Mixing of the levels of a compound nucleus in the field of a light wave

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The mixing of the levels of a compound nucleus in the field of a high-intensity light wave is considered. The cross sections for inelastic neutron scattering and radiative capture are computed with allowance for this effect. The effect of the electron shell on the effective charge for the dipole interaction between the neutron + nucleus system and the external electromagnetic field is taken into consideration.

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1. INTRODUCTION

The effect of resonance excitation of atoms in a laser-radiation field has been fairly well studied from both the theoretical and experimental points of view.[1] In the case of a nucleus the resonance excitation of the levels near the ground state was accomplished with the aid of the Mössbauer effect. There are only incoherent-radiation sources for this energy region. The situation is different when the levels of a compound nucleus are excited, e.g., in reactions with neutrons. In the case of heavy nuclei (the rare earths, the actinides), for excitations corresponding to the binding energy of the neutron, the distance between the levels of the compound nucleus is close in order of magnitude to the energies of laser-light quanta. Therefore, we can, in principle, raise the question of resonance transitions between two levels of a compound nucleus, one of which is excited in the course of the neutron capture. In this case levels with opposite parities, which may differ significantly in the partial nuclear-decay probabilities associated with them, will, as a rule, be mixed in an external resonance electromagnetic field.

In earlier papers,[2,3] we attempted to estimate the probability for mixing of the levels of a compound nucleus in the presence of an external electromagnetic field. We assumed that the electron shell of the atoms had no effect on the probability for resonance mixing of the levels of a compound nucleus, i.e., that the frequency of the electromagnetic field was not equal to the electron-transition frequency. Furthermore, the spin of the neutron was assumed to be equal to zero. These two assumptions are rather far removed from what the real situation is, especially, the neglect of the effect of the electron shell. It had been shown earlier in another connection[4] that, in the case of electric dipole nuclear transitions with low frequency ($\omega_{\text{nucl}} \lesssim \omega_{\text{el}}$, the electron-transition frequency), the electron shell of the atom has an appreciable effect on the nuclear-transition probabilities.

In the present paper we investigate the role of the electron shell in the processes of resonance mixing of the levels of a compound nucleus, and also compute the probability for this mixing.

2. EFFECT OF THE ELECTRON SHELL

The Hamiltonian of an atom + neutron system in an external electromagnetic field has the form

$$H=\sum_{i=1}^{Z'}\left(\mathbf{p}_i+\frac{e}{c}\mathbf{A}\right)^2(2m)^{-1}+\sum_{i=1}^{Z'}V(|\mathbf{r}_i-\mathbf{r}_{\text{nucl}}|)$$
$$+\mathbf{p}_n^2/2M+V(|\mathbf{r}_n-\mathbf{r}_{\text{nucl}}|)+\left(\mathbf{p}_{\text{nucl}}-\frac{Ze}{c}\mathbf{A}\right)^2(2MA)^{-1}+H_{int}, \tag{1}$$

where m and M are respectively the electron and neutron masses, $\mathbf{p}_i$ is the electron momentum operator, $\mathbf{p}_n$ is the incident-neutron momentum operator, $\mathbf{p}_{\text{nucl}}$ is the nucleus momentum operator, MA is the mass of the nucleus, $V(|\mathbf{r}_n-\mathbf{r}_{\text{nucl}}|)$ is the effective neutron–nucleus interaction potential, $V(|\mathbf{r}_i-\mathbf{r}_{\text{nucl}}|)$ is the potential energy operator of the atom, and H_{int} is the operator for the interaction of the neutron with the nucleons of the nucleus. The H_{int} interaction leads to the excitation of more complicated configurations, e.g., three-quasiparticle configurations.

Let us write the vector potential of the external electromagnetic field in the dipole approximation in the form

$$\mathbf{A}=\mathbf{A}_0\cos\omega t. \tag{2}$$

We transform to a system of coordinates fixed to the nucleus; to do this, we introduce the relative coordinates and the coordinate of the center of gravity:

$$\xi_i=\mathbf{r}_i-\mathbf{r}_{\text{nucl}},\quad \xi_n=\mathbf{r}_n-\mathbf{r}_{\text{nucl}},$$
$$\mathbf{X}=\left(M\mathbf{r}_n+AM\mathbf{r}_{\text{nucl}}+m\sum_{i=1}^{Z'}\mathbf{r}_i\right)\Big/\left(Z'm+M(A+1)\right). \tag{3}$$

After simple transformations, the Hamiltonian (1) can be written in the form

$$H=H_{0a}+H_{0n}+V, \tag{4}$$

where H_{0a} and H_{0n} are respectively the Hamiltonians of the atom and neutron + nucleus system in the absence of an external electromagnetic field, and for the operator V we obtain the following expression:

$$V=-i\hbar\frac{e}{mc}\sum_{i=1}^{Z'}\frac{\partial}{\partial\xi_i}\mathbf{A}-i\hbar\frac{Ze}{MA}\frac{\partial}{\partial\xi_n}\mathbf{A}-\frac{\hbar^2}{MA}\sum_{i=1}^{Z'}\frac{\partial^2}{\partial\xi_i\,\partial\xi_n}. \tag{5}$$

