Absorption and stimulated emission of quanta of an external inhomogeneous electromagnetic field by a free electron

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An analysis is given of the possibilities of absorption or stimulated emission by a free electron of electromagnetic-field quanta, possibilities due to the inhomogeneity of the intensity distribution in space or to the limited duration of the pulse. The probabilities of multiphoton processes are found, and the nonrelativistic and relativistic limits are discussed. It is shown, in particular, that when a beam of relativistic electrons is at a small angle to the direction of propagation of a laser beam a very considerable energy broadening of the former may occur. This broadening is a linear function of the field strength and is due to its inhomogeneity or to the short duration of the pulse.

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I. INTRODUCTION

It is well known that a free electron can neither absorb nor emit a single photon or several identical photons. Such processes are forbidden because the laws of conservation of energy and of momentum cannot be simultaneously satisfield for them (in the absence of a third body). This conclusion is valid only for true photons, i.e., for plane waves infinite both in space and time. It is clear, however, that the electromagnetic radiation emitted by any real source is not, strictly speaking, a plane wave because the source is localized both in space and in time. When these factors are taken into account, the above restriction on absorption and stimulated emission of field quanta is, in general, removed. When the duration τ of the pulse of electromagnetic radiation is long in comparison with the time taken by the electron to traverse the region of space in which the field is concentrated, the restricted duration of the pulse can be neglected in the first approximation, i.e., it may be assumed that $\tau = \infty$. Spatial localization of the field is then the dominant factor. For a monochromatic electromagnetic wave of frequency ω, propagating along the z axis and restricted in the transverse directions x and y, the electric field can be written in the form

$$\vec{\mathscr{E}}=\vec{\mathscr{E}}_0 f(x,\ y)\sin(\omega t-kz),\qquad(1)$$

where $k=\omega/c$ and $f(x,y)$ is the spatial amplitude envelope, normalized to unity at the maximum ($f_{\max}=1$). We note that the representation of the field by (1) is approximate because the function $\mathscr{E}(x,y,z,t)$ given by (1) does not, strictly speaking, satisfy the Maxwell equations. However, (1) is valid in free space if the transverse dimension d of the beam, which is a measure of the rate of variation of the continuous function $f(x,y)$, is much greater than the wavelength $\lambda = 2\Pi c/\omega$ of the radiation. The relation $\lambda/d \ll 1$ is then an indication of the precision with which (1) can be used.

Absorption or stimulated emission of field quanta $\hbar\omega$ becomes possible when an electron interacts with the inhomogeneous field $\mathscr{E}$ given by (1) because the laws of conservation of momentum in the transverse directions need not then necessarily be satisfied. It is clear that the probabilities of such processes will not be small when the function $f(x,y)$ is sufficiently sharp, since the opposite limit of completely contiuous dependence of $\mathscr{E}$ on the transverse coordinates x and y corresponds to the transition to a plane wave. It follows that, in free space, we have two opposite situations, namely, either the function $f(x,y)$ is smooth ($d \gg \lambda$), so that the field can be taken in the form given by (1), or $\mathscr{E}$ is a rapidly-varying function of x and y and gives rise to an appreciable absorption or emission of photons. Calculations show that these two conditions are not contradictory and can both be satisfied in the relativistic limit of electron energy. For nonrelativistic electrons in free space, the condition that the function $f(x,y)$ be sufficiently sharp cannot be satisfied simultaneously with the condition that $d \gg \lambda$. However, when the field $\mathscr{E}$ is produced not in a vacuum but on the separation boundary between two media, a sharp discontinuity can occur on this boundary. For example, when the skin-layer depth δ is small, both the field $\mathscr{E}$ and the function $f(x,y)$ rise rapidly over the short distance δ on the surface of the metal from zero to some finite value. Calculations show that, in this case, the probability of emission or absorption of field quanta $\hbar\omega$ need not be small for moderate values of the field strength $\mathscr{E}_0$.

Finally, if the pulse length τ is small in comparison with the time taken by an electron to traverse the region in which the field is localized, the evolution of the field $\mathscr{E}$ in time becomes more important than the spatial inhomogeneity. This means that the space envelope $f(x,y)$ in (1) must be replaced by the time envelope $f(t)$. The latter may be due to, for example, the rise and fall of the field amplitude as the pulse of radiation crosses the focal region. Here again, field quanta $\hbar\omega$ can be absorbed or emitted when the radiation pulse interacts with relativistic electrons. We note that, for the sake of simplicity, we shall confine ourselves to planar geometry, i.e., assume that $f(x,y)=f(x)$, when we examine the distribution of the field in space. Analysis of the one-dimensional space envelope $f(x)$ and time envelope $f(t)$ is mathematically very similar. The final general solutions can be obtained by a

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simple redesignation of the parameters. Nevertheless, these two cases are physically distinct both in the formulation of the problem and in the final solution and in the conditions for its validity.

The present paper is concerned with the set of problems outlined above. In all cases, we shall find the solution of the problem, i.e., the expressions for the probabilities of absorption or stimulated emission of several field quanta $\hbar\omega$.

