Loss and capture of electrons by fast ions and atoms of helium in various media

I. S. Dmitriev, N. F. Vorob'ev, Zh. M. Konovalova, V. S. Nikolaev, V. N. Novozhilova, Ya. A Teplova, and Yu. A. Faĭnberg

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We have measured cross sections for loss and capture of one and two electrons by ions and atoms of helium with velocities $v = 8 \cdot 10^8$, $1.2 \cdot 10^9$, and $2 \cdot 10^9$ cm/sec on passage through helium, nitrogen, neon, and argon and have calculated cross sections for capture of an electron by the ions He^{+2} and He^+ in the region $v = 2 \cdot 10^8 - 10^{10}$ cm/sec in collisions with atoms of practically all elements with nuclear charge Z_t from 1 to 100. We have established that for helium atoms with $v = (3-12) \cdot 10^8$ cm/sec in the region $Z_t \leq 18$ the dependence of the experimental cross sections for electron loss on Z_t is close to step-like and the dependence of the electron capture cross sections on Z_t for all helium ions, at least for velocities v from $3 \cdot 10^8$ to $2 \cdot 10^9$ cm/sec in the region of Z_t from $v/2v_0$ to 100, is oscillatory. We have shown that corresponding oscillations occur also in the values of the equilibrium charge fractions and the average charge of the fast helium ions.

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I. INTRODUCTION

In previous experimental studies¹ of the cross section for loss and capture of electrons by ions of nitrogen and neon in various gases it was observed that at an ion velocity $v \sim 10^9$ cm/sec the dependence of these cross sections on the nuclear charge of the atoms of the medium Z_i is nonmonotonic: the cross sections for capture of electrons in neon are substantially greater than in nitrogen and argon, and the cross sections for loss of electrons in neon are somewhat less than in nitrogen. On increase or decrease of the ion velocity these anomalies in neon disappeared. A nonmonotonic dependence of cross sections on Z_t for $v > v_0 = 2.19 \cdot 10^8$ cm/sec has been noted also for several lighter ions and atoms. As a result of this, to study of the dependence of the cross sections on Z_t in the present work we have determined experimentally the cross sections for loss and capture of one and two electrons by the ions He^{+2} and He^{+} and by He atoms in helium, nitrogen, neon, and argon at $v = 8.10^8$, $1.2.10^9$, and $2 \cdot 10^9$ cm/sec and have carried out theoretical calculations of the cross sections for capture of an electron by the ions He^{+2} and He⁺ in collisions with atoms of almost all elements with Z_t from 1 to 100 at v from v_0 to $60v_0$. In previous experiments²⁻¹⁴ on the determination of such cross sections for ions and atoms of helium in the region $v \ge 3v_0$ in neon, only the cross section for loss of one electron by helium atoms was determined. To check the results of the cross-section study, we have considered the correspondence of the cross sections with the experimental data on the equilibrium charge composition of beams of helium ions which have passed through various targets.^{6,14–22} Some of the results have been reported in a previous article.23

2. DESCRIPTION OF EXPERIMENTS AND CALCULATIONS

2.1. Experimental method

Cross sections for loss and capture of electrons σ_{ik} , where *i* and *k* are the initial and final charges of the particles, were determined in an experimental apparatus which differed from that described previously³ mainly in the fact that for making measurements with neutral particles an additional magnetic separator was included. We extracted from the 72-cm cyclotron He⁺ ions with $v = 8 \cdot 10^8$ cm/sec and He⁺² ions with $v = 1.2 \cdot 10^9$ and 2.10^9 cm/sec. Atoms and ions with other charges at the same velocities v were obtained by passing the ion beam through a celluloid film of thickness $2-3 \mu g/cm^2$. The errors in the resulting cross sections σ_{ik} , which were determined mainly by the errors in determination of the thickness of the gas target and by the statistical spread of the results of individual measurements, amounted as a rule to 10-15% for $\sigma_{i,i\pm 1}$ and 20-25% for $\sigma_{i,i\pm 2}$.

Since in beams of fast helium ions a certain fraction of them α are in metastable states, the values σ_{01} and σ_{02} obtained from experiments should differ somewhat from the cross sections σ_{01}^0 and σ_{02}^0 for unexcited atoms: $\sigma_{0k}/\sigma_{0k}^0 = 1 + \alpha \left[\sigma_{0k}^m/\sigma_{0k}^0 - 1 \right]$, where σ_{0k}^m is the cross section for metastable particles. From the experimental values $\alpha = 26 + 4\%$ obtained in Ref. 11 at $v \approx 7.10^8$ cm/sec, the dependence of α on v which follows from calculations of the cross sections for capture of an electron into various states of the helium atom carried out Ref. 24 and in the present work, and from the ratio of the cross sections σ_{01}^m and σ_{01}^0 calculated in Ref. 25, it follows that the values of σ_{01} for $Z_t = 2, 7$, and 10 should exceed the values of σ_{01}^0 by 10–15% at $v = 8 \cdot 10^8$ cm/sec and by 6-8% at $v = 2 \cdot 10^9$ cm/sec. The excess of the values of σ_{01} in argon is respectively 12–18% and 7–10%. The relative error in the cross section σ_{02} turns out to be the same if in accordance with the estimates obtained in Ref. 26 we assume that $|\sigma_{02}^m/\sigma_{02}^{09}-1| \le 0.8$. Thus, the deviation of the values found for σ_{01} and σ_{02} from σ_{01}^0 and σ_{02}^0 in most cases does not exceed the error given above for the cross sections.

2.2. Calculation of electron-capture cross sections

Cross sections for electron capture $\sigma_{i,i-1}$ were calculated in the Oppenheimer-Brinkman-Kramers (OBK) approximation,²⁷ in which the cross section $\sigma(a \rightarrow c)$ for capture of an

electron from an initial state a of a hydrogen-like system with nuclear mass A into a final state c of a fast particle with mass C is represented (with use of atomic units in which $\mu = e = \hbar = 1$) in the form

$$\sigma(a \to c) = (4\pi v_a^2)^{-1} \int_{K_a - K_c}^{K_a + K_c} [I_a + Q_a^2/2]^2 |\Phi_a(\mathbf{Q}_a)|^2 |\Phi_c(\mathbf{Q}_c)|^2 dQ_a^2,$$
(2.1)

where $\Phi_a(\mathbf{Q}_a)$ and $\Phi_c(\mathbf{Q}_c)$ are the wave functions of the initial and final states of the electron in the momentum representation;

$$\mathbf{Q}_{a} = \frac{A}{A+1} \mathbf{K}_{a} - \mathbf{K}_{c}, \quad \mathbf{Q}_{c} = \mathbf{K}_{a} - \frac{C}{C+1} \mathbf{K}_{c}$$

is the change of the momenta of particles A and C as a result of the collision;

$$\mathbf{K}_{a} = C (A+1) (A+C+1)^{-1} \mathbf{v}_{a}, \quad \mathbf{K}_{c} = A (C+1) (A+C+1)^{-1} \mathbf{v}_{c}$$

are the momenta of the relative motion of the colliding particles before and after the collision; \mathbf{v}_a and \mathbf{v}_c are the corresponding velocities; I_a and I_c are the binding energies of the electron in the states a and c.

