Quantum effects in two-dimensional superconducting films at $T > T_c$

M. E. Gershenzon, V. N. Gubankov, and Yu. E. Zhuravlev

Institute of Radio Engineering and Electronics, USSR Academy of Sciences
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The magnetic-field and temperature dependences of the resistance of thin aluminum and niobium films are investigated at temperatures $T/T_c > 1$ (T_c is the superconducting-transition temperature). The anomalous behavior for the film resistances can be described adequately by the theory of weak localization and interelectron interaction effects in two-dimensional disordered systems. The absolute values and temperature dependences of the interelectron coupling constant and inelastic-scattering time of the conduction electrons in the films are determined.

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I. INTRODUCTION

Considerable interest has been aroused recently by quantum effects in disordered systems with metallic type of conduction—"weak" localization of electrons[1] and enhancement of electron-electron interaction in the presence of impurity scattering.[2] These effects lead to the appearance of corrections to the classical conductivity, which depend in anomalous fashion on the temperature, magnetic field, and a number of other factors.[3] The temperature and field dependences of the quantum corrections are determined in this case by the relation between the system dimensions and the characteristic lengths dealt with in the theory. The characteristic scale for localization effect is the length over which the electron diffuses during the time τ_φ of phase relaxation of its wave function, i.e., $L_\varphi = (D\tau_\varphi)^{1/2}$, where D is the diffusion coefficient. For interaction effects this scale is the coherence length in the normal metal, $L_T = (\hbar D/kT)^{1/2}$. At present the most thoroughly studied features of the low-temperature behavior of the conductivity of two-dimensional disordered systems (for which the thickness $d \ll L_\varphi,\ L_T$), with thin films of normal metals and semiconductor inversion layers as the examples. It was shown (see, e.g., Refs. 4 and 5 and the references therein) that the anomalous dependences of the resistance of such objects on the temperature and on the magnetic field are due to a joint manifestation of the localization and the interaction, and are well described by the existing theory. Since the concrete forms of the manifestations of quantum effects are influenced by various relaxation processes of the electron wave function, an experimental study of these effects makes it possible to determine a large number of conduction-electron energy and sin relaxation times.[4–7]

Allowance for the interaction between electrons and disordered systems leads to the appearance of two types of corrections to the resistance, corresponding to the interaction between electrons with close energies and small total momentum (the so-called Cooper channel) and to interaction between the electrons with small energy and momentum difference (the so-called diffusion channel).[8] Additionally distinguished in the Cooper interaction channel are the Maki-Thomson corrections corresponding to electron scattering by superconducting fluctuations.[8,9] (This mechanism acts also in the case of repulsion between electrons, when there is no superconducting transition.[9]) The relation between the contributions of different quantum effects to the temperature and field dependences of the resistance of disordered systems depends on the mechanisms that destroy the coherence of the conduction-electron wave function, and also on the singularities of the interelectron interaction. Thus, for example, the magnitude of the corrections to the conductivity, which are due to interaction in the Cooper channel and to electron scattering by superconducting fluctuations, is determined by the value of the interelectron interaction at a small total momentum $g(T, H)$, which is positive for repulsion between the electrons and negative for attraction. Therefore the anomalous magnetoresistance (AMR) of two-dimensional films of normal metals, for which $|g| \ll 1$, turns out to be connected upon satisfaction of the condition $H \ll kT/g_L\,\mu_B,\ mc/e\tau$ (g_L is the Lande factor of the conduction electrons, μ_B is the Bohr magneton, and τ is the momentum relaxation time) mainly with the localization effects, while the contribution of the Cooper interaction channel turns out to be insignificant.[5] (Under the restriction imposed above on the value of H, the magnetic field does not influence the interaction in the diffusion channel.) At the same time, in experiments on p-type inversion channels, in which the localization effects are suppressed by a strong spin-orbit interaction, it becomes possible to observe AMR due to the influence of the magnetic field on the Maki-Thomson corrections.[10]

The features of the quantum effects in superconductors at temperatures above critical are governed by the fact that as T approaches T_c the effective interelectron interaction should increase drastically, and this is equivalent to an increase of the absolute value of the interelectron-interaction constant. In this case the relations between the contributions of various quantum effects may turn out to be substantially different than for normal metals. The earlier experimental studies of the region of strong localization, with three-dimensional granulated aluminum films as the example, did not reveal these features.[11–13] We report here a study of the magnetoresistance and of the temperature dependence of the resistance of two-dimensional superconducting films at $T/T_c \gtrsim 1$. The study of the magnetoresistance in wide ranges of H and T permits an unambiguous separation of the contributions of different quantum effects and reveals singularities

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connected with the enhancement of the effective interelectron interaction at T approaches T_c. The analysis of the experimental data, first, is evidence of the adequacy of the description, by the theory,[8,9,14] of the behavior of the conductivity of thin superconducting films as functions of the magnetic field and the temperature at $T/T_c \gtrsim 1$. Second, it makes it possible to determine the absolute values and temperature dependences of the interelectron-interaction constant and of the electron inelastic scattering time τ_ϵ for the investigated films. The obtained values of τ_ϵ agree with the data obtained earlier in experiments on the study of nonequilibrium processes in thin superconducting films. Preliminary results of these investigations were published in Ref. 15.