The solutions given below are approximate and valid only under certain restrictions on the field strength $\mathscr{E}_0$ even though the solutions are found without using a perturbation theory in the field $\mathscr{E}_0$. We shall discuss the physical restrictions imposed on the field parameters by the conditions for the validity of these solutions. We shall also consider the possibility of and conditions for the experimental observation of the effects.

2. GENERAL SOLUTIONS

Neglecting spin effects, we start with the Klein-Gordon equation for an electron in the field $\mathscr{E}$ (1) in planar geometry:

$$\left\{m^2+\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial z^2}+2ie\frac{\mathscr{E}_0}{\omega}f(x)\cos(\omega t-kz)\frac{\partial}{\partial x}+\frac{e^2\mathscr{E}_0^2}{\omega^2}f^2(x)\cos^2(\omega t-kz)\right\}\Psi=0, \tag{2}$$

where the field vector $\vec{\mathscr{E}}$ is assumed to lie in the x,y plane and $\hbar=c=1$.

Let us now represent the electron wave function in the form of a superposition of plane waves:

$$\Psi=\sum_{n=-\infty}^{+\infty}C_n(x)N_pe^{i(\mathbf{pr}-\varepsilon t)}e^{-in(kz-\omega t)}, \tag{3}$$

where $\varepsilon=(\mathbf{p}^2+m^2)^{1/2}$, $\mathbf{p}=\{p_x,0,p_z\}$ is the initial momentum of the electron as $(x\to-\infty)$, and N_p is a normalizing constant.

The required functions $C_n(x)$ have the meaning of probability amplitudes for absorption $(n<0)$ or stimulated emission $(n>0)$ of $|n|$ photons $\hbar\omega$ when the transverse coordinate of the electron is equal to x. $|C_n(\infty)|^2$ are the probabilities of these processes after the electron has passed through the layer of electromagnetic radiation. When $|n|$ photons are absorbed (emitted), the electron energy changes by $n\hbar\omega$, and its momentum along the z axis changes by $n\hbar k$. It follows that, in principle, the absorption or emission of quanta can be detected experimentally by measuring the energy or the angular distribution of scattered electrons. The energy distribution should have a principal maximum at $\varepsilon'=\varepsilon$ as well as additional peaks separated from the principal peak by $n\hbar\omega$. As for the angular distribution, the absorption and emission of photons splits the initial beam into a "fan." It is clear that this redistribution of electrons can be observed if the resulting effective width of the energy distribution $(\sim|n|_{\max}\hbar\omega)$, or the angular width of the "fan," is greater than the initial energy of angular width of the electron beam.

Substitution of the expansion (3) in (2) yields the following equations for the functions $C_n(x)$:

$$ip_x\frac{dC_n}{dx}-n(kp)C_n+\frac{e\mathscr{E}_0}{2\omega}p_xf(x)(C_{n+1}+C_{n-1})=-\frac{d^2C_n}{dx^2}+i\frac{e\mathscr{E}_0}{2\omega}f(x)\frac{d}{dx}(C_{n+1}+C_{n-1})+\frac{e^2\mathscr{E}_0^2}{4\omega^2}f^2(x)\left(C_n+\frac{1}{2}(C_{n+2}+C_{n-2})\right), \tag{4}$$

where $(kp)=\omega(\varepsilon-p_z)$.

We shall seek the solution of these equations by assuming that the functions $C_n(x)$ are slowly varying and that the field $\mathscr{E}_0$ is not too high. These approximations will be formulated more rigorously in the next section, where their validity will be examined. Assuming for the moment that these conditions are satisfied, and neglecting all terms on the right-hand side of (4), we obtain the following simpler form for the functions $C_n(x)$:

$$i\frac{dC_n}{dx}=nqC_n-\beta f(x)(C_{n+1}+C_{n-1}), \tag{5}$$

where

$$q=\frac{(kp)}{|\mathbf{p}|\cos\theta},\quad \beta=\frac{e\mathscr{E}_0}{2\omega},\quad \cos\theta=\frac{p_x}{|\mathbf{p}|}. \tag{6}$$

The boundary conditions for (5) are $C_n(-\infty)=\delta_{n,0}$.

The set of equations given by (5) is a special case of the Raman-Nath equations.[1] The appearance of these equations is typical of problems in which an electron interacts with periodic perturbations.[2] The particular feature of (5) is the absence of anharmonism, i.e., the absence of a term $\propto n^2$. Comparison with the exact equations (4) shows that the absence of anharmonism from (5) is unconnected with the approximations made above, and is more readily a general property of the interaction of an electron with one inhomogeneous electromagnetic wave. In more complicated systems, e.g., when there are two waves, or a wave and an undulator field, the anharmonic term quadratic in n will necessarily appear in the Raman-Nath equations.[2]

In the absence of anharmonism, the Raman-Nath equations (5) can be solved exactly. Let us introduce the function

$$g(x,\xi)=\sum_n e^{in\xi}C_n(x), \tag{7}$$

which, by definition, must be periodic in ξ:

$$g(x,\xi\pm2\pi)=g(x,\xi)$$

and must satisfy the boundary condition $g(-\infty,\xi)=1$.