From Eq. (2.1) we obtain for the cross section of electron capture from the subshell $n_a l_a$ of the target atom to the state $n_c l_c$ of the fast particle (after averaging over the magnetic quantum numbers m_a and summation over m_c)

$$\sigma(n_a l_a \rightarrow n_c l_c) = 2\pi v_c^{-2} N_a \theta_c B J; \qquad (2.2)$$

here

$$\begin{split} B &= 4^{l_a+l_c} (2l_c+1) \left(l_a! l_c! \right)^2 n_a n_c \frac{(n_a-l_a-1)! (n_c-l_c-1)!}{(n_a+l_a)! (n_c+l_c)!} \\ &\left(\frac{n_a}{Z_a}\right) \left(\frac{n_c}{Z_c}\right)^3 \\ J &= \int_{\tau}^{1} (1-x^2)^{l_a} (1-y^2)^{l_c} (1-y)^4 \left[\left(\frac{Z_a}{n_a}\right)^2 (1+x) + 2I_a (1-x) \right]^2 \\ &\times [G_{n_a-l_a-1}^{l_a+1} (x)]^2 [G_{n_c-l_c}^{l_c+1} (y)]^2 dx, \\ x &= \left[Q_a^2 - \left(\frac{Z_a}{n_a}\right)^2 \right] \left[Q_a^2 + \left(\frac{Z_a}{n_a}\right)^2 \right]^{-1} , \\ &\gamma &= \left[q_a^2 - \left(\frac{Z_a}{n_a}\right)^2 \right] \left[q_a^2 + \left(\frac{Z_a}{n_a}\right)^2 \right]^{-1} \\ y &= B_-/B_+, \quad B_{\pm} = q_c^2 - q_a^2 \pm \left(\frac{Z_c}{n_c}\right)^2 + \left(\frac{Z_a}{n_a}\right)^2 \left(\frac{1+x}{1-x}\right), \\ &q_a^2 &= (I_c - I_a + v^2/2)^2 v^{-2}, \quad q_c^2 &= (I_c - I_a - v^2/2)^2 v^{-2}, \end{split}$$

where $G_n^{\lambda}(x)$ are Gegenbauer polynomials; Z_a and Z_c are the effective charges of the nuclei A and C, which enter into Φ_a and Φ_c , and N_a is the number of electrons in the shell $n_a l_a$ of the target atoms; θ_c is the relative number of unfilled states $n_c l_c$ of particle C.

For the total cross section for charge exchange $\sigma_{i,i-1}^{OBK}$ we have

$$\sigma_{i,i-1}^{OBK} = \sum_{n_a l_a} \sum_{n_c l_c} \sigma(n_a l_a \rightarrow n_c l_c).$$
(2.3)

In calculation of the cross sections σ_{21}^{OBK} and σ_{10}^{OBK} with a BÉSM-6 computer the summation in (2.3) was carried out over all states with $n_c \leq 10$; the contribution of the terms with $n_c > 10$ which were not taken into account did not exceed 0.01%. The values of Z_a and Z_c were chosen according to Slater's rules. Values of I_a were taken from the table of Lotz.²⁸ Values of I_c for $n_c = 1$ and 2 were taken from the NBS tables,²⁹ and for $n_c \geq 3$ they were assumed to be hydrogen-like.

3. RESULTS OF STUDY OF ELECTRON LOSS

3.1. Comparison of cross sections with results of other studies

Our experimental electron-loss cross sections σ_{12} , σ_{01} , and σ_{02} per atom of the medium are given in Fig. 1 as a function of Z_t . In the same figure we have shown all experimental values known to us of these cross sections in various media from Refs. 5, 6, 8, 10, 11, 13, and 14 and also theoretical cross sections obtained in the Born approximation^{30,31} and in the approximation of free collisions.²⁵

The difference between the cross sections σ_{12} , σ_{01} , and σ_{02} obtained in the present work and those obtained in Refs. 5, 6, 8, and 11 does not exceed as a rule the error indicated for the measurements. The values of σ_{12} and σ_{01} for $Z_t = 2$ and 7 at $v = 8 \cdot 10^8$ cm/sec from Ref. 13 are on the average 30% below those obtained in the present work and in Refs. 5, 6, and 8, and the values of σ_{02} for $Z_t = 1$ and 2 from Ref. 13 at the same velocity v are double those obtained in Ref. 8 for $Z_t = 1$ and in the present work for $Z_t = 2$. This deviation of the results of Ref. 13 from other data may be due to the failure in Ref. 13 to make the necessary allowance for the change of the charge of the medium.



FIG. 1. Electron-loss cross sections σ_{12} , σ_{01} , and σ_{02} for He⁺ ions and He atoms as a function of Z_i for $v = 8 \cdot 10^8$ cm/sec (a), 1.2 \cdot 10^9 cm/sec (b), and 2 · 10⁹ cm/sec (c). Experimental cross sections: \bullet —present work, \bigcirc —Ref. 5, ∇ —Ref. 6, \bigtriangleup —Refs. 8, 10, and 11, \square —Ref. 13, \blacksquare —for $v = 1.24 \cdot 10^9$ cm/sec in carbon from Ref. 14, (\bigtriangleup)—according to the ratios σ_{01} (Li)/ σ_{01} (He) and σ_{01} (Na)/ σ_{01} (Ne) from Ref. 11 which do not depend on v. Theoretical cross sections: +—Born approximation, $^{30,31} \times$ —free-collision approximation for $Z_i = 2$, 10, 18, 36 and 54 (Ref. 25); dashed lines—calculation with Eq. (3.1).

At all velocities the values obtained in the present work for σ_{12} and σ_{01} at $Z_t = 2$ agree within 10–15% with the values calculated in the Born approximation.^{30,31} The same correspondence is observed also between the Born and experimental values of σ_{12} and σ_{01} for $Z_t = 1$ from Ref. 6, which at $v = (7-9)\cdot 10^8$ cm/sec are appreciably underestimated (see Fig. 4a in Ref. 30).¹⁾

The experimental values of σ_{01} for $Z_t = 2$, 10, and 18 at $v = (8-20) \cdot 10^8$ cm/sec agree within 10-15% with the values calculated in Ref. 25 in the approximation of free collisions with use of the best data on the electron-scattering cross sections. The values of σ_{01} and σ_{12} for $Z_t \ge 7$ differ by no more than 1.5-2 times from those calculated according to the Bohr formula⁵:

$$\sigma_{i,i+1} = \pi a_0^2 N_i Z_i^{\gamma_i} v_0^2 / v u, \qquad (3.1)$$

where N_i is the number of electrons in the ion with charge *i*, *I* is their binding energy, μ is the electron mass, $a_0 = 5.29 \cdot 10^{-9}$ cm, and $u = (2I/\mu)^{1/2}$.