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Here the first term describes the interaction of the electrons with the external electromagnetic field, the summation being over all the electrons in the atom ($Z' \leqslant Z$); the second term describes the interaction of the system neutron + nucleus with the **A** field; and the third term describes the correlation between the motion of the electron + nucleus system and that of the neutron. This interaction is the result of the separation of the center of mass of the atom + neutron system and the going over to the coordinates of the neutron and the electrons relative to the nucleus. In Ref. 4 a similar interaction between the nucleons of the nucleus and the electron shell arises from the Coulomb energy as a result of an expansion in powers of the small parameter $|\mathbf{r}_{\text{nucl}}|/|\mathbf{r}_{\text{el}}|$. Therefore, this interaction is proportional to the square of the charge, but the interaction considered in the present paper does not depend on the charge at all.

The quantity Ze/A should be interpreted as the effective charge for the electric dipole transitions in the neutron + nucleus system. If the effect of the electron shell is insignificant, then only this charge needs to be considered. Let us consider the effect of the electron shell on the effective charge. To do this, let us compute the matrix element of the transition between the compound-nucleus level for the s neutron and the corresponding p level with emission or absorption of a quantum of the external electromagnetic field. We shall assume that the electron shell of the atom remains in the process in the ground state. Then we obtain in the lowest orders of perturbation theory the expression

$$V_{\lambda n}=\frac{Ze}{2MAc}(\mathbf{p}_n\mathbf{A}_0)_{\lambda n}-\frac{e}{2mMAc}\sum_m\Big(\sum_{i=1}^{Z'}\mathbf{p}_i\mathbf{p}_n\Big)_{\lambda n,0m}\times\Big(\sum_{i=1}^{Z'}\mathbf{p}_i\mathbf{A}_0\Big)_{m0}(E_m-\hbar\omega-E_0)^{-1}-\frac{e}{2mMAc}\sum_m\Big(\sum_{i=1}^{Z'}\mathbf{p}_i\mathbf{A}_0\Big)_{0m}\times\Big(\sum_{i=1}^{Z'}\mathbf{p}_i\mathbf{p}_n\Big)_{\lambda n}(E_m+E_\lambda-E_0-E_n)^{-1},\qquad(6)$$

where $\mathbf{p}_i = -i\hbar\partial/\partial\xi_i$, $\mathbf{p}_n = -i\hbar\partial/\partial\xi_n$, the sum is over all the excited electron states of the atom, E_m and E_0 are respectively the excited- and ground-state energies of the atom, E_n is the energy of the incoming neutron, and E_λ is the energy of the compound-nucleus level at which the neutron is captured. If the absorption of the neutron is accompanied by the absorption of a photon, then $E_\lambda = E_n + \hbar\omega$. Substituting this value of E_λ into (6), and using the relation

$$\mathbf{p}_{12}=im(E_1-E_2)\mathbf{r}_{12}/\hbar,$$

we obtain ($A_0 = cE_0/\omega$)

$$V_{\lambda n}=i\frac{Ze}{2A}(\xi_{nz}E_0)_{\lambda n}-i\frac{em}{2\hbar A}(\xi_{nz}E_0)_{\lambda n}\times\Big(\sum_m{}'\frac{\omega_{m0}^2}{\omega_{m0}-\omega}|(\xi_z)_{m0}|^2+\frac{\omega_{m0}^2}{\omega_{m0}+\omega}|(\xi_z)_{m0}|^2\Big)\qquad(7)$$

Here, for simplicity, we consider the case in which the z axis is oriented along the vector $\mathbf{E}_0$. Let us transform the expression in the round brackets. Let us separate out in this expression the ω-independent term, and, using the well-known sum rule for the square of the matrix element of the dipole operator,[5] reduce it to the ratio Z'/Z, where Z' is the number of electrons in the atom (ion) and Z is the nuclear charge. Let us express the other term, which is proportional to ω^2, in terms of the dipolar polarizability of the atom (ion)[1]:

$$\beta(\omega)=\frac{e^2}{\hbar}\sum_m{}'|(\xi_z)_{m0}|^2\Big(\frac{1}{\omega_{m0}-\omega}+\frac{1}{\omega_{m0}+\omega}\Big)\qquad(8)$$

As a result, we obtain an expression for the effective transition matrix element for the neutron + nucleus system in the form

$$V_{\lambda n}=i\frac{Ze}{2A}(\xi_{nz}E_0)_{\lambda n}\Big(\frac{Z-Z'}{Z}-\frac{m\omega^2}{\hbar c}\frac{137}{Z}\beta(\omega)\Big)=i\frac{Ze}{2A}(\xi_{nz}E_0)_{\lambda n}\Delta.\qquad(9)$$

