron angular momentum and depends strongly on the features of the reflection of the electrons from the surface of the metallic particle. It is shown that the corresponding effective electron mean free path varies between a value on the order of the particle size (in the case of diffuse reflection) and a value corresponding to the electron mean free path in a bulk specimen (for specular reflection from the surface of a spherical particle). The relaxation of the electron angular momentum in a particle whose boundary consists of randomly oriented facets is also considered. The results explain the anomalously large infrared-absorption cross section.

PACS numbers: 78.30.Er, 41.10.Hv

1. INTRODUCTION

Infrared absorption of finely dispersed metallic systems is distinguished by a number of features because of the small size of the absorbing particles. The behavior of a system of electrons in an electromagnetic field of frequency ω is governed by two parameters: the electron mean free path l_e and the electron path per field oscillation v_F/ω. At low temperatures, both these parameters significantly exceed the particle size in the far infrared region (particle size is several tens to a few hundred angstrom). Therefore the interaction between the metallic particles and a low-frequency field is determined by the paths of the electrons over a longer period of time, during which the electrons succeed in repeatedly colliding with the surface of the particles. Naturally, under such conditions the relation between the field and current will be nonlocal. Hence, it is clear why the classical theory, based as it is on a local relation between the field and current in particles, and successfully used to describe the electromagnetic properties of finely dispersed metallic systems in the optical frequency region, where $v_F/\omega \ll a$ always (a is the particle size), is not capable of explaining the greater absorption observed in the far infrared region. The divergence between the theoretical and experimental absorption coefficients is several orders of magnitude.[1–3]

The classical theory, in which the permittivity of a particle is described by the Drude formula and the electron mean free path is assumed to be simply limited by the particle size (i.e., $l_e \sim a$), leads to the following expression for the absorption coefficient:[4]

$$\alpha = f\frac{\omega^2}{c^2}\left(\frac{9c}{\omega_p^2\tau} + \frac{a^2\omega_p^2\tau}{10c}\right), \qquad (1)$$

where $\tau \sim a/v_F$ is the electron mean free time, f is the fraction of volume occupied by the particles in the system, and ω_p is the plasma frequency.

The first term in (1) corresponds to electric-dipole absorption, and the second to magnetic-dipole absorption. For typical metal parameters, the magnetic-dipole absorption makes the main contribution at $a \gtrsim 40$ Å. The parameter τ is chosen artifically and with a degree of uncertainty in this theory mainly because electron collisions with the particle surface cannot be successfully taken into account within the framework of the local theory.

Below we develop a theory based on a nonlocal relation between current and field in a metallic particle. It will be shown that the absorption coefficient has the form (1), though the relaxation times occurring in the electric dipole and magnetic dipole terms differ greatly. In the first case, $\tau = \tau_p$ is always on the order of a/v_F and corresponds in sense to the relaxation time of the electron momentum. In the second case, τ is equal to the relaxation time of the angular momentum τ_m of the electron. This time is basically governed by the reflection of the electron from the boundary and by the shape of the particle. In the case of diffuse reflection, it is on the order of a/v_F, but in the other limiting case, in the case of specular reflection in a spherical particle, angular momentum conservation in collision with the boundary allows the angular momentum to change only as a result of scattering of the particle inside the volume. Consequently, τ_m is equal to the electron mean free time in the bulk, and at low temeprатures may greatly exceed a/v_F. In real situations, it is obvious that τ_m can also greatly exceed a/v_F, since the particles are nearly spherical in shape and have a crystal structure close to perfect.[5]

Below we will consider three cases, two limiting cases of a diffusely and specularly reflecting boundary, as well of a

0038-5646/83/080409-05$04.00

boundary consisting of randomly inclined specularly reflecting facets. The entire approach is based on the classical kinetic equation. Quantum effects are discussed only briefly. The treatment is also limited to the linear-response theory and, consequently, to sufficiently weak fields.