The coefficients $C_n(x)$ are then given by the Fourier integral

$$C_n(x)=\frac{1}{2\pi}\int_{-\pi}^{\pi}d\xi g(x,\xi)e^{-in\xi}. \tag{8}$$

The relationship between Ψ and $g(x,\xi)$ can now be readily found by comparing the above function with the original expansion (3) of the wave function Ψ:

$$\Psi=N_pe^{i(\mathbf{pr}-\varepsilon t)}g(x,kz-\omega t). \tag{9}$$

The physical significance of $g(x, \xi)$ in (7) is now clear from (9): $g(x, \xi)$ is the modulating function that depends on the coordinate x and the phase $kz - \omega t$ of the wave, and results from the interaction between the electron and the wave field. When this interaction is absent, we have $g \equiv 1$ and the electron wave function Ψ given by (9) is a plane wave. When the field is uniform, we have $f(x) \equiv 1$, and the function $g(x, \xi)$ is independent of x and depends only on the phase $\xi = kz - \omega t$. The explicit form of the function $g(\xi)$ in this case enables us to reproduce the well-known Volkov functions (3) for an electron in the field of a plane electromagnetic wave (4). In an inhomogeneous field, the modulating function g depends on two variables, and the determination of its explicit form is essentially the solution of our problem because the determination of the coefficients $C_n(x)$ given by (8) for known $g(x, \xi)$ is an elementary problem.

The equation for the modulating function $g(x, \xi)$ follows from the definition (7) and from Eq. (5):

$$i\left(\frac{\partial}{\partial x}+q\frac{\partial}{\partial \xi}\right)g(x,\xi)=-2\beta f(x)\cos\xi\, g(x,\xi). \tag{10}$$

It is readily seen that Eq. (10) can be solved exactly and that when the boundary condition for $g(x, \xi)$ is taken into account the solution assumes the form

$$g(x,\xi)=\exp\left\{2i\beta\int_{-\infty}^{x}dx' f(x')\cos[q(x'-x)+\xi]\right\}. \tag{11}$$

When the probabilities $C_n(x)$ are calculated from (8), it is convenient to dispose of the term $-qx$ in the argument of the cosine in (11) by substituting the new integration variable $\xi' = \xi - qx$. Fourier expansion of the periodic function of ξ' followed by integration with respect to ξ' then yields

$$C_n(x)=\exp\left\{in\,\mathrm{arctg}\frac{\int_{-\infty}^{x}dx' f(x')\sin qx'}{\int_{-\infty}^{x}dx' f(x')\cos qx'}\right\}\times i^n e^{-inqx}J_n\left(2\beta\left|\int_{-\infty}^{x}dx' f(x')e^{iqx'}\right|\right). \tag{12}$$

Finally, the asymptotic probabilities of multiphoton emission or absorption after the electron has traversed the region in which the field is localized take the form

$$w_n=|C_n(\infty)|^2=J_n^2\left(2\beta\left|\int_{-\infty}^{+\infty}dx f(x)e^{iqx}\right|\right). \tag{13}$$

We note that, subject to the approximations introduced above, Eq. (13) gives $w_n = w_{-n}$, i.e., the probabilities of absorption or stimulated emission of $|n|$ quanta are equal. This means that, on average, there is no change in the electron energy. It is clear that this is valid so long as the energy change $|n|\hbar\omega$ is small in comparison with the initial energy ε [see condition (28) below]. When this condition is not satisfied, the terms on the right-hand side of (4) must be taken into account more accurately. In particular, unless these terms are included it is impossible to predict when absorption or stimulated emission of photons will predominate. This problem is of independent interest, but will not be examined here.

3. PHOTON MULTIPLICITY. RELATIVISTIC AND NONRELATIVISTIC APPROXIMATIONS

The derived general formula (13) expresses the probability of multiphoton emission or absorption, w_n, in terms of the Fourier transform of the field spatial envelope $f(x)$. The parameter q has the significance of momentum transferred along the x axis and determines the characteristic dimension $d_c = 1/q$ with which we must compare the transverse dimension d of the region in which the radiation is localized. When $d \ll d_c$, Eq. (13) assumes the form

$$w_n=J_n^2\left(2\beta\int_{-\infty}^{+\infty}dx f(x)\right)=J_n^2\left(\frac{e\mathscr{E}_0}{\hbar\omega}\int_{-\infty}^{+\infty}dx f(x)\right). \tag{14}$$

The argument of the Bessel function and, hence, the maximum number $n_{\max}$ of quanta $\hbar\omega$ that can be absorbed or emitted by the electron are given by the following order-of-magnitude expression:

$$n_{max}\sim\beta d=e\mathscr{E}_0 d/\hbar\omega. \tag{15}$$

The physical significance of $n_{\max}$ (15) is that this is the ratio of the work done on the electron by the field $\mathscr{E}_0$ over the distance d to the energy $\hbar\omega$ of the quantum. When $d \lesssim d_c$ is not inconsistent with the original condition $d \gg \lambda$, the formula (15) yields the correct estimate for $n_{\max}$, which may be large for very moderate values of the field $\mathscr{E}_0$. Particular estimates of $n_{\max}$ will be given in Sec. 4 after we have examined the validity of (13) and the upper bound of $\mathscr{E}_0$.