2. ASPECTS OF THE THEORY

We describe concretely the results of the theory as applied to the case of a two-dimensional metal film with effective interelectron attraction. The temperature dependence of the resistance of superconducting films in the interval $T_c \lesssim T \ll T_D$ (T_D is the Debye temperature), where the $R(T)$ dependence due to scattering by phonons can be neglected, should be determined by quantum effects. These effects are electron localization, interaction between electrons in the diffusion and Cooper channels, as well as electron scattering by superconducting fluctuations. (The temperature and field dependences of the Aslamazov-Larkin fluctuation contribution can be neglected under the condition $T - T_c \gg \hbar/k\tau_\varphi$ which is realized in experiment[14]; it will be shown below that for the investigated films the temperature interval $T - T_c \sim \hbar/k\tau_\varphi$ does not exceed 0.1 K.) The sum of the temperature dependences of the quantum corrections to the conductivity at $T - T_c \gg \hbar/k\tau_\varphi$ and $H = 0$ can be represented in the form[2,14]:

$$\sigma(T_2)-\sigma(T_1)=\frac{e^2}{2\pi^2\hbar}\left\{\beta(T_2)\ln\left[\frac{k}{\hbar}T_2\tau_\varphi(T_2)\right]\right.$$
$$\left.-\beta(T_1)\ln\left[\frac{k}{\hbar}T_1\tau_\varphi(T_1)\right]+\ln\left[\frac{\ln(T_c/T_2)}{\ln(T_c/T_1)}\right]+\ln\frac{T_2}{T_1}\right\}+\Delta\sigma^L. \quad (1)$$

Here σ is the conductivity per square of the film surface, and is a function of g and is tabulated in Ref. 9. The curly brackets contain the sum of the contributions due to the interelectron interaction: the first and second terms correspond to the Maki-Thomson corrections, while the third and fourth correspond to the Cooper and diffusion interaction channels. [A factor $(1-F)$ that should precede the diffusion-channel contribution in (2) (see Ref. 2) has been left out because $F \ll 1$ for the investigated films.] The expression for the localization contribution, with account taken of the spin-orbit interaction, is of the form[8]

$$\Delta\sigma^L=\frac{e^2}{2\pi^2\hbar}\left\{\frac{3}{2}\ln\left[\frac{\tau_\varphi^*(T_1)}{\tau_\varphi^*(T_2)}\right]-\frac{1}{2}\ln\left[\frac{\tau_\varphi(T_1)}{\tau_\varphi(T_2)}\right]\right\}. \quad (2)$$

The electron wave-function phase relaxation time τ_φ and the time τ_φ^* modified to allow for the spin-orbit interaction are determined from the relations[16]

$$\tau_\varphi^{-1}=\tau_\epsilon^{-1}+2\tau_s^{-1}, \quad \tau_\varphi^{*-1}=\tau_\epsilon^{-1}+{}^2/_3\tau_s^{-1}+{}^4/_3\tau_{so}^{-1}, \quad (3)$$

where τ_s is the time of elastic scattering with spin flip on the paramagnetic impurities, and τ_{so} is the time of spin relaxation on account of the spin-orbit interaction in elastic scattering of the electrons.

With decreasing temperature, the interelectron interaction in the diffusion and Cooper channels should increase the resistance, and scattering of the electrons by the superconduction fluctuations should lower the resistance. The sign of the localization contribution is determined by the relation between τ_φ and τ_{so} at $\tau_\varphi \gg \tau_{so}$ the localization effects should decrease R with decreasing T, and R should increase with decreasing T at $\tau_\varphi \ll \tau_{so}$.

The effect of the magnetic field on the quantum corrections to the resistance should lead to the appearance of an anomalous magnetoresistance that manifests itself in classically weak magnetic field, and is anisotropic with respect to the relative orientations of the magnetic field and the film plane. At $T - T_c \gg \hbar/k\tau_\varphi$ it is necessary to take into account the field dependences of all the conductivity corrections listed above, with exception of the contribution of the diffusion interaction channel, namely under the conditions realized in our experiments this contribution should be independent of H.

The field dependence of the localized contribution to the conductivity of two-dimensional systems upon satisfaction of the condition $H \ll kT/g_L\mu_B, \hbar c/2e(\max[d,l])^2$ (l is the electron mean free path and d is the film thickness), in a field perpendicular to the plane of the film (H takes the following form[8]:

$$\sigma(H_\perp)-\sigma(0)\equiv\Delta\sigma(H_\perp)$$
$$=\frac{e^2}{2\pi^2\hbar}\left\{\frac{3}{2}f\left(\frac{4eDH_\perp}{\hbar c}\tau_\varphi^*\right)-\frac{1}{2}f\left(\frac{4eDH_\perp}{\hbar c}\tau_\varphi\right)\right\}, \quad (4)$$

$$f(x)=\ln x+\psi(1/2+1/x),$$

where $\psi(y)$ is the logarithmic derivative of the Γ function.

