Numerical simulation of a Langmuir collapse

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The evolution of an isolated Langmuir caviton (low-density region in which Langmuir oscillations are localized) is studied. For this purpose, the Vlasov equations for electrons and ions are solved by a numerical method for the case of two spatial variables. It is shown that under certain conditions caviton collapse takes place and enhances considerably the oscillating electric field. It is shown that under typical conditions there is no time for the collapse to become supersonic. When the caviton is compressed to a size less than $10r_D$ the energy of the oscillations localized in it is transferred almost completely to the fast electrons.

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1. INTRODUCTION

The first study of the collapse of Langmuir waves[1] was published in 1972. It was shown there that the development of modulation instability[2] leads to localization of oscillations of the electric field and to formation of cavitons, which are regions with decreased plasma density. Cavitons collapse within a finite time, and during the concluding stage of this collapse (Langmuir collapse) the energy of the oscillations trapped in the caviton is dissipated and the electrons are heated.

It is easily understood that Langmuir collapse should play a most important role in the physics of plasma turbulence. Collapse ensures an effective mechanism of absorption of energy from long-wave Langmuir oscillations in a collisionless transparent plasma, and therefore determines the character of the interaction of the plasma with high-power electron beams and electromagnetic radiation. It is important that as a result of the collapse the energy of the Langmuir oscillations is transferred not to all electrons, but only to a small group of them. This circumstance must be taken into account in the theory of collective methods of plasma heating.

We note that collapse takes place also for other branches of the plasma-oscillation spectra, particularly for electromagnetic[3] and lower hybrid[4] waves. Thus, investigation of wave collapse is an essential part of many problems in the theory of collisionless plasma.

A direct experimental observation of the collapse of an isolated caviton entails considerable difficulties because of its small size (tens to hundreds of Debye radii r_D) and short lifetime (hundreds of plasma periods $2\pi/\omega_p$). It was possible, however, to observe in experiment[5,6] structures that can be naturally interpreted as collapsing cavitons. The appearance of non-Maxwellian accelerated electrons in experiments on laser and beam heating of a plasma can also be regarded in many cases as a macroscopic consequence of collapse.

The heretofore performed theoretical investigations of Langmuir collapse[7–12] were based on time-averaged dynamic equations.[1] These equations are not valid, however, during the concluding stage of the caviton evolution, when the Landau damping sets in and the energy of the Langmuir oscillations becomes dissipated. A correct approach to the study of the concluding stage of the collapse calls for solving the complete system of Vlasov's equations for electrons and ions, and in view of the complexity of the problem this can be done only by numerical methods. In this situation, principal significance attaches to numerical simulation of the collapse by the method of particles. In this method, the model of the plasma is a system of a large number of charged "macroparticles," i.e., the simulation is carried out from first principles. This makes it possible to regard the numerical experiment as a certain analog of a physical experiment. We note that the method of particles turned out to be quite effective for the investigation of plasma turbulence and of the associated problems of absorption and scattering of light in a plasma (see Refs. 13–15).

Attempts to investigate Langmuir collapse by numerical methods were undertaken earlier in Refs. 16–18. The results of numerical simulation, carried out in these studies, do not agree fully with those ideas concerning collapse which came into being following the publication of Refs. 1 and 7–12. The possible causes of this discrepancy will be discussed below (see Sec. 4). Here we note only that the main problem of the kinetic description of the evolution of an isolated Langmuir caviton was not solved in Refs. 16–18.

The present paper is devoted to a numerical simulation of Langmuir collapse by the method of particles. We investigate in detail the evolution of a caviton in two-dimensional geometry. The choice of this case is due to the fact that an adequate investigation of the real three-dimensional collapse problem is beyond the capabilities of modern computers. The calculations performed show clearly the entire sequence of events that takes place in collapse: the caviton contraction accompanied by a considerable growth of the oscillating electric field, the dissipation of the oscillation energy during the concluding stage of the collapse, and the formation of "tails" of fast electrons. The qualitative picture of the collapse which follows from the calculations agrees with the results of Refs. 1 and 7–12.