We note that (14) and the estimate (15) for $n_{\max}$ are universal at $d \lesssim d_c$ in the sense that they do not depend on the electron energy or the geometry of the experiment, i.e., on the angle θ. Only the transferred momentum, i.e., the characteristic length $d_c = 1/q$, depends on the energy ε and the angle θ (see below).

Finally, we note that (14) is equivalent to the results obtained by Raman and Nath in the case of scattering of light by ultrasonic waves[1] (see also Ref. 5). When $d \gg d_c$, the Fourier transform of the envelope $f(x)$ is small. For the Gaussian curve

$$f(x)=e^{-x^2/d^2} \tag{16}$$

the Fourier transform is exponentially small:

$$\int_{-\infty}^{+\infty}dx e^{iqx}f(x)=d\pi^{1/2}\exp\left(-\frac{q^2d^2}{4}\right). \tag{17}$$

For other forms of $f(x)$, the Fourier transform may take the form of a power function rather than an exponential. For example, when $f(x) = e^{-|x|/d}$ we have

$$\int_{-\infty}^{+\infty}dx f(x)e^{iqx}=\frac{2d}{1+q^2d^2}. \tag{18}$$

As an example, we also note that when

$$f(x)=\begin{cases}\frac{1}{2}(1+\cos(x/d)), & |x|<\pi d,\\ 0, & |x|>\pi d,\end{cases} \tag{19}$$

the Fourier transform of the envelope is

$$\int_{-\infty}^{+\infty} dx f(x) e^{iqx} = \frac{\sin \pi q d}{q(1-q^2d^2)}. \tag{20}$$

Whenever $qd \gg 1$, i.e., $d \gg d_c$, we have

$$\left| \int dx f(x) e^{iqx} \right| \ll d.$$

Consequently, when $d \gg d_c$, the argument of the Bessel functions in (13) is smaller than at $d \lesssim d_c$ [Eq. (14)]. In the limit as $d \to \infty$, the argument tends to zero and $w_n \to \delta_{n,0}$, i.e., in the limit of an absolutely smooth envelope, the probabilities of absorption and emission of field quanta tend to zero, as they should.

For finite d but $d \gg d_c$, the order of smallness of the Fourier transform of the envelope $f(x)$ depends on the form of the function $f(x)$ in an essential way. Hence, the estimate of the magnitude of the argument of the Bessel functions and, hence, of the photon multiplicity of absorption and emission, will be different for radiation pulses with different space-envelope profiles. For example, for the Gaussian profile (16) the argument of the Bessel functions in (13) is exponentially small at $d \gg d_c$, so that the absorption or emission of field quanta is hardly possible in this case in moderate fields. On the other hand, for pulses of the form (18) and (19), the Fourier transform of $f(x)$ for $d \gg d_c$ decreases as a power function. Although the photon multiplicity $n_{\max}$ under these conditions is lower than at $d \lesssim d_c$ (15), it is nevertheless possible, in general, to have $n_{\max} > 1$ even for $d \gg d_c$ (see below, Sec. 4).

Let us now examine the dependence of the parameters q and d_c on the electron energy and on the geometry of the experiment. Starting with the definition of q in (6), we can readily verify that, in the nonrelativistic limit $v \equiv |\mathbf{p}|/m \ll c$

$$q = \frac{\omega}{v \cos\theta}, \quad d_c = \frac{v \cos\theta}{\omega}. \tag{21}$$

The maximum value of the parameter d_c in (21) is reached at $\theta = 0$, i.e., when the electron propagates at right-angles to the light beam and $(d_c)_{\max} = v/\omega \ll \lambda$. Hence, it follows that, when $d \gg \lambda$, it is always the case that $d \gg d_c$ in the nonrelativistic approximation. When $v \ll c$ in free space, the Fourier transform of the envelope $f(x)$ is always small in comparison with d.

Let us now suppose that the electron energy is high, so that $\gamma = \varepsilon/mc^2 \gg 1$. The most effective interaction between relativistic electrons and an electromagnetic wave occurs when the directions of the electron momentum $\mathbf{p}$ and the wave vector $\mathbf{k}$ are close to one another. We shall therefore suppose that θ is close to $\pi/2$: $\chi \equiv \pi/2 - \theta \ll 1$. If we adopt these approximations, we obtain

$$q \approx \frac{\pi}{\lambda\gamma} \frac{1+\chi^2\gamma^2}{\chi\gamma}, \quad d_c = \frac{\lambda\gamma}{\pi} \frac{\chi\gamma}{1+\chi^2\gamma^2}. \tag{22}$$

When $\chi\gamma \sim 1$ we have $d_c \sim \gamma\lambda \gg \lambda$. It follows that it is possible to have the situation where

$$d_c \sim \gamma\lambda \gtrsim d \gg \lambda. \tag{23}$$

The Fourier transform of the envelope $f(x)$ is then not small ($\approx d$) and the photon multiplicity $n_{\max}$ is given by (15).