A similar functional dependence of $H_\perp$ is possessed by the AMR connected with the influence of the magnetic field on electron scattering from superconducting fluctuations[9]:

$$\Delta\sigma(H_\perp)=-\frac{e^2}{2\pi^2\hbar}\beta(T,H)f\left(\frac{4eDH_\perp}{\hbar c}\tau_\varphi\right), \quad (5)$$

where $\beta(T,H)$ is a function of $g(T,H)$. In the case of attraction between the electrons the interelectron interaction constant satisfies the expressions[8]

$$g^{-1}(T,H)=\begin{cases}\dfrac{1}{\bar\lambda}+\ln\dfrac{\gamma T_D}{\pi T}=-\ln\dfrac{T}{T_c}, & H\ll\dfrac{ckT}{eD},\\[2ex] \dfrac{1}{\bar\lambda}+\ln\dfrac{ckT_D}{eDH}, & H\gg\dfrac{ckT}{eD},\end{cases} \quad (6)$$

where $\bar\lambda$ is the dimensionless bare constant of the interaction and $\ln\gamma\approx 0.577$. By virtue of satisfaction of the condition $\tau_\varphi \gg \hbar/kT$, which is necessary for the theory of Ref. 8 to be valid, there should exist a magnetic-field region

$$\hbar c/4eD\tau_\varphi \ll H \ll ckT/eD,$$

in which, on the one hand, the temperature dependence of g is still not suppressed by the field, and on the other the argument of the function f is larger than unity. According to (5), in this interval of H the contribution of the superconducting fluctuations to the AMR should start to predominate as T approaches T_c.

Consideration of the interaction in the Cooper channel leads to the following dependence[8]:

$$\Delta\sigma(H_\perp)=-g(T,H)\frac{e^2}{2\pi^2\hbar}\xi\left(\frac{2eDH_\perp}{\pi ckT}\right),\qquad \xi(x)=\begin{cases}0.3x^2, & x\ll 1\\ \ln x, & x\gg 1\end{cases}. \tag{7}$$

We note that the conditions for the suppression of $|g|$ by the magnetic field and for the appearance of appreciable arguments of the function ξ coincide.

3. EXPERIMENTAL PROCEDURE

The objects of the investigation were thin aluminum and niobium films obtained by high-frequency evaporation of aluminum 99.999% pure and niobium 99.9999% pure in an atmosphere of argon 99.995% pure. Preliminary evacuation to a pressure $P\approx 2\times 10^{-6}$ mbar was effected with a turbomolecular pump, and the evaporation was at a voltage ≈ 3 kV on the electrodes and a discharge current 5 mA/cm^2. The aluminum films were deposited at $P_{\rm Ar}\approx 5\times 10^{-3}$ mbar on room-temperature glass substrates at a rate 450 Å/min. These films had a thickness 30–150 Å, a resistance per surface square $R_\square = 4$ to 500 Ω, and a critical temperature $T_c \approx 1.6-2.1$. determined from the midpoint of the resistive transition. To obtain high values of T_c of the niobium films, they were deposited on sapphire substrates heated to 600° C. At an argon pressure 3×10^{-2} mbar, the rate of deposition of the niobium films was 600 Å/min. For thin ($d\lesssim 500$ Å) Nb films one observes a characteristic $T_c(d)$ dependence[17] connected with the proximity of the film to the surface layer in which the superconductivity is weakened by the increased oxygen content. The investigated niobium films were 70 to 120 Å thick, and a decrease of d from 120 to 70 Å lowers T_c approximately from 7 to 2 K.

The electron mean free path was determined from the relation $\rho l=$ const, where $\rho = R_\square d$ is the resistivity of the film. (For Al we have $\rho l = 9\cdot 10^{-12}$ Ω·cm, 18 and for Nb (Ref. 19) $\rho l = 3.7\cdot 10^{-12}$ Ω·cm). For aluminum films the values of d and l are close, pointing to a predominant diffuse surface scattering of the electrons. The values of l for Nb range approximately from 5 to 20 Å and turn out to be much less than d. The electron diffusion coefficient is given by $D = 1/3 l v_F$, where v_F is the Fermi velocity, equal to 1.36×108 cm/sec and 0.65×108 cm/sec for aluminum[20] and niobium[19] respectively. To illustrate the results, the table lists the values of d, l, $R_\square$(10 K), and T_c for several Al and Nb films chosen from a large number of investigated objects.

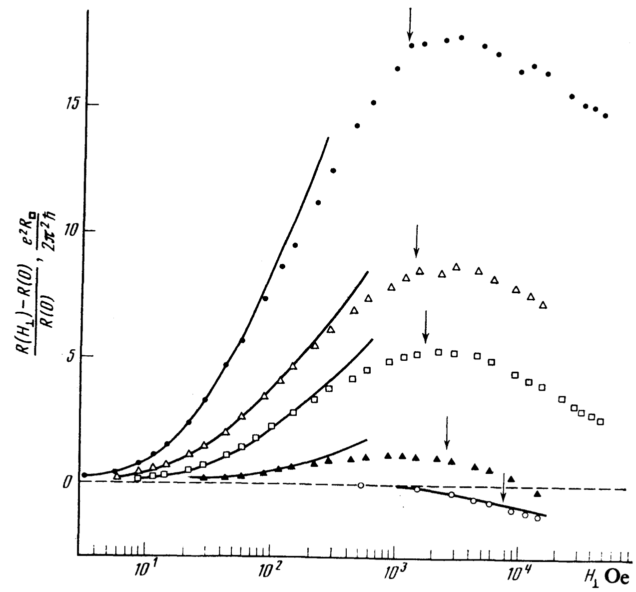

FIG. 1. Plots of $R(H_\perp)$ for aluminum films Al 1. Solid lines—plots of

$$\Delta R/R = -\beta^*(T)\frac{e^2R_\square}{2\pi^2\hbar} f\left(\frac{4eDH_\perp}{\hbar c}\tau_\varphi\right),$$

corresponding to the values of $g(T)$ and $\tau_\varphi(T)$ shown in Figs. 3 and 5. The arrows show the values of $H_\perp$ for which $H_\perp = ckT/eD$. Corresponding to the $R(H_\perp)$ plots are the following values of T: ● — 2.8 K, △ — 3.5 K, □ — 4.2 K, ▲ — 6.7 K, ○ — 20 K.