Estimates of the second term in (9) in the static limit yield a value of 10^{-4} for nonalkali neutral atoms and a value roughly an order of magnitude higher for the alkali atoms. When allowance is made for the ω dependence of the dynamical polarizability, the corresponding term can be much greater than the term in the static case. Thus, for example, for the cesium atom and a neodymium laser the value of the polarization term can be as high as 10^{-2}. In the case of ions the first term, as a rule, predominates. Thus, the electron shell has quite an appreciable effect on the effective charge for the electric dipole transitions in the neutron + nucleus system in a laser-radiation field. Notice that this conclusion is valid in the nonresonance—with respect to the electrons—approximation (i.e., in the case when $\omega_{m0} - \omega \gg \Gamma_m$, where Γ_m is the width of the corresponding atomic level.

3. PROBABILITY FOR MIXING OF THE LEVELS OF A COMPOUND NUCLEUS

For the subsequent calculations it is convenient to express the nuclear transition operator in (9) in terms of the spherical harmonics $Y_{lm}(\theta,\varphi)$:

$$\hat{V}=e_tE_0\xi_n\sqrt{\frac{\pi}{3}}\Big[\cos\theta_0Y_{10}+\frac{1}{\sqrt{2}}\sin\theta_0(Y_{1-1}\exp(i\varphi_0)-Y_{11}\exp(-i\varphi_0))\Big]\qquad(10)$$

where $e_t = Ze/A$, θ_0 is the angle between $\mathbf{E}_0$ and $\mathbf{p}_n$, φ_0 is the corresponding azimuthal angle, and θ is the angle between $\mathbf{p}_n$ and ξ_n.

We shall compute the cross section for an inelastic scattering in which the neutron undergoes a transition from the state with initial energy ε_p into the state with energy $\varepsilon_p \pm \hbar\omega$, using perturbation theory. Then the cross section for inelastic scattering of the neutron in a laser-radiation field can be written in the form

$$d\sigma_{\lambda n}=\frac{2\pi M}{\hbar^2k_0}\sum_1|V_{\lambda n}|^2\delta(\varepsilon_p\pm\hbar\omega-E_1),\qquad(11)$$

where $k_0^2 = 2M\varepsilon_p/\hbar^2$ is the square of the wave vector of the incident neutrons; the summation is over all the final states.

In the case of slow-neutron scattering by nuclei we can limit ourselves in the initial state to the consideration of the s and p waves. We shall assume that the target nucleus has zero spin; then the initial-state wave function in the region $r_n > R$ (the nuclear radius) will have the form (we assume, for simplicity, that the neutron is a spinless particle)

$$\varphi_{\text{init}}=\exp(i\delta_0(k_0))[\cos\delta_0(k_0)j_0(x_0) -\sin\delta_0(k_0)\eta_0(x_0)]P_0(\cos\theta) +3i\exp(i\delta_1(k_0))[\cos\delta_1(k_0)j_1(x_0) -\sin\delta_1(k_0)\eta_1(x_0)]P_1(\cos\theta), \quad (12)$$

where $x_0 \equiv k_0\xi_n$; j_0, j_1, η_0 and η_1 are spherical Bessel and Neumann functions of orders zero and one; $P_0(y)$ and $P_1(y)$ are the Legendre polynomials. The phases of the s and p waves for the neutron energy corresponding to k_0 are given by the following expressions[6]:

$$\exp(2i\delta_{0,1}(k_0)) =\exp(2i\delta_{0,1}^{\text{pot}}(k_0))[1-i\Gamma_{n0,1}/(\varepsilon_p-E_{0,1}+{}^1/_2 i\Gamma_{0,1})], \quad (13)$$

where $\Gamma_{n,0,1}$ are the elastic s- and p-resonance widths for the neutrons, $E_{0,1}$ are the positions of the s and p resonances, ε_p is the energy of the incoming neutron, and $\delta_{0,1}^{\text{pot}}$ are the potential scattering phases of the s and p waves at the neutron energy corresponding to k_0. The wave function φ_{fin} of the scattered neutron will also have the form of a superposition of s and p waves. The expression for φ_{fin} has the form of (12) with x_0 replaced by $x_1 \equiv k_1\xi_n$, where $k_1^2 = 2M(\varepsilon_p \pm \hbar\omega)/\hbar_2$, and the scattering phases δ_0 and δ_1 evaluated at the neutron energy corresponding to k_1.