2. NONLOCAL PARTICLE CONDUCTIVITY

For a system of electrons the kernel of the conductivity operator which governs the integral relation between the current and field in a metallic particle, is of the form

$$\sigma^{ij}(\mathbf{r},\mathbf{r}',t-t')=\frac{3\omega_p^2}{(4\pi)^2}\int d\mathbf{n}\,d\mathbf{n}'n^in'^jP(\mathbf{r},\mathbf{n},t|\mathbf{r}',\mathbf{n}',t') \quad (2)$$

in a quasiclassical description of the electron motion. Here $P(\mathbf{r},\mathbf{n},t|\mathbf{r}',\mathbf{n}',\mathbf{t}')$ is the probability density of observing at a time t an electron at the point $\mathbf{r}$, moving in direction $\mathbf{n}$, if it was located at the point $\mathbf{r}',\mathbf{n}'$ of the phase space at the instant t'. Integration in (2) is with respect to the directions $\mathbf{n}$ and $\mathbf{n}'$. If the electron mean free path $l_e \ll a$ or if the frequency is high enough so that $t-t'\sim 1/\omega \approx a/v_F$, the probability P decreases rapidly with increasing distance $|\mathbf{r}-\mathbf{r}'|$, and we obtain for σ^{ij} a local expression described by the Drude formula if it is assumed that the field varies slowly at these distances $|\mathbf{r}-\mathbf{r}'|$ at which σ^{ij} differs appreciably from zero. In the opposite case ($l_e \gg a$ and $a\omega \ll v_F$) it is necessary to take into account electron collisions with the surface of the particle. If the particle is a sphere, allowance for the collisions with the boundary leads to the following equation for P:

$$P(\mathbf{r},\mathbf{n},t|\mathbf{N}_0,\mathbf{n}_0,t_0)=P_0(\mathbf{r},\mathbf{n},t|\mathbf{N}_0,\mathbf{n}_0t_0)$$
$$+\int d\mathbf{N}\frac{W(\theta,\theta')}{4\cos\theta}P\left(\mathbf{r},\mathbf{n},t|\mathbf{N},\mathbf{n}_0,t_0+\frac{a}{v_F}|\mathbf{N}-\mathbf{N}_0|\right), \quad (3)$$

where $\mathbf{N}_0$ is a unit vector that indicates the point on the sphere at which a particle with coordinates $\mathbf{r}'$ and $\mathbf{n}'$ at an initial moment of time $t'<t_0$ arrives at time t_0. The probability P_0 describes the free motion of a particle after a single collision with the boundary at point $\mathbf{N}_0$, after which it acquires a momentum in the direction $\mathbf{n}_0$. The second term in (3) takes into account multiple collisions. The angle θ is equal to the angle between the direction $\mathbf{N}_0$ and the chord connecting the directions $\mathbf{N}$ and $\mathbf{N}_0$, while the function $W(\theta,\theta')$ is the particle scattering probability in the direction θ relative to the normal if the particle is incident to the surface at an angle θ'. In the case of diffuse scattering, W is independent of θ' and equal to $\pi^{-1}\cos\theta$. The probability P satisfying (3) is only that part of the total probability of particle transition between states $\mathbf{n}$, $\mathbf{r}$, and $\mathbf{n}'$, $\mathbf{r}'$ which is due to collisions with the boundary. The path that directly connects the points $\mathbf{r}$ and $\mathbf{r}'$ also contributes to the total probability. In the case of diffuse scattering, we find from (2) and (3) that

