Contribution to the theory of electromagnetic showers in crystalline media

A. I. Akhiezer and N. F. Shul'ga

Physicotechnical Institute, Ukrainian Academy of Sciences, Kharkov
(Submitted 19 January 1983)
Zh. Eksp. Teor. Fiz. **85**, 94–108 (July 1983)

Evolution of electromagnetic showers in crystalline media is considered. It is shown that a shower can develop in a crystal over a much shorter length than in an amorphous medium. It is established that at sufficiently high particle energies the cascade functions in the crystal differ only by a scale factor from those in an amorphous medium. The cascade functions are found for the region of relatively low photon energies, when coherent effects manifest themselves in the shower development only when particles radiate in the crystal. The probability of appearance of a large number of shower particles over a radiation length in a polycrystal is estimated.

PACS numbers: 61.80. – x

§1. INTRODUCTION

The cascade theory of electromagnetic showers, first proposed by Bhabha and Heitler[1] and by Carlson and Oppenheimer[2] and subsequently developed by Landau and Rumer[3] and by Tamm and Belen'kiĭ,[4] is applicable to the case when the substance in which the shower develops is amorphous. In this case, according to the equations of cascade theory, the shower develops over the radiation length.

The present paper is devoted to a theoretical investigation of the development of electromagnetic showers in crystalline media.

It was shown in Refs. 5–7 that when high-energy particles interact with crystals coherent and interference effects can be caused by radiation and production of electron-positron pairs, and that owing to these effects the probabilities of radiation and pair production at high energies can exceeed considerably in crystals the corresponding probabilities for an amorphous medium. It is clear that coherent and interference effects should manifest themselves also in the presence of electromagnetic showers in crystals.

The first attempt to study the development of electromagnetic showers in crystalline media was undertaken in Ref. 6 in connection with observation, in cosmic rays, of several events with anomalously large numbers of particles (electron-positron pairs) on the radiation length[8–11] (the so-called Schein showers). Namely, it was assumed that these events are due to the fact that the shower develops in a polycrystal rather than in an amorphous medium. In this case, however, by far not all the possibilities were investigated, and the cases considered led in essence, for a shower passing through a polycrystal, to the same results as in an amorphous medium.

It was suggested in Ref. 12 that the Schein anomalous showers are connected with the radiation of electrons and positrons channeling in a polycrystal.

An investigation of the spatial distribution of the ionization in cascades produced by high-energy muons in lead has revealed the so-called "short" cascades, i.e., cascades in which particle absorption is faster than in ordinary electromagnetic cascades.[17] In the study of the factors that lead to such cascades, attention was called to the fact that in the experiment the shower developed not in an amorphous medium but in a polycrystal.[17]

It was shown in Ref. 18 that coherent effects can occur in the development of a shower in a polycrystal only if the polycrystal grain dimensions are large enough. That is to say, it is necesary that the shower have time to develop within the confines of individual grains of the polycrystal. Attention was called in Refs. 18 and 19 to the fact that coherent effects can arise in the course of shower development not only when the particles interact with the polycrystal, but also in the case when the shower develops in a single crystal.

We develop in the present paper a theory of electromagnetic showers in crystalline media, with consistent account taken of coherent effects in the interaction of the particles with the crystal-lattice atoms. We show that owing to these effects the length over which the shower develops in the crystal can be much shorter than the radiation length, and that coherent effects in the development of the shower in the crystal can take place at energies attainable with modern accelerators.

In §2 we present the general equations of the cascade theory of showers and discuss the peculiarities of the solution of these equations in the case of particles moving in an amorphous medium, in a crystal, and in an intense external field. In addition, we obtain in this section equations for the case when production of electron-positron pair can be disregarded and when the characteristic photon-emission frequencies are low.

In Sec. 3 is determined the cascade function of the electrons at low energy transfers and when the production of electron-positron pairs is neglected. It becomes possible in this case to consider from a unified point of view such questions as the ionization energy lost by fast particles in a medium, the electron energy losses in a synchrotron with fluctuations taken into account, and the formation of photon showers in single crystals.

In §4 we investigate the development of electromagnetic showers when particles move near the crystallographic axes in the absence of channeling, and show that if the energy of the particles that participate in the shower are high enough the shower functions in a crystal differ only by a

 0038-5646/83/070055-08$04.00

scale factor from the shower functions in an amorphous medium.

In §5 is investigated shower development in crystals at relatively low photon energies, when the influence of the production of electron-positron pairs on the shower development can be disregarded.

In §6 we consider shower development in polycrystals.

§2. BASIC EQUATIONS OF CASCADE THEORY

We denote by $\Gamma(\omega)d\omega$ and $\Pi(\varepsilon)d\varepsilon$ the numbers of photons and electrons and positrons in the energy intervals $(\omega, \omega+d\omega)$ $(\varepsilon, \varepsilon+d\varepsilon)$ at a penetration depth t of the particles in the medium. It is known then[3] that the development of an electromagnetic shower in a medium at high energies is determined by the system of equations

$$\frac{d\Pi(\varepsilon)}{dt}=2\int_{\varepsilon}^{\infty}du\Gamma(u)\gamma(u,\varepsilon)+\int_{\varepsilon}^{\infty}du\Pi(u)\pi(u,u-\varepsilon)-\int_{0}^{\varepsilon}du\Pi(\varepsilon)\pi(\varepsilon,\varepsilon-u), \tag{2.1a}$$

$$\frac{d\Gamma(\omega)}{dt}=\int_{\omega}^{\infty}du\Pi(u)\pi(u,\omega)-\int_{0}^{\infty}du\Gamma(\omega)\gamma(\omega,u). \tag{2.1b}$$

Here $\pi(u,\omega)$ and $\gamma(\omega,u)$ are the probabilities, per unit length, of emission of a photon of energy ω by an electron of energy ε and of formation, by the photon, of an electron-positron pair with electron energy[1)] u. These quantities are connected with the cross sections $d\sigma_\gamma(u,\omega)$ for photon emission and $d\sigma_\pm(\omega, u)$ for electron-positron pair production by the relations

$$\pi(u,\omega)=n\frac{d\sigma_\gamma(u,\omega)}{d\omega},\qquad \gamma(\omega,u)=n\frac{d\sigma_\pm(\omega,u)}{du}, \tag{2.2}$$

where n is the number of atoms per unit volume.

Equations (2.1) are independent of where the shower develops, in an amorphous or crystalline medium. This factor governs only the structure of the quantities π and γ. Equations (1.1) can be used also when the shower develops in a given external field, in which case it is necessary to know only the corresponding probabilities π and γ.

