Field effects in impurity-containing metals

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The temperature dependent part of the conductivity of metals containing impurities and placed in strong electric fields is investigated. It is shown that besides the usual heating of the electron subsystem there exists one more field effect due to the acceleration of the electrons during scattering. The contribution of the diffusive modes of the electron-density oscillations to the resistance is then suppressed, and the corresponding correction to the conductivity vanishes, $\Delta\sigma \propto E^{-2}$. Simultaneously eliminated are the divergences that take place as $T \to 0$ in two- and one-dimensional systems. The situation is analyzed in which heating the electron subsystem and suppression of the contribution of the diffusive mode take place simultaneously. It is shown that the correction to the conductivity is a nonmonotonic function of the electric field and can reverse sign.

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1. INTRODUCTION

The electric conductivity of metals is calculated as a rule only in the linear approximation in the electric field. This approximation is valid if the electrons are not heated or accelerated over the electron mean free path. Since the relaxations of the electron momentum and of their energy is usually characterized by different lengths l_p and l_ε, we have two inequalities:

$$|e|El_p \ll \hbar/\tau,\ kT, \tag{1}$$

$$|e|El_\varepsilon \ll kT, \tag{2}$$

where e is the electron charge, τ is the momentum relaxation time, and T is the temperature of the system in equilibrium. At low temperature the energy dissipation from the electron subsystem becomes very small, and condition (2) may become violated at relatively low electric field strengths. In a more rigorous treatment of the conditions under which the electron subsystem heating sets in, the balance is considered of the power generated by the electric field and dissipated in the electron-phonon interaction.[1-3] This question is discussed in greater detail in Sec. 3.

In the foregoing arguments we neglected the acceleration of the electron during the very act of scattering, i.e., we assumed the inequality

$$l_p \gg l_s \tag{3}$$

(l_s is the scattering length, i.e., the characteristic size of the scattering object), which coincides with one of the conditions for the applicability of the quasiclassical Boltzmann equation for a description of kinetic processes in metals (see, e.g., Ref. 4 as well as the discussion in Ref. 5). If the inequality (3) does not hold, we are no longer justified in using the Boltzmann equation to calculate the electric resistance, and must resort a more adequate procedure, say the one proposed in Ref. 5. We must then re-examine the conditions under which the approximation linear in the electric field is valid. Obviously, we need besides (1) and (2) one more conditions that follows from the following requirement: the quasiparticle must not acquire over the scattering length l_s an energy higher than kT or $\hbar/\tau$, i.e.,

$$|e|El_s \ll kT,\ \hbar/\tau. \tag{4}$$

In impurity-containing metals, at the low-temperature limit $kT \ll \hbar/\tau$, a unique resistance mechanism appears and is due to interference between the electron-impurity and electron-electron interactions.[6] It is due to scattering of the electrons by the diffusive modes of the electron-density oscillations. The wavelength of the diffusive mode plays in this case the role of the scattering length

$$l_s \propto \tau v (\tau kT/\hbar)^{-1/2}. \tag{5}$$

With decreasing temperature this value increases, and this can lead to violation of condition (4). One should expect here the appearance of nonlinear field effects of a type different from ordinary electron-subsystem heating already discussed in the literature.[1-3]

We shall show that conditions (2) and (4) can be violated simultaneously, and both field effects must be considered in this case simultaneously. We are not always able, however, to use the results of Refs. 1 and 2, for at temperature $T < \hbar T_D/\tau\varepsilon_F$ (T_D is the Debye temperature and ε_F is the Fermi energy) the energy dissipation due to electron-phonon interaction can no longer be described by the Boltzmann equation. An analysis using a quantum kinetic equation is needed in this case.[5]

We emphasize that in impurity-containing metals the resistance at low temperatures is determined principally by the scattering of the electrons by static defects, whereas inelastic processes such as electron-phonon or electron-electron interaction lead to the appearance of small temperature-dependent corrections. It is easily understood that in practice it is practically impossible to observe any nonlinear field effects in the residual resistance. When temperature-dependent contributions are studied, however, we encounter an entirely different situation, and should expect strong deviations from Ohm's law; these will be considered below.

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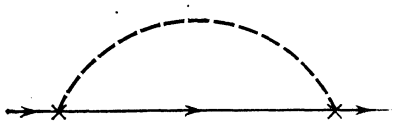

FIG. 1. Diagram for mass operator corresponding to scattering of electrons by static defects.