We shall compute the radial matrix element of the transition operator (10) between the wave functions φ_{init} and φ_{fin} in the squarewell model. Using the well-known quantum-mechanical formula

$$(\ddot{\xi}_n)_{12}=(E_1-E_2)^2(\xi_n)_{12}/\hbar^2, \quad (14)$$

as well as the relation

$$M(\ddot{\xi}_n)_{\lambda n}=-(\nabla_n U)_{\lambda n}, \quad (15)$$

which is shown in the Appendix to be valid for the Hamiltonian (4), we obtain for the differential cross section for neutron scattering in a laser-radiation field the expression

$$d\sigma_{\lambda n}/d\Omega_{k_1}=\left(\frac{e_i E_0}{\hbar\omega}\right)^2\left(\frac{U_0}{\hbar\omega}\right)^2\frac{1}{k_0k_1}\Delta^2 \times[|A|^2\cos^2\theta_0+|B|^2(\cos\theta_0\cos\theta_1 +\sin\theta_0\sin\theta_1\cos(\varphi_1-\varphi_0))^2 -2(\operatorname{Re}A\operatorname{Re}B+\operatorname{Im}A\operatorname{Im}B)(\cos\theta_0\cos\theta_1 +\sin\theta_0\sin\theta_1\cos(\varphi_1-\varphi_0))\cos\theta_0], \quad (16)$$

where

$$A=\frac{k_1}{k_0}(R+a_0(k_1))\frac{\delta_1(k_0)}{k_0R}, \quad B=(R+a_0(k_0))\frac{\delta_1(k_1)}{k_1R}, \quad (17)$$

U_0 is the depth of the well, θ_1 is the angle between $\mathbf{k}_0$ and $\mathbf{k}_1$, and $a_0(k_0)=\delta_0(k_0)/k_0$, $a_0(k_1)=\delta_0(k_1)/k_1$ are the s-wave neutron scattering lengths at ε_p and $\varepsilon_p \pm \hbar\omega$ respectively.

Integrating the expression (16) over the scattering angle, we obtain the total cross section for inelastic neutron scattering in a laser-radiation field:

$$\sigma_{\lambda n}=\left(\frac{e_i E_0}{\hbar\omega}\right)^2\left(\frac{U_0}{\hbar\omega}\right)^2|\Delta|^2\left[|A|^2\cos^2\theta_0+\frac{1}{3}|B|^2\right]\frac{4\pi}{k_0k_1}. \quad (18)$$

Let us consider in detail the two simple cases in which one of the terms in the expression (18) is greater than the other. Let the initial neutron energy ε_p be such that it practically coincides with the s-resonance energy E_0 of the compound nucleus, while the scattered-neutron energy lies in the p-resonance region ($\varepsilon_p \pm \hbar\omega \approx E_1$). Then, since the resonance scattering phase is usually much greater than the potential scattering phase, $|B|^2 \gg |A|^2$ and

$$\sigma_{\lambda n}^{(1)}\approx\left(\frac{e_i E_0}{\hbar\omega}\right)^2\left(\frac{U_0}{\hbar\omega}\right)^2|\Delta|^2|B|^2\frac{4\pi}{3k_1k_0}, \quad (19)$$

$$|B|^2\approx\frac{\Gamma_{n0}^2(k_0)/k_0^2}{4(\varepsilon_p-E_0)^2+\Gamma_0^2(k_0)}\,\frac{\Gamma_{n1}^2(k_1)/(k_1R)^2}{4(\varepsilon_p\pm\hbar\omega-E_1)^2+\Gamma_1^2(k_1)}.$$

In the second case $\varepsilon_p \approx E_1$ and $\varepsilon_p \pm \hbar\omega \approx E_0$, i.e., the initial neutron energy is close to the p resonance, while the scattered-neutron energy is close to the s resonance. Then the total scattering cross section has the form

$$\sigma_{\lambda n}^{(2)}\approx\left(\frac{e_i E_0}{\hbar\omega}\right)^2\left(\frac{U_0}{\hbar\omega}\right)^2|\Delta|^2\cos^2\theta_0|A|^2\frac{4\pi}{k_0k_1},$$

$$|A|^2\approx\frac{k_1^2}{k_0^2}\,\frac{\Gamma_{n0}^2(k_1)/k_1^2}{4(\varepsilon_p\pm\hbar\omega-E_0)^2+\Gamma_0^2(k_1)}\,\frac{\Gamma_{n1}^2(k_0)/(k_0R)^2}{4(\varepsilon_p-E_1)^2+\Gamma_1^2(k_0)}. \quad (20)$$

Thus, as follows from a comparison of the formulas (19) and (20), in the second case (transition from the p into the s state) the total cross section depends on the angle θ_0 between the vectors $\mathbf{k}_0$ and $\mathbf{E}_0$, and, when $\theta_0 = \pi/2$, the total inelastic neutron scattering cross section tends to zero. At the same time, in the first case (transition from the s into the p state) the total cross section does not depend on this angle at all. Let us estimate the ratio of the total cross sections at exact resonance, when in the first case the initial neutron energy coincides with the s-resonance energy and the scattered neutron energy coincides with the location of the p resonance (let, for definiteness, the transition from the s into the p state be accompanied by the emission of a photon). Then the transition from the p into the s state is accompanied by the absorption of a photon:

$$\frac{\sigma_{\lambda n}^{(2)}}{\sigma_{\lambda n}^{(1)}}\sim 3\cos^2\theta_0\frac{\varepsilon_p+\hbar\omega}{\varepsilon_p}>1, \quad (21)$$

if $\cos^2\theta_0 \sim 1$.

