An experimental investigation of the thermomechanical circulation effect in the subcritical flow regime of He II

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The thermomechanical circulation effect has been studied quantitatively in the temperature range 1.40 to 2.17 K. The theoretical predictions for the subcritical flow regime have been verified.

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1. The thermomechanical circulation effect,[1,2] which is one of the macroscopic examples of the manifestation of the quantization of circulation in He II, consists of the following. Suppose that an annular vessel, filled with He II, consists of two "large" volumes connected by two capillaries. If different temperatures T_1 and T_2 are maintained in the two large volumes, then a flow of the n and s components arises in the capillaries, and to achieve the condition for quantization of the circulation of the velocity of the superfluid part of the liquid over any closed circuit, a superfluid flow circulating around the ring must arise with a large-volume velocity v_{s0}

$$v_{s0}=\frac{\rho_n}{\rho_s}\frac{\rho\sigma\Delta T(s_1-s_2)}{8\pi\eta l_0\xi},\quad \xi=1+\frac{s_0}{l_0}\left(\frac{l_1}{s_1}+\frac{l_2}{s_2}\right),\qquad (1)$$

where ρ, ρ_n, ρ_s are the densities of liquid helium and of its n and s components, σ is the entropy per unit mass, η is the viscosity of He II, $\Delta T = T_2 - T_1$ is the temperature difference across the ends of the capillaries, ρ_0 is the overall length of the large volumes, ρ_1 and ρ_2 are the capillary lengths, s_0, s_1, s_2 are the cross sections of the large volume and of the first and second capillaries.

The thermomechanical circulation effect was revealed by the reaction of the walls of the ring hung from a torsion balance.[3] The aim of the present work is a quantitative study of the effect in the temperature range 1.40 to 2.17 K and verification of the correctness of the theoretical predictions for subcritical flow of He II.

2. The thermomechanical circulation effect was generated in an annular vessel of mean radius 2 cm, made from a polythene tube with inner diameter 5 mm. The total length of the large volumes 1 and 2 (see Fig. 1,a) was $l_0 = 3$ cm. They were filled with 300 μm diameter nylon threads to a filling factor of 0.5. A 5.5 Ω constantan wire heater 3 was placed in one of the large volumes. The second large volume was provided with a 2 mm diameter tube 4, open to a He II bath which served to fill the ring with helium and also acted as a cold finger, maintaining the bath temperature in this part of the ring. Lid 5 was fixed onto tube 4 to compensate the constant mechanical moment which can arise as a result of a current of helium flowing out of the ring. The first capillary 6 had length $l_1 = 4.5$ cm and was filled with 45 μm diameter glass fibers, the second capillary 7 had $l_2 = 4.5$ cm and was filled with 15 μm diameter glass fibers. On the assumption of quadratic close packing of the fibers, the cross section of each pore in the first capillary $s_1 = 4.3\times10^{-6}$ cm^2, and the total cross section of these pores $\Sigma s_1 = 0.043$ cm^2. The corresponding values for the second capillary were $s_2 = 2.1\times10^{-7}$ cm^2 and $\Sigma s_2 = 0.043$ cm^2. The mass of the ring was 11 g. The cross sections and lengths of the channels in the large volumes and of the capillaries were chosen to widen as much as possible the range of the subcritical flow regime studied.

The ring was firmly fixed to a glass rod 8 (Fig. 1,b) and suspended from the trosion fiber 9. The current was led to heater 3 by contactless means: electromagnetic oscillations of frequency $\sim 10^5$ Hz were excited in an "external" oscillating circuit 10 by a GZ-33 audio frequency generator. The "internal" circuit 11, inductively coupled to it and having heater 3 in its circuit, was fixed to the balance and an emf was induced in it.

The oscillations were recorded automatically using a system described previously,[4] and also using an improved version described below. Light from source 12 (Fig. 1,b)

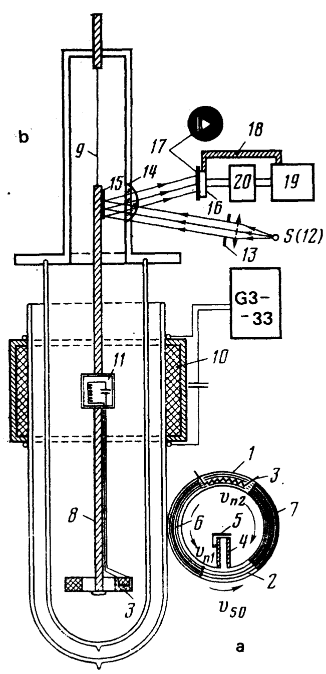

FIG. 1. a) Arrangement of ring; b) arrangement of apparatus and of the automatic recording of oscillations of the system.

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passes through a narrow vertical diaphragm 13, a thin lens 14 and falls on mirror 15 fixed to the glass rod. The narrow beam of light reflected from the mirror falls on the photocell 17 and the recorder 19 fixed on the carriage 18. When the system is at rest, the light spot is focused onto the center of the mask which is in the middle of the recorder scale, together with the photocell and the carriage. To maintain the carriage in this position, a signal from the photocell, suitably amplified, is fed to the input of the recorder. Displacement of the light spot along the mask increases (decreases) the voltage (imbalance signal) at the recorder input, and the carriage moves in the corresponding direction. Synchronization of the motion of the carriage and light spot was achieved by setting the gain of the amplifier, and by using a recorder that traversed the whole scale in a time much less than the period of oscillation of the system.