2. DISRUPTION OF DIFFUSIVE MODES IN A STRONG ELECTRIC FIELD

We consider first nonlinear field effects due to acceleration of electrons in an electric field over the scattering length. We assume satisfaction of the conditions

$$|e|E\tau v/T \ll 1, \tag{6}$$

$$|e|E\tau v/T(\tau T)^{1/2} \gg 1. \tag{7}$$

To calculate the current in such a regime, we use the system of equations derived in Ref. 5 with the aid of the Keldysh technique[7]

$$(\varepsilon - \varepsilon_{\mathbf{p}} - \Sigma_r) G_r = 1, \tag{8}$$

$$(\mathbf{Ev}) \frac{\partial n(\varepsilon)}{\partial \varepsilon} \frac{\partial}{\partial \varepsilon_{\mathbf{p}}} \left[\eta A + 2 \operatorname{arctg} \frac{2\eta}{\Gamma} \right] = \frac{\Gamma F - \Omega A}{2}. \tag{9}$$

Here $G_r(\mathbf{p},\varepsilon)$ and $\Sigma_r(\mathbf{p},\varepsilon)$ are nonequilibrium retarded Green functions and the mass operator,

$$\begin{aligned} A &= -2 \operatorname{Im} G_r, \quad \Gamma = -2 \operatorname{Im} \Sigma_r, \\ \eta &= \varepsilon - \varepsilon_{\mathbf{p}} - \operatorname{Re} \Sigma_r, \quad n(\varepsilon) = (1+e^{\beta\varepsilon})^{-1}, \\ \beta &= (kT)^{-1}, \quad \mathbf{v} = \partial \varepsilon_{\mathbf{p}} / \partial \mathbf{p}, \end{aligned} \tag{10}$$

F and Ω are "kinetic" Green functions and mass operator written in accord with Ref. 5. The definition used by Keldysh[7] differs by a factor i. In the derivation of (8) and (9) the electric field was assumed small in the sense of inequality (6). No corrections containing explicitly the large parameter (7) appear in these equations. They do appear however, as nonlocal terms in the calculation of the mass operator.

We assume the presence of only two types of interaction in the system, electron-electron and electron-impurity. The latter determines the residual resistance, whose calculation calls for the use of the mass operator shown in Fig. 1. At low temperatures the electron-electron interaction interferes strongly with the electron-impurity interaction, and this leads to the appearance (in the approximation linear in E) of negative corrections to the resistance.[6] Corresponding to this process is the diagram shown in Fig. 2 for the retarded mass operator. To avoid excessive complication of the calculation, we neglect below Fermi-liquid effects and assume a contact electron-impurity interaction potential:

$$u(\mathbf{r}) = u\delta(\mathbf{r}). \tag{11}$$

A complete calculation would lead only to a renormalization of the quantities in the final answer, without changing the result qualitatively. The procedure in such a calculation can be understood from Refs. 5 and 8. The wavy line in Fig. 2 corresponds to the retarded Coulomb function

$$\begin{aligned} U_r(\mathbf{q},\omega) &= V_q \left[1 + \frac{1}{2} V_q \pi_r(\mathbf{q},\omega) \right]^{-1} \\ &= \frac{4\pi e^2}{\varkappa^2} \frac{(\tau q v)^2/3 - i\tau\omega}{(\tau q v)^2/3}, \\ \pi_r(\mathbf{q},\omega) &= \pi_r(\mathbf{q},0) \frac{(\tau q v)^2/3}{(\tau q v)^2/3 - i\tau\omega}, \\ V_q &= \frac{4\pi e^2}{q^2}, \quad \varkappa^2 = 8\pi e^2 N(0), \end{aligned} \tag{12}$$

where $N(0)$ is the electronic density of states on the Fermi surface. The numbers on the diagram correspond to the matrix indices of the Keldysh technique in the triangular representation. Near the left-hand Coulomb vertex are given two variants of index arrangement. It is easy to verify that the expressions corresponding to diagrams with any other arrangement of the indices are either equal to zero or are small to the extent that the impurity density c is low.

We show now how we arrived at the index arrangement shown in Fig. 2. The extreme left (1) and extreme right (2) indices are fixed, since we are interested in the retarded mass operator $\Sigma_{12} = \Sigma_r$. We move along the electron line from right to left. On passing through the cross (electron-impurity vertex) the index must change, i.e., the index on the left of the extreme-right cross is 1. Since $G_{11} \equiv 0$, the index before the second cross on the right must be 2. Using this reasoning, we arrange uniquely the indices on the entire electron line all the way to the right-hand Coulomb vertex. Now we have two possibilities. We can place on the Coulomb line the index 1 or 2. If the index is 1, the indices on the electron lines on both sides of the vertex must coincide, but if the index is 2, the indices on the electron lines must be different.[7] We write first the index 1. We can then continue to arrange the indices from the left from the Coulomb vertex, in the same manner

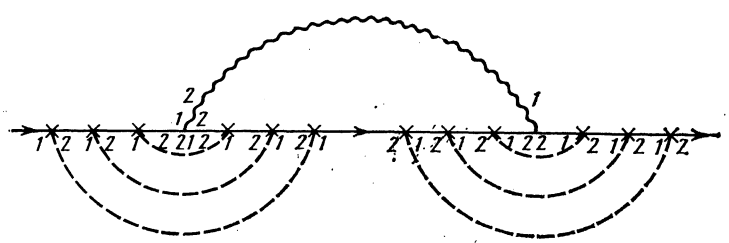

FIG. 2. Diagram for mass operator Σ_r^Q corresponding to electron scattering by diffuse modes of electron-density oscillations. The wavy line corresponds to the Coulomb function (12).