When the relativistic electron travels at an angle to the light beam, the path length increases (compared with the case $\theta = 0$) but the electron velocity also increases (compared with the nonrelativistic case $v \ll c$). Equations (22) and (23) show that the velocity increase is the more important. If, for the nonrelativistic electron, the beam of electromagnetic radiation is wide, $d \gg d_c$ at high energies $\gamma \gg 1$ and for oblique propagation $\chi \sim 1/\gamma \ll 1$, the increase $d \propto \gamma$ [Eq. (22)] ensures that the beam becomes thin for the electron. The function $f(x)$ is then sufficiently sharp to ensure highly effective multiphoton absorption and stimulated emission field quanta $\hbar\omega$.

We note that all the conclusions relating to estimates of the characteristic length d_c as well as the definition (6) of q and formulas (21) and (22) can be obtained directly from the laws of conservation of energy without solving the problem. As already noted, the physical significance of the parameter q in Eq. (5) and in all the subsequent solutions is that q is the momentum transferred from the electron to or drawn from the field in the direction of the x axis. The z component of the momentum, p_z and the electron energy ε then satisfy the conservation laws

$$p_z' = p_z \pm \omega, \quad \varepsilon' = \varepsilon \pm \omega. \tag{24}$$

Hence, it is readily shown that the change in the momentum along the x axis is given by

$$q \equiv |\Delta p_x| = |p_x' - p_x| = |[p_x^2 \pm 2\omega(\varepsilon - p_z)]^{1/2} - p_x| \approx \omega \frac{\varepsilon - p_z}{p_x}, \tag{25}$$

which is in agreement with the definition (6).

We note that the quantity q [Eq. (25)] differs by the factor $|\mathbf{p}|/p_x = 1/\cos\theta$ from the minimum transferred momentum $q_{\min}$ in the theory of bremsstrahlung of a photon by an electron.[6] In the relativistic limit and for $\theta = 0$, we have $q = q_{\min}$ and, as is well known, $q_{\min} = \omega/v$. When $\gamma \gg 1$, $\chi \ll 1$, Eq. (25) leads directly to (23) and all the conclusions relating to the characteristic size d_c formulated above.

4. CONDITIONS OF VALIDITY AND OF OBSERVATION

After the representation (1), which is valid at $d \gg \lambda$, has been used for the electromagnetic field $\mathscr{E}$, the only approximation introduced was the transition from the exact equations (4) to the approximate equations (5). Let us now estimate the conditions for the validity of this transition, using the explicit form of the solutions (12). As already noted, the transition from (4) to (5) is justified if the functions $C_n(x)$ are sufficiently smooth and the field $\mathscr{E}$ not too strong.

It follows from (12) that the rate of change of the functions $C_n(x)$ is characterized by the parameter Δx, which has the dimension of length and is given by

$$\Delta x = \min\left\{ d, \frac{1}{|n|q} = \frac{d_c}{|n|}, \frac{1}{\beta} \right\}. \tag{26}$$

The condition that the second derivative d^2C_n/dx^2 on the right-hand side of (4) be small in comparison with $p_x dC_n/dx$ is

$$p_x \Delta x \gg 1. \tag{27}$$

We shall now examine the significance of this condition in the nonrelativistic and relativistic approximations, and for different relationships between the parameters in (26).

1. In the nonrelativistic approximation ($v \ll c$) we have $d_c = v/\omega$, so that $d \gg d_c$ in all cases, and

$$\Delta x = \min\{d_c/|n|, \beta^{-1}\}.$$

Suppose now that $\beta d_c \ll 1$, i.e., $(ev\mathscr{E}_0/\hbar\omega^2) \ll 1$ and $\Delta x = d_c = v/\omega$. For $\theta = 0$, the condition $p_x \Delta x \gg 1$ gives

$$v/|n|\omega \gg \lambda_{dB} = \hbar/mv \quad \text{or} \quad |n|\hbar\omega \ll mv^2, \tag{28}$$

i.e., in a weak field, when $|n| = 1$, the de Broglie wavelength must be much greater than the path length traversed by the electron in one period, or the quantum energy $\hbar\omega$ must be much less than the kinetic energy of the electron, which is usually the case.

If, on the other hand, $\beta d_c \gg 1$, i.e., the field $\mathscr{E}_0$ is sufficiently strong:

$$ev\mathscr{E}_0/\hbar\omega^2 \gg 1, \tag{29}$$

the condition (27) assumes the form

$$v_E = e\mathscr{E}_0/m\omega \ll v. \tag{30}$$

Here v_E is the amplitude of the electron oscillation velocity in the field of the wave, which, according to (30), must be much smaller than the translational velocity v of the electron. By virtue of (28), condition (30) is not in conflict with the assumption (29). The inequality (30) defines the upper limit for $\mathscr{E}_0$ in the case of nonrelativistic electrons. This is not a very stringent limitation. For example, when $\omega = 3\times 10^{15}$ s^{-1}, $v = 3\times 10^8$ cm/s, the inequality given by (30) yields $\mathscr{E}_0 < 6\cdot 10^8$ V/cm, whereas, for $\omega = 10^{14}$ s^{11} and $v = 3\times 10^9$ cm/s, we have $\mathscr{E}_0 < 2\cdot 10^8$ V/cm. As for the estimated possible number n_{max} of emitted or absorbed quanta, it has already been noted that, for $v \ll c$, which is the case we are considering here, this number is sensitive to the shape of the space envelope $f(x)$. When $f(x)$ is a Gaussian, the effect is exponentially small, so that, in free space, the absorption or emission of field quanta is practically impossible. For $f(x)$ of the form given by (18), we have

$$n_{max} \sim \frac{e\mathscr{E}_0 v^2}{\hbar\omega^3 d} = \frac{v_E}{v}\frac{mv^3}{\hbar\omega^2 d}. \tag{31}$$