$$\sigma^{ij}(\mathbf{r},\mathbf{r}',\omega)=\frac{3\omega_p^2}{\pi v_Fa^2}\int\frac{d\mathbf{n}\,d\mathbf{n}'}{(4\pi)^2}n^in'^j\exp\left[i\frac{\omega}{v_F}(l+l')\right]$$
$$\times\sum_{l,m}\frac{Y_{lm}\left(\dfrac{\mathbf{r}-\mathbf{n}l}{a}\right)Y_{lm}^*\left(\dfrac{\mathbf{r}'+n'l'}{a}\right)}{1-F_l} \quad (4)$$
$$+\frac{3\omega_p^2}{(4\pi)^2v_F}\frac{(\mathbf{r}-\mathbf{r}')^i(\mathbf{r}-\mathbf{r}')^j}{|\mathbf{r}-\mathbf{r}'|^4}\exp\left[i\frac{\omega}{v_F}|\mathbf{r}-\mathbf{r}'|\right],$$

where

$$l=(\mathbf{rn})+[(\mathbf{rn})^2+a^2-r^2]^{1/2},\quad l'=-(\mathbf{r}'\mathbf{n}')+[(\mathbf{r}'\mathbf{n}')^2+a^2-r'^2]^{1/2},$$
$$F_l=\frac{1}{2}\int P_l(\cos\theta)\exp\left(\frac{2i\omega}{v_F}a\sin\frac{\theta}{2}\right)\sin\theta\,d\theta.$$

3. DIPOLE ABSORPTION

In finely dispersed systems with a low concentration of metallic particles, absorption of electromagnetic radiation is determined basically by the absorption properties of the individual particles. In a dipole approximation, the absorption coefficient is

$$\alpha=\frac{3\omega f}{ca^3}\operatorname{Im}(\chi_m+\chi_e), \quad (5)$$

where χ_m and χ_e the magnetic and electric polarizabilities of the particles. For the sake of simplicity, unity permitivity and unit magnetic permeability of the environment are assumed. To compute the electric polarizability, it is necessary to solve Poisson's equation inside the particle; the induced charge is nonlocally connected with the potential $\varphi(\mathbf{r})$ by (4). Expanding the potential in spherical functions, we obtain in the dipole approximation ($l=1$) an equation for the radial part of the potential:

$$\frac{d}{dr}r^2\frac{d}{dr}\varphi(r)-2\varphi(r)=\frac{4\pi i}{\omega}\int dr'r^2r'^2K(r,r')\varphi(r'), \quad (6)$$

$$K(r,r')=2\pi\int P_1(\cos\theta)\frac{\partial^2}{\partial r^i\,\partial r^j}\sigma^{ij}(\mathbf{r},\mathbf{r}',\omega)\sin\theta\,d\theta. \quad (7)$$

The integration in the latter equation is with respect to the angle between the directions $\mathbf{r}$ and $W\mathbf{r}'$. The potential $\varphi(r)$ must be matched to the radial part of the potential outside the sphere, which has the form

$$\varphi^0=\chi_eE_0/r^2-rE_0, \quad (8)$$

where E_0 is the external electric field. When matching the potentials, it is required that the potential and its radial derivative be continuous. Note that the continuity of the normal component of the electric field, but not of the induction, follows from the microscopic description of the fields. But if the permittivity of the environment is not unity, and if we also take into account the contribution of intraband transitions to the permittivity of the metal, the induction associated with the corresponding polarizability must be continuous.

In (6), we may expand the kernel K in powers of the small parameter $\omega a/v_F$. Here the zeroth term of the expansion vanishes identically owing to the absence of current through the particle surface, while the first term corresponds to static screening and contributes to the right side of (6) a term equal to $\varphi(r)r_s^{-2}$, where r_s is the screening radius. Thus, to obtain $\operatorname{Im}\chi$ it is necessary to expand K to the second degree in ω inclusive. We denote this term of the expansion by $\gamma(r,r')\omega^2$. From (6)–(8) and from the boundary conditions it follows that when $a\gg r_s$ we have

$$\chi_e=a^3+12\pi i\omega r_s^2\int dr\,dr'r^2r'^2\gamma(r,r')\exp\left(\frac{r+r'-2a}{r_s}\right). \quad (9)$$