as we did from the right. It turns out then that only retarded Green functions $G_{21}\equiv G_r$ are located on the entire electron line to the right of the left Coulomb vertex. Consequently each rung of the right-hand ladder of impurity dashed lines is determined by the small integral

$$\int \frac{d^3p}{(2\pi)^3} G_r(p-q)\,G_r(p) \sim 1/\tau\varepsilon_F \sim c. \tag{13}$$

We neglect the contribution of such diagrams. Thus, only the index 1 can be placed on the Coulomb line near the right-hand vertex. Since $U_{11}\equiv 0$, the index 2 should be placed near the left-hand vertex. As a result, on the section between the right-hand Coulomb vertex and the first left-hand cross we find an electron line with ends marked by the indices 2, and all the vertices are nondiagonal relative to the Keldysh indices. It is easy to verify that on this line it is possible to place only in some one spot the Green function

$$G_{22}\equiv -iF = -iSA. \tag{14}$$

To the left of this function will then be located only the functions $G_{21}\equiv G_r$, and to the right $G_{12}\equiv G_a$. To avoid the appearance of a rung of the type (13) in either ladder, the function G_{22} must be placed directly to the right or to the left of the left Coulomb vertex. The result is the index arrangement indicated in Fig. 2. The kinetic equation (9) contains also the mass operator

$$\Sigma_{11}{}^{Q} = -i\Omega^{Q}. \tag{15}$$

The diagram for Σ_{11}^{Q} is obtained from Fig. 2 by interchange the indices next to the extreme-right cross.

When the nonequilibrium mass operator is calculated in the momentum representation, nonlocal field terms appear.[5] The reason is that the Green functions depend in this case not only on the difference of the coordinates, but also on their sum:

$$G(p, X) = \int d^4\xi \exp[-i(p+eA_X)\xi]\,G(x, x'). \tag{16}$$

Here $X = (x + x')/2$, $\xi = x - x'$, while $A_X = (A_X,\ \varphi_X)$ is the 4-vector-potential of the electric field and depends on the 4-coordinate X. The nonlocal terms appear in the calculation of the Fourier transform of the convolution of two functions $f(x,x')$ and $g(x,x')$ of the type (16)

$$\int d^4\xi \exp[-i(p+eA_X)\xi] \int d^4x_1 f(x, x_1)\, g(x_1, x')$$

$$= \exp\left[i\frac{\mathbf{E}}{2}\hat{\mathbf{\Pi}}\right](f(p), g(p)), \tag{17}$$

$$\hat{\mathbf{\Pi}}(f(p), g(p)) = \frac{\partial f}{\partial \mathbf{p}}\frac{\partial g}{\partial \varepsilon} - \frac{\partial f}{\partial \varepsilon}\frac{\partial g}{\partial \mathbf{p}}.$$

Equation (17) is a generalization of Eq. (2.11) of Ref. 5 to include the case of arbitrarily large electric fields. We assume here that A_X depends on X linearly, i.e., the field is constant and uniform.

We obtain now the explicit form of the expression for Σ_r^Q (Fig. 2) with allowance for the nonlocal corrections. This calls for finding the Fourier transform of a convolution of the type (17), but containing a large number of functions. The operator $\hat{\Pi}$ must be understood in this case as follows. We take all possible arrangements of the pair of differential operators $\partial/\partial\mathbf{p}$ and $\partial/\partial\varepsilon$ on the Green functions that make up the electron line on Fig. 2. These operators must act only on different Green functions, and the sign is determined by their relative placement: plus if the operator $\partial/\partial\mathbf{p}$ is to the left of the operator $\partial/\partial\varepsilon$, and minus in the opposite case. The operator Π does not act on the electron functions in the closed loops—the corresponding contributions are zero. It is necessary to retain in the expression for Σ_r^Q only the following arrangements of the operators $\partial/\partial\mathbf{p}$ and $\partial/\partial\varepsilon$. The derivative $\partial/\partial\varepsilon$ acts on the function S that enters in G_{22} [see (14)] adjacent to the left Coulomb vertex. This can be taken to be the equilibrium function

$$S(\varepsilon) = 1 - 2n(\varepsilon),$$

for when account is taken of the integration of the electron momentum over the angles, the nonequilibrium corrections to S in the corresponding rung of the ladder vanish. The derivative $\partial/\partial\mathbf{p}$ acts only on the Green functions that enter in the left ladder, or on the Green function placed between two ladders. It is easily understood that each differentiation of the ladder Green functions increases the order of the diffusive pole. The field corrections turn out in this case to be large in terms of the parameter (7). When differentiating the Green function contained between the ladders, it suffices to retain only the first derivative, since the higher derivatives are small in terms of the parameter (6). All other arrangements of the operators $\partial/\partial\mathbf{p}$ and $\partial/\partial\varepsilon$ make contributions that are small in the impurity density.