4. RESONANCE CROSS SECTION FOR NEUTRONS WITH SPIN

We shall consider in this section the effect of the neutron spin on the process of inelastic scattering in a laser-radiation field. To avoid unwieldy formulas, let us consider a target nucleus with spin $I = 0$, and limit ourselves to the case in which the energy of the incoming neutrons is close to the s-resonance energy of the compound nucleus and the energy of the inelastically scattered neutrons coincides with the position of the p state. Then we can, on the basis of the preceding analysis, limit ourselves in the initial-state wave function to only the s wave:

$$\varphi_{\text{init}}=\sqrt{4\pi}\sin(x_0+\delta_0(k_0))Y_{00}(\theta,\varphi)\chi(\sigma)/x_0. \qquad (22)$$

Here $\chi(\sigma)$ is the spin wave function of the neutron. Let us expand the wave function of the final state with $l = 1$ (the p state) in terms of the wave functions of the total angular momentum j in this state.[6] Since the target nucleus has zero spin, the total spin in the final state can have the values $\frac{3}{2}$ and $\frac{1}{2}$; therefore, the wave function of the final state with $l = 1$ in the region $x_1 \ll 1$ can be written in the form: for the $+\frac{1}{2}$ spin component

$$\Phi^{1/2}=i\frac{\sqrt{4\pi}}{x_1^2}\Big\{\sin\delta_{3/2}(k_1)\sqrt{2}\Big[\sqrt{\frac{2}{3}}Y_{10}\chi\Big(\frac{1}{2}\Big)+\sqrt{\frac{1}{3}}Y_{11}\chi\Big(-\frac{1}{2}\Big)\Big]$$
$$+\sin\delta_{1/2}(k_1)\Big[\sqrt{\frac{1}{3}}Y_{10}\chi\Big(\frac{1}{2}\Big)-\sqrt{\frac{2}{3}}Y_{11}\chi\Big(-\frac{1}{2}\Big)\Big]\Big\} \qquad (23)$$

and for $m = -\frac{1}{2}$

$$\Phi^{-1/2}=i\frac{\sqrt{4\pi}}{x_1^2}\Big\{\sqrt{2}\sin\delta_{3/2}(k_1)\Big[\sqrt{\frac{1}{3}}Y_{1-1}\chi\Big(\frac{1}{2}\Big)+\sqrt{\frac{2}{3}}Y_{10}\chi\Big(-\frac{1}{2}\Big)\Big]+\sin\delta_{1/2}(k_1)\Big[\sqrt{\frac{2}{3}}Y_{1-1}\chi\Big(\frac{1}{2}\Big)-\sqrt{\frac{1}{3}}Y_{10}\chi\Big(-\frac{1}{2}\Big)\Big]\Big\}. \qquad (24)$$

Here $\delta_{3/2}(k_1)$ and $\delta_{1/2}(k_1)$ are the scattering phases for a neutron in the states with total spins $j = \frac{3}{2}$ and $j = \frac{1}{2}$ respectively. In the expressions (23) and (24) the spherical harmonics Y_{lm} depend on the angle θ' (the angle between $\mathbf{k}_1$ and $\mathbf{r}_n$). Let us use the transformation formulas given in Ref. 7, and go over to the angles θ and θ_1 (θ is the angle between $\mathbf{k}_0$ and $\mathbf{r}_n$; θ_1, the angle between $\mathbf{k}_0$ and $\mathbf{k}_1$).

Using the square-well model and the corresponding relations (11) and (15), we obtain for the differential cross section for inelastic neutron scattering in a laser-radiation field the following expression:

$$d\sigma_{\lambda n}/d\Omega={}^1/_9\Big(\frac{e_tE_0}{\hbar\omega}\Big)^2\Big(\frac{U_0}{\hbar\omega}\Big)^2\frac{|\Delta|^2}{k_0k_1}\frac{1}{(k_1R)^2}|R+a_0(k_0)|^2$$
$$\times\Big[|2\delta_{3/2}(k_1)+\delta_{1/2}(k_1)|^2(\cos\theta_0\cos\theta_1+\sin\theta_0\sin\theta_1\cos(\varphi_0-\varphi_1))^2$$
$$+|\delta_{3/2}(k_1)-\delta_{1/2}(k_1)|^2\Big(\sin^2\theta_1\cos^2\theta_0+\frac{1}{2}\sin^2\theta_0(1+\cos^2\theta_1)$$
$$+\cos\theta_0\sin\theta_0(\sin\theta_1+\sin\theta_1\cos\theta_1)\Big)\Big]. \qquad (25)$$