The samples obtained from the films by photolithography were strips 10 μm wide, either single or assembled into a meander, with a total length from 3 mm to 20 cm. The sample resistance was measured in direct current with a digital voltmeter of relative accuracy $\approx 10^{-6}$. The temperature dependence of the resistance was measured in the interval from 1.5 to 300 K; the magnetoresistance was determined at $T=15$ to 30 K in magnetic fields of strength up to 60 kOe. A detailed description of the procedure is given in Ref. 5.

4. DEPENDENCE OF THE RESISTANCE ON THE MAGNETIC FIELD

We start the description of the experimental results with an examination of the data on the magnetoresistance in a field perpendicular to the film plane. Figure 1 shows[1)] typical $R(H_\perp)$ plots obtained for aluminum films at different T for sample Al 1 (the samples are numbered as in the table). At temperatures $T\gtrsim 15$ K the magnetoresistance is negative and the plots of $R(H_\perp)$ are close in form to the corresponding plots typical of normal metals (see, e.g., Ref. 5). At lower

TABLE I

Sample	d, Å	l, Å	$R_\square$(10K), Ω	T_c, K	τ_ε^e(10 K), sec
Al1	85	53	19.9	1.806	$1.52\cdot 10^{-10}$
Al2	35	23	112	2.01	$3.69\cdot 10^{-11}$
Nb1	120	17	18.1	7.0	$1.65\cdot 10^{-10}$
Nb2	70	8	67.0	1.85	$5.56\cdot 10^{-11}$

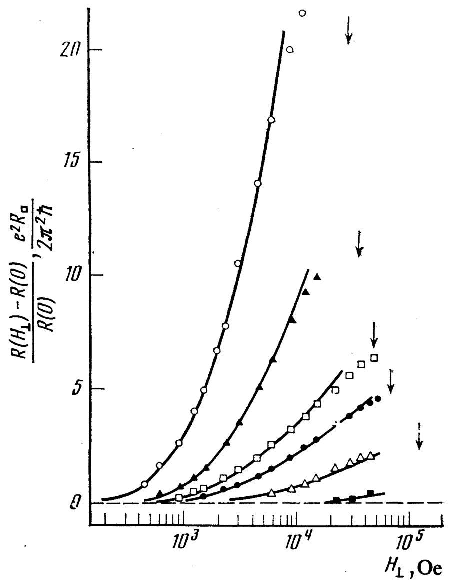

FIG. 2. Plots of $R(H_\perp)$ for niobium films Nb 2. The solid lines are plot of the relation

$$\frac{\Delta R}{R} = -\beta^*(T)\frac{e^2R_\square}{2\pi^2\hbar} f\left(\frac{4eDH_\perp}{\hbar c}\tau_\varphi\right),$$

and correspond to the values of $g(T)$ and $\tau_\varphi(T)$ shown in Figs. 4 and 6. The arrows show the values of $H_\perp$ for which $H_\perp = ckT/eD$ is satisfied. Corresponding to the $R(H_\perp)$ plots are the following values of T: ○ — 2.5 K, ▲ — 3 K, □ — 4.2 K, ● — 5.5 K, △ — 15 K, ■ — 20 K.

temperatures in weak fields, a positive magnetoresistance sets in and increases rapidly as T approaches T_c. With increasing H the growth of the resistance slows down and at certain values of H the $R(H_\perp)$ plots go through a maximum. The magnetic length $L_H = (\Phi_0/2\pi H)^{1/2}$ (Φ_0 is the magnetic-flux quantum) corresponding to such a field is approximately equal to the coherence length $L_T(H \sim ckT/eD)$ of the normal metal. With further increase of H a logarithmic decrease of the resistance, $\Delta R/R = \alpha(e^2R_\square/2\pi^2\hbar)\ln H$, is observed where the coefficient α depends little on temperature and is approximately equal to unity.

A family of $R(H_\perp)$ plots typical of niobium films at different T is shown in Fig. 2 (sample Nb2). In weak magnetic fields the $R(H_\perp)$ plots are similar in shape to those observed for aluminum films. The small values of l of the investigated Nb films causes the condition $H_\perp = ckT/eD$ to be satisfied for these samples at larger values of $H_\perp$ than for Al films, and it is impossible to track the course of $R(H_\perp)$ at $H_\perp \gg ckT/eD$. In contrast to the aluminum films, a positive magnetoresistance is observed for niobium films even at appreciable deviation from T_c.