We obtain thus for the retarded nonequilibrium mass operator the equation

$$\Sigma_r{}^{Q} = \frac{i}{2}\int \frac{d^4q}{(2\pi)^4}\frac{U_r(q)}{1-\zeta_q}\left[1 - ie\mathbf{E}\frac{\partial}{\partial\mathbf{p}}\frac{\partial}{\partial\varepsilon}\right] G_a(p-q)$$

$$\times \exp(-ie\mathbf{E}\hat{\varphi})\,[S(\varepsilon) - S(\varepsilon-\omega)]/(1-\zeta_q), \quad q = (\mathbf{q}, \omega). \tag{18}$$

Here

$$\zeta_q = u^2\int \frac{d^3p}{(2\pi)^3} G_r(p)\,G_a(p-q) \approx 1 - (\tau qv)^2/3 + i\tau\omega,$$

$$\hat{\varphi} = \frac{\partial}{\partial\mathbf{q}}\frac{\partial}{\partial\varepsilon}, \quad \frac{1}{\tau} = 2\pi N(0)\,u^2 \quad (\tau qv, \tau\omega \ll 1).$$

It is understood that the operator $\partial/\partial\varepsilon$ in (18) acts only on the function $S(\varepsilon)$–$S(\varepsilon-\omega)$. The electric current is given by

$$\mathbf{j} = e\int \frac{d^4p}{(2\pi)^4}\frac{\mathbf{p}}{m}[A(p)\,\delta S(p) + S(\varepsilon)\,\Delta A(p)], \tag{19}$$

$$\Delta A(p) = i(G_r{}^2\Sigma_r{}^{Q} - G_a{}^2\Sigma_a{}^{Q}).$$

Equation (19) is obtained from Eq. (3.7) of Ref. 5 if it is recognized that in the considered case the nonequilibrium part of the spectral function $A(p)$ is determined exclusively by the nonlocal field corrections.[8] For convenience, we have retained in (19) also the equilibrium part of the function $A(p)$, which drops out in the integration over the angles of the vector $\mathbf{p}$. The nonequilibrium correction

$$\delta F = A(p)\,\delta S$$

is obtained by iteration from the kinetic equation (9). The first iteration corresponds to allowance, in the collision integral, for only the scattering of electrons by static defects (see Fig. 1):

$$A(p)\,\delta S_1(p) = 2\tau \frac{\partial}{\partial \varepsilon_p}\left(\eta A + 2\,\mathrm{arctg}\,\frac{2\eta}{\Gamma}\right)\frac{\partial n(\varepsilon)}{\partial \varepsilon}(\mathbf{Ev}). \qquad (20)$$

By calculating the corresponding contribution to the current we can find the standard equation for the residual resistance.

We take now into account the processes shown in Fig. 2, and obtain the second iteration correction:

$$A(p)\,\delta S_2(p) = -A\delta S_1\Gamma^Q - A\delta S_1\delta\Gamma + i\frac{\partial S}{\partial \varepsilon}\mathbf{E}\frac{\partial \Gamma^Q}{\partial \mathbf{p}}A. \qquad (21)$$

Here $\Gamma^Q = -2\,\mathrm{Im}\,\Sigma_r^Q$, and $\delta\Gamma$ is the correction to the imaginary part of the diagram Fig. 1; this correction is due to the fact that the electron Green function is changed by the Coulomb process by an amount

$$\delta G_r = G_r^2\Sigma_r^Q. \qquad (22)$$

The third term in (21) appears when account is taken of the nonlocal corrections to the mass operator Ω^Q. When calculating the second term in (19) it is necessary to consider separately the terms of zeroth and first order in $-ie\mathbf{E}(\partial/\partial\mathbf{p})(\partial/\partial\varepsilon)$ in (18). The contribution of the linear term to the current cancels exactly the contribution of all the terms in (21). (When using the Kubo formula, this circumstance corresponds to cancellation of all the diagrams that do not contain a dividing ladder,[8] which follows from the electron-number conservation law.) Thus, to find for the total electric current the correction due to the Coulomb interaction, it suffices to consider the expression

$$\delta\mathbf{j} = -2\,\mathrm{Im}\left\{e\int \frac{d^4\mathbf{p}}{(2\pi)^4}\frac{\mathbf{p}}{m}S(\varepsilon)G_r^2(p)\Sigma_r^Q(p)\right\}. \qquad (23)$$

It is assumed here that in Eq. (18) for the mass operator Σ_r^Q are retained only terms of zeroth order in $-ie\mathbf{E}(\partial/\partial\mathbf{p})(\partial/\partial\varepsilon)$. Strictly speaking, it is necessary to retain in (23) also only the terms odd in $-i\mathbf{E}\hat{\varphi}$. However, we incur no error by retaining all the terms during the intermediate stages of the calculations, since they drop out of the final answer upon integration over the angles of the vectors $\mathbf{p}$ and $\mathbf{q}$.

