# Nonlinear transit resonance in semiconductors with strong inelastic scattering of carriers

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A "transit" regime of electron motion in a semiconductor at low temperature, when the dominant scattering mechanism is spontaneous emission of optical phonons of frequency $\omega_0$ by electrons, is possible. In this case the electrons execute in momentum space, under the influence of the electric and magnetic fields, a cyclic motion from the point $\mathbf{p}=0$ to the boundary of the passive region ($\varepsilon=\hbar\omega_0$) along a so-called principal trajectory. We develop in this paper a mathematical method of taking into account the influence of the alternating electric field on the indicated transit motion. In the approximation where the scattering of the electrons in the passive region is weak ($\varepsilon<\hbar\omega_0$) and the penetration into the active region ($\varepsilon>\hbar\omega$) is small we obtain a trajectory equation which is investigated in detail for the case of crossed electric and magnetic fields. It is shown that electron bunching is possible even in the region of weak alternating electric fields.

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## 1. INTRODUCTION

Recent experiments on AgBr at helium temperatures[1] have made popular a kinetic model with pronounced dynamics of electron motion in momentum space; this model was expounded in detail in Refs. 3 and 4. Its gist is that at low temperatures ($kT\ll\hbar\omega_0$) where $\omega_0$ is the frequency of the optical phonon) and under the conditions

$$\tau^+\ll\hat{\tau}\ll\tau^- \tag{1.1}$$

($\tau^+$ is the characteristic time of emission of an optical phonon by an electron in the active region $\varepsilon>\hbar\omega_0$, $\tau^-$ is the characteristic time of electron scattering in the passive region $\varepsilon<\hbar\omega_0$, $\tau$ is the time of flight of the electron from $\mathbf{p}=0$ to the boundary of the passive region) all the electrons are concentrated at the so-called invariant trajectories. These are either the principal trajectory along which the electrons execute cycles of acceleration from the point $\mathbf{p}=0$ to the passive-region boundary, or trajectories that lie entirely in the passive region (in crossed fields $\mathbf{E}_0\perp\mathbf{H}$ the aggregate of the latter is called the "spindle" region). All the kinetic properties of a semiconductors are in fact determined by the dynamics of the electron motion on these invariant trajectories, as well as by their distribution. Connected with the spindle region is the idea of obtaining the inverse distribution function and realizing microwave generation.[4] It was recently confirmed in experiment.[5] Connected with the principal trajectory is the idea of transit resonance investigated in the linear approximation, in the presence of a constant electric field, in Refs. 6–8. It was found that, in the region of the transit frequency $\hat{\omega}=2\pi/\hat{\tau}$ or its harmonic $\omega_l=\hat{\omega}l$ ($l=1, 2,...$), a weak alternating electric field jointly with weak mechanisms (electron scattering and small penetration of the electrons into the active region) simulates the distribution of the electrons on the principal trajectory. This gives rise to transit resonance and makes possible the appearance of negative differential conductivity (NDC) on one of the wings of the resonance curve. This linear theory of transit resonance is based in fact on expression in terms of the small parameter $E/E_0$ (where $E_0$ is the constant electric field and $E$ is the amplitude of the alternating electric field) and is therefore valid in the region $E\ll E_0$.

It was shown in Ref. 9 that turning on a constant magnetic field $\mathbf{H}\perp\mathbf{E}_0$ alters the transit resonance substantially. The magnetic field bends the principal trajectory, and now the alternating electric field changes the trajectory length periodically. As a result a strong mechanism begins to act on the modulation of the electron distribution function, namely scattering of the electron by optical phonons. The resonance is increased by $\xi^{-1}$ times ($\xi=\max[\hat{\tau}/\tau^-, (\tau^+/\hat{\tau})^{2/3}]$) and consequently the nonlinear phenomena should come into play already in the region of weak alternating fields

$$\xi E_0\ll E\ll E_0. \tag{1.2}$$

This circumstance allows us to construct an analytic nonlinear theory of transit resonance. It is shown in the present paper that when the conditions (1.1) are satisfied total bunching of the electrons takes place on the principal trajectory, with increasing $E$, i.e., the electron distribution function is transformed on the principal trajectory into narrow peaks that travel in synchronism with the alternating electric field. This leads to saturation of the transit resonance and to strong generation of the higher harmonics of the current. We note that a search for nonlinear high-frequency phenomena in the transit regime is under way at present also by numerical-modeling methods (see, e.g., Ref. 10).