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2. WHY IS A NUMERICAL SIMULATION OF THE LANGMUIR COLLAPSE NECESSARY?

It was noted above that the collapse concept was created on the basis of averaged dynamic equations obtained in Ref. 1

$$\Delta\left(i\frac{\partial\psi}{\partial t}+\frac{3}{2}\omega_p r_D^2\Delta\psi\right)=\frac{\omega_p}{2n_0}\operatorname{div}(\delta n\nabla\psi), \quad (1)$$

$$\frac{\partial^2\delta n}{\partial t^2}-c_s^2\Delta\delta n=\frac{1}{16\pi M}\Delta|\nabla\psi|^2. \quad (2)$$

Here $\psi(r, t)$ is the complex envelope of the high-frequency electric potential,

$$E=-{}^1/_2\nabla[\psi\exp(-i\omega_p t)+\text{c.c.}],$$

δn is the slow variation of the plasma density, c_s is the speed of ion sound, n_0 is the equilibrium plasma density, and M is the ion mass. The system (1), (2) has two integrals of motion: the number of Langmuir quanta

$$I_1=\frac{1}{8\pi\omega_p}\int dV|\nabla\psi|^2$$

and the "Hamiltonian"

$$I_2=\int dV\left\{\frac{3\omega_p r_D^2}{64\pi}|\Delta\psi|^2+\frac{\delta n}{16\pi n_0}|\nabla\psi|^2+\frac{1}{2}Mn_0v^2+\frac{1}{2}Mc_s^2\frac{\delta n^2}{n_0}\right\}, \quad (3)$$

where v is the hydrodynamic velocity of the plasma. The total plasma energy is connected with the integrals I_1 and I_2 by the relation

$$\mathscr{E}=\omega_p I_1+I_2.$$

Investigation of the system (1), (2) shows[1] that the sufficient condition for collapse is that the Hamiltonian I_2 be negative. In the static limit, when the ions move at subsonic velocities and the derivative with respect to time can be neglected in (2), this statement has the character of a mathematical theorem. In the general case, variational estimates were found,[7,8] self-similar solutions[1,7,8] describing the collapse of the caviton were obtained, and a numerical solution[7–9] of Eqs. (1) and (2) confirmed the qualitative collapse picture proposed in Ref. 1 (in particular, the existence of a self-similar regime).

It is important, however, that during the concluding stage of the collapse, when the dimension of the caviton reaches several Debye radii, Eqs. (1) and (2) no longer hold. In this region there is dispersion of the oscillations, the electronic nonlinearities disregarded in (1) and (2) become substantial, and it becomes necessary to take into account the interaction of the waves with the particles. This raises a number of questions, which can be answered only by numerical simulation. It is necessary first to show that the caviton actually contracts to dimensions comparable with the Debye radius, despite the change in the dispersion law at $kr_D\sim 1$ and the influence of the electronic nonlinearities, which generally speaking could stop the collapse. A fundamentally important question is which part of the energy trapped in the collapse process is transferred to the particle, in other words, what is the efficiency of dissipation in collapse.

After "burning out" of the Langmuir oscillations, what is left of the caviton is a region of perturbed density, which vanishes within acoustic times. It is possible that it is precisely these density fluctuations (and not the evolution of the collapsing caviton) which is more observable in experiment. In addition, it became clear recently that conversion of the Langmuir and electromagnetic waves by short-wave sound is an effective wave-dissipation mechanism.[19] The collapse is one of the mechanisms that generates short-wave sound, and to estimate the efficiency of the generation it is necessary to know the minimum dimension of the caviton and the extent of variation of the density. Finally, one of the principal microscopic consequences of collapse is generation of fast electrons. The study of this process is also quite difficult without numerical simulation.

Thus, there are many phenomena whose understanding calls for a detailed investigation of the concluding stage of Langmuir collapse, and this is possible only within the framework of numerical simulation.