We substitute (18) and (12) in (23) and recognize that the operator $\exp[-ie\mathbf{E}(\partial/\partial\mathbf{q})\times(\partial/\partial\varepsilon)]$, which acts on the a function of q, means a shift by the vector $-ie\mathbf{E}\partial/\partial\varepsilon$. Integrating with respect to $\mathbf{p}$, we obtain

$$\delta\mathbf{j} = e\tau\,\mathrm{Im}\left\{i\int d\varepsilon S(\varepsilon)\int \frac{d^4q}{(2\pi)^4}\frac{\tilde{\mathbf{q}}}{\tilde{q}^2}\frac{S(\varepsilon)-S(\varepsilon-\omega)}{(\tau qv)^2/3 - i\tau\omega}\right\}, \qquad (24)$$

where $\tilde{\mathbf{q}} = \mathbf{q} + i\mathbf{a}$, $\mathbf{a} = e\mathbf{E}\partial/\partial\varepsilon$. Integrating over the angles of the vector $\mathbf{q}$, we find

$$\delta\mathbf{j} = \frac{e\tau}{4\pi^3}\,\mathrm{Im}\left\{i\int d\varepsilon S(\varepsilon)\int d\omega\int_0^\infty dq q^2 F(q,\mathbf{a})\frac{S(\varepsilon)-S(\varepsilon-\omega)}{(\tau qv)^2/3 - i\tau\omega}\right\}, \qquad (25)$$

where

$$F(q,\mathbf{a}) = -\frac{\mathbf{a}}{2qa^2}\left\{iqa - \frac{q^2+a^2}{4}\,\mathrm{Ln}\left(\frac{q+ia}{q-ia}\right)^2\right\} \qquad (26)$$

(Ln denotes the principal value of the logarithm). In the weak-field limit $q \gg a$, i.e., when an inequality inverse to (7) is satisfied, we have

$$F(q,\ \mathbf{a}) = i\mathbf{a}/3q^2.$$

Substituting this result in (26) and performing the remaining integrations, we obtain the known expression for the Coulomb part of the resistance of an impurity metal.[6]

Of greater interest to us now, however, is the opposite limiting case, when inequality (7) holds, i.e., $q \ll a$. Then

$$F(q,\ \mathbf{a}) = -i\mathbf{a}/a^2. \qquad (27)$$

In the derivation of (27) with the operator $\partial/\partial\varepsilon$ acting on a real function, it is possible to act as with a real number. We transform the integral with respect to ω in (25) by parts in a way as to obtain the second derivative of the integrand with respect to ω. We then substitute (26) in (25) and obtain the correction to the electric current in a sample of unit volume, due to Coulomb processes (Fig. 2), in the strong field limit:

$$\delta\mathbf{j} = \frac{3\sqrt{3}}{10\sqrt{2}\pi^2}B_{5/2}\frac{c^{3/2}\varepsilon_F^{\,5}}{\hbar^4 v^3}\left(\frac{kT}{\varepsilon_F}\right)^{7/2}\frac{\mathbf{E}}{E^2}. \qquad (28)$$

Here B_n is a positive number defined by the integral

$$B_n = \int_0^\infty dx x^n \frac{d}{dx}\left(\frac{x}{1-e^x}\right). \qquad (29)$$

This result is easy to generalize to the case of quasi-two-dimensional and quasi-one-dimensional systems, in which one or two dimensions are smaller than the length of the diffusive mode. In this case the vector $\mathbf{q}$ must be regarded as two- or one-dimensional, respectively. In the strong-field limit the function (27) retains the same form regardless of dimensionality. After simple calculations we obtain

$$\delta\mathbf{j} = \frac{3B_2}{16\pi}\frac{c\varepsilon_F^{\,2}}{\hbar^3 v^2}\left(\frac{kT}{\varepsilon_F}\right)^3\frac{\mathbf{E}}{E^2} \qquad (30)$$

in the two-dimensional case and

$$\delta\mathbf{j} = \frac{\sqrt{2}B_{1/2}}{\sqrt{3}\pi}\frac{c^{1/2}\varepsilon_F^{\,3}}{\hbar^2 v}\left(\frac{kT}{\varepsilon_F}\right)^{5/2}\frac{\mathbf{E}}{E^2} \qquad (31)$$

in the one-dimensional case.

The physical meaning of the results (29)–(31) can be understood in the following manner. In strong electric field, when the condition (7) is satisfied, the electron motion is determined by the drift. No diffusive modes of corresponding length can be formed, and their contribution to the electron scattering is suppressed. The degree of this suppression is determined by the square of the parameter in (7), since the current is an even function of the electric field. The current corrections necessitated by the Coulomb mechanism tend therefore to zero as E^{-1} with increasing electric field. It is interesting that the electric field, suppressing the diffusive modes, eliminates simultaneously also the divergences that take place at $T \to 0$ in two- and one-dimensional systems.[9]