## 2. ELEMENTARY MODEL

We consider in this section a simple model that explains the bunching in an alternating electric field. We assume that there is no scattering in the passive region and that the electron reaching the boundary of the passive region emits instantaneously an optical phonon and jumps over to the point $\mathbf{p}=0$. We consider only electrons that move along the principal trajectory. Let $\hat{\tau}(t)$ be the time of flight, along the principal trajectory, of an electron that reaches the boundary of the passive region at the instant $t$. We can then naturally

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write the equation

$$t_{n+1}=t_n+\hat{\tau}(t_{n+1}), \tag{2.1}$$

which connects the successive instants of emission of optical phonons by the electron.

Although the arguments that follow are suitable for any case when the length of the principal trajectory varies sinusoidally with time, we consider for the sake of argument the specific case of crossed static and electric fields, $\mathbf{E}_0 \perp \mathbf{H}$. Assume that we have an alternating electric field $\mathbf{E}(t) = \mathbf{E} \times \cos\omega t$. The field $\mathbf{E}$ lies in a plane perpendicular to $\mathbf{H}$ and makes an angle $\chi$ with $\mathbf{E}_0$. Solving the trajectory equations in an approximation linear in the alternating electric field, we can easily show that in the vicinity of the principal resonance, when

$$\omega = \frac{1}{\tau}(2\pi + \Omega), \quad |\Omega| \ll 1, \tag{2.2}$$

the following relations hold:

$$\hat{\tau}(t) = \bar{\tau} + \Delta\hat{\tau}(t), \tag{2.3}$$

$$\Delta\hat{\tau}(t) = C\cos(\omega t - \psi). \tag{2.4}$$

Here

$$\tau = \frac{2p_c}{eE_0}\arcsin\frac{p_0}{2p_c} \tag{2.5}$$

is the time of flight of the electron over the principal trajectory in the absence of an alternating electric field ($p_c = mcE_0/H$, $p_0 = (2m\hbar\omega_0)^{1/2}$), and

$$C = \frac{2\,\mathrm{tg}(\omega_c\bar{\tau}/2)}{|\omega^2 - \omega_c^2|}(\omega^2\sin^2\chi + \omega_c^2\cos^2\chi)^{1/2}\frac{E}{E_0}, \tag{2.6}$$

$$\psi = \mathrm{arctg}\left(\frac{\omega}{\omega_c}\mathrm{tg}\,\chi\right), \quad \omega_c = \frac{eH}{mc}. \tag{2.7}$$

In this case we have a sinusoidal change $\Delta\hat{\tau}(t)$ of the time of flight, whose phase relative to the phase of the alternating field depends substantially on the polarization of this field.

Substituting now (2.4) in (2.1), introducing the dimensionless time $\varphi = \omega t - \psi$, and putting $\alpha = \omega C$ we obtain the equation

$$\varphi_{n+1} = \varphi_n + 2\pi + \Omega + \alpha\cos\varphi_{n+1}. \tag{2.8}$$

This is the one-dimensional transformation for the quantity $\varphi_n$, which is the phase of the alternating electric field at the instant of emission of a phonon by an electron. Naturally, the electrons will be bunched when this transformation has a limit point. Let us investigate this transformation graphically. It is shown in Fig. 1 for two fundamentally different cases: $|\Omega| < \alpha$ and $|\Omega| > \alpha$. The dashed trajectory shows the process of multiple application of the indicated transformation. We see that in the case $|\Omega| < \alpha|$ there exists a limit point

$$\varphi^* = \arccos(-\Omega/\alpha), \tag{2.9}$$

and thus any initial electron distribution in the phases on the principal trajectory will contract into a narrow peak at $\varphi = \varphi^*$. For the case $|\Omega| > \alpha$ there is no limit point, and we have, as seen from Fig. 1b, motion with mixing. Averaging over many electrons we obtain in this case a certain modulation of the electron density on the trajectory, inasmuch, as