We find for γ from (4) and (7) that

$$\gamma(r,r')=(8\pi r_s^2 v_F r r')^{-1}\left(\frac{23}{9}+\ln\frac{r+r'}{|r-r'|}\right).$$

Thus, we finally obtain

$$\operatorname{Im}\chi_e=\frac{3a^3\omega}{\omega_p^2\tau_p},\quad \tau_p^{-1}=\frac{v_F}{6a}\left[\frac{23}{9}+\ln\left(3.6\frac{a}{r_s}\right)\right].\qquad(10)$$

It is clear from (1), (10), and (5) that the electric-dipole part of the absorption has the same form as in the local theory. A consistent nonlocal theory made it possible to determine the effective mean free time of the electron. Despite the logarithmic term in (10), this time is on the order of a/v_F if the particles are not too large. For example, if $a = 300$ Å and $r_s = 1$ Å, we have $\tau_p^{-1} = 1.6 v_F/a$.

Note that this result is not the same as that achieved with a quantum-mechanical computation of the electric-dipole absorption[6] in the frequency region where the quantum size effects are insignificant and the result of a quantum-mechanical computation should conicide with the classical result, i.e., for $\omega \gg \hbar(k_F m a^3)^{-1}$. The reason for different form of the dependence of the absorption on a is that in Ref. 6 the quasiclassical matrix elements were calculated without allowance for the fact that the screening potential acts on the electron only in a narrow region $\sim r_s$ near the surface of the particle.

Note that in electric dipole absorption importance may attach also to quantum effects other than the quantum size effects. The latter, obviously, can be accounted for by merely multiplying Eq. (10) by an oscillating dimensionless function determined by the static level of the system.[7] In particular, since the screening length in metals is on the order of the electron wavelength, it is important to take into account the behavior of the electron wave function near the surface of the particles.

4. MAGNETIC-DIPOLE ABSORPTION

Magnetic-dipole absorption is due to eddy currents excited in metallic particles by the solenoidel component of the electric field. Magnetic dipole oscillations in a metallic particle indicate the presence of an electric field component proportional to the spherical vector harmonic

$$Y^M_{1,\pm1}\left(\frac{\mathbf{r}}{r}\right)=2^{-1/2}\left[\frac{\mathbf{r}}{r}\nabla Y_{1,\pm1}\left(\frac{\mathbf{r}}{r}\right)\right].$$

This function determines the angular dependence of the electric field, while the radial dependence is determined by the function $e(r)$ which satisfies inside the particle the equation

$$\frac{d}{dr}r^2\frac{d}{dr}e(r)-\left(2-\frac{\omega^2}{c^2}r^2\right)e(r)$$

$$=-\frac{4\pi i\omega}{c^2}\int dr' r^2 r'^2\sigma^M(r,r',\omega)\,e(r'),\qquad(11)$$

$$\sigma^M(r,r',\omega)=\int d\mathbf{N}\,d\mathbf{N}'\,Y^{Mi}_{10}(\mathbf{N})\,\sigma^{ij}(\mathbf{r},\mathbf{r}',\omega)\,Y^{*Mj}_{10}(\mathbf{N}').\qquad(12)$$

Outside the particle, $e(r)$ is given by

$$e^0(r)=\frac{2\omega}{cr^2}\chi_m E_0-\frac{\omega}{c}rE_0.$$

As can be easily verified, only the part corresponding to the free motion of the electron (without any collisions with the boundary) i.e., the second term in (4), contributes to σ^M. This is because σ^M is proportional to the correlation function the angular momentum of the electron or, in other words, to the mean value of the angular momentum at the time, t under the condition that the electron had some prescribed angular momentum at the instant t'. In diffuse scattering, the electron has a zero mean angular momentum after the very first collision with the boundary.