We analyze first the behavior of the AMR at fields $H_\perp \gg ckT/eD$. At an appreciable deviation from T_c the behavior of the superconducting films should not differ substantially from the behavior of normal metals because of the weakness of the effective interelectron interaction: $|g| \ll 1$ according to (6). In this case the magnetoresistance in the region of weak magnetic field should be due mainly to localization effects. The different magnetoresistance signs observed in this temperature region for Al and Nb films attest to the fact that for Al films the spin-orbit interaction at $T \gtrsim 20$ K ceases to influence the form of the localization contribution to the AMR ($\tau_\varphi \ll \tau_{so}$, $\tau_\varphi \sim \tau_\varphi^*$), whereas for Nb films, in the entire temperature range $T \lesssim 30$ K, the spin-orbit interaction is strong ($\tau_{so} \ll \tau_\varphi$, $\tau_\varphi \gg \tau_\varphi^*$)—see Eq. (4). As T approaches T_c the values of $|g|$ increase. It becomes essential here to take into account the magnetoresistance connected with the influence of the magnetic field on the Maki-Thomson corrections. We note that the AMR due to localization can be represented in the region of weak fields in the form:

$$\Delta\sigma(H_\perp) \approx \frac{e^2}{2\pi^2\hbar}\begin{cases} -\frac{1}{2} f(4eDH_\perp\tau_\varphi/\hbar c), & \tau_\varphi \gg \tau_\varphi^*, \\ f(4eDH_\perp\tau_\varphi/\hbar c), & \tau_\varphi \approx \tau_\varphi^*. \end{cases}$$

$$H \ll \hbar c/4eD\tau_\varphi^*, \qquad (8)$$

therefore at any ratio of τ_φ and τ_φ^* the sum of the contributions to the AMR from localization and superconducting fluctuations is described at $H_\perp \ll \hbar c/4eD\tau_\varphi^*$ by a single function f, and the coefficient of this function (β^* in our notation) is equal to $(\beta + 1/2)$ at $\tau_\varphi \gg \tau_\varphi^*$ and to $(\beta - 1)$ at $\tau_\varphi \approx \tau_\varphi^*$. The corresponding relations that represent the sum of the localization and fluctuation contributions to the AMR are shown in Figs. 1 and 2 for the weak-field region by thick lines. The adjustment parameters needed to compare theory and experiment are the values of β^* and τ_φ shown in Figs. 3–6. The obtained values of τ_φ in the entire temperature range correspond to the condition $\tau_\varphi(T) \gg \hbar/kT$, which in accord with the results of the theory[9] [see Eq. (5)] leads to an increasing role of the superconducting fluctuations to the AMR as T_c is approached.

The values of $\tau_\varphi(T)$ for Al films are comparable with the value $\tau_{so} \approx (1-2)\cdot 10^{-11}$ sec expected for such films (see, e.g., Ref. 21) at $T \lesssim 10$ K. Therefore at $T \lesssim 10$ K and $T \gtrsim 10$ K we have respectively $\beta^*(T) \approx \beta(T) + 1/2$ and $\beta^*(T) \approx \beta(T) - 1$. For Nb films, for which $\tau_\varphi^* < \tau_\varphi$ in the entire temperature interval, it can be assumed that $\beta^*(T) \approx \beta(T) + 1/2$ at all values of T. Figure 5 shows the values of $\beta(T)$ obtained in this manner for the sample Al 1.

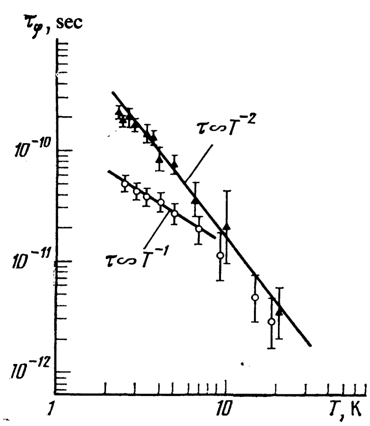

FIG. 3. Plots of $\tau_\varphi(T)$ for aluminum samples: ▲ — Al 1, ○ — Al 2.

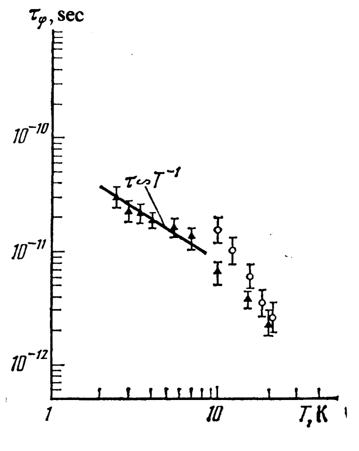

FIG. 4. Plots of $\tau_\varphi(T)$ for niobium samples: ○ — Nb 1, ▲ — Nb 2.

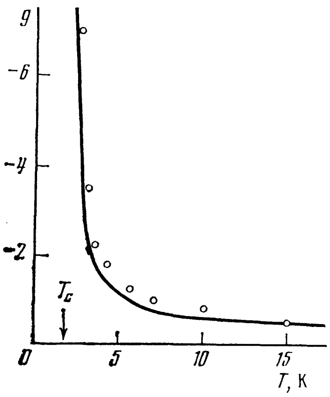

FIG. 6. Plot of $g(T)$ for sample Nb 2. Solid line—theoretical plot of $g(T)$.

Figures 5 and 6 show plots of $g(T,H=0)$ obtained for aluminum and niobium films from the values of $\beta(T)$ found by using the function $\beta(g)$ tabulated in Ref. 9. The experimental plots of $g(T)$ and $\beta(T)$, for both aluminum and niobium films, are in good agreement with the theoretical ones calculated with the aid of independently determined values of T_c. Thus, an analysis of the AMR in the field region $H < ckT/eD$, carried out in a wide temperature range, yields the absolute values and the temperature dependences of the interelectron interaction constant $g(T,H=0)$ and of the phase relaxation time $\tau_\varphi(T)$ of the electron wave function.