In the case of an amorphous medium the values of π and γ are determined by the known Bethe and Heitler equations[3,7]:

$$\begin{aligned}\pi_{\text{B-H}}(\varepsilon,\omega)&=\frac{1}{L}\frac{\varepsilon^2+(\varepsilon-\omega)^2-{}^2/_3\varepsilon(\varepsilon-\omega)}{\varepsilon^2\omega}\\ \gamma_{\text{B-H}}(\omega,\varepsilon)&=\frac{1}{L}\frac{\varepsilon^2+(\omega-\varepsilon)^2+{}^2/_3\varepsilon(\omega-\varepsilon)}{\omega^3},\end{aligned} \tag{2.3}$$

where

$$L^{-1}=4e^6Z(Z+1)nm^{-2}\ln(183Z^{-1/3}),$$

L is the radiation length, $z|e|$ is the charge of the atom of the medium, and m is the electron mass. Inasmuch as in the case of an amorphous medium L is the only quantity with the dimension of length in Eqs. (2.1), it is natural for the shower to develop in an amorphous medium over a length of the order of the radiation length.

When fast particles move in crystals the quantities π and γ are much more complicated functions of the particle energy than in an amorphous medium.[7] The reason is the presence of coherence and interference effects that manifest themselves in the motion of high energy particles at small angles to the crystal axes or planes. Owing to these effects, the probabilities of radiation and pair production in the crystal can exceed considerably those in an amorphous medium. Consequently the development of electromagnetic shower in a crystal can also proceed more intensively than in an amorphous medium.

The study of photon emission by fast particles in a crystal is similar in many respect to the problem of photon emission by high-energy particles in an external macroscopic field.[20] Thus, in a number of cases the probability of formation of electron-positron pairs in a crystal turns out to be much less than the photon-emission probability. The situation is similar as a rule even in strong external fields. In these cases the cascade-theory equations become much simpler:

$$\frac{d\Pi(\varepsilon)}{dt}=\int_{\varepsilon}^{\infty}du\Pi(u)\pi(u,u-\varepsilon)-\int_{0}^{\varepsilon}du\Pi(\varepsilon)\pi(\varepsilon,\varepsilon-u), \tag{2.4a}$$

$$\frac{d\Gamma(\omega)}{dt}=\int_{\omega}^{\infty}du\Pi(u)\pi(u,\omega). \tag{2.4b}$$

We see that here the distribution in electron energy does not depend on the photon distribution and is determined by the first equation of (2.4). Finding from it the function we can obtain in accord with (2.4b) the distribution in photon energy $\Gamma(\omega)$.

Equation (2.4a) can obviously be rewritten in the form

$$\frac{d\Pi(\varepsilon)}{dt}=\int_{0}^{\infty}d\omega[\Pi(\varepsilon+\omega)\pi(\varepsilon+\omega,\omega)-\Pi(\varepsilon)\pi(\varepsilon,\omega)]. \tag{2.5}$$

If the characteristic photon-emission frequencies are low $(\omega\ll\varepsilon)$, we have

$$\pi(\varepsilon+\omega,\omega)\Pi(\varepsilon+\omega)\approx\Pi(\varepsilon)\pi(\varepsilon,\omega)+\omega\frac{\partial}{\partial\varepsilon}[\Pi(\varepsilon)\pi(\varepsilon,\omega)]+\frac{1}{2}\omega^2\frac{\partial^2}{\partial\varepsilon^2}[\Pi(\varepsilon)\pi(\varepsilon,\omega)]+\dots \tag{2.6}$$

and Eq. (2.5) takes the form

$$\frac{d\Pi(\varepsilon)}{dt}=\frac{\partial}{\partial\varepsilon}[\Pi(\varepsilon)\bar{E}(\varepsilon)]+\frac{1}{2}\frac{\partial^2}{\partial\varepsilon^2}[\Pi(\varepsilon)\bar{E}^2(\varepsilon)], \tag{2.7}$$

where

$$\bar{E}(\varepsilon)=\int_0^\infty d\omega\omega\pi(\varepsilon,\omega),\qquad \bar{E}^2(\varepsilon)=\int_0^\infty d\omega\omega^2\pi(\varepsilon,\omega). \tag{2.8}$$

If the total energy lost by the particle to radiation is low, the quantities $\pi(\varepsilon+\omega,\omega)$ and $\pi(\varepsilon,\omega)$ in (2.5) can be replaced by $\pi(\varepsilon_0,\omega)$, where ε_0 is the initial energy of the electron. In this case (2.5) takes the form

$$\frac{d\Pi(\varepsilon)}{dt}=\int_0^\infty d\omega\pi(\varepsilon_0,\omega)[\Pi(\varepsilon+\omega)-\Pi(\varepsilon)]. \tag{2.9}$$

An equation of this type was first obtained and used by Landau[21] to determine the distribution function of fast particles that lose energy to ionization in a thin layer of matter.

We note that the Landau equation (2.9) goes over at $\omega \ll \varepsilon$ into Eq. (2.7) if ε replaced by ε_0 in the expressions $\overline{E}(\varepsilon)$ and $\overline{E}^2(\varepsilon)$ in this equation. Clearly, such a replacement is valid only for particles passing through a thin layer of matter.

Equations (2.4)–(2.9) are simpler than the inital Eqs. (2.1) of cascade theory, and will therefore be the starting point of our analysis.

§3. CASCADE ELECTRON FUNCTION IN THE CASE OF SMALL ENERGY TRANSFERS

A formal solution of the Landau equation (2.9), satisfying the condition

$$\Pi(\varepsilon)|_{t=0} = \delta(\varepsilon - \varepsilon_0),$$

is known to be of the form

$$\Pi(\varepsilon)=\frac{1}{2\pi i}\int\limits_{-i\infty+\sigma}^{i\infty+\sigma} dp\exp[p(\varepsilon_0-\varepsilon)-tW(p)], \tag{3.1}$$

where

$$W(p)=\int\limits_0 d\omega\pi(\varepsilon_0,\omega)(1-e^{-p\omega}),$$

$\delta(\varepsilon-\varepsilon_0)$ is a delta function, and $\sigma > 0$.