3. In response to the change in angular momentum of the circulating superfluid flow, which occurs on switching on the thermal power, the ring which was at rest should go into oscillatory motion with amplitude φ:

$$\varphi_0 = \frac{2\pi\rho_s lR \sum s_0}{\theta f} v_{s0}, \qquad (2)$$

where $l = l_0 + l_1 + l_2$ is the perimeter of the ring, $\theta = 70$ s is the period of oscillation of the system, $f = 0.32$ dyne·cm·rad^{-1} is the torsional constant of the balance, $\Sigma s_0 = 0.1$ cm^2 is the total cross section of the pores in the large volume. It is assumed in deducing Eq. (2) that when the power is turned on, the ring almost instantaneously receives from the liquid an angular momentum

$$\mathscr{L} = \rho_s lRv_{s0} \sum s_0,$$

to which corresponds an energy of the ensuing oscillations of the system $f\varphi^2/2 = \mathscr{L}^2/2I$, where I is the moment of inertia of the system. In addition, it is assumed that the moments of the viscous forces in the capillaries cancel one another.

The dependence of the amplitude of oscillation of the ring, φ_0, on the power Q dissipated in the heater is shown in Fig. 2 for various bath temperatures (from 1.40 to 2.17 K)

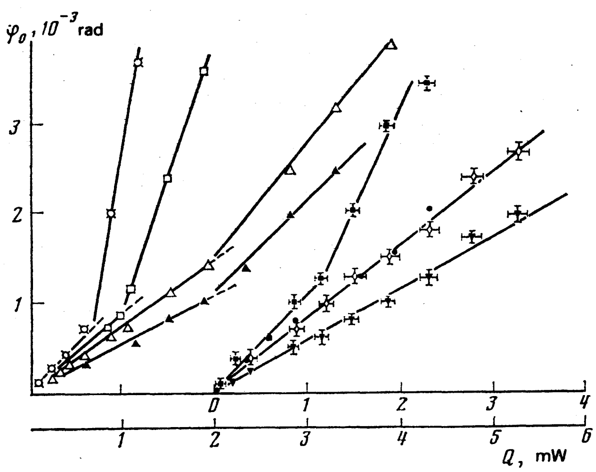

FIG. 2. Dependence of amplitude of oscillations φ_0 on the power Q dissipated in the heater, for various bath temperatures: ○—1.40 K, ×—1.46 K, □—1.55 K, ■—1.74 K, ▲—1.90 K △—1.96 K, ●—2.06,K, ◇—2.10 K, ▼—2.15 K.

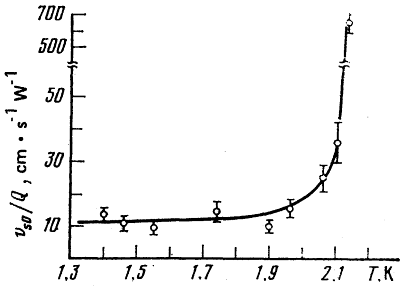

FIG. 3. $v_{s0}/Q = f(T)$. Solid curve—calculated from Eq. (5), points—experimental values.

maintained to an accuracy of $\pm 5\times 10^{-4}$ K. The initial linear parts of the curves correspond to the subcritical flow regime. Past the kink, the supercritical flow regime is evidently realized, for which the ring receives additional momentum due to formation and existence of quantized vortices. The absence of a kink in the curves corresponding to $T > 2.06$ K means that the critical regime is not reached within the range of heating powers used.

On the assumption of convective heat transfer in He II in the stationary regime we have

$$Q = \rho\sigma T\left(v_{n1}\sum s_1 + v_{n2}\sum s_2\right), \qquad (3)$$

where v_{n1} and v_{n2} are the rates of flow of the n component in the capillaries corresponding to the Poiseuille law:

$$v_{n1,\,n2} = \rho\sigma\Delta T s_{1,\,2}/8\pi\eta l_{1,\,2}. \qquad (4)$$

Taking account of Eq. (3) and (4), we obtain for Eq. (1)

$$v_{s0} = \frac{\rho_n}{\rho_s\sigma T\rho} kQ,$$
$$k = (s_1 - s_2) \Big/ \left(\frac{s_1\sum s_1}{l_1} + \frac{s_2\sum s_2}{l_2}\right). \qquad (5)$$

The existence of many pores is taken into account in ξ in the following way:

$$\xi = 1 + \frac{\sum s_0}{l_0}\left(\frac{l_1}{\sum s_1} + \frac{l_2}{\sum s_2}\right).$$

The solid line in Fig. 3 shows the dependence $v_{s0}/Q = f(T)$ calculated from Eq. (5). The coefficient k depends on the possible assumptions about the form of the packing density of the fibers in the capillaries. The value $k = 4$, corresponding to "moderately dense" packing (quadratic) was used to construct the full line in Fig. 3. The maximum packing density corresponds to $k = 4.3$. Values of $v_{s0}/Q = f(T)$ obtained from the experimental results on the initial parts of the curves shown in Fig. 2 (subcritical regime) are represented by the points in Fig. 3. The good agreement between the calculated values and those obtained from the experimental results make it possible to assert that a circulating superfluid flow does actually arise in the ring, bearing out the principle of quantization of circulation, and taking place in the subcritical flow regime in complete accord with theoretical predictions.

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