Matching e^M and de^M/dr at the boundary and setting $\omega = 0$ in $\sigma(r,r',\omega)$, thereby discarding small terms on the order of $\omega a/v_F$, we obtain

$$\chi_m=\frac{2\pi i\omega}{3c^2}\int^a dr\,dr'\,r^3 r'^3\sigma^M(r,r')=\frac{ia^5\omega_p^2\tau_m\omega}{30c^2},\qquad(13)$$

where $\tau_m = \frac{5}{8} a v_F^{-1}$. As expected, in diffuse scattering $\tau_m \sim a/v_F$.

5. SPECULAR REFLECTION

As we have seen in the preceeding section, reflection of electrons by particle boundaries is of significance in magnetic dipole absorption. To graphically demonstrate this fact, let us consider the case of specular reflection.

We first transform the expression (13) for χ_m. Using (12) and (2), and also the fact that

$$r\mathbf{n}\mathbf{Y}_{10}{}^M=-\frac{1}{\sqrt{2}p_F}\mathbf{M}\nabla Y_{10}=-\frac{i}{p_F}\left(\frac{3}{8\pi}\right)^{1/2}M^z,$$

where $\mathbf{M}$ is the angular momentum, we may express χ in terms of the correlation function of the angular momenta thus:

$$\chi_m(\omega)=\frac{ia^3\omega_p^2\omega}{2c^2p_F^2}\int\langle M^z(t)M^z(0)\rangle e^{i\omega t}\,dt$$

$$=\frac{ia^3\omega_p^2\omega}{6c^2p_F^2}\int\langle\mathbf{M}(t)\mathbf{M}(0)\rangle e^{i\omega t}\,dt.\qquad(14)$$

In specular reflection, the angular momentum of the electron is conserved in each collision with the particle surface. Therfore, $\mathbf{M}(t) = \mathbf{M}(0)$ and $\chi_m(\omega) \to \infty$ as $\omega \to 0$. However, collisions within the particle induce relaxation of the angular momentum. These collisions may be taken into account if we set

$$\langle\mathbf{M}(t)\mathbf{M}(0)\rangle=\langle M^2(0)\rangle e^{-t/\tau_b}.\qquad(15)$$

In this case the relaxation time τ_b is determined by the electron's mean free path in the bulk "specimen",[1)] i.e., $\tau_b \sim l_e/v_F$. From (14) and (15) it follows that

$$\operatorname{Im}\chi_m=a^5\omega_p^2\tau_b\omega/30c^2(1+\tau_b\omega).$$

Thus, $\tau_m \approx \tau_b$ if $\tau_b\omega \lesssim 1$, and magnetic dipole absorption turns out to be greater in the case of specular reflection by comparison with the classical result if $\tau_b \gg a/v_F$ or $l_e \gg a$. At the same time, it can be easily verified that electric dipole absorption remains of the same order of magnitude as in diffuse reflection.

6. FACETED SURFACE

It follows from the preceding section that in magnetic dipole absorption the effective mean free time of an electron may be determined by finding the time dependence of the correlation function of the angular momenta. Let us consider this problem for a model of the boundary that corresponds to some extent to the actual situation. Namely, the boundary is assumed to consist of plane facets with linear dimensions much less than that of the particle. It is assumed that the facets are randomly inclined relative a spherical surface, and that the random angle θ between the normal **N** to the surface of an individual facet and the normal to the spherical surface passing through the facet is distributed according to the law[2)]

$$dp=w(\theta)\,d\Omega,$$

where $d\Omega$ is an element of a solid angle and $w(\theta)=(\delta^2\pi)^{-1}\exp(-\theta^2/\delta^2)$.