3. HEATING OF ELECTRON SUBSYSTEM

In the low-temperature region $T \ll T_D$ the nonlinear field effects can be due also to heating of the electron subsystem. The difference between the "electronic" temperature Θ and the lattice temperature T is determined from the balance of the energy generated by the electric field and dissipated from the electron subsystem into the phonon subsystem.[1-3] In many cases there is realized in metals and in degenerate semiconductors the so-called energy-control system, wherein the interelectron collisions determine the energy distribution among the electrons, but do not influence the momentum relaxation. The relation between the relaxation times in the energy-control regime are the following:

$$\tau_{eph}^{-1} \ll \tau_{ee}^{-1} \ll \tau^{-1}. \tag{32}$$

Here τ_{ee}^{-1} is the frequency of the electron-electron collisions, τ_{eph}^{-1} the frequency of the electron-phonon collisions, and τ^{-1} the frequency of the impurity-impurity collisions that cause the momentum relaxation. At temperatures $T < kT_D^2/\varepsilon_F$ the condition (32) is satisfied, since

$$\tau_{ee}^{-1} \sim \frac{T^2}{\varepsilon_F^2}, \quad \tau_{eph}^{-1} \sim \frac{T_D}{\varepsilon_F}\frac{T^3}{T_D^3}. \tag{33}$$

This statement, which is valid for pure metals, requires additional examination in the case of metals with impurities. The point is that at low temperatures, alongside the "pure" electron-phonon interaction processes (Fig. 3a), a substantial role is assumed by electron scattering by the vibrating impurities (Figs. 3b–d). The contribution of the pure processes ($\sim T^3/T_D^3$) dominates at temperatures $T > cT_D$, when the contribution of the impurity electron-phonon interaction is $\sim cT^2/T_D^2$. At temperatures $T < cT_D$ the situation is less obvious, since it is now necessary to take into account also the contribution of numerous interfering electron-impurity and electron-phonon interactions.[8,10] The leading processes in this case are those shown in Fig. 3e–3g. Summing the contribution corresponding to them with the contribution of the processes shown in Fig. 3b–3d, we can verify that they cancel one another and make a zero contribution to the energy relaxation. A similar cancellation takes place also in the electric resistance,[8] as well as in the polarization operator that determines the damping and the renormalization of the long-wave phonons.[10] It would be necessary now in principle to consider the weaker processes shown in Fig. 2, in which the Coulomb line is replaced by a phonon line and the vertices are replaced by pure and impurity (Fig. 3) electron-phonon vertices. It can be easily verified, however, that the arrangement of the Keldysh indices does not change in such diagrams, which will therefore include only retarded (D_r) or advanced (D_a) phonon Green functions that do not contain the temperature of the phonon subsystem. This means that only momentum relaxation, but not energy dissipation, takes place in such processes. These processes do depend, however on the electron temperature and are in this sense inelastic.

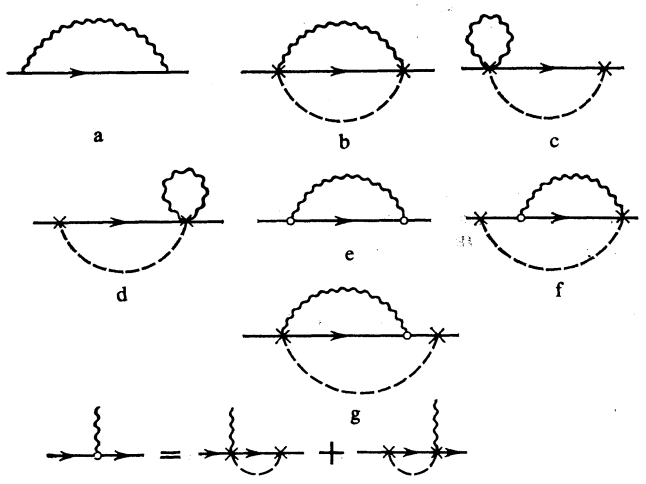

FIG. 3. Diagram for the mass operators corresponding to different electron-phonon interaction processes. A wavy line denotes here a phonon Green function.

Thus, in the entire low-temperature region, energy exchange between the electron and phonon subsystems is determined by pure electron-phonon interaction processes (Fig. 3a). If relation (32) is satisfied, the electron subsystem becomes thermalized and to find the quasi-equilibrium electron temperature Θ we must use the energy-balance equations.[1-3] It must be remembered here that at $T, \Theta < cT_D$ the Boltzmann quasi-classical equation no longer holds, and to describe the energy exchange between the electron and phonon subsystems we must use the more general quantum-kinetic equation (9).