If the characteristic frequencies of photon emission by an electron in an external field are small compared with the electron energy, the function $W(p)$ can be expanded in powers of p. Retaining the first two terms of the expansion we find that

$$W(p)=p\overline{E}_0-{}^1/_2p^2\overline{E_0^2}, \tag{3.2}$$

where $\overline{E}_0 = \overline{E}(\varepsilon_0)$ and $\overline{E}_0{}^2 = \overline{E}^2(\varepsilon_0)$. We then obtain for $\Pi(\varepsilon)$

$$\Pi(\varepsilon)=(2\pi\overline{E_0^2}t)^{-1/2}\exp\{-(\varepsilon_0-t\overline{E}_0-\varepsilon)^2/2t\overline{E_0^2}\}. \tag{3.3}$$

It is clear that the same expression can be obtained from (2.7) by replacing in the latter $\overline{E}(\varepsilon)$ and $\overline{E}^2(\varepsilon)$ by $\overline{E}_0$ and $\overline{E}_0^2$ respectively.

The quantities $(t\overline{E}_0)$ and $(t\ \overline{E_0^2})$ in (3.3) determine the mean values of the particle energy lost to radiation and the mean fluctuations of the energy lost to radiation at the depth t.

In the particular case of motion of a relativitic electron along a circle in a magnetic field, the function $\pi(\varepsilon,\omega)$ takes the form[22]

$$\pi_H(\varepsilon,\omega)=\frac{3^{1/2}}{2\pi}\frac{e^3H}{m\omega}F\left(\frac{\omega}{\omega_H}\right),\quad F(\xi)=\xi\int\limits_\xi^\infty d\zeta K_{5/3}(\zeta), \tag{3.4}$$

where H is the magnetic field strength,

$$\omega_H = 3\varepsilon^3/2m^2R_H,$$

$R_H = \varepsilon/eH$ is the radius of the electron orbit and $K_{5/3}(\zeta)$ is a Bessel function of imaginary argument.

According to (2.8) we have then

$$\overline{E}_0=\frac{2e^4H^2}{3m^2}\left(\frac{\varepsilon_0}{m}\right)^2,\quad \overline{E_0^2}=\frac{55}{3^{1/2}\cdot 24}\frac{e^4H^2}{m^2R_H}\left(\frac{\varepsilon_0}{m}\right)^5. \tag{3.5}$$

Knowing $\overline{E_0^2}$ we easily find the fluctuation of the radius of an electron orbit in a synchrotron

$$\Delta\overline{R_H^2}=\overline{R_H^2}-(\overline{R}_H)^2=\frac{55}{3^{1/2}\cdot 24}\frac{e^2}{m^2R_H}\left(\frac{\varepsilon_0}{m}\right)^5 t. \tag{3.6}$$

(This expression for first obtained by Sokolov and Ternov[23] by another method[2)]).

If we disregard in (2.7) the fluctuations of the electron energy losses, (2.7) takes the form

$$\frac{d\Pi(\varepsilon)}{dt}=\frac{\partial}{\partial\varepsilon}[\overline{E}(\varepsilon)\Pi(\varepsilon)]. \tag{3.7}$$

The solution of this equation can be obtained for an arbitrary value of the electron energy loss in an external field (see, e.g., Ref. 24):

$$\Pi(\varepsilon)=\frac{1}{\overline{E}(\varepsilon)}\delta\left(t-\int\limits_\varepsilon^{\varepsilon_0}\frac{du}{\overline{E}(u)}\right). \tag{3.8}$$

Knowing $\Pi(\varepsilon)$, we can find according to (2.4b) the change of the number of photons with penetration depth t:

$$\Gamma(\omega)=\int\limits_0^t dt'\pi(\bar{\varepsilon}(t'),\omega),\qquad \bar{\varepsilon}(t)=\int\limits_0^\infty d\varepsilon\varepsilon\Pi(\varepsilon), \tag{3.9}$$

where $\bar{\varepsilon}(t)$ is the mean electron energy at the depth t. The quantity $\bar{\varepsilon}(t)$ satisfies according to (3.8) the equation

$$d\bar{\varepsilon}(t)/dt=-\overline{E}(\varepsilon). \tag{3.10}$$

In the particular case of an ultrarelativistic electron moving in an electric field of intensity $\mathbf{E}$ the value of E is given by 22

$$\overline{E}(\varepsilon)=2e^4\mathbf{E}_\perp{}^2\varepsilon^2/3m^4, \tag{3.11}$$

where $\mathbf{E}_\perp$ is the $\mathbf{E}$ component orthogonal to the electron velocity. In this case, according to (3.10),

$$\bar{\varepsilon}(t)=\varepsilon_0\left\{1+\varepsilon_0\int\limits_0^t dt'(2e^4\mathbf{E}_\perp{}^2/3m^4)\right\}^{-1}. \tag{3.12}$$

This formula was first obtained by Pomeranchuk.[25] It is valid if the characteristic frequencies of the radiation are low and, in addition, the electron-positron pair-production probability is negligibly small.

§4. EVOLUTION OF ELECTROMAGNETIC SHOWER PRODUCED IN A CRYSTAL BY A PARTICLE MOVING ALONG A CRYSTALLOGRAPHIC AXIS

Proceeding to the investigation of electromagnetic-shower evolution in crystalline media, we consider first the case when there is no particle channeling in the crystal. To this end it is necessary that the angle ψ between the incident beam and the crystallographic axis (the z axis) exceed the critical axial-channeling angle $\psi_c = (4Ze^2/\varepsilon d)^{1/2}$, where d is the distance between the atoms along the z axis.[26]

The cross section for emission of a relativistic particle in a crystal is determined by the following general formula[7]:

$$d\sigma=d\sigma_{coh}+d\sigma_1, \tag{4.1}$$

where $d\sigma_{coh}$ and $d\sigma_1$ are respectively the coherent and noncoherent parts of the radiation cross sections. In the absence of channeling $d\sigma_1$ hardly differs from the cross section for electron scattering by an isolated atom. The coherent part of the radiation cross section in the absence of channeling is given by

$$\frac{d\sigma_{coh}}{d\omega}=\frac{(2\pi)^3}{\Delta}\sum_{g}|S(g)|^2\exp(-\bar{u}^2g^2)\frac{d^4\sigma_{B-H}}{dg\,d\omega}, \qquad (4.2)$$

where Δ is the volume of the unit cell of the crystal, $S(\mathbf{g})$ is the structure factor, $\mathbf{g}$ are the reciprocal-lattice vectors, $\overline{u^2}$ is the mean square of the thermal vibrations of the lattice atoms, and $d^4\sigma_{B-H}/d\mathbf{g}d\omega$ is the differential cross section, with respect to the momentum transfers and the frequencies, for electron emission by a single atom of the crystal. The summation over g in (4.2) is carried out with account taken of the condition $g_\| \geqslant \sigma$, where $\delta = m^2\omega/2\varepsilon(\varepsilon-\omega)$ and g is the component of $\mathbf{g}$ parallel to the incident-electron momentum. This equation is valid if the conditions[20] $\psi \gg \psi_c$ and $Ze^2/m\psi d \ll 1$ are satistied (the condition that the radiation be dipolar).