It is also assumed that the electrons are reflected specularly from the facets and that $\delta^2\ll 1$. The boundary condition for an electron distribution function that satisfies the kinetic equation has the form (see, e.g., Ref. 8)

$$-f^-(\mathbf{n}')n_z'=\int W(\mathbf{n}',\mathbf{n})n_zf^+(\mathbf{n})\,d\mathbf{n}, \tag{16}$$

where f^+ and f^- are the electron distribution functions before and after the electrons collide with the boundary. The z axis is directed along the inward normal to the spherical surface at the point of contact. The scattering probability W satisfying the normalization condition and the detailed balancing principle has the form

$$W(\mathbf{n}',\mathbf{n})=C\delta(\tilde{\mathbf{n}}'-\mathbf{n})+w(\theta)n_z', \tag{17}$$

where $\tilde{\mathbf{n}}'=\{n_x',n_y',-n_z'\}$, and

$$C=1-\int w(\theta)n_z'\,d\Omega. \tag{18}$$

In the case of specular reflection from the facets the angle θ is related to the vectors $\mathbf{n}$ and $\mathbf{n}'$ by the equations

$$\mathbf{N}=(\mathbf{n}-\mathbf{n}')/|\mathbf{n}-\mathbf{n}'|,\quad N_z=\cos\theta. \tag{19}$$

If $\delta^2\ll 1$, values of $\mathbf{n}$ close to $\tilde{\mathbf{n}}'$ make the major contribution to the integral (16). Then the integral boundary condition may be reduced to a differential one by expanding $n_zf^+(\mathbf{n})$ near $\tilde{\mathbf{n}}'$. From (16)–(19), we obtain

$$f^+(\mathbf{n})-f^-(\tilde{\mathbf{n}})=\delta^2\left(\Delta_n-\frac{\partial^2}{\partial\varphi^2}\right)[f^+(\mathbf{n})n_z], \tag{20}$$

where Δ_n is the angular part of the Laplacian operator and φ is an angle which determines the direction of the projection of the vector $\mathbf{n}$ on the tangential surface at the considered point of the spherical surface. It is best to select the following coordinates, which determine the state of the electron (see Fig. 1): the spherical angles of the vector **M** (ψ and α), the angle of inclination θ of the chord along which the electron moves between two collisions, the coordinate x of the electron on the chord, and the angle ω which determines the position of the chord in the plane of motion of the electron. Three of these variables (ψ, α, and θ) are slow in the sense that slow diffuse spreading (to the extent that δ is small) of the distribution function with respect to these variables is produced by collisions with the surface. In specular reflection, ψ, α, and θ are integrals of the motion, and the distribution function with respect to these variables does not change form with time. The other two variables (x and ω) are fast variables. Rapid cyclic motion with times $\sim a/v_F$ with respect to these variables occurs, and it is therefore natural to average over them if we are interested in the behavior of f at $t\gg a/v_F$ (averaging over the initial values x' and ω' is also assumed). Averaging over to α is also possible, since χ_m can be determined simply by finding the correlation function $\langle M^z(t)M^z(0)\rangle$ (see (14)), and the component M^z is independent of the angle α.

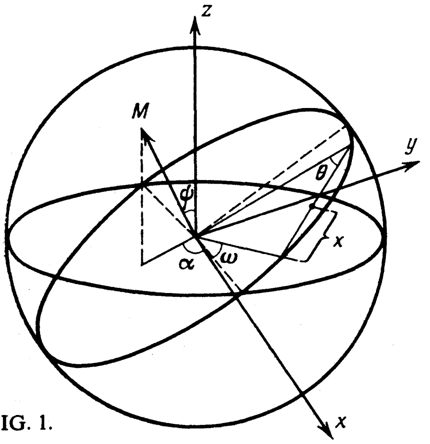

FIG. 1.