3.2. The relation between the cross sections for electron loss by fast particles in different media

At all velocities studied, the cross sections σ_{12} , σ_{01} , and σ_{02} in helium, nitrogen, neon, and argon rise monotonically with increase of Z_t . At $v = 8 \cdot 10^8$ and $1.2 \cdot 10^9$ cm/sec the values of $\sigma_{i,i+1}$ in helium, nitrogen, neon, and argon are in the ratio of 0.26:1:1.2:1.8, and at $v = 2 \cdot 10^9$ cm/sec their dependence on Z_t is somewhat enhanced. With increase of Z_t the cross sections σ_{02} increase more rapidly than the cross sections σ_{01} . As a result for the ratio of the cross sections $\zeta = \sigma_{02}/\sigma_{01}$ at $Z_t = 2, 7, 10$, and 18 with accuracy 20–30% we have respectively $\zeta = 0.03, 0.09, 0.11, \text{ and } 0.13$. Thus, for atoms and ions of helium having velocity $v = (8-20) \cdot 10^8$ cm/ sec no reduction of the cross sections in neon $\sigma_{i,i+1}$ (Ne) relative to the cross sections in nitrogen $\sigma_{i,i+1}(N_2)$ is observed. However, at $v = (2.5-4.5) \cdot 10^8$ cm/sec according to the data of Ref. 11 the values of σ_{01} (Ne) for helium atoms are 10–15% smaller than the values of $\sigma_{01}(N_2)$. Furthermore, in the region of still lower velocities at $v = (0.8-1.7) \cdot 10^8$ cm/sec according to the data of Ref. 32 the values of $\sigma_{01}(Ne)$ again become 10–30% greater than $\sigma_{01}(N_2)$.

The lower values of $\sigma_{i,i+1}$ in neon than in nitrogen according to Ref. 1 are due to the fact that in neon atoms the electrons of the outer shell have 1.5 times greater orbital velocities $u_t(Ne)$ than the values $u_t(N_2)$ in atoms and molecules of nitrogen. The relation $\sigma_{i,i+1}(Ne) < \sigma_{i,i+1}(N_2)$ according to the estimates of Ref. 1 should be observed for ions with

$$u < 2[u_t(Ne)u_t(N_2)]^{\frac{1}{2}} \approx 4v_0$$

in the region of v/u values satisfying the condition

$$K(u/4v_0) < v/u < K, \tag{3.2}$$

where $K \sim 1$. From the experimental data for multiply charged ions¹ it follows that K = 1-1.3, while from the cross sections σ_{01} for helium atoms we obtain $k \approx 2$. The ratio $\sigma_{01}(\text{Ne})/\sigma_{01}(\text{N}_2) = 0.85-0.90$ for helium atoms is observed in



FIG. 2. Experimental cross sections for electron loss $\sigma_{i,i+1}$ by the simplest atomic systems as a function of Z_i . For H and $H^-: \bigcirc \bigoplus$ —Ref. 34, $\triangle \implies$ — Ref. 33; for He (1s2s): $\bigoplus \bigcirc \implies \square$ —Refs. 10 and 11; for He (1s²): $\bigoplus \square \bigoplus \bigcirc$ — Ref. 10, $\bigtriangledown \square$ —present work, (\square)—according to the *v*-independent ratios σ_{01} (Li)/ σ_{01} (He) and σ_{01} (Na)/ σ_{01} (Ne) from Ref. 11. Near the lines we have indicated the values of *v* in units of 10⁸ cm/sec.

the region of values of v/u from $0.6u/v_0$ to $1.5u/v_0$. Therefore we should expect that for He⁺ ions the ratio $\sigma_{12}(\text{Ne})/\sigma_{12}(\text{N}_2) \approx 0.9$ should be observed at $v \approx 6 \cdot 10^8$ cm/sec.

If we consider the experimental data on the cross sections σ_{01} for atoms of helium in lithium and sodium vapor,¹¹ we can conclude that at $v = 8 \cdot 10^8$ and $1.2 \cdot 10^9$ the dependence of σ_{01} on Z_t is close to step-like: on increase of Z_t from 2 to 3 and from 10 to 11 the values of σ_{01} rise rapidly, while in the regions $Z_t = 3-10$ and $Z_t > 11$ they change much more slowly (Fig. 1). Since with decrease of v from $1.2 \cdot 10^9$ to $2.5 \cdot 10^8$ cm/sec according to the experimental data from Refs. 6, 8, 11, and 32 the cross-section ratio $\sigma_{01}(\text{Li})/\sigma_{01}(N_2)$ increases by almost a factor of two while the ratios $\sigma_{01}(\text{Li})/2$ $\sigma_{01}(\text{He})$ and $\sigma_{01}(\text{Na})/\sigma_{01}(\text{Ne})$ remain approximately constant, the dependence of the cross sections σ_{01} on Z_i for $v = (2.5-4) \cdot 10^8$ cm/sec becomes nonmonotonic. A nonmonotonic dependence of the cross sections $\sigma_{i,i+1}$ on Z_i for $v \sim 3.10^8$ cm/sec occurs also for atoms and negative ions of hydrogen^{33,34} and metastable helium atoms,^{10,11} and in the last two cases, which are distinguished by a low electron binding energy, it is expressed more clearly and has an oscillatory nature (Fig. 2).

The increased values of $\sigma_{i,i+1}$ (Li) and $\sigma_{i,i+1}$ 0(Na), like the increased values of the ratios $\sigma_{i,i+1}$ (N₂)/ $\sigma_{i,i+1}$ (Ne), are due to the smaller screening of the Coulomb field of the nuclei by the external electrons in the atoms of the media considered, as a result of the lower orbital velocities u_i of the electrons. Therefore for ions with $u \leq 2 [u_i(\text{Li})u_i(\text{N}_2)]^{1/2} \approx 2.5v_0$ we should expect qualitatively about the same regularities in the cross-section ratios $\sigma_{i,i+1}$ (Li)/ $\sigma_{i,i+1}$ (N₂) as in the ratios $\sigma_{i,i+1}$ (N₂)/ $\sigma_{i,i+1}$ (Ne).

4. RESULTS OF STUDY OF CHARGE EXCHANGE

4.1. Comparison of experimental cross sections with results of other studies

The measured electron-capture cross sections σ_{10} , σ_{21} , and σ_{20} per atom of the medium are shown in Fig. 3. In that



FIG. 3. Electron-capture cross sections σ_{21} , σ_{20} , and σ_{10} for helium ions as a function of Z_i for $v = 8 \cdot 10^8$ cm/sec (a), $1.2 \cdot 10^9$ cm/sec (b), and $2 \cdot 10^9$ cm/ sec (c). Experimental cross sections: \bullet —present work, \blacksquare —Ref. 2, \bigcirc — Refs. 3 and 4, \bigtriangledown —Refs. 6 and 7, \bigcirc —Ref. 9, \Box —Ref. 12, \bullet —for $v = 1.24 \cdot 10^9$ cm/sec in carbon from Ref. 14. Theoretical cross sections: $+ - \text{CBA}^{35,36}$; $\times - \text{CDWA}^{37}$; solid curves— $0.18\sigma_{i,i-1}^{0\text{BK}}$, dot-dash curves— $0.18\sigma_{i,i-1}^{0\text{BK}}(n)$ for n = 1-5: dotted curve— $3.5 \cdot 10^{-3}\sigma_{21}^{0\text{BK}}$; dashed straight lines: 1 - Eq.(4.1), 2 - Eq.(4.2). LABEL: 1) cm²/atom

figure we have also shown all other experimental values of these cross sections known to us in various media from Refs. 2-4, 6, 7, 9, 12, and 14, the theoretical cross sections σ_{21} and σ_{20} for $Z_t = 1$ and 2 from Refs. 35–38, the values of σ_{21} and σ_{10} for $Z_t \ge 2$ given by the simple formulas from Ref. 39, and the cross sections σ_{21}^{OBK} and σ_{10}^{OBK} .