The cross section for the production of an electron-positron pair by a photon in a crystal in the absence of channeling and when the condition $Ze^2/m\psi d \ll 1$ is satisfied is determined by equations similar to (4.1) and (4.2). It is necessary to make in (4.1) and (4.2) the substitutions $\omega \to -\omega$, $\varepsilon \to -\varepsilon_+$, $\varepsilon' = \varepsilon - \omega \to \varepsilon_-$ and to multiply the cross section (4.1) by

$$(\varepsilon_+/\omega)^2(d\varepsilon_+/d\omega),$$

where ε_- and ε_+ are the energies of the electron and positron.

These expressions for the cross sections for radiation and pair production should be substituted in Eq. (2.1), which determines the shower development in a crystal. The equations obtained in this manner are very complicated and can be solved in general form only with a computer. We shall be interested mainly in how the coherent effects manifest themselves in the course of shower development in a crystal. It is well known that all the coherent effects in radiation and electron-positron-pair production come into play for particles moving near crystal axes or crystal planes. A particularly simple case is that when the particles move in the crystal near one of the crystal axes but far from close-packed crystal planes. In this case simple expressions can be obtained for the cross sections for coherent radiation and pair production in the crystal. We begin with consideration of just this case.

When fast particles move in the crystal at a small angle ψ to the crystal axis z but far from close-packed crystal planes, the coherent part of the radiation cross section (4.2) can be represented in the form[20]

$$\frac{d\sigma_{coh}}{d\omega}=\frac{8Z^2e^6}{m^2\omega}\frac{\delta}{d}\frac{\varepsilon'}{\varepsilon}\sum_{g_z}\int d^2g\frac{g_\perp^2}{g_\|^2}\cdot$$
$$\times\frac{\exp(-\bar{u}^2g^2)}{(g^2+R^{-2})^2}\left[1+\frac{\omega^2}{2\varepsilon\varepsilon'}-2\frac{\delta}{g_\|}\left(1-\frac{\delta}{g_\|}\right)\right], \qquad (4.3)$$

where

$$g_\perp^2=g_x^2+g_y^2,\quad g_\|=g_z+g_x\sin\psi\geqslant\delta,$$

R is the screening radius of the atom, and $d^2g = dg_x dg_y$. A similar formula determines also the cross section for electron-positron pair production in a crystal.

Equation (4.3) shows that the periodicity of the atom arrangement in the crystal along the z axis affects strongly the radiation by the electron if the conditions $\delta^{-1} \gtrsim d/2\pi$ and $\psi \lesssim R/d$. are satisfied. The first of these conditions means that within the limits of the coherence length $l = -1$ there are many crystal atoms; the second condition stipulates that the collisions of the electron with atoms located along the z axis be correlated. If $\delta^{-1} \gg d/2\pi$ and $\psi \ll R/d$, the main contribution to (4.3) is by the term with $g_z = 0$. In this case the radiation cross section (4.3) takes at $\overline{u^2} = 0$ the form

$$d\sigma_{coh}\approx d\sigma_{B-H}\begin{cases} kR/\psi d, & \delta R/\psi \ll 1,\\ k/\delta d, & \delta R/\psi \gg 1,\end{cases} \qquad (4.4)$$

where $k = \pi/2\ln(183Z^{-1/2})$.

Figure 1 shows the results of a numerical integration of the cross section, differential with respect to frequency, for the emission of an electron (positron) in a tungsten crystal, (4.1), with $d\sigma_{coh}$ determined from (4.3). The calculations were performed for a fixed angle $\psi = 10^{-3}$ rad between the electron momentum and the crystal axis $\langle 111\rangle$ at $T = 293$ K. The ordinate is the function f connected with the cross section (4.1) by the relation $\omega d\sigma/d\omega = (Z^2e^6/m^2)f$, and the abscissa is ω/ε.

The numbers at the curves correspond to different electron energies ε in GeV, for which the calculations were made. The dash-dot curve corresponds to the cross section of electron radiation on a single atom; the dashed curve corresponds to the incoherent part of the radiation cross section (4.1).

The corresponding functions for the cross sections for electron-positron pair production by a photon in a tungsten crystal are shown in Fig. 2. The numbers at the curves denote the photon energies ω in GeV.

The plots presented show that the differential cross sections for radiation and electron-positron pair production exceed substantially, in a wide energy interval, the corresponding values for an amorphous region. For this reason the showers should develop more rapidly in a crystal than in an amorphous medium.

The results show also that if the condition $\delta R/\psi \ll 1$ are satisfied up to values $\omega \sim \varepsilon$, the probabilities of radiation and pair production in a crystal differ only by a numerical factor

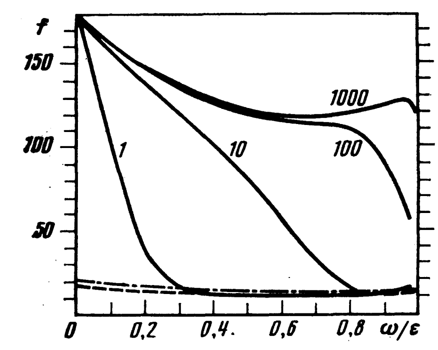

FIG. 1.

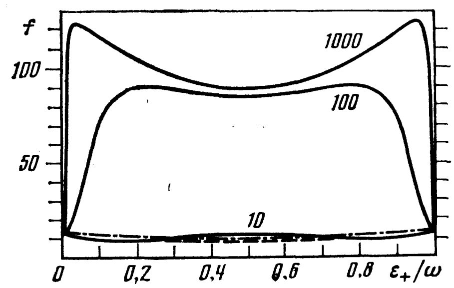

FIG. 2

$A \sim R/\psi d$ from the corresponding probabilities in an amorphous medium:

$$\pi \approx A\pi_{B-H}, \quad \gamma \approx A\gamma_{B-H}. \tag{4.5}$$

If $\overline{u^2} = 0$, then $A = kR/\psi d$. The corresponding shower function for a crystal can be obtained in this case directly from the shower functions of the amorphous medium by replacing in the latter the radiation length L by the modified radiation length $L^* = L/A$. In this case the shower will develop in the crystal over a length smaller by a factor $A \gg 1$ than in an amorphous medium. We emphasize that this situation obtains only when the coherence lengths of the radiation $l = \delta^{-1}$ and of the electron-positron pair production $l_\pm = 2\varepsilon_+\varepsilon_-/m^2\omega$ are large enough: $l \gg R/\psi$ and $l_\pm \gg R/\psi$. The quantity A has then a simple physical meaning—it is of the order of the number of atoms in an individual chain of crystal atoms on the coherence length ($A \sim R/\psi d$ atoms of the chain are contained in the coherence lengths l and $l_\pm$ at $l \gg R/\psi$ and $l_\pm \gg R/\psi$).