Comparing (25) with (16), we note that allowance for the neutron spin leads to a new angular dependence if the scattering phases $\delta_{3/2}$ and $\delta_{1/2}$ differ from each other; in the opposite case when $\delta_{3/2}\approx\delta_{1/2}$ the cross section (25) coincides with that term in the cross section (16) which is proportional to $|B|^2$. Let us integrate (25) over the scattering angle and obtain the total cross section:

$$\sigma_{\lambda n}=\frac{1}{27}\frac{4\pi}{k_0k_1}\Big(\frac{e_tE_0}{\hbar\omega}\Big)^2\Big(\frac{U_0}{\hbar\omega}\Big)^2|(R+a_0(k_0)|^2\frac{|\Delta|^2}{(k_1R)^2}$$
$$\times[|2\delta_{3/2}(k_1)+\delta_{1/2}(k_1)|^2+2|\delta_{3/2}(k_1)-\delta_{1/2}(k_1)|^2]. \qquad (26)$$

Notice that, when $\delta_{3/2}\approx\delta_{1/2}$, the expression (26) for the cross section coincides with the previously obtained analogous expression (19). In the resonance approximation, in which the inelastically-scattered-neutron energy $\varepsilon_p \pm \hbar\omega \approx E_1$, the resonance energy of a compound nucleus with total spin $\frac{3}{2}$, the expression for the cross section can be represented in the form

$$\sigma_{\lambda n}^{3/2}=\frac{2}{3}\frac{\pi}{k_0^2}\frac{\Gamma_{np}(^3/_2)\Gamma_n(^3/_2k_1)}{(\varepsilon_p\pm\hbar\omega-E_{3/2})^2+{}^1/_4\Gamma^2(^3/_2,k_1)}|\Delta|^2,$$
$$\Gamma_{np}=\frac{1}{3}\frac{k_0}{k_1}\Big(\frac{e_tE_0}{\hbar\omega}\Big)^2\Big(\frac{U_0}{\hbar\omega}\Big)^2|R+a_0(k_0)|^2\Gamma_n(^3/_2,k_1)$$
$$\times\Big(\frac{\varepsilon_p\pm\hbar\omega}{1\text{ eV}}\Big)^{1/2}. \qquad (27)$$

Here $\overline{\Gamma}_n(\frac{3}{2},k_1)$ is the reduced neutron width of the $\frac{3}{2}$-resonance. Similarly, if $\varepsilon_p \pm \hbar\omega\approx E_{1/2}$ $(\delta_{1/2}\gg\delta_{3/2})$, then

$$\sigma_{\lambda n}^{1/2}=\frac{\pi}{k_0^2}\frac{1}{3}\frac{\Gamma_{np}(^1/_2)\Gamma_n(^1/_2k_1)}{(\varepsilon_p\pm\hbar\omega-E_{1/2})^2+{}^1/_4\Gamma^2(^1/_2,k_1)}|\Delta|^2. \qquad (28)$$

From a comparison of the expressions (27) and (28) for the cross sections, we can see that the ratio of the cross sections for neutron scattering via the resonance with total spin $\frac{3}{2}$ and via the resonance with spin $\frac{1}{2}$ can be written as

$$2\overline{\Gamma}_n(^3/_2)\Gamma_n(^3/_2)\Gamma(^1/_2)/\overline{\Gamma}_n(^1/_2)\Gamma_n(^1/_2)\Gamma(^3/_2),$$

so that the cross sections differ by a factor of two if all the widths Γ are of the same order of magnitude.

In conclusion, let us note that allowance for the neutron spin does not give rise to significant changes in the expression for the cross section; a dependence on the angle between $\mathbf{E}_0$ and $\mathbf{k}_0$ does not arise in this case, too.

5. INDUCED NUCLEAR REACTIONS

The total level width Γ for the compound nucleus is usually determined by the processes of elastic scattering and radiative capture of the neutron. In heavy nuclei, to these processes is added fission, so that $\Gamma=\Gamma_n+\Gamma_\gamma+\Gamma_f=\Gamma_n+\gamma$, where γ is the width characterizing the decay of the compound-nucleus level via the radiative and fission channels. With allowance for this observation, the cross section σ_c for induced nuclear reactions (fission, radiative capture) for the process of neutron capture at the s resonance can be written as

$$\sigma_{cs}=\frac{\pi}{k_0^2}|\Delta|^2\frac{\Gamma_{np}\gamma}{(\varepsilon_p\pm\hbar\omega-E_1)^2+{}^1/_4\Gamma_1^2}. \qquad (29)$$