We consider now the field region $H > ckT/eD$. At these values of H the temperature dependence of the magnetoresistance, which is negative in the case of attraction between the electrons, should be suppressed.[8] As a result of the suppression of $|g|$ by the field, Eqs. (6) and (7) for the AMR, due to the effect of the field on the Maki-Thomson corrections and to the interaction in the Cooper channel contain values of β and $|g|$ that do not exceed unity. For the investigated Al film the condition $H \approx \hbar c/4eD\tau_\varphi^*$ is satisfied in approximately the same field region as $H \approx ckT/eD$, therefore at $H \gg ckT/eD$ and $\hbar c/4eD\tau_\varphi^*$ the $R(H_\perp)$ dependence determined by all the quantum effects can be represented in the form

$$\frac{\Delta R}{R} = (1-\beta(H)-g(H))\frac{e^2 R_\square}{2\pi^2\hbar}\ln H. \qquad (9)$$

Recognizing that for Al the bare constant is $\bar{\lambda} \approx 0.3$, we obtain at $H = 20$ kOe, in accord with (6) and (9), $g \approx -0.80$, $\beta \approx 0.73$, and $\alpha = 1 - \beta - g \approx 1.07$. The obtained value of α agrees with the experimentally determined values of α, which depend little on T. (For the given sample the values of α were varied from 1.1 at $T = 4.2$ K to 1.3 at $T = 2.6$ K.) Thus, inasmuch as in strong fields the contributions of the Cooper channel of the interelectron interaction and of the superconducting fluctuations to the AMR are small and practically cancel each other, we arrive again at a situation wherein the field dependence of the resistance, just as in normal-metal films, is due mainly to localization effects.

Without dwelling in detail on the discussion of the anisotropy of the observed magnetoresistance, we note that the change of the form of the $R(H)$ dependence with change of orientation of H relative to the film plane from perpendicular to parallel is analogous to that observed in two-dimensional films of normal metals[5] and takes place in accord with the results of the theoretical analysis in Ref. 22.

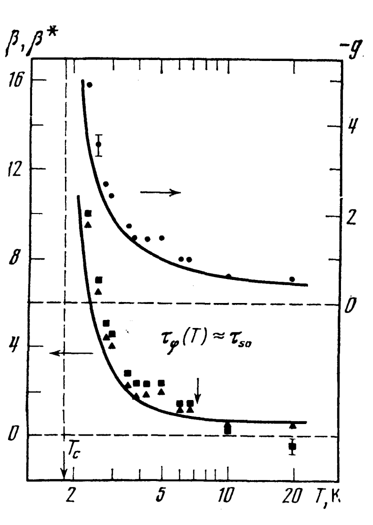

FIG. 5. Temperature dependences of β (▲), β^* (■) and $-g(H=0)$ (●) for Al 1 sample. Solid lines—theoretical plots of $g(T)$ and $\beta(T)$.

5. TEMPERATURE DEPENDENCE OF THE RESISTANCE

$R(H)$ plots typical of most investigated aluminum and niobium film and measured at $H = 0$ and $H = 30$ kOe, when the conditions $H \gg ckT/eD$ and $c\hbar/4eD\tau_\varphi^*$, are satisfied in the temperature region $T \lesssim 40$ K, are shown in Fig. 7 (Sample Al 1). The observed temperature dependences of the resistance can be explained within the framework of the theory of analogous effects by resorting to data obtained from the analysis of the magnetoresistance. Indeed, at $T \lesssim 10$ K, where the relation $\tau_\varphi \gg \tau_{so}$ is satisfied for sample Al 1 [the first term in (2) is equal to zero], localization effects lead to a decrease of R with decrease of T, and in the case $\tau_\varphi \propto T^{-2}$ the temperature dependence of the contributions due to the localization and to the diffusion channel of the interaction cancel each other. Therefore at $T \lesssim 10$ K the observed $R(T)$ dependence should be determined by the scattering of the electrons by superconducting fluctuations and by the interaction in the

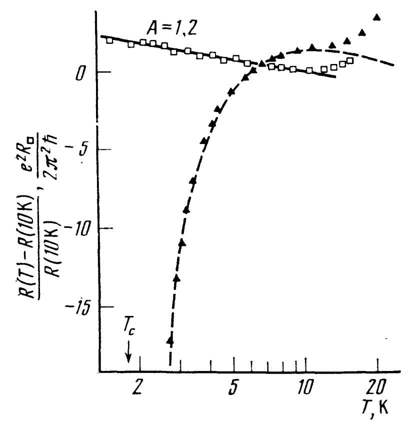

FIG. 7. Plot of $R(T)$ obtained for Al 1 at $H = 0$(▲) and $H = 30$ kOe (□). Dashed line—theoretical plot of Eq. (1).