The factor

$$\frac{mv^3}{\hbar\omega^2 d} = \frac{mv^2}{\hbar\omega}\frac{d_c}{d}$$

may be not small. For example, when $v = 3\times 10^9$ V/cm, $\omega = 10^{14}$ s^{-1}, and $d = 10^{-1}$ cm, the value of this factor is 30. This means that $n_{max} \sim 1$ when $v_E/v \sim 1/30$, i.e., when $\mathscr{E}_0 \sim 10^7$ V/cm.

When $f(x)$ has the form given by (19), we find, with the aid of (20), that

$$n_{max} \sim \frac{e\mathscr{E}_0 v^3}{\hbar\omega^4 d^2} = \frac{v_E}{v}\frac{mv^4}{\hbar\omega^3 d^2}. \tag{32}$$

The last expression contains the additional small factor $d_c/d = v/\omega d \ll 1$ as compared with (31). This means that the factor

$$\frac{mv^4}{\hbar\omega^3 d^2} = \frac{mv^2}{\hbar\omega}\left(\frac{d_c}{d}\right)^2$$

may be not small only at lower frequencies. For example, when $\omega = 3\times 10^{11}$ s^{-1}, $d = 1$ cm, and $v = 3\times 10^9$ cm/s, we have $n_{max} \sim 3\times 10^3\,(v_E/v)$, i.e., $n_{max} \sim 1$ for $v_E \sim 3\times 10^{-4}\,v$ or $\mathscr{E}_0 \sim 6\cdot 10^5$ V/cm.

The above estimates show that, in principle, nonrelativistic electrons in free space can be used to observe multiphoton emission or absorption by having an electron beam cross a beam of electromagnetic radiation. However, these estimates are very sensitive to the shape of the space envelope and, in particular, the above processes cannot be realized when $f(x)$ has the Gaussian shape (16). A somewhat different possible experimental scheme with nonrelativistic electrons, which involves the use of the abrupt change in the field on the boundary of a metal, will be examined in Sec. 6.

2. Let us now suppose that the electron has high energy, $\gamma \gg 1$, and that its direction of motion lies close to the direction of the wave vector **k**, $\chi \sim 1/\gamma \ll 1$. It has already been shown in Sec. 3 that, in this case, a possible (and the most interesting) combination of parameters is that for which (23) is satisfied and, consequently,

$$\Delta x = \min\{d, \beta^{-1}\}. \tag{33}$$

Suppose, to begin with, that $\beta d \ll 1$, i.e., the field $\mathscr{E}_0$ is weak: $e\mathscr{E}_0 d/\hbar\omega \ll 1$. Condition (27) then assumes the form

$$\chi\gamma\frac{d}{\hbar/mc} \equiv \chi\gamma\frac{d}{\lambda_C} \sim \frac{d}{\lambda_C} \gg 1, \tag{34}$$

i.e., the transverse dimension of the beam must be much greater than the Compton wavelength of the electron $\lambda_C 3\times 10^{-11}$ cm, which is always the case.

For stronger fields, for which $e\mathscr{E}_0 d/\hbar\omega \gg 1$, condition (27) gives

$$e\mathscr{E}_0/m\omega \equiv v_E/c \ll \chi\gamma \sim 1. \tag{35}$$

This inequality sets the upper limit for the field, This limit is never exceeded for existing sources of radiation (for example, when $\omega = 10^{15}$ s^{-1}, we have $\mathscr{E}_0 \ll 2\cdot 10^{10}$ V/cm or $I \ll 10^{17}$ W/cm^2).

When the conditions given by (23) are satisfied, the probability w_n of multiphoton processes and the photon multiplicity n_{max} are given by (14) and (15). Let us estimate the maximum energy emitted or absorbed by an electron in the limit of validity of the approximation of a thin layer of radiation, i.e., for $d \sim d_c$:

$$\delta\varepsilon \sim n_{max}\hbar\omega = e\mathscr{E}_0 d_c \sim \gamma\lambda e\mathscr{E}_0 = 2\pi\varepsilon v_E/c. \tag{36}$$