Similarly, we can derive an expression for the induced-nuclear-reaction cross section for the case in which the neutron is captured at the p resonance; from (20) it follows that

$$\sigma_{cp}=\frac{\pi}{k_0^2}3\cos^2\theta_0|\Delta|^2\frac{\tilde{\Gamma}_{np}\gamma}{(\varepsilon_p-E_1)^2+{}^1/_4\Gamma_1^2}$$

$$\tilde{\Gamma}_{np}=\frac{1}{3}\frac{k_1}{k_0}\left(\frac{e_tE_0}{\hbar\omega}\right)^2\left(\frac{U_0^2}{\hbar\omega}\right)^2|R+a_0(k_1)|^2\,\bar{\Gamma}_{n1}(k_0)\left(\frac{\varepsilon_p}{1\text{ eV}}\right)^{1/2}. \tag{30}$$

Knowing the width γ for the compound-nucleus level in question, we can, by estimating the quantities Γ_{np} and $\tilde{\Gamma}_{np}$, find the quantity σ_c.

6. DISCUSSION OF THE RESULTS. CONCLUSION

Above we determined the neutron-scattering and reaction cross sections in the field of an electromagnetic wave whose frequency is equal to the separation of the compound-nucleus levels with opposite parities. But a number of conditions must be fulfilled to be able to observe the effect of laser radiation on the cross section for interaction of a neutron with nuclei.

1. The neutron must be scattered by the nucleus of an ion. For example, atoms of the rare-earth elements and the actinides, when implanted in a dielectric, exist in an ionic state with charge $Z-Z'\sim 3$; therefore, the effective charge of the neutron + nucleus system is, according to (18), reduced by a factor of 20–30 as a result of the effect of the electron shell. But for the ion to be regarded as isolated, the condition $\omega\gg\omega_{\rm opt}$, where $\omega_{\rm opt}$ is the characteristic frequency of the optical phonons, must be fulfilled; otherwise, according to the conclusion drawn in Sec. 2, the effective charge can drop down to 10^{-4}.

Let us, moreover, note that the proposed method of computing the cross sections is valid if the condition $\hbar\omega\gg\Gamma_0$ is also fulfilled.

2. The scattering, accompanied by the absorption or emission of a quantum of the electromagnetic field, of an s-wave neutron near an isolated p resonance is highly improbable, since in this case the s-wave neutron undergoes only potential scattering, and does not form a compound nucleus in the potential-well model. The corresponding probability in this model is estimated in Ref. 8.

3. Estimates with the aid of the formulas (19) and (20) show that the cross section for s-wave neutron scattering accompanied by a transition of the neutron into the p state is of the same order of magnitude as the corresponding cross section for the case in which the p-wave neutron goes over into the s state. But the total cross section for capture in the case of the s resonance of the compound nucleus is, as a rule, greater than the corresponding cross section in the case of the neighboring p level. Therefore, it is more expedient to have the p level of the compound nucleus in the initial state, since we can use a thicker target in that case. Let us also emphasize that the transition from the $s(p)$ level of the compound nucleus into another state, which may be forbidden or of low probability in the absence of an electromagnetic field (the partial (n,γ) transition, for example), can become allowed to the extent of the mixing of the compound-nucleus levels after the field has been switched on.

The case in which the external electromagnetic wave is at resonance with some electron transition in the atom (ion) is of interest. Such a resonance in the Rabi regime ($\gamma_{\rm el}\ll\Delta\omega$, the electromagnetic field width) is considered in Ref. 2, where it is shown that in this case the high-frequency electromagnetic field inside the electron shell can be an appreciably amplified field. Resonance in the case when the Rabi conditions are not fulfilled requires a more detailed analysis.

In conclusion, let us give an estimate for the power of the electromagnetic field in the specific case of the La^{139} nucleus ($Z-Z'\sim 3$). In this nucleus there is a p level with energy $E_1=0.734$ eV and an s level with energy $E_0=-37.5$ eV. Although this s level is, in energy terms, located quite far from the p level, it has a fairly large width ($\Gamma_\gamma^s=45\times10^{-3}$ eV and $g\bar{\Gamma}_n^s=0.115$ eV, and therefore the effect of this level is felt quite far beyond the location of the p level. For the p level $\Gamma_\gamma^p\approx 56\times10^{-3}$ eV and $g\bar{\Gamma}_n=3.2\times10^{-8}$ eV. For a neodymium laser ($\hbar\omega=1.17$ eV) with power 30 kW/cm^2 the stimulated-capture cross section ≈ 2 b for neutrons with energy $\varepsilon_p=0.734+\hbar\omega$. In this case we assume that the charge of the La^{139} ion is equal to three, and that the light wave propagates in a direction perpendicular to the direction of motion of the neutrons. Thus, even with allowance for the electron screening, the stimulated-neutron-capture effect can be observed at moderate laser-radiation powers.