3. DIFFICULTIES OF NUMERICAL SIMULATION

Adequate simulation of Langmuir collapse is at the limit of the capabilities of the modern computer technology. The difficulties that arise are due mainly to the discrete character of the model. To obtain as large an inertial interval as possible, the calculation must be started with the lowest oscillation density W compatible with the collapse criterion

$$W/nT>(kr_D)^2.$$

Since $k_{\min}=2\pi/L$, where L is the size of the simulation region, it is necessary to use in the calculations a grid with a maximum possible number of nodes. To suppress the parasitic shortwave computational instabilities with a wave vector that differs from k by a value equal to the reciprocal-lattice vector, the mesh of the grid must not exceed the Debye radius. In our calculations we used a grid $64r_D\times 64r_D$, ensuring a minimum value $(kr_D)^2\simeq 10^{-2}$.

The characteristic level of Langmuir turbulence should be much higher than the level of the thermal fluctuations. The latter is inversely proportional to the number N_d of particles in the Debye cell. In practice N_d cannot be less than 20. Thus, even in the two-dimensional case, simulation of Langmuir collapse on a grid $(64r_D)^2$, calls for $\sim 10^5$ macroparticles, which is a rather stringent requirement on the memory and operating speed of the computer. Solution of a three-dimensional problem on a $(64r_D)^3$ grid calls for 10^7 particles, which makes three-dimensional calculation exceedingly difficult at the contemporary level of computer technology.

Our numerical experiments were performed on a multiprocessor computer.[20] A feature of this computer is the possibility of distributing in parallel the algorithms both among the arithmetic processors and among parallel-operating devices that are of different types. This circumstance turned out to be most important when solving problems in plasma kinetics by the particle method. The program that realizes the algorithm of this method in the computer is described in

Ref. 21. To write the program we used certain results of the interesting paper by Kamimura *et al.*[22] The use of parallel distribution has made it possible to achieve an inner-cycle time of 30 μsec/particle with a total computation time that depends linearly on the product of the number of particles by the number of time steps. When simulating Langmuir collapse with 2×10^5 microparticles, the performance of 10^3 time steps ($\sim 200\ \omega_p^{-1}$) the required computer time was approximately two hours.

4. CHOICE OF INITIAL CONDITIONS

In Eqs. (1) and (2), on the analysis of which the concept of Langmuir collapse is based, the fast and slow motions are explicitly separated. The collapse criterion $I_2<0$ is expressed likewise in the corresponding variables. In a numerical simulation by the particle method, there is no such separation of variables and one specifies the initial density and velocity distributions of all the plasma particles. Because of the large ratio of the ion and electron masses, it can be assumed that the slow ion-sound motions should correspond to quasineutral variation of the density $\delta\bar{n}$, and the separation of the charges $\delta\tilde{n}$ causes high-frequency Langmuir oscillations. Since the only negative term in the integrand of Eq. (3) can be the one with $\delta\bar{n}|\nabla\psi|^2$, we must have $\delta\bar{n}\neq0$ in the initial condition to which $I_2<0$ corresponds. Thus, the initial particle-density distribution should take the form

$$n_e=n_0+\delta\bar{n}+\delta\tilde{n},\quad n_i=n_0+\delta\bar{n}. \tag{4}$$

If, as was done in Refs. 16–18, the initial ion distribution density is regarded as spatially homogeneous, $\Delta\bar{n}\equiv0$, it follows from this directly that $I_2>0$. Of course, in this case I_2 can become negative in certain regions during the course of the evolution. However, in accordance with the statements made above, there is in practice no inertial interval for such a perturbation. To speak of collapse in this situation is meaningless. Local maxima of the field, which were observed in numerical experiments,[16] can be easily explained as being due to interference of plasma waves that cannot leave the calculation region because of the periodic boundary conditions.