In practise, relativistic electron beams have an energy spread $\Delta\varepsilon \ll \varepsilon$ which, of course, is much greater than the energy of the optical quantum $\hbar\omega$. It follows that it is not possible to observe the discrete structure, with spacing $\hbar\omega$, in the energy distribution of the scattered electrons. However, it is possible to observe the additional broadening of the electron energy distribution, the scale of which, $\delta\varepsilon$ (36), is a linear function of the field $\mathscr{E}_0$. The condition that will ensure that this additional broadening will be appreciable is that $\delta\varepsilon$ (36) must be greater than the initial width $\Delta\varepsilon$ of the energy distribution, i.e.,

$$v_E/c > \Delta\varepsilon/2\pi\varepsilon. \quad (37)$$

Condition (37) is not inconsistent with the limitation on the field strength (35), so that it would seem that this type of experiment is realizable. All the necessary conditions are satisfied, for example, when $\omega = 10^{15}\,\mathrm{s}^{-1}$ ($\lambda = 2\times10^{-4}$ cm), $\gamma \sim 1/\chi \sim 10^2$, and $d \sim d_c \sim 2\times10^{-2}$ cm, we have $\Delta\varepsilon/\varepsilon \sim 10^{-2}$ if the length of the caustic (region of focusing) is $l \sim d/\chi \sim 2$ cm and the field strength is $\mathscr{E}_0 > 3\cdot10^7$ V/cm, i.e., when $I > 10^{11}$ W/cm^2 and the pulse length is $\tau > d/c\chi \sim 10^{-10}$ s.

When we examined the conditions governing the transition from (4) to (5), we analyzed only the requirement that the functions $C_n(x)$ must be sufficiently slow-varying, i.e., that the first term on the right-hand side of (4) must be small. It is readily verified that, if we explicitly demand that the other discarded terms must be small, this does not lead to new restrictions but simply reduces to the inequalities (30) and (35), which set the upper limit on the field strength in the nonrelativistic and relativistic limits.

5. SHORT PULSES OF ELECTROMAGNETIC RADIATION

Let us now suppose that the pulse length τ is small in comparison with the time taken by an electron to traverse the interaction region:

$$\tau < d/v\cos\theta. \quad (38)$$

In this case, the space envelope $f(x)$ in the Klein-Gordon equation (2) must be replaced with $f(t)$. The coefficients C_n in the expansion (3) are now functions of time, and satisfy equations similar to (5). The probabilities w_n of multiphoton processes, found by analogy with the foregoing procedure, are determined by (13) in which x must be replaced with t and the parameters q and β must be multiplied by the velocity v_x of the electron in the direction of the x axis. Instead of the characteristic length d_c we now have the characteristic time

$$t_c = \frac{1}{qv_x} = \begin{cases} \omega^{-1} & \text{for} \quad v \ll c, \quad \theta = 0, \\ \dfrac{2}{\omega}\dfrac{\gamma^2}{1+\gamma^2\chi^2} & \text{for} \quad \gamma \gg 1, \quad \chi \ll 1. \end{cases} \quad (39)$$

In the nonrelativistic limit, we have $\tau \gg t_c$ in all cases, so that the argument of the Bessel function, which determines w_n, is relatively small. Once again, the possibility of detection of multiphoton emission and absorption will depend on the shape of the envelope $f(t)$.

At high energies, the time t_c is found to increase rapidly ($\propto \gamma^2$ for $\chi\gamma \sim 1$), and the situation may arise for which

$$t_c \sim \gamma^2/\omega \gtrsim \tau \gg 1/\omega. \quad (40)$$

The expressions for the probabilities w_n and photon multiplicity $n_{\max}$ then assume the form

$$w_n = J_n^2\left(\frac{e\mathscr{E}_0\chi c}{\hbar\omega}\int_{-\infty}^{+\infty} dt f(t)\right), \quad (41)$$

$$n_{max} \sim e\mathscr{E}_0 l\chi/\hbar\omega, \quad (42)$$

where $l = c\tau$ is the length of the train of electromagnetic radiation. Although the formula (42) contains the small factor $\chi \ll 1$, the value of $n_{\max}$ can, nevertheless, be quite high. For example, at the limit of validity of the first of the conditions in (40), we have for $\tau \sim t_c \sim \gamma^2/\omega$ and for $\chi \sim 1\gamma$

$$n_{max} \sim ec\mathscr{E}_0\gamma/\hbar\omega^2. \quad (43)$$

The parameter $n_{\max}$ (43) is a measure of the strong nonlinearity of the function $w_n(\mathscr{E}_0)$, and can be large for moderate values of the field $\mathscr{E}_0$.

Finally, the condition restricting the maximum value of $\mathscr{E}_0$, introduced in the formulation of the problem with a time envelope $f(t)$, becomes still less stringent than (35):

$$e\mathscr{E}_0/m\omega \equiv v_E/c \ll \gamma/\chi \sim \gamma^2. \quad (44)$$

Consider an example: when $\omega = 10^{15}\,\mathrm{s}^{-1}$ and $\gamma = 10^2$, the pulse length should be $\tau \sim t_c \sim 10^{-11}$ s. The minimum transverse size of the beam is given by (38): $d > c\tau\chi \sim 3\cdot10^{-3}$ cm. The additional energy spread in the electron beam that arises as a result of the interaction with the short pulse of radiation can be readily estimated with the aid of (43), and its order of magnitude is given by the same expression (36) as in the case of a long pulse. Consequently, when $\Delta\varepsilon/\varepsilon \sim 10^{-2}$, the quantity $\delta\varepsilon$ becomes, as before, greater than $\Delta\varepsilon$ for $v_E/v > 10^{-3}$ or $\mathscr{E}_0 > 3\cdot10^7$ V/cm.