The experimental single-charge-exchange cross sections σ_{21} and σ_{10} found in the present work for $Z_t = 2, 7, and$ 18 at $v = 8.10^8$ and $1.2.10^9$ cm/sec agree within 15-25% with those obtained in Refs. 3 and 9 except for the cross section σ_{10} for $Z_t = 18$ at $v = 8 \cdot 10^8$ cm/sec, which is 1.7 times higher than that given in Ref. 3. The double-chargeexchange cross sections σ_{20} are higher than those given in Ref. 4 by 20% for $Z_t = 2$ and by 60% for $Z_t = 7$. The value of σ_{20} for $Z_t = 7$ is close to that given in Ref. 7, but the values of σ_{20} from Ref. 7 for all media are 1.8–2 times greater than the values of σ_{20} from Ref. 4, while the values of σ_{21} and σ_{10} from Refs. 6 and 7 are 15-30% smaller than ours. The values of σ_{21} and σ_{10} from Ref. 12 are lower than ours by a larger factor, namely by 30-55%. The reduction of the cross sections σ_{21} and σ_{10} in Refs. 6, 7, and 12 and the significantly larger increase of the cross sections σ_{20} in Ref. 7, like the similar deviations found in Ref. 13 as mentioned above for the electron-loss cross sections, are apparently the result of neglecting the change in the charge of the fast particles in multiple collisions with target atoms. Under conditions similar to those discussed, in which the measured cross sections σ_{21} and σ_{10} are several times smaller than the cross sections for the reverse transitions σ_{12} and σ_{01} , it becomes especially important to take these collisions into account. In the present work and in Refs. 3–5 secondary collisions have been taken into account as completely as possible. Therefore the experimental values of σ_{21} , σ_{10} , and σ_{20} from the present work and from Refs. 3, 4, and 9 must be considered preferable. The spread of the cross sections from these studies, as a rule, corresponds to the usual accuracy 10–15% for $\sigma_{i,i-1}$ and 20–25% for $\sigma_{i,i-2}$.

The experimental values of σ_{21} from Refs. 3 and 9 and the present work at $v = (8-12) \cdot 10^8$ cm/sec are 1.5 times greater than those calculated in the Coulomb Born approximation (CBA) for $Z_t = 1$ (Ref. 35) and practically coincide with the latter for $Z_t = 2.^{36}$ Values of σ_{21} calculated in the continuum distorted-wave approximation (CDWA)³⁷ for the same Z_t and v are 1.5–2 times smaller than those calculated in the CBA and smaller than the experimental values by 1.5–3 times. However, the value of σ_{20} calculated in the CDWA for $Z_t = 2$ and $v = 8.10^8$ cm/sec (Ref. 38), which is four times larger than that calculated in the CBA, practically coincides with the experimental values from Ref. 4 and the present work. Thus, values of σ_{21} obtained in the CBA and values of σ_{20} calculated in the CDWA turn out to be closer to the experimental values.

Values of σ_{21}^{OBK} and σ_{10}^{OBK} calculated in the present work in the region $v = 8 \cdot 10^8 - 2 \cdot 10^9$ cm/sec exceed the experimental cross sections σ_{21} and σ_{10} by 5–15 times. However, the ratio of the experimental cross sections $\sigma_{i,i-1}$ to $\sigma_{i,i-1}^{OBK}$ for He⁺² nuclei does not change greatly and, as a rule, differs from the value 0.18 by no more than 20–25%, while for He⁺ ions it decreases from about 0.2 to 0.07 with increase of Z_t from 2 to 18.

The experimental values of σ_{21} and σ_{10} for $Z_t = 2, 7, 10$, and 18 for $Z_t > v/2v_0$ are as a rule respectively from 1.5 to 5 times greater than those calculated according to the Bohr formula for the cross sections for charge exchange of light nuclei with charge $Z < v/v_0$ in heavy media³⁹:

$$\sigma_{i, i-1} = 4\pi a_0^2 Z^5 Z_t^{\nu_s} (v_0/v)^6$$
(4.1)

and according to Eq. (4.2) for ions with charges i = (0.3 - 0.6)Z (Ref. 39):

$$\sigma_{i, i-1} = 4\pi a_0^{2} i^3 Z_t^{\prime \prime_3} (v_0/v)^5.$$
(4.2)

4.2. Relation between experimental charge-exchange cross sections in different media

Our experimental data show that at an ion velocity $v = 8 \cdot 10^8$ cm/sec the cross sections σ_{21} , σ_{20} , and σ_{10} change substantially nonmonotonically with change of Z_t . With increase of Z_t from 2 to 10 the values of $\sigma_{i,i-1}$ and σ_{20} increase respectively by 5.4 and 20 times, and on increase of Z_t from 10 to 18 the values of σ_{21} and σ_{20} decrease 2.3–2.5 times and the value of σ_{10} by 4 times. Here the ratios of the cross sections in neon and argon $\sigma_{i,i-1}$ (Ne)/ $\sigma_{i,i-1}$ (Ar) for He⁺² and He⁺ ions turn out to be practically the same as for N⁺² and N⁺ ions, respectively.¹ The cross-section ratio $\eta = \sigma_{20}/\sigma_{21}$ depends practically monotonically on Z_t : for $Z_t = 2, 7, 10$, and 18 we have respectively $\eta = 0.004, 0.010, 0.023$, and 0.021.

With increase of v the ratios between the cross section

 σ_{21} in nitrogen, neon, and argon change rather rapidly and at $v = 1.2 \cdot 10^9$ and $2 \cdot 10^9$ cm/sec they become qualitatively different, namely 1:5:5 and 1:1.2:4.2 instead of 1:2.3:1 at $v = 8 \cdot 10^8$ cm/sec. In the region $v < 8 \cdot 10^8$ cm/sec the nonmonotonic nature of the dependence of the cross sections σ_{21} and σ_{10} on Z_t is preserved, but for $v = (1.6-3) \cdot 10^8$ cm/sec the cross sections in neon are reduced.⁴⁰⁻⁴² For protons and hydrogen atoms, for which the values of $\sigma_{i,i-1}$ in neon are known, ^{33,34} at $v \le 6 \cdot 10^8$ cm/sec and for protons also at $v \ge 1.4 \cdot 10^9$ cm/sec (Ref. 33) the ratios of the values of $\sigma_{i,i-1}$ in different media are qualitatively the same as for He⁺² and He⁺ ions, respectively. Thus, a nonmonotonic dependence of the electron-capture cross sections on Z_t with substantially increased cross sections in neon at $v \sim 10^9$ cm/sec is observed for many ions of light elements with $Z \le 10$.