The initial density profiles in our calculations were chosen in the following manner:

$$\delta\bar{n}=\varepsilon_1[\cos(2\pi x/L)+\cos(2\pi y/L)],$$
$$\delta\tilde{n}=\varepsilon[\cos(\pi x/L)+\cos(\pi y/L)].$$

The initial microscopic velocities of the electrons and ions were assumed equal to zero. The parameter ε_1 was chosen such that at $t=0$ the high-frequency electric field connected with $\delta\tilde{n}$ by the Poisson equation $\nabla^2\psi=-4\pi e\delta\tilde{n}$, satisfy the condition

$$|\nabla\psi|^2/16\pi T=-\delta\bar{n},$$

i.e., that the ponderomotive forces balance the variation of the thermal pressure. In the numerical calculations we varied the electron and ion temperatures, the parameter ε, and the mass ratio: $100\leqslant M/m\leqslant800$.

5. RESULTS OF CALCULATIONS

Our calculations pertain to two-dimensional collapse. From the physical point of view, this case is important in principle. It is known[2] that the Langmuir turbulence can be qualitatively regarded as a gas of quasiparticles (plasmons) with attraction. In view of the conservation of the number of plasmons, the frequency shift (in the static limit) increases with decreasing caviton dimension l, in proportion to l^{-d}, where d is the dimensionality of space. The dispersion increment to the frequency is proportional to l^{-2}. Therefore in the one-dimensional case the contraction is always stopped, in the three-dimensional case the role of the nonlinearity in the contraction process increases, while the two-dimensional case is intermediate: the qualitative character of the evolution of the caviton is determined in this case by parameters that enter in the initial conditions. We note that in the three-dimensional situation the collapse criterion is weaker than in the two-dimensional one: collapse is possible also at $I_2>0$.[1]

The threshold value of the parameter ε obtained from the condition $I_2=0$ turned out (at $T_i=0$) to be $\varepsilon_c\sim0.01$. Calculation of a variant with $\varepsilon=0.01$ did not yield a clear picture of the collapse. The distribution of the oscillation energy turned out to be qualitatively similar to that obtained in Ref. 17 at $I_2>0$. In variants with $\varepsilon=0.025, 0.04$, and 0.05 a clear picture of the collapse was observed, followed by "burning out" of the isolated caviton. Figures 1 and 2 show the spatial profiles of the energy density of the Langmuir oscillations and of the ion density at several successive instants of time. The small caviton satellites observed in the figures are not significant and are apparently due to the choice of the boundary condition. In the calculations carried out for an isothermal plasma, these satellites are much less pronounced. The caviton contracts with acceleration, and then after a certain limiting size is reached and the Landau damping sets in, the energy contained in the caviton "burns out" rapidly. This is clearly seen from Fig. 3, which shows the time dependence of the energy density of the oscillations at the center of the caviton. We note that the energy characteristics obtained from the numerical calculation by the method of particles have a rapidly oscillating (with frequency $2\omega_p$) component, therefore the plots of the corresponding quantities, given in papers on numerical simulation, have a characteristic smeared-out form. Of physical meaning are the time-averaged quantities, which are in fact presented in the present paper. The averaging was carried out over an interval $\Delta t=6\omega_p^{-1}$ in accord with

$$\bar{f}=(\Delta t)^{-1}\int_{t-\Delta t}^{t} f\,dt.$$

The energy of the oscillations trapped in a collapsing caviton is transferred almost completely to the electrons. This can be seen from Fig. 4, which shows the temporal evolution of the energy of the Langmuir oscillations. It can be seen that the absorption of the energy by the particles proceeds very rapidly, and the energy fraction ($\sim50\%$) remaining during the concluding stage belongs to oscillations that do not land in the caviton.