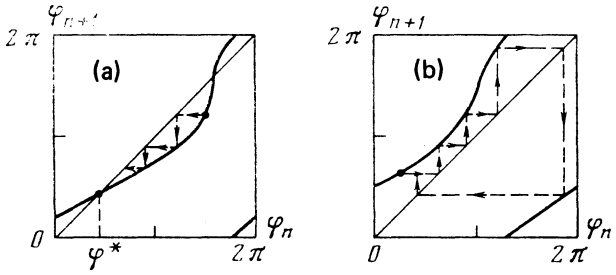


FIG. 1. Graphic representation of the transformation (2.8): a—$|\Omega| < \alpha$, b—$|\Omega| > \alpha$.

seen from the figure, the phase jumps are not uniformly distributed.

It is natural to expect both the scattering of the electrons in the passive region and their penetration into the active region to spoil the bunching. The electron distribution function on the principal trajectory is established as the result of a competition of these mechanisms with the bunching mechanism described above. But to determine this distribution function we need a kinetic analysis, which will be developed in the sections that follow.

## 3. EQUATION FOR THE PRINCIPAL TRAJECTORY

For the analysis of electron bunching on the principal trajectory, the kinetic equation can be considerably simplified and reduced to some simpler equation for the $J(t)$ of the electrons that leave the passive region at a given instant. In addition, the spindle region can be neglected. Since the electrons rotate in this region at the cyclotron frequency, which differs from the transit frequency, and therefore make no contribution to the transit resonance. The redistribution, due to the pulsation of the spindle region in the alternating field, of the electrons on the principal trajectory is negligible, for in final analysis it is determined by the scattering in the passive region.

Once $J(t)$ is obtained, all the kinetic properties of the semiconductor can be studied in the approximation of an infinitely narrow principal trajectory, using $J(t)$ as the source of the electrons for the principal trajectory. To study the nonlinear phenomena in the interval of (1.2) of the alternating fields it suffices to obtain the indicated equation in the approximation linear in the small parameters $E/E_0$, $\hat{\tau}/\tau^-$, and $\tau^+/\hat{\tau}$, the contribution of each of which to the sought equation can be obtained independently. We take into account first the contribution from the change, due to the alternating electric field, of the length of the principal trajectory. If $J(t)$ is the number of electrons that leave the principal trajectory at a given instant of time, and $\hat{\tau}(t)$ is the time that it took these electrons to travel along this trajectory, the integral

$$N = \int_0^{\hat{\tau}(t)} d\tau\, J(t-\tau) \tag{3.1}$$

is proportional to the total number of electrons on the principal trajectory and is therefore a constant. Differentiating (3.1) with respect to $t$ and using the notation of (2.3), we

obtain the equation

$$J(t)=J(t-\hat{\tau}(t))-J(t-\tau(t))\frac{d\hat{\tau}(t)}{dt}$$
$$\approx J(t-\hat{\tau})-\frac{d}{dt}\{\Delta\hat{\tau}(t)J(t-\hat{\tau})\}. \quad (3.2)$$

To obtain the sought equation for $J(t)$ it is necessary to add to (4.2) the contributions from the electron scattering in the passive region and from the electron penetration into the active region.

The contribution from the scattering in the passive region is easily obtained in the limit of an infinitely narrow principal trajectory, as was done in Ref. 9. This leads to the appearance of a certain integral term that takes into account the departure and arrival of the electrons from and to the principal trajectory, since each electron that leaves the principal trajectory as a result of scattering returns to it with a time of the order of $\hat{\tau}$. However, since we confine ourselves to the resonant region, where the role of the indicated scattering reduces only to a broadening of the resonance curves, we approximate the aforementioned term in the simplest manner by adding to (3.2) a relaxation term of the form $-\beta(J-1)$, where $\beta=\hat{\tau}/\tau^-$ and the following normalization condition is assumed:

$$\int_0^{\hat{\tau}} d\tau\, J(t-\tau)=\hat{\tau}. \quad (3.3)$$

Allowance for the penetration of the electrons into the active region is a more complicated matter, since the swelling of the principal trajectory calls for setting up a certain integral equation for the transverse profile of the principal trajectory. The procedure for this is given in the Appendix, where the trajectory equation is obtained also in final form:

$$J(t)=\left\{J\left[1-\frac{d\hat{\tau}(t)}{dt}\right]-\beta[J-1]+a\frac{dJ}{dt}+b\frac{d^2J}{dt^2}\right\}\Big|_{t-\hat{\tau}(t)}. \quad (3.4)$$

The coefficients $a$ and $b$ are proportional to the small parameter $(\tau^+/\hat{\tau})^{2/3}$. The last two terms have a simple physical meaning. The term containing the first derivative is analogous to the second term of (4.2) and takes into account the increase of the transit time as a result of penetration of the electrons into the active region. The small dephasing that results from the fluctuation of the penetration depth is taken into account by the diffusion term, which contains the second derivative.

It will be shown below that in the vicinity of the "transit" resonance the characteristic behavior of the function $J(t)$ is determined not by the small parameters $\Omega$, $E/E_0$, $\tau^+/\hat{\tau}$, and $\hat{\tau}/\tau^-$ themselves, but by their ratios. Therefore an expression for the distribution function in terms of $J(t)$, and also expressions for the different kinetic coefficients, can be obtained in the zeroth approximation in all the indicated parameters, i.e., in the approximation with only one infinitely narrow principal trajectory defined in the static electric and static magnetic field. In this approximation, the electron distribution function normalized to unity takes the form of a wave traveling on the principal trajectory, i.e.,

$$f(\mathbf{p},t)=J\left(t-\frac{\Phi}{\omega_c}\right)\delta(p_z)\delta[p_x^2+(p_y+p_c)^2-p_c^2]\frac{1}{\hat{\tau}}, \quad (3.5)$$

where

$$\Phi=\operatorname{arctg}\frac{p_x}{p_y+p_c}, \quad E_0\|x, \quad H\|z. \quad (3.6)$$

Using (3.5) it is eady to express any kinetic characteristic of the electron system in terms of the function $J(t)$. For example, the expression for the current in the case of steady-state periodic motion takes the form

$$j_x=\frac{nep_c}{m\hat{\tau}}\int_0^{\hat{\tau}} d\tau J(t-\tau)\sin\omega_c\tau, \quad (3.7)$$

$$j_y=\frac{nep_c}{m\hat{\tau}}\int_0^{\hat{\tau}} d\tau J(t-\tau)(\cos\omega_c\tau-1). \quad (3.8)$$

The function $J(\Phi/\omega_c)$ itself is the instantaneous distribution of the electrons on the principal trajectory.

## 4. DISTRIBUTION OF ELECTRONS

Equation (3.4) can be solved in the general case only numerically. A qualitative picture of the behavior of $J(t)$ can, however, be easily obtained by considering limiting cases.

We consider first the case of instantaneous emission of an optical phonon by an electron, i.e., we put $a=b=0$ in (3.4). Then, using the substitution $\varphi=\omega t-\psi$ and introducing the notation $J_n=J\left(\frac{\varphi_n+\psi}{\omega}\right)$, we can represent Eq. (3.4) in the form of the following two-dimensional transformation:

$$J_{n+1}=J_n(1+\alpha\sin\varphi_{n+1}-\beta)+\beta, \quad (4.1)$$
$$\varphi_{n+1}=\varphi_n+2\pi+\Omega+\alpha\cos\varphi_{n+1}. \quad (4.2)$$