Thus, at relativistic energies, and despite the difference between the physical mechanisms involved, the restricted pulse length τ (in the picosecond range) and the restricted transverse size of the laser beam (in the nanosecond range) leads to the same energy broadening of the electron beam, other things being equal.

6. ABSORPTION AND EMISSION OF PHOTONS DURING THERMIONIC EMISSION BY A METAL IN THE FIELD OF AN ELECTROMAGNETIC WAVE

Let us now return to nonrelativistic electrons and suppose that the duration of the radiation pulse τ is large ($\tau > d/v$). We can then imagine the following experimental situation. The region $x < 0$ is occupied by a metal, and thermionic emission of electrons takes place into the space $x > 0$. An electromagnetic wave propagates along the surface of the metal in the free space $x > 0$. Let us suppose that the skin-layer depth δ is small in comparison with v/ω and that the parameter d that represents the thickness of the layer of electromagnetic radiation in the direction $x > 0$ is large in comparison with v/ω. We may then suppose that the field $\mathscr{E}$ undergoes a sharp change at $x = 0$. The Schroedinger equation for $x < 0$ must be solved without the field and, for $x > 0$, with the field $\mathscr{E}$ [Eq. (1)]. The two solutions must then be joined. It is readily verified that the solution of the problem leads to a result which is analogous to (13) except that, instead of the complete Fourier transform of the envelope $f(x)$, we now have an integral over a half-space:

$$w_n = J_n^2\left(2\beta\left|\int_0^{\infty} dx e^{iqx} f(x)\right|\right). \quad (45)$$

The presence of the sharp change in the field at $x = 0$ ensures that the argument of the Bessel functions in (45) is not exponentially small even for the Gaussian envelope $f(x)$ of the form given by (16):

$$\left|\int_0^\infty e^{iqx} f(x)\,dx\right| = \left|\frac{d\pi^{1/2}}{2} e^{-q^2d^2/4}\left(1+\Phi\left(\frac{iqd}{2}\right)\right)\right| \approx \frac{1}{q} = \frac{v}{\omega}, \tag{46}$$

where $\Phi(u)$ is the probability integral[7] and we have used the asymptotic behavior of $\Phi(u)$ for large values of the imaginary argument $|u| \gg 1$, $\Phi(iu) \sim e^{u^2}/u\pi^{1/2}$. The result (46) for $d \gg v/\omega$ is universal and is independent of the shape of the function $f(x)$. The photon multiplicity obtained from (45) and (46) is

$$n_{max} \sim ev\mathscr{E}_0/\hbar\omega^2 = mv^2 v_E/\hbar\omega v. \tag{47}$$

According to (47), the parameter $n_{\max}$ and the average change in the electron energy $n_{\max}\hbar\omega$ may not be small, since $mv^2 \gg \hbar\omega$. Two points must be borne in mind in this connection. Firstly, the condition that the skin-layer depth δ must be small in comparison with v/ω can readily be satisfied only for frequencies smaller than the effective collision frequency in the metal, ν_{eff}, i.e., for $\omega \lesssim 10^{13} - 10^{14}\,\text{s}^{-1}$. Secondly, the field $\mathscr{E}$ should not produce an appreciable heating of the electrons in the metal within the interval τ since, otherwise, this effect may mask the change in the energy of the electrons due to the direct interaction with the field.

The condition $v_E \ll v$ given by (30) ensures that the maximum change in the energy of the electron $n_{\max}\hbar\omega \sim mv^2(v_E/v)$ is much greater than the increase in the energy due to gradient forces,[8] which is equal to the energy $mv_E^2/2$ of the oscillations of the electron in the field of the wave. It follows that, when $v_E \ll v$, the acceleration of electrons in the inhomogeneous field $\mathscr{E}$ by the gradient forces will not mask the above quantum effect whereby wave photons are absorbed when the field is inhomogeneous.

The experiment discussed above is very similar to the well-known experiment reported by Lompré *et al.*[9] However, the mechanism discussed here for the absorption of field quanta does not correspond directly to the conditions in Ref. 9. In the latter experiment, the laser beam lay above the surface of the metal at a distance $\sim d$ above it. The field $\mathscr{E}$ on the surface was therefore practically zero, and the function $f(x)$ was smooth throughout. Moreover, in contrast to the assumption (28) above, the quantum energy $\hbar\omega$ in Ref. 9 was higher than the electron energy. The question as to whether the absorption of field quanta due to spatial or temporal inhomogeneity is possible for $\hbar\omega > \varepsilon$ will be examined separately elsewhere.

7. CONCLUSIONS

The foregoing analysis shows that there are certain specific conditions under which the spatial inhomogeneity of the field of an electromagnetic wave, or the restricted duration of a pulse of radiation, may be responsible for the absorption or emission of several field quanta $\hbar\omega$ by a free electron. In principle, this effect can be observed in the various experimental arrangements described above. Experiments with relativistic electrons, in which one observes the broadening of the electron energy distribution by the field, are physically the most interesting. Of course, from the practical point of view, an increase in the energy spread of an electron beam is an undesirable effect. Nevertheless, it seems to us that the experimental verification of this effect would be interesting from the standpoint of the physics of the interaction between strong electromagnetic radiation and free electrons.

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