4.3. Oscillations of the charge-exchange cross sections with change of the medium

The ratios between the experimental charge-exchange cross sections in helium, nitrogen, neon, and argon mentioned above, which are qualitatively different at different velocities v, are confirmed by calculations of these cross sections in the OBK approximation; whereas for He⁺ ions the agreement is only qualitative, for He^{+2} ions it is quantitative (Fig. 3). Thus, there is a basis for using this approximation in discussing the dependence of these cross sections on Z_{i} . Calculations in the OBK approximation show that the dependence of the cross sections $\sigma_{i,i-1}$ on Z_i is oscillatory, and the experimentally observed sharply nonmontonic dependence of these cross sections on Z_t at $Z_t = 2, 7, 10$, and 18 and the qualitative changes in the ratio between the values of $\sigma_{i,i-1}$ in nitrogen, neon, and argon on change of v are only the consequence of these oscillations, which shift toward larger Z_t with increase of v. For example, in the cross section σ_{21} at $v = 8 \cdot 10^8$ cm/sec the first four maxima should be observed at $Z_t = 3$, 10, 30, and 55, and for $v = 2.10^9$ cm/sec they should be observed at $Z_i = 6$, 16, 36, and 70 (Figs. 3 and 4). For $v \gtrsim 3v_0$ these maxima occur at atoms of the medium for which the binding energy I_n of the electrons of one of the shells with principal quantum number n turns out to be close to $I_v/3$, where $I_v = \mu v^2/2$. Capture of electrons from states with other n in these cases makes a small contribution to the cross section (Fig. 3), so that each of the five successive maxima of the dependence of $\sigma_{i,i-1}$ on Z_t (for $v \sim 10^9$ cm/sec) is due to a maximum of the partial cross sections for electron capture respectively from the atomic shells K, L, M, N, and 0.

The largest reductions, up to three times in the cross sections $\sigma_{i,i-1}$ in the transition from the maximum to the closest minimum at larger Z_i , should be observed at $v = (8-12)\cdot 10^8$ cm/sec. At higher velocities the amplitudes of the oscillations decrease and as a result at $v \sim 4 \cdot 10^9$ cm/sec the dependence of σ_{21} on Z_i should be close to step-like, and for $v \gtrsim 8 \cdot 10^9$ cm/sec it should be practically smooth. With decrease of v from $8 \cdot 10^8$ to $2 \cdot 10^8$ cm/sec the amplitude of the oscillations also decreases, and for $v \approx (2-5)\cdot 10^8$ cm/sec small additional maxima corresponding to electron capture from states with definite n and l values appear in the dependence.



FIG. 4. Cross sections for charge exchange of helium nuclei σ_{21} as a function of Z_i for various nuclear velocities v. Experimental values of σ_{21} : — present work, — Ref. 3, ∇ —Ref. 7, \triangle —Ref. 9, \square —Ref. 40, \blacksquare —Ref. 41, \blacktriangle —Ref. 42. Theoretical cross sections: solic curves— $K\sigma_{21}^{OBK}$, dashed curves— $K\sigma_{21}^{OBK}(n)$ for n = 1-5 with K = 0.4 for $v = 2.6 \cdot 10^8$ cm/sec and K = 0.18 for $v \ge 8 \cdot 10^8$ cm/sec. Near the curves we have given the values of v in units of 10⁸ cm/sec. LABEL: 1) cm²/atom

dence of σ_{21} on Z_t (Fig. 4). In the region $Z_t > v/2v_0$ the cross sections σ_{21} oscillate about average values proportional to Z_t^r , where the exponent *r* is small and on increase of *v* from $8 \cdot 10^8$ to $8 \cdot 10^9$ cm/sec it changes approximately from 0.7 to 1.6, whereas in the region $Z_t < v/2v_0$ the cross sections σ_{21} increase much more rapidly with increase of Z_t : $r = d \log \sigma_{21}/d \log Z_t \approx 4-4.5$.

The weakening and then the disappearance of the oscillations discussed with increase of v is due to the influence of three factors acting in the same direction. First-the coming together of the maxima in the dependence of $\sigma_{i,i-1}$ on $\log Z_i$ with increase of v as the result of decrease of the influence of internal and external screening on the effective nuclear charge Z_{nl} and on the binding energy I_{nl} of the electrons in the initial state with increase of Z_i . Second-the broadening of the curves of the partial cross sections $\sigma_{i,i-1}(n)$ for electron capture from states with given n as a function of $\log Z_t$ with increase of Z_t as the result of filling of all states with a given n. Finally the third factor is a broadening of the same curves as a result of decrease of the ratios I_c/I_v and $Z_c/v_0/v$ with increase of v, where I_c and Z_c are the electron binding energy and the effective nuclear charge for the electron in the final state. With increase of v from $8 \cdot 10^8$ to $4 \cdot 10^9$ cm/sec the action of the first factor is approximately double the combined action of the other two. Therefore the oscillations in the dependence of $\sigma_{i,i-1}$ on Z_t for $v \leq 10^9$ cm/sec are the result primarily of the fact that the relative reduction of the values of Z_{nl} and I_{nl} for electrons of the outer shells as the result of the presence of other electrons in the atom increases

with increase of *n*. This factor alone is sufficient to produce an *LM* minimum at $v = 8 \cdot 10^8$ cm/sec near $Z_t = 18$ between the maxima for capture of *L* and *M* electrons, but the combined influence of all three factors is necessary for formation of a *KL* minimum near $Z_t = 5$.

5. OSCILLATIONS OF THE EQUILIBRIUM CHARGE FRACTIONS OF FAST IONS

5.1. The correspondence between the cross sections for loss and capture of electrons and the equilibrium charge fractions of helium ions

The electron loss and capture cross sections σ_{ik} determine the charge composition of an ion beam passing through a gas. Therefore oscillations in the charge-exchange cross sections should lead to oscillations of the equilibrium charge fractions F_i and of the average charge of the ions $\overline{\tau} = \Sigma i F_i$, which could be determined in experiments which do not depend on the experimental determination of the cross sections σ_{ik} . To check the results of the study of the charge-exchange cross sections, we therefore used the values of σ_{ik} obtained in the present work by solving the system of equations^{33,39}

$$F_i \sum_{k \neq i} \sigma_{ik} = \sum_{k \neq i} F_k \sigma_{ki}, \qquad \sum_i F_i = 1, \qquad (5.1)$$

where *i* and *k* take on values 0, 1, and 2, to calculate the values of F_i and \overline{i} and compared them with the experimental values.