The second of these equations coincides, naturally, with Eq. (2.8) used in the analysis of the elementary model. Since we restrict the phase to the interval from 0 to $2\pi$, the term $2\pi$ in the right-hand side of (4.2) will be disregarded. It is convenient to investigate the solution of the foregoing system graphically, by drawing the phase portrait in the $(J,\varphi)$ plane. Since each discrete transformation causes only a small change of the variables (it is proportional to the small parameters $\alpha$, $\beta$, and $\Omega$), we can replace the discrete transformation by a continuous one, replacing the system (4.1)–(4.2) by the system of differential equations

$$\frac{dJ}{dn}=(\alpha\sin\varphi-\beta)J+\beta, \quad \frac{d\varphi}{dn}=\Omega+\alpha\cos\varphi, \quad (4.3)$$

whose phase portrait is constructed by standard methods. Such phase portraits are shown in Fig. 2. Figure 2a shows the case $|\Omega|>\alpha$, when the system has no singular points, and any initial distribution (shown in the figure by the dashed line) moves during a time $n$ along the trajectories and hugs the periodic trajectory shown by the thick line in the figure. Thus, a periodically modulated distribution function is established in the course of time in the case $|\Omega|>\alpha$.

Two qualitatively different phase portraits are possible in the case $|\Omega|<\alpha$, depending on the ratio $\alpha/(\Omega^2+\beta^2)^{1/2}$. Figure 2b shows the case $\alpha<(\Omega^2+\beta^2)^{1/2}$, when there are

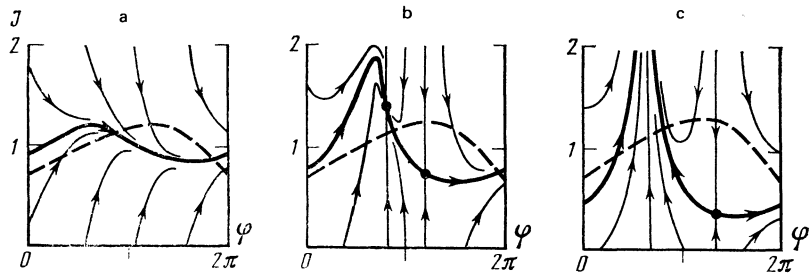


FIG. 2. Phase portraits of the system of equations $a$—$|\Omega| > \alpha$, b—$(\Omega^2+\beta^2)^{1/2} > \alpha > |\Omega|$, c—$\alpha > (\Omega^2+\beta^2)^{1/2}$.

two singular points in the region $J>0$, one a node and the other a saddle. It can be seen from the figure that in this case any initial distribution (dashed curve) contracts to one of the separatrices (thick curve) of the saddle point. Comparing this phase portrait with Fig. 2a we see that a tendency to bunching sets in with increasing $\alpha$. Figure 2c shows the phase portrait for the case $\alpha > (\Omega^2+\beta^2)^{1/2}$, when both singular points become saddles, one of which drops into the region $J<0$. It can be deduced from the figure that any initial distribution contracts to the separatrix of the saddle located in the region $J>0$. This separatrix is not bounded. Consequently, at $^*\alpha > (\Omega^2+\beta^2)^{1/2}$ the scattering in the passive region cannot smear out the singularity in the distribution function of the bunched electrons. The singularity is always smeared as a result of the finite penetration of the electrons into the active region. This can be verified by considering the other limiting case, namely, neglecting the electron scattering in the passive region. Putting $\beta = 0$, we find from Eq. (3.4) that the function $J(t)$, which is periodic and has the period of the field, must satisfy the differential equation

$$\frac{d}{dt}\left\{aJ+b\frac{dJ}{dt}-C\cos(\omega t-\psi)J-\frac{\Omega}{\omega}J\right\}=0. \tag{4.4}$$

This equation is solved in quadratures. But we shall not write down these relatively straightforward but cumbersome equations, but present some characteristic curves in Fig. 3. It is seen from this figure that at $\Omega = 0$ there is a relatively narrow but closed peak at $\varphi = \pi/2$. When the detuning is taken into account, the peak broadens and shifts to $\pi$ or to 0, depending on the sign of $\Omega$.