Cooper channel. The contributions of these effects to the resistance are of opposite sign and increase monotonically in absolute value with decreasing temperature. The Maki-Thomson corrections that lead to a decrease of R and to a lowering of T begin to predominate at $T \lesssim 3T_c$ for sample Al 1, and at $T = 3$ K they account already for 80% of the resistance change [referred to R (10 K)]. The sum of all the quantum effects, calculated from Eqs. (1) and (2), is shown dashed in Fig. 7. The calculation was performed using the experimentally obtained $\beta(T)$ dependence as well as the relation $\tau_\varphi \approx 1.7 \cdot 10^{-9}$ sec·K^2/$(T\,[\mathrm{K}])^2$, which approximates well the experimentally obtained values of τ_φ (see Fig. 3). The value of τ_{so} used in the calculation of the localization contribution amounted to 1×10^{-11} sec. The theoretical $R(T)$ plot was matched to the experimental one at the point corresponding to $T = 10$ K, where the contribution made to the resistance by scattering from phonons becomes negligible. A good quantitative agreement between theory and experiment can be traced all the way to temperatures satisfying the condition $T - T_c \approx \hbar/k\tau_\varphi$ (see Fig. 8). For the sample Al 1 ($T_c = 1.806$ K) this condition is satisfied at $T \approx 1.83$ K, and

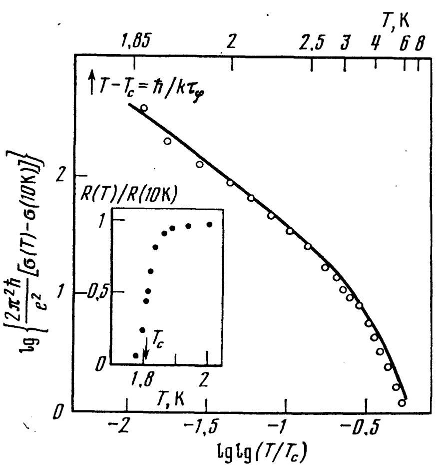

FIG. 8. Plot of $\Delta\sigma(T)$ obtained for Al 1 sample at $H = 0$. Solid line—theoretical plot of (1). The inset shows the $R(T)$ plot for the Al 1 sample near T_c.

the resistance change $\Delta R = R(1.83\ \mathrm{K}) - R(10\ \mathrm{K})$ corresponding to this temperature amounts to $\approx 13\%$ of R (10 K), i.e., $(\Delta R/R)(2\pi^2\hbar/e^2R_\square) = -530$.

In the temperature region where the condition $\tau_\varphi \ll \tau_{so}$ is satisfied ($T \gtrsim 10$ K for samples Al 1 and Al 2), the localization contribution to the resistance increases with decreasing T. In addition, at a considerable distance from T_c ($T \gtrsim 5T_c$), the Maki-Thomson corrections become small. At a considerable distance from T_c the theory predicts therefore an increase of the resistance with decreasing temperature (the section of the dashed curve at $T \gtrsim 10$ K on Fig. 7). The nonmonotonic $R(T)$ dependence can be tracked with high-resistance aluminum film as an example. For these films the phonon contribution to the resistance appears only at $T \gtrsim (25-30)$ K. Thus, for the sample Al 2 an increase of resistance with decreasing T was observed in the interval $T = (13-24)$ K.

In a strong magnetic field ($H \gg ckT/eD, c\hbar/4eD\tau_\varphi^*$) the temperature dependences of the contributions of all the quantum effects, except for the interaction in the diffusion channel, should be completely suppressed. In strong fields one observes accordingly

$$\frac{R(T_1)-R(T_2)}{R(T_1)} = A\frac{e^2R_\square}{2\pi^2\hbar}\ln\frac{T_2}{T_1}$$

with a value of A close to unity (see Fig. 7). The values of A of the investigated Al and Nb films were in the interval (1.2–1.6).

We note that to observe niobium films with $d \lesssim 50$ Å, in which the superconductivity turns out to be fully suppressed on account of the high oxygen content, the resistance increases logarithmically with decreasing T, and the logarithm is preceded by a coefficient $A \approx 1$ that does not depend on the value of H (Ref. 23). For such films, just as for normal-metal films, $|g| \ll 1$ at any temperature and the $R(T)$ dependence is determined mainly by effects of localization and interaction in the diffusion channel. The field-independent values of $A \approx 1$ indicate here that the temperature dependence of the localization contribution is suppressed by processes of scattering with spin flip.

6. ELECTRON ENERGY RELAXATION TIMES

We discuss now the obtained temperature dependences of the phase relaxation time τ_φ. The very fact that τ_φ depends on temperature is evidence that in practically the entire temperature interval $T \approx (1.5-30)$K the phase relaxation time of the wave function of the electron coincides with the time τ_ϵ of the inelastic scattering of the electron, since τ_ϵ should not depend on T [See Eq. (3)]. The observed $\tau_\varphi(T)$ plots for a the investigated samples with relatively large values of d and l turn out to be close in form to $\tau_\varphi \propto T^{-2}$ in the entire temperature range, just as in the case of films of normal metals with comparable values of the parameters (Refs. 5 and 7). With decreasing d and l in the low-temperature region, the form of the $\tau_\varphi(T)$ dependences changes— a temperature interval in which $\tau_\varphi \propto T^{-1}$, which increases with decreasing d and l, appears. Such a change in the form of the $\tau_\varphi(T)$ plot is illustrated in Figs. 3 and 4, where plots of $\tau_\varphi(T)$

for Al and Nb samples with different d and l are shown. It must be noted that the measured values of τ_φ in aluminum and niobium films with d and l of the same order, as well as in copper, gold, and silver films,[5,7] are practically the same. With decreasing film, τ_φ has a tendency to decrease.

The electrons can be inelastically scattered by phonons (τ_ϵ^{ph}) electrons (τ_ϵ^e), and vibrating impurity ions (τ_ϵ^i). It will be shown below that these mechanisms lead in the case of the investigated films to different power-law dependences of $\tau_\varphi(T)$. Therefore the observed change of the exponent of the $\tau_\varphi(T)$ dependence points to the onset of a different inelastic scattering process.