From the experimental cross sections σ_{ik} we calculated values of F_i for $z_i = 2, 7, 10$, and 18 at $v = 8 \cdot 10^8, 1.2 \cdot 10^9$, and $2 \cdot 10^9$ cm/sec. The values of σ_{20} in argon at $v = 8 \cdot 10^8$ cm/sec and of σ_{21} in helium at $v = 2 \cdot 10^9$ cm/sec were taken respectively from Refs. 7 and 9, and for σ_{21} in helium at $v = 1.2 \cdot 10^9$ cm/sec we used the geometric mean of the values from Ref. 3 and Ref. 9. For $v = 1.2 \cdot 10^9$ and $2 \cdot 10^9$ cm/sec, owing to lack of experimental values of σ_{10} and σ_{20} , we calculated only F_2 and F_1 . In addition, values of F_1 were calculated for almost

all values of Z_t from 1 to 100 for the same velocities v, and also at $v = 6.9 \cdot 10^8$, 2.4 \cdot 10^9, 3.4 \cdot 10^9, and 4.4 \cdot 10^9 cm/sec on the basis of the cross sections σ_{21} and σ_{10} calculated in the OBK approximation and semiempirical values of the remaining cross sections. As the cross sections $\sigma_{i,i-1}$ we took the values $0.16\sigma_{i,i-1}^{OBK}$ for $v = 6.9 \cdot 10^8$ cm/sec and values $0.18\sigma_{i,i-1}^{OBK}$ for $v \ge 8.10^8$ cm/sec, while as the cross sections $\sigma_{i,i+1}$ we took the values shown in Fig. 1 by the solid lines for $v = (8-20) \cdot 10^8$ cm/sec and semiempirical values determined in a similar manner for $v = 6.9 \cdot 10^8$ cm/sec. Values of σ_{12} for $v > 2 \cdot 10^9$ cm/sec at $Z_t \ge 3$ were determined from Eq. (3.1). The cross sections σ_{02} and σ_{20} were determined from the values $\zeta = \sigma_{02}/\sigma_{01}$ and $\eta = \sigma_{20}/\sigma_{21}$. It was assumed that $\zeta = 0.012 Z_t$ for $Z_t \leq 10$ and $\zeta = 0.12$ for $Z_t \geq 10$, and that $\eta = 0.002 Z_t$ for $Z_t \leq 10$ and $\eta = 0.02$ for $Z_t \geq 10$, which corresponds to the experimental values of η for $v = 8 \cdot 10^9$ cm/ sec. The values of F_0 calculated in this way for $v \ge 1.2 \cdot 10^9$ cm/ sec give an upper limit of the values of F_0 , since with increase of v the values of η do not increase,⁴ and the values F_0 calculated with $\eta = 0$ give their lower limit (Fig. 5).

For $v \ge 8 \cdot 10^8$ cm/sec the relations (5.2) which follow from solution of the system (5.1) are valid with accuracy 5% or better for $F_0 \le 0.04$, $\eta < 0.03$, and $\varkappa = \sigma_{12}/\sigma_{10} \ge 10$, namely

$$F_{0} = v_{0}v_{1}(1+v_{1})^{-1}(1+\varkappa\eta)(1+\xi)^{-1}, \quad F_{1} = v_{1}(1+v_{1})^{-1},$$

$$F_{2} = (1+v_{1})^{-1}, \quad \overline{i} = 1+F_{2} = 2-v_{1}(1+v_{1})^{-1},$$
(5.2)

where $v_0 = \sigma_{10}/\sigma_{01}$ and $v_1 = \sigma_{21}\sigma_{12}$. From this we have $\delta F_1 = F_2 \delta v_1$ and $\delta F_2 = F_1 \delta v_1$ for the relative errors δF_i of the quantities F_1 and F_2 calculated from σ_{ik} , so that the values of δF_1 are as a rule 15–20%, and the values of δF_2 at $v = 8 \cdot 10^8$ and 1.2 \cdot 10^9 cm/sec do not exceed respectively 5–8% and 1–2%. The values of δF_0 for $v = 8 \cdot 10^8$ cm/sec when $\eta \varkappa \lesssim 1.5$ amount to about 30%. The experimental values of F_i have as a rule several times smaller errors.

The calculated values of F_i for $v \ge 8 \cdot 10^8$ cm/sec and val-



FIG. 5. Charge fractions F_i and average charge 7 of helium ions as a function of Z_i . Experimental data: ∇ —Ref. 6, \triangle —Ref. 15, \square —Refs. 16 and 17, \diamond — Ref. 18, \blacksquare —Ref. 14, \blacktriangle —Ref. 19, \bigvee —Ref. 20, \bullet + —Ref. 21, \times —Ref. 22. Results of calculations: \bigcirc —from the experimental cross sections $\sigma_{i,i}$, solid curves—from the cross sections $K\sigma_{i,i-1}^{OBK}$ and semiempirical values of the other cross sections; dot-dash curves—variant of calculation of F_0 with $\sigma_{20} = 0$. Near the curves we have indicated the velocity of the ions in units of 10^8 cm/sec.

ues of 7 for $v = 6.9 \cdot 10^8$ cm/sec are shown in Fig. 5. This figure shows also all values known to us of F_i and 7 obtained as the result of direct measurements of the equilibrium charge fractions F_i for helium ions which have passed through various gases^{6,15-18} and thin solid targets, ^{14,19,20} and also for helium ions scattered by thick solid and molten targets heated to high temperatures to clean their surfaces²¹ and targets with a continually renewed surface.²² A combined discussion of the experimental values of F_i and $\tilde{1}$ for gaseous and condensed media is justified by the fact that the change of the charge fractions with increase of the thickness of a solid (carbon) target for helium ions turns out to be practically the same as in a gas.^{14,43}

It can be seen from Fig. 5 that the values of F_2 and F_1 calculated from the experimental cross sections σ_{ik} differ from the experimental values of F_2 and F_1 from Ref. 15 respectively by 3-6% and 10-20%, and from the values obtained in Refs. 6 and 18 by 8-10% and 25-30%. The only exception is the value of F_1 for $Z_t = 7$ at $v = 1.2 \cdot 10^9$ cm/sec, which is half that given in Ref. 15. The value of F_0 calculated for $Z_{t} = 2$ at $v = 8 \cdot 10^{8}$ cm/sec differs from the experimental values from Refs. 6 nd 18 by 25 and 50%, and the value of F_0 for $Z_t = 7$ is higher than those obtained in Ref. 18 for $Z_t = 7$ and 8 respectively by 3.5 and 1.2 times. Thus, with the exception of single cases, the values of F_i found from Ref. 15 correspond to the experimental cross sections σ_{ik} found in the present work, and the values of F_i from Refs. 6 and 18 differ from the values calculated from σ_{ik} by amounts which are 1.5 times greater than the errors of the calculated F_i values.

5.2. Oscillations of the charge fractions and of the average charge of ions

Calculation of charge fractions on the basis of theoretical charge-exchange cross sections shows that the dependence of the values of F_i and $\overline{\tau}$ on Z_i should be oscillatory, the maxima in the dependence of the cross sections $\sigma_{i,i-1}$ on Z_i corresponding to minima in the values of F_2 and $\overline{\tau}$ and to maxima in the values of F_1 and F_0 . For $v \approx (7-8) \cdot 10^8$ cm/sec the values of $\overline{\tau}$ and F_2 at neighboring extrema differ respectively by 10–15% and 25–30%, the values of F_1 by 2–3 times, and the values of F_0 by up to 10 times. With increase of v the depth of the oscillations decreases. However, for $v \approx 2 \cdot 10^9$ cm/sec the oscillations in the values of F_1 and F_0 are still rather large: on increase of Z_i roughly from 18 to 28 and from 40 to 60 the values of F_i are reduced by 1.8–3 times. Oscillations in the values of F_i practically disappear for $v > 4 \cdot 10^9$ cm/sec (Fig. 5).