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APPENDIX

We can write the Hamiltonian of the neutron + nucleus system in the form

$$H_{0n}=H_0+H_{int},$$

where H_0 describes the motion of the neutron in the self-consistent field of the nucleus, while $H_{\rm int}$ describes the interaction of the neutron with the more complicated configurations, e.g., the two particles–one hole configuration.

The matrix element of the dipole operator for the neutron + nucleus system is equal to

$$\mathbf{r}_{12}=\frac{1}{\omega^2M}\dot{\mathbf{p}}_{12}=\frac{i}{\hbar\omega^2M}(H_{0n}\mathbf{p}-\mathbf{p}H_{0n})_{12}$$
$$=-\frac{1}{M\omega^2}(\nabla_nU)_{12}-\frac{1}{M\omega^2}(\nabla_nH_{int})_{12}, \tag{A.1}$$

where 1 and 2 denote the eigenfunctions of the operator H_{0n}. Let us estimate the contribution of the second term in (A.1). The nuclear interaction $H_{\rm int}$ of the neutron with the rest of the nucleons of the nucleus attenuates over distances of the order of a $\sim r_0=RA^{-1/3}$. Therefore, we have the estimate

$$|(\nabla_nH_{int})|\sim H_{int}/r_0,$$

and, consequently,

$$(\nabla_n H_{int})_{12} \sim \frac{1}{r_0}\left(\frac{\mathbf{r}_n}{r_n} H_{int}\right)_{12}. \quad \text{(A.2)}$$

The width W characterizing the decay of the single-particle state n into more complicated states (the "mixing" width) is defined as follows:

$$W = 2\pi |(H_{int})_{12}|^2 \rho_2,$$

where ρ_2 is the energy density of the complicated states. Consequently, we have

$$|(H_{int})_{n2}| \sim (W/\rho_2)^{1/2}.$$

Separating the single-particle components in the wave functions 1 and 2, we obtain

$$|(\nabla_n H_{int})_{12}| \sim r_0^{-1} |C_{1,2}| (W/\rho_2)^{1/2}, \quad \text{(A.3)}$$

where $C_{1,2}$ are the amplitudes of the admixtures of the single-particle neutron configurations in the states 1 and 2. On the other hand, for a potential of rectangular shape we have

$$|(\nabla_n U)_{12}| \sim |C_1 C_2 \varphi_1(R) \varphi_2(R)| U_0, \quad \text{(A.4)}$$

where $\varphi_{1,2}$ are the single-particle components of the wave functions of the corresponding states $C_{1,2} \sim (\rho_{1,2} W)^{-1/2}$ of the compound nucleus. Finally, comparing (A.3) and (A.4), and bearing in mind the fact that $\varphi_{1,2}(R) \sim R^{-1/2}$, we obtain the estimate

$$|(\nabla_n H_{int})_{12}| |(\nabla_n U)_{12}|^{-1} \sim \frac{WR}{r_0 U_0} \sim \frac{WA^{1/3}}{U_0}. \quad \text{(A.5)}$$

Thus, the role of the interaction H_{int} in the case of electric dipole transitions between the compound-nucleus levels is, to the extent that the imaginary part of the optical potential is small compared to the shell spacing, minor.

[1] N. B. Delone and V. P. Kraĭnov, Atom v sil'nom svetovom pole (The Atom in a Strong Light Field), Atomizdat, Moscow, 1978.

[2] D. R. Zaretskiĭ and V. V. Lomonosov, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 541 (1979) [JETP Lett. **30**, 508 (1979)].

[3] D. R. Zaretskiĭ and V. V. Lomonosov, Zh. Eksp. Teor. Fiz. **81**, 429 (1981) [Sov. Phys. JETP **54**, 229 (1981)].

[4] M. Leon and R. Seki, Nucl. Phys. A **298**, 333 (1978).

[5] H. A. Bethe and E. E. Salpeter, Quantum Mechanics of One- and Two-Electron Systems, Springer Verlag, Berlin, 1958 (Russ. Transl., Fizmatgiz, Moscow, 1960, p. 420).

[6] L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika (Quantum Mechanics), Nauka, Moscow, 1977, p. 636 (Eng. Transl., 3rd ed., Pergamon, Oxford, 1977).

[7] A. R. Edmonds, Angular Momentum in Quantum Mechanics, Princeton University Press, Princeton, 1957 (Russ. Transl., in: Deformatsiya atomnykh yader (Deformation of Atomic Nuclei), IL, Moscow, 1958, p. 305).

[8] A. L. Dzyublik, Zh. Eksp. Teor. Fiz. **82**, 977 (1982) [Sov. Phys. JETP **55**, 573 (1982)].

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