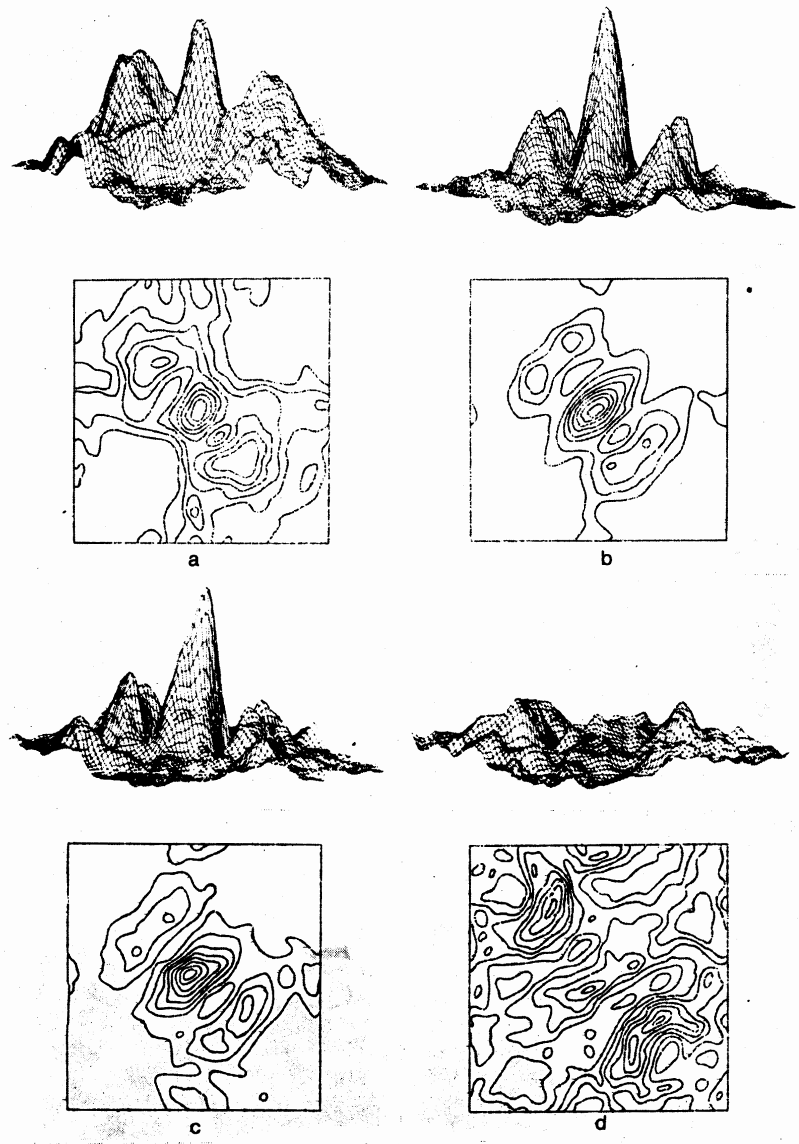

FIG. 1. Spatial profiles and level lines of the quantity $|E|^2/8\pi nT$ at different instants of time: a) $\omega_p t = 45$, b) $\omega_p t = 65$, c) $\omega_p t = 90$, d) $\omega_p t = 130$.

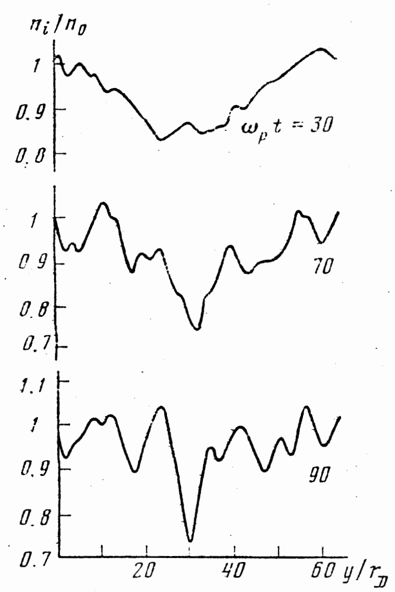

FIG. 2. Distribution of the ion density in the section $x = L/2$.

It follows from Fig. 5 that the energy of the oscillations is absorbed by rapid electrons. The anisotropic character of the heating is due to the anisotropy of the caviton, which can be clearly seen from Fig. 1. The shape of the caviton is similar to that obtained from the numerical solution of the averaged equations ($l_{\parallel} \approx 2l_{\perp}$) (Ref. 7).