The experimental data on the values of F_i and \overline{i} for gaseous media with $Z_t = 1, 2, 7, 10, 18$, and 36 at $v = (5-8) \cdot 10^8$ cm/sec from Refs. 6 and 15–18 confirm the existence of minima in the dependence of F_2 and \overline{i} on Z_t and of maxima in the values of F_1 and F_0 at $Z_t = 2$ and 10, which are due to the first two maxima in the dependence of $\sigma_{i,i-1}$ on Z_t . For $v \approx 2 \cdot 10^8$ cm/sec, as follows from Refs. 16 and 17, the ratio of the experimental values of F_2 for $Z_t = 10$ and $Z_t = 7$ and 18 is reversed: the values of F_2 for $Z_t = 10$ exceed the values of F_2 for $Z_t = 7$ and 18 by 1.6 and 2.4 times respectively, which also agrees with the dependence of $\sigma_{i,i-1}^{OBK}$ on Z_t .

From the calculations it follows that for $v \approx (7-8) \cdot 10^8$ cm/sec the third and fourth minima in the dependence of \overline{i} on Z_i , which are due to maxima of the M and N electron capture cross sections, should be observed at $Z_{1} \approx 28$ and 50. The experimnental data obtained in Refs. 21 and 22 for the average charge of helium ions scattered by certain solid and liquid targets with Z_{1} from 6 to 82 indicate existence of minima in the values of 7 in the region $Z_1 \approx 29$ and at Z_1 values between 50 and 73 and $Z_t > 79$ (Fig. 5). The reduction of the experimental values of \overline{i} at $Z_i = 29$ in comparison with the values at $Z_{1} = 18$ and 37 and of the values of 7 at $Z_{1} = 73$ and 82 with respect to the value of 7 at $Z_1 = 78$ amount to 8–15%, which is 1.5-2.5 times smaller than the reduction of the calculated values of 7 at the minima. Thus, the experimental data on F_i and \overline{i} at $v \approx (5-8) \cdot 10^8$ cm/sec confirm the conclusion that there is an oscillatory dependence of these quantities on Z_t over the entire region of values of Z_t from 1 to 82.

A different ratio is observed between the calculated and experimental values of F_1 and F_0 obtained in Ref. 20 at $v = 2.4 \cdot 10^9$ and $3.4 \cdot 10^9$ cm/sec for $Z_t = 6, 13, 28, 47, and 79$. The experimental values of F_1 and F_0 rise monotonically with increase of Z_t , while the calculations indicate the existence of oscillations, with variation of the values of F_1 by 1.5– 2 times and of the values of F_0 by 3–5 times. It is evident from Fig. 5 that for solution of the question of existence or nonexistence of these oscillations in the dependence of F_1 and F_0 on Z_t for $v > 2.10^9$ cm/sec the number of targets used in the experiment must be increased.

In connection with the discussion of the values of F_i and 7 for condensed media it must be noted that in the transition from gaseous media to solids no significant variations in the experimental values of F_i and 7 for ions of helium and hydrogen is observed (Fig. 5). This result is in agreement with the fact that the cross sections for loss and capture of electrons by these particles on passage through solids turn out to be practically the same as in a rarefied medium (see Refs. 14, 43, and 44 and Figs. 1 and 3). The closeness of these cross sections is due to the fact that fast ions with small charges capture electrons preferentially into the lowest excited state, as a result of which in the first approximation the presence of excited particles in the ion beam can be neglected.

Consideration of the experimental values of F_i and \overline{i} for ions and atoms of hydrogen³⁴ and heavier ions⁴⁵ shows that these quantities change nonmonotonically with change of Z_t also for other ions. In a beam of ions and atoms of hydrogen with $v \sim 6 \cdot 10^8$ cm/sec, for example, as in a beam of helium ions having the same velocity, the values of F_0 are maximal for $Z_t = 2$ and 10 and minimal for $Z_t = 1, 3, and 18$. Similar relations between the values of F_i for $Z_i = 1, 2, 7.10, 18$, and 36 are observed also for ions of iodine with charges $i \leq \overline{i}$ for $v \approx 1.3 \cdot 10^9$ cm/sec. The ratio between the values of $\bar{1}$ for the same ions in different gases, like the ratio between values of F_i for $i > \overline{i}$, is reversed: they are minimal at $Z_i = 2, 10, \text{ and } 36$ and maximal at $Z_t = 1$, 7, and 18. Thus, the oscillatory dependence of the charge fractions F_i and the average charge \vec{i} of ions on Z_t due to the similar dependence on Z_t of the charge-exchange cross sections $\sigma_{i,i-1}$ is a rather general phenomenon.

6. CONCLUSION

The results of the present work show that in the dependence of the electron-capture cross sections on the nuclear charge Z_{t} of the atoms of the medium there are oscillations which arise as a consequence of the shell structure of the atoms and are manifest both in experiments on measurement of the cross sections for electron capture by fast ions and in experiments on determination of the equilibrium charge state of beams of fast particles. With increase of the ion velocity v the extrema in the electron-capture cross sections $\sigma_{i,i-1}$ are displaced toward larger Z_i . Therefore the anomalously high values of the cross sections for capture of electrons from neon atoms at $v = 8 \cdot 10^8$ cm/sec are replaced by reduced values of these cross sections at $v = 3 \cdot 10^8$ cm/sec and 2.10⁹ cm/sec. At ion velocities $v \ge 4.10^9$ cm/sec, as the calculations show, the oscillations in the electron-capture cross sections should disappear. The appearance of these oscillations at $v \leq 10^9$ cm/sec is due mainly to the enhancement of the influence of inner and outer screening on the average orbital velocity and binding energy of the electrons of the outer shells with increase of their principal quantum number n, as a result of which the relative difference between these quantities for states with neighboring n values increases. Incomplete filling of the outer electron shells leads to a deepening of the minima in the dependence of the cross sections $\sigma_{i,i-1}$ on Z_i .

For $v \approx 3.10^8$ cm/sec oscillations due to the shell structure of the atoms of the medium are observed also in the electron-loss cross sections $\sigma_{i,i+1}$. However, the extrema in the dependence of these cross sections on Z_t does not shift towards larger Z_t with increase of the particle velocity v, and the oscillatory dependence in the region $Z_t \leq 18$ goes over into a step-like dependence, and then into a smoother dependence at low particle velocities than is the case for electroncapture cross sections. The appearance of these oscillations and their disappearance on increase of the particle energy is due to the fact that in the transition from inert gases to the neighboring alkali elements, i.e., with increase of Z_t by unity, the relative increase of the electron-loss cross section turns out to be rather large and to be only weakly dependent on the particle velocity v, whereas the cross sections for electron loss in the inert gases at low velocities v rise with increase of Z, much more slowly than at high particle energies.

The features noted in the behavior of the electron loss and capture cross sections are present not only in ions and atoms of helium, but also in many other fast atoms and ions, since the causes of their appearance are rather general.