We introduce the function

$$\rho(\theta,\psi)=\int dx\,dx'\langle f\rangle, \tag{21}$$

where the angular brackets denote averaging over ω, ω', α, or α'. From the kinetic equation,

$$\frac{\partial f}{\partial t}+v_F\frac{\partial f}{\partial x}=0,$$

which describes the change of the distribution function between two collisions with the surface ($0<x<2a\cos\theta$), it follows that

$$\frac{\partial\rho}{\partial t}=v_F\int_0^{2a\cos\theta}dx'(\langle f^-\rangle-\langle f^+\rangle). \tag{22}$$

Hence, from (20), changing to new variables and averaging over ω and α, we obtain

$$\frac{2a}{v_F\delta^2}\frac{\partial\rho}{\partial t}=\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}\rho+\frac{\mathrm{ctg}^2\theta}{2\sin\psi}\frac{\partial}{\partial\psi}\sin\psi\frac{\partial}{\partial\psi}\rho+2\rho. \tag{23}$$

The initial condition for Eq. (23) has the form

$$\rho|_{t=0}=\delta(\theta-\theta')\delta(\psi-\psi'). \tag{24}$$

The boundary condition follows from (23) if we require that there be no diffusion flow through the boundary of the range of θ ($0<\theta<\pi/2$), i.e.,

$$\rho(\pi/2,\psi)=0. \tag{25}$$

The solution of (23) satisfying conditions (24) and (25) has the form

$$\rho=-\frac{1}{2}\sum_{n,l}\exp(-\Omega_{nl}t)P_{n+\mu}^{-\mu}(\cos\theta)P_{n+\mu}^{-\mu}(\cos\theta')P_l(\cos\psi)P_l(\cos\psi')[1-(-1)^n], \tag{26}$$

where

$$\Omega_{nl}=\frac{\delta^2 v_F}{2a}[(n+\mu)(n+\mu+1)-\mu^2-2],\quad \mu=\left[\frac{l(l+1)}{2}\right]^{1/2},$$

l and n are positive integers, while $P_{n+\mu}^{-\mu}$ and P_l are Legendre functions. Since $M^2 \propto \sin\theta\cos\psi$, only the term with l, $n = 1$ contributes to the correlation function. Thus,

$$\tau_m=\Omega_{11}^{-1}={}^2/_3(a/v_F)\delta^{-2}. \tag{27}$$

Obviously, it should be kept in mind that actually, $\tau_m^{-1} = \Omega_{11} + v_F/l_e$, and that if $\delta \ll a/l_e$, when $\Omega_{11} \ll v_F/l_e$, the effective mean free time τ_m is determined by collisions within the particles and is equal to the electron's mean free time in the bulk specimen.

The author thanks V. M. Agranovich for valuable discussions.

[1)] Bear in mind that this "specimen" must simulate the internal structure of the particles, i.e., contain the same density of the electron-scattering electrons defects.

[2)] In fact, the actual form of the function $w(\theta)$ is not important. What is important is that $w(\theta)$ decrease sufficiently rapidly for values of θ greater than some (small) angle.

[1] C. G. Granquist, R. A. Buhrman, J. Wyns, and A. J. Sievers, Phys Rev. Lett. **37**, 625, 1976.

[2] N. E. Russel, J. C. Garland, and D. B. Tanner, Phys. Rev. **B23**, 632, 1981.

[3] G. L. Carr, R. L. Henry, N. E. Russel, J. C. Garland, and D. B. Tanner, Phys. Rev. **B24**, 777, 1981.

[4] L. D. Landau and E. M. Lifshitz, Elektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), Moscow, Nauka, 1982 [Pergamon].

[5] C. G. Granquist and O. Hunderi, Phys. Rev. **B16**, 3513, 1977.

[6] A. A. Lushnikov, V. V. Maksimenko, and A. Ya. Simonov, Fiz. Tverd. Tela **20**, 505, 1978 [Sov. Phys. Solid State **20**, 292 (1978)].

[7] L. P. Gor'kov and G. M. Eliashberg, Zh. Eksp. Teor. Fiz. **48**, 1407 1965 [Sov. Phys. JETP **21**, 940 (1965)].

[8] E. M. Lifshitz and L. P. Pitaevskiĭ, Physical Kinetics, Pergamon, 1981.

Translated by Robert H. Silverman