The power generated in the electron subsystem by the electric field is

$$P_1 = E^2/\rho_0, \tag{34}$$

where ρ_0 is the residual resistivity of the metal. The power dissipated from the electron subsystem in electron-phonon collisions

$$P_2 = \int \frac{d^4p}{(2\pi)^4} \varepsilon J(p), \tag{35}$$

is expressed in terms of the collision integral

$$J(p) = \Sigma^+(p)G^-(p) - \Sigma^-(p)G^+(p), \tag{36}$$

which determines the energy flux from the electron into the phonon subsystem. We have used here a different form of the collision integral, equivalent to (9). The mass operators in (36) correspond to the diagrams of Fig. 3a and take the form

$$\Sigma^{\pm}(p) = i\int \frac{d^4q}{(2\pi)^4} |g(q)|^2 D^{\pm}(q) G^{\pm}(p-q), \tag{37}$$

where $g(q)$ is the matrix element of the electron phonon interaction in a pure metal,[1]

$$D^{\pm}(q) = \pm i\pi[{}^1/_2 \mp ({}^1/_2 + N(q))][\delta(\omega-\omega_q) - \delta(\omega+\omega_q)],$$
$$G^{\pm}(p) = \pm i[{}^1/_2 \mp ({}^1/_2 - n(p))]A(p), \tag{38}$$

$N(q)$ and $n(p)$ are the phonon and electron distribution functions. Under conditions when the electron subsystem is ther-

malized, $N(q)$ and $n(p)$ are respectively Bose and Fermi functions corresponding to the phonon (T) and electron (Θ) temperatures. The spectral function

$$A(p)=\frac{\hbar/\tau}{(\varepsilon-\varepsilon_p)^2+\hbar^2/4\tau^2} \tag{39}$$

takes into account the finite lifetime of the electrons. The phonon damping in the considered temperature region can be neglected.

After carrying integration with respect to the momentum p and the energies ε and ω in (35) and (37) we obtain for P_2, in the case of three-dimensional systems, the expression

$$P_2=N(0)\int\frac{d^3q}{(2\pi)^3}|g(q)|^2\frac{\omega_q^2}{qv}\operatorname{arctg}(\tau qv)$$
$$\times[N_\Theta(\omega_q)-N_T(\omega_q)]. \tag{40}$$

The power-balance condition $P_1=P_2$ leads to an equation that describes the quantities E, Θ, and T. Transforming to a dimensionless electron-phonon interaction constant

$$\bar g=g(p_F)N(0), \tag{41}$$

this equation take place at temperatures Θ, $T\ll cT_D$ the form

$$\frac{E^2}{\rho_0}=\frac{2\tau\varepsilon_F\bar g^2}{\hbar^2}A_dk^2\frac{\Theta^{d+3}-T^{d+3}}{T_D^{d+1}}. \tag{42}$$

We have generalized here Eq. (40) to include the one- and two-dimensional cases ($d=1$, 2, and 3 is the dimensionality index). It is assumed also that the leading role is played by scattering from acoustic phonons, i.e., $g^2(q)\propto q$;

$$A_d=\int_0^\infty\frac{x^{d+2}}{e^x-1}dx. \tag{43}$$

At temperatures Θ, $T\gg cT_D$ (but Θ, $T\ll T_D$) we obtain

$$\frac{E^2}{\rho_0}=\frac{\pi}{2}\bar g^2\frac{A_{d-1}}{\hbar}k^2\frac{\Theta^{d+2}-T^{d+2}}{T_D{}^d}. \tag{44}$$

We have assumed here that in quasi-two-dimensional and quasi-one-dimensional systems one or two dimensions become smaller than the wavelength of the phonon. In this case the electron momentum on the Fermi surface remains three-dimensional. If, however, the dimensionality of the Fermi momentum is also lowered, it is impossible to obtain a universal relation such as (42) or (44), and the balance equations for systems with different dimensionalities take the form

$$\frac{E^2}{\rho_0}=\begin{cases}\dfrac{\pi}{2}\bar g^2A_2\dfrac{k^2}{\hbar}\dfrac{\Theta^5-T^5}{T_D{}^3}, & d=3,\\ \bar g^2A_1\dfrac{k^2}{\hbar}\dfrac{\Theta^4-T^4}{T_D{}^2}, & d=2,\\ \bar g^2A_{-1}k^2\dfrac{\Theta^2-T^2}{2\tau\varepsilon_F}, & d=1.\end{cases} \tag{45}$$

It is interesting to note that in the one-dimensional case of (45) the right-hand side contains a dependence of τ, and the power transferred from the electrons to the lattice decreases steeply at a low defect density $[(\tau\varepsilon_F)^{-1}\sim c\ll 1]$.

Equation (42) is valid if $\Theta\ll cT_D$, consequently

$$\frac{E^2}{\rho_0}\ll c^{d+2}T_D{}^2\frac{k^2}{\hbar}. \tag{46}$$

On the other hand, a noticeable heating of the electron subsystem means that $\Theta\gg T$, i.e.,

$$\frac{E^2}{\rho_0}\gg c^{d+2}T_D{}^2\left(\frac{T}{cT_D}\right)^{d+3}\frac{k^{d+3}}{\hbar}. \tag{47}$$