The foregoing formulas are valid if the angle ψ between the direction of electron motion in the crystal and the crystal axis changes little in the course of shower development. The change of the angle ψ is governed by two factors, by the photon radiation (in this case the change of the angle ψ is of the order of m/ε) and by multiple scattering of the electron by the thermal vibrations of the lattice atoms (and then the change of the angle ψ is of the order of

$$\Delta\psi \sim (\varepsilon_s/\varepsilon)(t/L)^{1/2},$$

where $\varepsilon_s^2 = 4\pi m^2/e^2$). For the formulas given above to be valid it is therefore necessary to satisfy the conditions

$$\psi^2 \gg \max\left(\frac{\varepsilon_s^2}{\varepsilon^2}\frac{t}{L}, \frac{m^2}{\varepsilon^2}\right). \tag{4.6}$$

If these conditions are not satisfied it is necessary, for shower development in a crystal, to take into account the redistribution of the particles with respect to the angles.

§5. SHOWER DEVELOPMENT IN A CRYSTAL AT RELATIVELY LOW PHOTON ENERGIES

In the preceding section we gave general formulas for the cross sections for radiation and pair production in a crystal and considered the shower development under conditions when the coherent effects manifested themselves substantially both in radiation and in electron-positron pair production. This situation obtains if the coherence lengths $l = \delta^{-1}$ and $l_\pm = 2\varepsilon_+\varepsilon_-/m^2\omega$ of radiation and pair production exceed considerably the lattice constant a. There exists, however, a wide range of particle energies in which coherent effects manifest themselves only for particle radiation in a crystal, but not for electron-positron pair production. This situation obtains if the energy of the photons that participate in the shower is relatively low, namely, it is necessary to satisfy the condition $\omega \lesssim m^2 a$. This condition means that the coherence length of the electron-positron pair production process be small compared with the lattice constant a. The radiation coherence length can then be large compared with a, and this is precisely the case that we shall consider.

If $l_\pm \lesssim a$, we can put in Eqs. (2.1), with good accuracy, $\gamma(\omega,\varepsilon) \approx \gamma_{B-H}(\omega,\varepsilon)$. As for the radiation, it is necessary in this case to take into account in the cross section both the term that describes the coherent effects in radiation and the term that describes incoherent effects in radiation. The latter term, as already noted, can be replaced with good accuracy by the cross section for electron radiation on an isolated atom. The quantity $\pi(\varepsilon,\omega)$ then takes the form

$$\pi(\varepsilon,\omega) \approx \pi_{B-H}(\varepsilon,\omega) + \pi_{coh}(\varepsilon,\omega), \tag{5.1}$$

where $\pi_{coh} = n(d\sigma_{coh}/d\omega)$.

The quantities $\gamma(\omega,\varepsilon)$ and $\pi(\omega,\varepsilon)$ given above must be substituted in Eqs. (2.1) that determine the shower development in a crystal. We assume next that the following conditions are satisfied:

$$m^2R/\psi \gg \varepsilon \gg m^2 d, \quad \psi \ll R/d$$

and that the shower develops at relatively small depths $t < L$. We can then neglect the production of electron-positron pairs and incoherent photon emission, i.e., the shower development is determined only by one function π_{coh}. The problem of finding the cascade functions of the particles in a crystal reduces then to finding the cascade functions in the presence of an external field.

To verify this, we estimate the average electron-energy losses in a crystal per unit length, due to coherent and incoherent effects in the radiation:

$$\bar{E}(\varepsilon) = \bar{E}_{coh}(\varepsilon) + \bar{E}_1(\varepsilon). \tag{5.2}$$

The value of the incoherent energy loss is given by

$$\bar{E}_1(\varepsilon) \approx \int_0^\varepsilon d\omega\, \omega\pi_{B-H}(\varepsilon,\omega) = \varepsilon/L. \tag{5.3}$$

The energy loss due to the coherent effect in emission is determined by the formula

$$\bar{E}_{coh}(\varepsilon) = \int_0^\varepsilon d\omega\, \omega\pi_{coh}(\varepsilon,\omega) = \eta\frac{\varepsilon}{m^2 d}\frac{\varepsilon}{L}, \tag{5.4}$$

where

$$\eta = -\frac{4\pi}{3\ln(183Z^{-1/3})}\left[1 + \left(1 + \frac{\overline{u^2}}{R^2}\right)\exp\frac{\overline{u^2}}{R^2}\operatorname{Ei}\left(-\frac{\overline{u^2}}{R^2}\right)\right] \tag{5.5}$$

and Ei (x) is the integral exponential function. [In the derivation of (5.4) we have neglected the terms that describe the recoil in emission; this is justified if the characteristic frequencies $\omega_{\rm char} \sim 2\varepsilon^2\psi/m^2R$ of the electron emission in the crystal are small compared with the electron energy ε.] At $\overline{u^2} \ll R^2$ the coefficient η is of the order of unity:

$$\eta \approx \frac{4\pi}{3\ln(183Z^{-1/3})}\left[\ln\frac{R^2}{\overline{u^2}}-1-C+o\left(\frac{\overline{u^2}}{R^2}\right)\right], \qquad (5.6)$$

where $C = 0.577$.

Comparing $\overline{E}_1(\varepsilon)$ and $\overline{E}_{\rm coh}(\varepsilon)$ we find that when the condition $\varepsilon \gg m^2 d$ is satisfied the electron energy loss in a crystal is due to coherent effects in emission. The average electron energy loss $t\overline{E}(\omega)$ in a crystal is of the order of the particle energy ε at depths $t \ll L$. Thus, if the shower is produced in the crystal by an electron of energy $\varepsilon \gg m^2 d$, the incoherent effects in emission can be disregarded at crystal thicknesses $t \ll L$, as well as the process of e^+e^- pair production. If the crystal thickness is small enough so that $t\overline{E}(\varepsilon) \ll \varepsilon$, the electron distribution in ε, with allowance for the fluctuations, will be determined by Eq. (3.3) in which we must substitute $\overline{E}(\varepsilon_0) = E_{\rm coh}(\varepsilon_0)$ and

$$\overline{E^2}(\varepsilon_0)=\frac{56\pi^{1/2}}{15\ln(183Z^{-1/3})}\frac{\varepsilon_0^2}{m^2Ld}\frac{\varepsilon_0^2\psi}{m^2(\overline{u^2})^{1/2}}. \qquad (5.7)$$

If, however, $t\overline{E}(\varepsilon) \sim \varepsilon$, the average electron energy loss at a depth t will be, in accord with (3.7) and (3.9),

$$\bar{\varepsilon}(t)=\varepsilon_0[1+(\eta\varepsilon_0 t/m^2Ld)]^{-1}. \qquad (5.8)$$

We note that $\overline{E}(\varepsilon)$ and $\bar{\varepsilon}(t)$ do not depend on the angle ψ between the electron momentum in the crystal and the crystal axis z. What depends on ψ is only the region of applicability of Eqs. (5.4) and (5.8): it is required that the characteristic emission frequencies be low enough, $\omega_{\rm char} \sim 2\varepsilon^2\psi/m^2R \ll \varepsilon$.