Consideration of the interelectron interaction with account taken of the impurity scattering in the two-dimensional case ($d \ll L_T$) leads to the following expression[24]:

$$(\tau_\epsilon^e)^{-1} = \frac{e^2 R_\square}{2\pi^2\hbar} kT \ln \frac{\pi\hbar}{e^2 R_\square} + \frac{\pi(kT)^2}{8\hbar E_F}, \qquad (10)$$

where E_F is the Fermi energy. The first and second terms in the right-hand side of (10) reflect contributions corresponding to scattering with small and large (compared with $\hbar/l$) momentum transfers. Estimates show that for the investigated films the values of τ_ϵ^e are determined by scattering with small momentum transfers, and consequently $\tau_\epsilon^e \propto T^{-1}$. The table lists the values of τ_ϵ^e (10 K) calculated from Eq. (10). Comparison with the experimental data shows that for thin and dirty films (samples Al 2 and Nb 2) at $T \lesssim 10$ K the temperature dependence of τ_φ agrees with the theoretical one for τ_ϵ^e, and the values of τ_ϵ^e calculated from Eq. (10) turn out to be several times smaller than the experimentally obtained values of τ_φ. For thicker films (samples Al 1 and Nb 1), both the temperature dependences of τ_φ and τ_ϵ^e and their absolute values are in poorer agreement at $T \gtrsim 1$ K, and for such films the role of the interelectron collisions and of the electron-energy relaxation is apparently small at temperatures higher than 1 K.

The results of the analysis of the electron-phonon mechanism of the energy relaxation depend on the ratio of the thermal-phonon wavelength λ_T to l, and also on the dimensionality of the phonon distribution function in the film. (The condition $\lambda_T \gtrsim d$ is realized for the investigated films at $T \lesssim 20-40$ K, therefore the phonons can be either two- or three-dimensional, depending on the matching of the film to the substrate.) In the temperature interval $(u/v_F)T_D \ll T \ll (u/v_F)(\hbar/k\tau)$ of interest to us (u is the speed of sound), scattering by three-dimensional phonons should yield

$$\tau_\epsilon^{ph} \sim \frac{1}{E_F\tau}\frac{T_D^3}{T^4},$$

and in the case of two-dimensional phonons[2)]

$$\tau_\epsilon^{ph} \sim \frac{1}{E_F\tau}\frac{T_D^2}{T^3}.$$

In either case the temperature dependence of τ_ϵ^{ph} turns out to be stronger than observed in experiment. At the same time, scattering by vibrating impurity fields in the case of a three-dimensional phonon spectrum should lead[25] to energy relaxation at a rate

$$\frac{1}{(\tau_\epsilon^i)} \sim \frac{1}{E_F\tau}\frac{T^2}{T_D}.$$

Estimates of the absolute values of τ_ϵ^i are in reasonable agreement with the values of τ_φ obtained in the temperature region where $\tau_\varphi \propto T^{-2}$. Thus, the $\tau_\varphi \propto T^{-2}$ dependence observed at high temperatures is apparently connected with scattering by vibrating impurities, whereas at low temperatures the principal role in the energy relaxation is played by electron-electron scattering. We note that the obtained values of $\tau_\epsilon(T)$ in thin Al and Nb films agree with the results obtained in the study of nonequilibrium processes in superconducting films of these metals.[26–29]

7. CONCLUSION

We note the main features of the low-temperature behavior of two-dimensional superconductors at $T - T_c \gg \hbar/k\tau_\varphi$ as functions of the magnetic field and the temperature. At $T \gg T_c$ the AMR of such films, just as of normal-metal films, is determined mainly by localization effects, owing to the smallness of $|g(T)|$. As T approaches T_c and with simultaneous satisfaction of the condition $H < ckT/eD$, the values of $|g|$ increase strongly, and this leads to a dominant contribution of the superconducting fluctuations to the AMR. In the strong-field region ($H > ckT/eD$) the temperature dependence of g is suppressed by the magnetic fields and the contributions to the AMR from the Cooper interaction channel and from the superconducting fluctuations, which are of opposite sign, cancel each practically completely. In this region of H the dependence of the resistance on the magnetic field is also connected with localization effects. The temperature dependence of the resistance at $H=0$ is due to effects of both the interelectron interaction and to localization. In this case the principal role is played at $T \lesssim 3T_c$, just as when the magnetoresistance is considered, by the Maki-Thomson corrections, which lead to an increase of R with decreasing temperature. In a strong magnetic field the $R(T)$ dependence is determined, just as in the case of films of normal metals, by the interaction in the diffusion channel.

Thus, the low-temperature dependence of the behavior of the conductivity of two-dimensional films of superconductors is well described, at temperatures above critical, by the existing theory of effects of localization and interelectron interaction in disordered system, and an experimental investigation of these effects makes it possible to determine the temperature dependence and the absolute values of the characteristic relaxation time of the electrons and the parameters of the interelectron interaction.

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[1)] At $\Delta R/R \ll 1$ it is possible to replace $\Delta\sigma$ in Eqs. (1), (2), (4), (5), and (7) by $\Delta R/RR_\square$. Accordingly, the values of $\Delta R/R$ in Figs. 1, 2, and 7 are renormalized to the value $e^2R_\square/2\pi^2\hbar$.

[2)] This circumstance was pointed out to us by Yu. M. Reĭzer.

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