Comparing (46) and (47), it can be understood that at $T\ll cT_D$ there exists a range of electric-field intensities at which both inequalities (46) and (47) are simultaneously satisfied. A situation may also arise with $T\ll cT_D$ but $\Theta\gg cT_D$. In this case Eqs. (42) and (43) must be used. The electrons are heated so strongly in fields

$$\frac{E^2}{\rho_0}\gg c^{d+2}T_D{}^2\frac{k^2}{\hbar}. \tag{48}$$

We compare now the criterion for the start of heating (46) with the criterion for the start of the electron acceleration in the electric field within the scattering length (7). The latter can be written in the form

$$\frac{E^2}{\rho_0}\gg\frac{k^3}{\hbar}\frac{T^3}{\varepsilon_F}. \tag{49}$$

It follows then from (47) and (49) that electron heating predominates under the condition

$$(T/cT_D)^d\ll c^{d-1}kT_D/\varepsilon_F, \tag{50}$$

which can be easily reconciled with the thermalization condition (32) and with the requirement $T<cT_D$.

In principle it is possible also to satisfy a relation inverse to (50). It is then difficult, however, to satisfy simultaneously the requirement $T<cT_D$ and the thermalization condition (32). It is possible, of course, to go over into a region of higher temperatures ($T>cT_D$). Then, at

$$T>T_D(kT_D/\varepsilon_F)^2 \tag{51}$$

the diffusive-mode suppression mechanism considered in Sec. 2 should be observed in pure form. Unfortunately, the main contribution to the resistance (apart from the residual one) is made in this temperature range not by the Coulomb mechanism but by scattering of electrons by vibrating defects, or even by pure electron-phonon interaction,[8] so that an experimental observation of the effect is substantially more difficult. Electron heating may turn out to be relatively weak in metallic glasses, in which there is one more relaxation mechanism connected with the inelastic scattering of the electrons by two-level systems.

In impurity metals one should more readily expect both effects, electron heating and suppression of diffusive modes, to play a substantial role simultaneously. We should then use the equations for the Coulomb and phonon parts of the conductivity and replace in them the thermostat temperature T by the quasi-equilibrium temperature Θ obtained for the electron subsystem from Eqs. (42), (44) or (45).

In the strong-field limit, when the conditions (50) and (46) are satisfied (i.e., $\Theta < cT_D$),

$$\Delta\sigma \propto E^{-5/6}, \tag{52}$$

but if $\Theta > cT_D$, the decisive contribution to the resistance is made by scattering of electrons from the vibrating impurities,[8,11] and

$$\Delta G \propto -E^{4/5}. \tag{53}$$

At $\Theta > c^{1/3}T_D$ the "pure" electron-phonon interaction begins to make the main contribution to the resistance (in our model $\rho_{ph} \sim (\Theta/T_D)^5$). The corresponding relations can be obtained for both the two-dimensional systems

$$\begin{aligned} &\Delta\sigma \propto E^{-4/5}, \quad \Theta < cT_D, \\ &\Delta\sigma \propto -E, \quad \Theta > cT_D, \end{aligned} \tag{54}$$

and for one-dimensional ones

$$\begin{aligned} &\Delta\sigma \propto E^{-3/4}, \quad \Theta < cT_D, \\ &\Delta\sigma \propto \sim -E^{\leqslant 4/3}, \quad \Theta > cT_D. \end{aligned} \tag{55}$$

The overall picture of the field dependence of the conductivity is the following (see Fig. 4). (We recall that we are dealing with a correction to the conductivity. The leading contribution is connected with the scattering of the electrons from the static defects, and obeys Ohm's law in the entire considered range of the electric field strengths.) Initially, in weak fields, $\Delta\sigma$ is positive and does not depend on E. The conductivity then increases because of the electron heating. When the suppression of the diffusive modes in the electric field comes into play, however, $\Delta\sigma$ reaches a maximum and begins to decrease. At $\Theta \gtrsim cT_D$ it reverses sign and begins to increase in absolute value on account of electron scattering by the vibrating impurities. On the whole, this picture does not depend on the dimensionality of the considered system, and only the exponents of E change on different sections of the plot of the conductivity against the electric field.

A sufficiently strong electric field should, in principle, destroy not only the diffusive modes but also the "Copper" ones.[12] However, the critical fields at which the nonlinear effects appear turn out to be much stronger,[13] i.e., $eEl_p > \tau^{-1}$. An analysis of this question can be carried out in analogy with an analysis of the suppression of diffusive modes. Then differentiation with respect to the momentum transfer likewise appears in the nonlocal terms. But the singularity does not become enhanced in the Copper channel, since it is connected with a small total momentum.

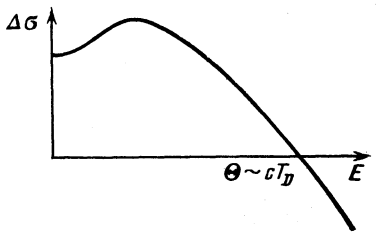

FIG. 4. Correction to conductivity vs electric field strength at a fixed phonon-subsystem temperature ($T < cT_D$).

[a] Strictly speaking, one should use here the impurity-metal matrix element obtained in the virtual-crystal approximation (see Ref. 10 for details).

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