So far we have considered coherent effects in shower development in the case when the particles move in the crystal near one of the crystal axes. As already mentioned, coherent effects appear also for particle motion near crystal planes. The qualitative picture of the shower development in the crystal is in this case the same as in particle motion near a crystal axis. We shall therefore not study in detail the shower development for particles moving near a crystal plane. We confine ourselves only to the case when the particle energy region is such that the effect of electron-positron pair production can be neglected, i.e., the case when the problem of finding the cascade functions in a crystal reduces to the problem of finding the cascade functions for particles moving in an external field. This case is of interest also because it is possible to trace in it clearly the effect of channeling in shower development in a crystal.

If a large number of atoms of the crystal plane is contained within the limits of the coherence length l and the angle θ between the incident beam and the plane is small enough, $\theta \ll R^2/a^2$, the motion and the emission of the particles in the crystal can be described by using the continuous-plane approximation—the crystal potential averaged over the coordinates of the atoms located in the crystal plane near which the particle moves. For many elements, the interplanar potential can be approximated with good accuracy by a parabolic function[27]

$$U(x)=4U_0x^2/d_p^2, \quad |x|\leqslant d_p/2,$$

where

$$U_0=2\pi\nu nRd_pZe^2,$$

x is the coordinate perpendicular to the plane, d_p is the distance between the planes, and ν is a numerical coefficient of the order of unity and is determined from the condition of best approximation of the interplanar potential by a parabolic function. If there is no particle channeling in the crystal, i.e., if $\theta \gg \theta_c$, where $\theta_c = (2U_0/\varepsilon)^{1/2}$ is the critical angle of planar channeling, and if the characteristic emission frequencies $\omega \sim 2\varepsilon^2\theta/m^2d_p \ll \varepsilon$ are low, the probability per unit length for coherent emission of an electron (positron) in a crystal is of the form.[20]

$$\pi_{coh}(\varepsilon,\omega)=\frac{16e^2U_0^2}{\pi^3m^2\theta d_p}\frac{1}{\omega}\frac{\delta'}{g\theta}\sum_n\frac{1}{n^4}\left[1-\frac{2\delta'}{ng\theta}\left(1-\frac{\delta'}{ng\theta}\right)\right], \qquad (5.9)$$

where

$$g=2\pi/d_p, \quad \delta'=\omega m^2/2\varepsilon^2 \text{ and } n\geqslant\delta'/g\theta.$$

The total loss of electron energy per unit path, due to coherent effects in emission, is according to (5.9)

$$\overline{E}_{coh}(\varepsilon)=\eta_p\frac{nR^2\varepsilon}{m^2}\frac{\varepsilon}{L}, \quad \eta_p=\frac{32\pi^2\nu^2}{9\ln(183Z^{-1/3})}. \qquad (5.10)$$

Comparing this expression for $\overline{E}_{\rm coh}(\omega)$ with $\overline{E}_1(\varepsilon)$, we find that at $10nR^2\varepsilon/m^2 \gg 1$ the main electron energy loss is due to coherent process in electron emission from crystal-plane atoms. In this case the electron distribution in energy at a depth t will be determined by the equations of §3, in which we must put $\overline{E}(\varepsilon) = \overline{E}_{\rm coh}(\varepsilon)$ and

$$\bar{\varepsilon}(t)=\varepsilon_0[1+(\eta_p nR^2\varepsilon_0 t/m^2L)]^{-1}. \qquad (5.11)$$

Substituting in (3.8) the so-obtained electron distribution in ϵ in the crystal, we can obtain the the photon distribution in the frequency ω at the depth t.

An important quantity that characterizes the process of electron emission in a crystal is the total number of photons emitted by an electron at a depth t. Let us estimate the length over which the number Γ of the emitted photons in the frequency interval $\Omega \leqslant \omega \leqslant \varepsilon_0$ ($\Omega \ll \varepsilon_0$) exceeds unity as the particles move near the crystal planes. The quantity Γ can be represented in the form

$$\Gamma=\Gamma_{coh}+\Gamma_1, \qquad (5.12)$$

where $\Gamma_{\rm coh}$ and Γ_1 are the numbers of photons produced on account of coherent and incoherent processes in radiation. If $\Omega \ll 2\varepsilon^2\theta/m^2d_p$ and $t \ll L$, we have according to (2.3)

$$\Gamma_1\approx t\int_\Omega^{\varepsilon_0} d\omega\,\pi_{B-H}(\varepsilon_0,\omega)=\frac{t}{L}\left(\frac{4}{3}\ln\frac{\varepsilon_0}{\Omega}+\frac{1}{6}\right). \qquad (5.13)$$

According to (5.9) the total number of photon coherently emitted in the case of motion near a crystal plane is determined by the expression

$$\Gamma_{coh}=t\int_\Omega^{\varepsilon_0} d\omega\,\pi_{coh}(\varepsilon_0,\omega)=t\frac{32e^2U_0^2}{3\pi^3m^2\theta d_p}. \qquad (5.14)$$

Comparing Γ_1 and $\Gamma_{\rm coh}$ we see that emission of more than one photon in a crystal on acccount of coherent processes takes place at a depth t much smaller than the radiation length, namely, $\Gamma \sim 1$ at $t \sim (\theta d_p{}^2/R^2)L$. We see thus that can develop a photon shower even in sufficiently thin single crystals ($t \ll L$).

The results above pertain to the case when there is no particle channeling in the crystal. We dwell now briefly on an elucidation of the role of the particle channeling in the development of an electromagnetic shower in a crystal.

In channeling, the particles do not pass at close distances to the nuclei of the lattice atoms, therefore for channeled particles the term with $d\sigma_1$ in the cross sections for radiation a pair production will be suppressed compared with the corresponding cross sections for an isolated atom. It turns out as a result that in channeling the role of coherent effects in the evolution of showers in crystals is enhanced.