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- ¹I. S. Dmitriev, Yu. A. Tashaev, V. S. Nikolaev, Ya. A. Teplova, and B. M. Popov, Zh. Eksp. Teor. Fiz. **73**, 1684 (1977) [Sov. Phys. JETP **46**, 884 (1977)].
- ²E. Rutherford, Phil. Mag. 47, 277 (1924).
- ³V. S. Nikolaev, I. S. Dmitriev, L. N. Fateeva, and Ya. A. Teplova, Zh. Eksp. Teor. Fiz. **40**, 989 (1960) [Sov. Phys. JETP **13**, 695 (1960)].
- ⁴V. S. Nikolaev, L. N. Fateeva, I. S. Dmitriev, and Ya. A. Teplova, Zh. Eksp. Teor. Fiz. **41**, 89 (1961) [Sov. Phys. JETP **14**, 67 (1962)].
- ⁵I. S. Dmitriev, V. S. Nikolaev, L. N. Fateeva, and Ya. A. Teplova, Zh.
- Eksp. Teor. Fiz. 42, 16 (1962) [Sov. Phys. JETP 15, 11 (1962)].
- ⁶L. I. Pivovar, V. M. Tubaev, and M. T. Novikov, Zh. Eksp. Teor. Fiz. 41, 26 (1961) [Sov. Phys. JETP 14, 20 (1962)].
- ⁷L. I. Pivovar, M. T. Novikov, and V. M. Tubaev, Zh. Eksp. Teor. Fiz. 42, 1490 (1962) [Sov. Phys. JETP 15, 1035 (1962)].
- ⁸P. Hvelplund, and E. Horsdal-Pedersen, Phys. Rev. A9, 2434 (1974).
- ⁹P. Hvelplund, J. Heinemeier, E. Horsdal-Pedersen, and F. R. Simpson,
- J. Phys. **B9**, 491 (1976).
- ¹⁰E. Horsdal-Pedersen, L. Larsen and J. V. Mikkelsen, J. Phys. B10, L669 (1977).
- ¹¹E. Horsdal-Pedersen, J. Heinemeier, L. Larsen, and J. V. Mikkelsen, J. Phys. B13, 1167 (1980).
- ¹²A. Itoh, M. Asari, and F. Fukuzawa, J. Phys. Soc. Japan 48, 943 (1980).
- ¹³A. Itoh, K. Ohnishi, F. Fukuzawa, J. Phys. Soc. Japan 49, 1513 (1980).
- ¹⁴N. Cue, N. V. de Castro-Faria, M. J. Gaillard, J. C. Poizat, and J. Remillieux, Nucl. Inst. Meth. **170**, 67 (1980).
- ¹⁵V. S. Nikolaev, I. S. Dmitriev, L. N. Fateeva, and Ya. A. Teplova, Zh. Eksp. Teor. Fiz. **39**, 905 (1960) [Sov. Phys. JETP **12**, 627 (1961)].
- ¹⁶W. Meckbach, and I. B. Nemirovsky, Phys. Rev. 153, 13 (1967)
- ¹⁷P. Torres, W. Meckbach, and A. Valenzuela, Phys. Rev. 183, 216 (1969).
- ¹⁸A. Itoh and F. Fukuzawa, J. Phys. Soc. Japan **50** 632 (1981).
- ¹⁹J. C. Armstrong, J. V. Mullendore, W. R. Harris, and J. B. Marion, Proc. Phys. Soc. 86, 1283 (1965).
- ²⁰A. Gladieux, A. Chateau-Thierry, and B. Delaunay, J. Phys. B12, 3591, (1979).
- ²¹Y. Kido, Y. Kanamori, and F. Fukuzawa, Nucl. Inst. Meth. 164, 565 (1979).
- (1979). ²²Y. Haruyama, Y. Kanamori, T. Kido, and F. Fukuzawa, J. Phys. **B15**, 779 (1982).
- ²³I. S. Dmitriev, N. F. Vorobiev, V. P. Zaikov, et al., J. Phys. B15, L351 (1982).
- ²⁴T. G. Winter and Ch. C. Lin, Phys. Rev. A12, 434 (1975).
- ²⁵D. P. Dewangan and H. R. J. Walters, J. Phys. B11, 3983 (1978).
- ²⁶I. S. Dmitriev, V. S. Nikolaev, Yu. A. Tashaev, and Ya. A. Teplova, Zh. Eksp. Teor. Fiz. **67**, 2047 (1974) [Sov. Phys. JETP **40**, 1017 (1975)].
- ²⁷M. R. C. McDaniell and J. P. Coleman, Introduction to the Theory of Ion-Atom Collisions, North-Holland Publishing Company, Amsterdam-London, 1970.
- ²⁸W. Lotz, J. Opt. Soc. Am. 60, 206 (1970).
- ²⁹C. E. Moore, Atomic Energy Levels, Vol. 1, Natl. Bur. Std. (US), Circ. 467, Washington, 1948.
- ³⁰I. S. Dmitriev, Ya. Zhileikin, and V. S. Nikolaev, Zh. Eksp. Teor. Fiz. 49, 500 (1965) [Sov. Phys. JETP 22, 352 (1966)].
- ³¹V. S. Nikolaev, V. S. Senashenko, V. A. Sidorovich, and V. Yu. Shafer, Zh. Tekh. Fiz. 48, 1399 (1978) [Sov. Phys. Tech. Phys. 23, 789 (1978)].
- ³²C. F. Barnett and P. M. Stier, Phys. Rev. 109, 385 (1958).
- ³³H. Tawara and A. Russek, Rev. Mod. Phys. 45, 178 (1973).
- ³⁴C. J. Andersen, R. J. Girnius, A. M. Howald, and L. W. Anderson, Phys. Rev. A22, 822 (1980).
- ³⁵Sh. Datta and S. C. Mukherjee, J. Phys. **B13**, 539 (1980).
- ³⁶S. C. Mukherjee, Kanika Roy, and N. C. Sil, J. Phys. B6, 467 (1973).
- ³⁷Dź. Belkić and R. Gayet, J. Phys. **B10**, 1911, 1923 (1977).
- ³⁸R. Gayet, R. D. Rivarola, and A. Salin, J. Phys. **B14**, 2421 (1981).
- ³⁹V. S. Nikolaev, Usp. Fiz. Nauk 85, 679 (1965) [Sov. Phys. Uspekhi 8, 269 (1965)].
- ⁴⁰S. K. Allison, Phys. Rev. 109, 76 (1958).
- ⁴¹R. A. Baragiola and I. B. Nemirovsky, Nucl. Inst. Meth. 110, 511 (1973).
- ⁴²R. W. McCullough, T. V. Goffe, M. B. Shah, M. Lennon, and H. Gilbody, J. Phys. **B15**, 111 (1982).
- ⁴³M. I. Gaillard, J. C. Poizat, A. Ratkowsky, I. Remillieux, and M. Auzas, Phys. Rev. A16, 2323 (1977).
- ⁴⁴R. C. Webber and C. Hojvat, IEEE Trans. Nucl. Sci. NS-26, 4012 (1979).
- ⁴⁵A. B. Wittkower and H. D. Betz, Atomic Data 5, 113 (1973).

¹⁾The Born values σ_{12} given in Fig. 1 for molecular hydrogen were obtained from those calculated for atomic hydrogen³⁰ by decreasing them by 7% in accordance with Eq. (11) from Ref. 31.

Translated by Clark S. Robinson