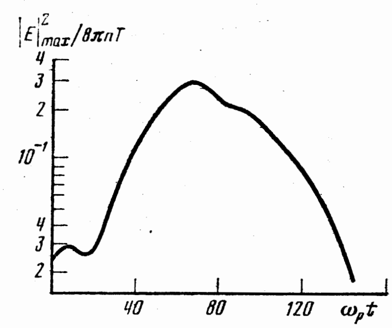

FIG. 3. Energy density of the oscillations at the center of the cavity as a function of the time.

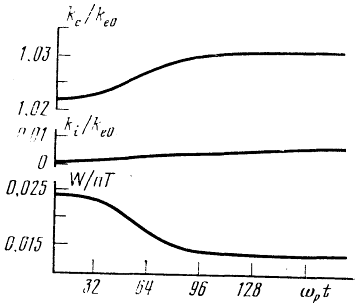

FIG. 4. Time variation of the total energy of the electrons k_e, of the ions k_i, and of the Langmuir oscillations.

A characteristic collapse feature that follows from the calculations is the rather large minimum size of the collapse, $l_\perp \approx 6r_D$, which does not depend on the initial amplitude of the oscillations. This result can be understood from the following simple considerations. The electric field inside the caviton varies approximately like cos $\omega_p t$. If the particles pass through the caviton within a time $\leqslant \pi/\omega_p$, they are accelerated and consequently absorb effectively the energy of the Langmuir oscillations. It is usually assumed on the basis of the numerical calculations that the intense damping is due to particles having velocities $v \sim (2-3)v_T$, and it is this which explains the size limit obtained from the calculation. We note that cavitons of just this size were observed in experiment[6] in beam excitation of oscillations in an isotropic plasma.

The energy trapped in the caviton in the course of the collapse does not leave the cavity prior to the start of the damping. Therefore, with increasing initial amplitude, the final amplitude also increases. Thus, at $\varepsilon = 0.02$ the maximum ratio $\eta = |E|^2_{max}/8\pi nT$ at the center of the caviton is 0.08, and at $\varepsilon = 0.05$ we have $\eta = 0.45$. The maximum density variations $\delta n/n_0$ amount in these cases to 0.18 and 0.27, respectively.

We note that at very high amplitude of the initial perturbation the picture of the collapse is no longer clear. An appreciable fraction of the particles is trapped in this case by the oscillations and it is possible that stationary structures analogous to the BGK solutions[23] are produced.

The calculation results shown in the figures were obtained for a mass ratio $M/m = 100$. The same versions, but with mass ratios 400 and 800, gave similar results. In particular, it was found that the collapse time is practically independent of the ratio M/m. This means that the collapse remains subsonic all the way to the concluding stage. The maximum variation of the density decreases somewhat with increasing mass ratio, and the energy fraction transferred to the ions remains small in all the variants (see Fig. 4). Thus, the picture of the evolution of Langmuir collapse, which follows from the numerical calculation, is in qualitative agreement with the results of Refs. 1 and 7–9, in which an averaged description was used.

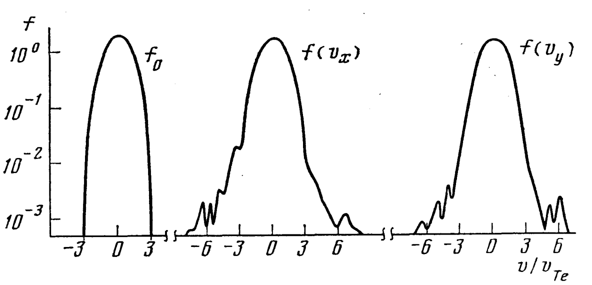

FIG. 5. Distribution function of the electrons; f_0—Maxwellian distribution at $t = 0$; $f(v_x)$ and $f(v_y)$ are the distributions at the instant of time $\omega_p t = 170$.

Simulation by the particle method makes it possible to describe more rigorously the concluding stage of the collapse. It shows, in particular, that the collapse does not manage to reach the ultrasonic regime, the minimum size of the caviton is of the order of $\sim 10r_D$, and the "burning out" of the energy trapped in the caviton is practically complete.

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