We consider now in greater detail shower development for channeled positrons moving along crystal planes that have a parabolic distribution of the interplanar potential.

In this case, in the dipole approximation,[13]

$$\pi(\varepsilon,\omega)\approx\pi_{coh}(\varepsilon,\omega)=\frac{4e^2U_0{}^2}{3m^2\theta_c d_p}\frac{1}{\omega}\zeta[1-2\zeta(1-\zeta)]\Theta(1-\zeta), \tag{5.15}$$

where $\zeta=\delta' d_p/2\theta_c$ and $\Theta(x)$ is the step function.

Substituting the radiation probability (5.15) in (2.4) we obtain equations that describe the development of a photon shower in a crystal by channeled positrons neglecting the process of formation of electron-positron pairs. For channeled positrons we have then

$$\bar{E}(\varepsilon)=8\eta_p\frac{nR^2\varepsilon}{m^2}\frac{\varepsilon}{L},\quad \bar{E}^2(\varepsilon)=\frac{7}{10}\frac{\varepsilon^2\theta_c}{m^2d_p}\bar{E}(\varepsilon) \tag{5.16}$$

and[3)]

$$\varepsilon(t)=\varepsilon_0[1+(8\eta_p nR^2\varepsilon_0 t/m^2L)]^{-1}. \tag{5.17}$$

The total number of photons emitted by channelled positrons at small depths is determined by the relation

$$\Gamma\approx t(8e^2U_0{}^2/9m^2\theta_c d_p). \tag{5.18}$$

From this relation we find that $\Gamma\sim 1$ at

$$t\sim(\theta_c d_p{}^2/R^2)L\ll L.$$

§6. SHOWER DEVELOPMENT IN POLYCRYSTALS

We have investigated above the evolution of showers in single crystals. It was shown that at high particle energies the shower can evolve in the crystal over short lengths

$$t\sim(\psi d/R)L\ll L.$$

In the case of a lead crystal, e.g., at $\varepsilon = 10^3$ GeV and $\psi = 10^{-4}$ rad, the length $L^* \sim (\psi d/R)L$ over which the shower evolves is of the order of $L^* \sim 10\ \mu$m. Under these conditions, obviously, for the effect to be observable it suffices for the crystal dimensions to be of the same order as L^*. In other words, there is no need for large crystals and we arrive at the problem of shower development not in a single crystal, but in a polycrystal whose grains (crystallites) should be of the order of or larger than L^*.

For shower development in a polycrystal, the cascade-theory equations (2.1) must be averaged over the crystallite orientations. If the crystallites are very small ($r \ll L^*$) the shower development in a polycrystal will not differ from that in an amorphous medium. Indeed, in this case the change of the shower functions within the confines of an individual crystallite will be negligible, and what should consequently be averaged in Eqs. (2.1) are the probabilities π_{coh} and γ_{coh}. The probabilities $\pi_{coh}(\varepsilon,\omega,\mathbf{q})$ and $\gamma_{coh}(\varepsilon,\omega,\mathbf{q})$, which are differential with respect to the momentum transfers and the energies, differ from the corresponding probabilities for an amorphous region only by the factor[7]

$$B=\frac{1}{N}\sum_{n,k}\exp\{i\mathbf{q}(\mathbf{r}_n-\mathbf{r}_k)\}, \tag{6.1}$$

where N is the number of atoms in the crystallites, $\mathbf{r}_n$ are the positions of the atoms in the lattice, and $\mathbf{q}$ is the momentum transfer. Averaging B over all the crystallite orientations we can easily show that $\bar{B}$ hardly differs from unity.[6,7] If, for example, the particle interaction takes place with an individual chain of N atoms of the crystal, which are disposed along the z axis, we have

$$\bar{B}=\frac{1}{4\pi N}\int do\sum_{n,k}\exp\{iqd(n-k)\cos\vartheta\}$$

$$=1+\frac{\pi}{qd}\left(1-\frac{qd}{2\pi}+2\left[\frac{qd}{2\pi}\right]\right), \tag{6.2}$$

where ϑ is the angle between $\mathbf{q}$ and the axis z of the chain and $[qd/2\pi]$ is the integer part of $qd/2\pi$. Since the main contribution to the cross sections for radiation and electron-positron pair production is made by values $R^{-1} \lesssim q \lesssim m$, the correction to unity in $\bar{B}$, due to the periodicity of the atoms in the chain, is negligibly small. The mean values of the probabilities of the radiation and pair production in the crystallite will be practically the same as the probabilities $\pi_{B\text{-}H}$ and $\gamma_{B\text{-}H}$ and, consequently, the evolution of the shower in the polycrystal will proceed as in an amorphous medium. In other words, in this case there should appear, in the crystal, over a length of the order of the radiation length L, at least one photon or one electron-positron pair. This result is valid of the crystallites are small enough, namely, if the grain dimensions r are small compared with the length L^* over which the shower develops in the crystal.

If, however, the crystallite size is comparable with the length L^*, this conclusion is wrong, for in this case the shower functions will vary significantly within the limits of individual crystallites. Over a length of the order of L there can then be produced not one particle but a large number of particles—photons and electron-positron pairs. Thus, for a shower to develop rapidly in a polycrystal it is necessary that its grain sizes be comparable with or larger than the length L^*.

For a large number of shower particles to be produced over a length L in a polycrystal it is necessary that over this length at least one crystallite with $r \gtrsim L^*$ have a small angle between the crystallographic axis (or plane) and the particle momentum. It is clear that over the length L such a collision of a particle with a crystallite will be random. We estimate in this connection the probability of the appearance of a large number of shower particles in a polycrystal over the radiation length.

Coherent effects in shower development in a crystal, as already mentioned, appear not only when the particles move at a small angle $\psi \ll R/d$ to one of the crystallographic axes, but also when the angle θ ($\theta \ll R^2/d_p^2$) between the particle momentum and one of the crystallographic planes is small. We estimate first the probability of the particle colliding over the length L with a crystallite of size $r \gtrsim L^*$ at a small angle ψ to its close-packet crystal axes. This probability is of the order of

$$W_r \sim N_r(2\psi^2)(L/r), \tag{6.3}$$

where N_r is the number of crystallographic axes, closely packed with atoms, in the crystallite. The quantity $2\psi^2$ is the ratio of the solid angle, in which the angle between the particle momentum and one of the close-packed crystal axes is less than ψ, to the total solid angle. The factor L/r is of the order of the number of crystals with which the particle collides over ;the length L. At $r \sim L^*$, where $L^* \sim (\psi d/R)L$, we have according to (6.3) $W_r \sim N_r(R\psi/d)$.

We obtain similarly the probability that over a length L the particle will collide with a crystallite of size $r \sim L_p^*$ at a small angle θ ($\theta \ll R^2/d_p^2$) to crystallographic planes closely packed with atoms:

$$W_p \sim N_p(4\theta/\pi)(L/r), \tag{6.4}$$

where N_p is the number of crystallographic planes closely packed with atoms. At $r \sim L_p^*$, where $L_p^* \sim (\theta d_p^2/R^2)L$, the sought probability

$$W_p \sim N_p(4R^2/\pi d_p^2)$$

does not depend on θ.

Comparing the quantities W_p and W_r we find that in the region of angles ψ and θ of interest to us ($\psi \ll R/d$ and $\theta \ll R^2/d_p^2$) we have $W_p \gg W_r$.

At crystallographic-plane orientations (100), (110), (111), (121), (112) and (211) at a small angle to the particle momentum, coherent effects in radiation are well observable in experiment (see, e.g., Refs. 30–32). Taking only these planes into account, we find that $W_p \sim 10^{-2}$.

[1)]We use a system of units in which the speed of light c and the Planck constant $\hbar$ are equal to unity.

[2)]The quantity $\overline{\xi^2}$ introduced in Ref. 23 is connected ΔR_H^2 by the relation $\overline{\xi^2} = \overline{\Delta R_H^2}/2$.

[3)]Similar results for the average energy loss of relativistic particles in a crystal in the case of channeling and of above-barrier motion were obtained in Refs. 12, 28, and 29.

[1]H. J. Bhabha and W. Heitler, Proc. Roy. Soc. **A159**, 432 (1937).
[2]A. G. Carlson and J. R. Oppenheimer, Phys. Rev. **51**, 220 (1937).
[3]L. D. Landau and U. Rumer, Proc. Roy. Soc. **A166**, 213 (1938).
[4]I. E. Tamm and S. Z. Belen'kiĭ, J. Phys. USSR **1**, 177 (1939).
[5]M. L. Ter-Mikaelyan, Zh. Eksp. Teor. Fiz. **25**, 296 (1953).
[6]H. Uberall, Phys. Rev. **103**, 1055 (1955).
[7]M. L. Ter-Mikaelyan, Vliyanie sredy na élektromagnitnye protsessy pri vysokikh energiyakh (Effect of Medium on Electromagnetic Processes at High Energies), Izd-vo Arm. SSR, 1969.
[8]M. Schein, D. M. Haskin, and R. G. Glasser, Phys. Rev. **95**, 855 (1954).
[9]A. Debenedetti, C. M. Garelli, L. Tallone, M. Vigone, and G. Wataghin, Nuovo Cimento **12**, 954 (1954).
[10]M. Koshiba and M. F. Kaplon, Phys. Rev. **100**, 327 (1955).
[11]L. Barbanti Silva, C. Bonacini, C. Depietri, *et al.*, Nuovo Cimento **3**, 1465 (1956).
[12]A. G. Bonch-Osmolovskii and M. I. Podgoretskiĭ, Yad. Fiz. **29**, 432 (1979) [Sov. J. Nucl. Phys. **29**, 216 (1979)].
[13]M. A. Kumakhov, Zh. Eksp. Teor. Fiz. **72**, 1489 (1977) [Sov. Phys. JETP **45**, 781 (1977)].
[14]A. I. Akhiezer, V. F. Boldyshev, and N. F. Shul'ga, Dokl. Akad. Nauk SSSR **236**, 830 (1977) [Sov. Phys. Dokl. **22**, 569 (1977)].
[15]V. A. Bazylev and N. K. Zhevago, Zh. Eksp. Teor. Fiz. **73**, 1697 (1977) [Sov. Phys. JETP **46**, 891 (1977)].
[16]A. D. Erlykin, A. K. Kulichenko, S. K. Machavarani, and S. I. Nikol'skii, Trudy FIAN **109**, 62 (1979).
[17]A. D. Erlykin, A. K. Kulichenko, S. I. Nikolsky, and A. P. Chubenko, FIAN preprint No. 80, M. 1981.
[18]A. I. Akhiezer and N. F. Shul'ga, Pis'ma Zh. Eksp. Teor. Fiz. **32**, 318 (1980) [JETP Lett. **32**, 294 (1980)].
[19]V. I. Kulubaba and G. D. Kovalenko, Zh. Eksp. Teor. Fiz. **78**, 433 (1980) [Sov. Phys. JETP **51**, 219 (1980)].
[20]A. I. Akhiezer and N. F. Shul'ga, Usp. Fiz. Nauk **137**, 561 (1982) [Sov. Phys. Usp. **25**, 541 (1982)].
[21]L. D. Landau, J. Phys. USSR **8**, 201 (1944).
[22]L. D. Landau and E. M. Lifshitz, Classical Theory of Fields, Pergamon, 1971.
[23]A. A. Sokolov and I. M. Ternov, Zh. Eksp. Teor. Fiz. **25**, 698 (1953).
[24]N. P. Kalashnikov, V. S. Remizovich, and M. I. Ryazanov, Stolknoveniya zaryazhennykh chastits v tverdykh telakh (Collisions of Charged Particles in Solids), Atomizdat, 1980.
[25]I. Ya. Pomeranchuk, Zh. Eksp. Teor. Fiz. **9**, 915 (1929).
[26]I. Lindhard, Usp. Fiz. Nauk **99**, 249 (1969) [Mat.-Fyz. Medd. Vid. Selsk. **34** (14), 1965].
[27]D. S. Gemmell, Rev. Mod. Phys. **46**, 129 (1974).
[28]M. I. Podgoretskiĭ, Yad. Fiz. **31**, 417 (1980) [Sov. J. Nucl. Phys. **31**, 218 (1980)].
[29]V. A. Bazylev and N. K. Zhevago, Rad. Eff. **54**, 221 (1981).
[30]I. I. Miroshnichenko, J. J. Murray, R. O. Avakyan, and G. H. Fighut, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 786 (1979) [JETP Lett. **29**, 722 (1979)].
[31]M. J. Alguard, R. L. Swent, R. H. Pantell, *et al.*, Phys. Rev. Lett. **42**, 1148 (1979).
[32]V. I. Vit'ko, I. A. Grishaev, G. D. Kovalenko, and B. I. Shramenko, Pis'ma Zh. Tekh. Fiz. **5**, 1291 (1979) [Sov. Tech. Phys. Lett. **5**, 541 (1979)].

Translated by J. G. Adashko