To conclude the analysis, let us estimate the dimensions of the peak for the case of exact resonance $\Omega = 0$. Confining ourselves to a small region $t_0 = \omega^{-1}(\psi + \pi/2)$ in the vicinity of the peak and expanding $\cos(\omega t - \psi)$ in a Taylor series, we

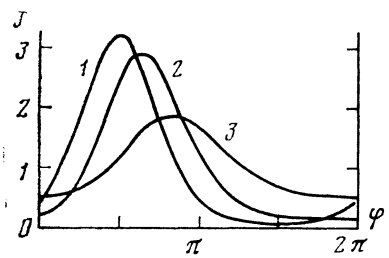


FIG. 3. Plots of the function $J(\varphi/\omega)$ for dominant penetration of the electrons into the active region: $\alpha = 3(\tau^+/\hat{\tau})^{2/3}$; 1—$\Omega = 0$, 2—$\Omega = 2(\tau^+/\hat{\tau})^{2/3}$; 3—$\Omega = 4(\tau^+/\hat{\tau})^{2/3}$.

obtain from (3.4), in the case when electron penetration into the active region dominates ($\beta = 0$)

$$J(t) \sim \exp\left[-\frac{\alpha}{2b}(t-t_0)^2\right] \tag{4.5}$$

and when scattering in the passive region dominates ($a = b = c$)

$$J(t) \sim |t-t_0|^{\beta/\alpha-1}. \tag{4.6}$$

For the characteristic width of the peak we have

$$\Delta(\omega t) = \begin{cases} \left(\frac{\omega^2 b}{\alpha}\right)^{1/2} \sim \left[\left(\frac{\tau^+}{\hat{\tau}}\right)^{2/3}\frac{E_0}{E}\right]^{1/2}, & \text{when } \beta=0; \\ \left(\frac{\beta}{\alpha}\right)^{1/2} \sim \left[\frac{\hat{\tau}}{\tau^-}\frac{E_0}{E}\right]^{1/2}, & \text{when } a=b=0. \end{cases} \tag{4.7}$$

Consequently, with increasing amplitude of the alternating electric field the electrons become bunched, and the scatter of the electron phase decreases in inverse proportion to the square root of the indicated amplitude.

## 5. DISCUSSION OF RESULTS

It was shown in the preceding sections that in the region of weak alternating electric fields total bunching of the electrons can be achieved. The electron distribution function is transformed in this case into a sequence of narrow bunches that travel along the principal trajectory in momentum space.

The manifestation of the investigated nonlinear phenomena in resonant absorption of an electromagnetic wave is quite trivial. An increase in the amplitude of the alternating electric field limits the resonance (since the amplitude of the alternating current cannot exceed $nep_0/2$) and broadens it. The NDC present in the linear region moves away from the resonance with increasing $E$ and goes out of the region of the considered approximation. It follows from (3.7) and (3.8) that with increasing $E$ the current tends to assume a certain limiting sawtooth shape. From the position $t_0 = (\psi + \pi/2)/\omega$ of the peak at resonance, as well as from the fact that $|\psi| \leqslant \pi/2$ according to (2.7), it follows that the phase difference between the alternating electric field and the first harmonic of the current cannot exceed $\pi/2$, i.e., no NDC can exist at the transit resonance ($\Omega = 0$). The clearest manifestation of the sawtooth shape of the current could apparently be intense generation of higher harmonics of the current.

We note in conclusion that at the present time, in our opinion, the most suitable subject in which to seek this

bunching of electrons is AgBr. At helium temperatures, according to Ref. 1, the time of electron scattering in the passive region is of the order of $\tau^- \sim 5\times 10^{-11}$ sec, $m = 0.3m_e$, and $\hbar\omega_0 \sim 17$ meV. At $E_0 = 150$ V/cm and $H = 550$ Oe the first transit resonance should be observed at the frequency $\nu \sim 30$ GHz. The time of flight already becomes shorter than the time of scattering in the passive region. According to Ref. 1 the value of $\tau^-/\tau^+$ should amount to two orders of magnitude.

## APPENDIX

We derive here the trajectory equation under the assumption that the electric and magnetic fields employed are static, and that there is no scattering of the electrons in the passive region.

Let the electron scattering in the active region be characterized by the probability of emission of an optical phonon

$$W(\mathbf{p}', \mathbf{p}) = \frac{1}{\tau^+ g(\hbar\omega_0)}\delta[\varepsilon(\mathbf{p}') - \varepsilon(\mathbf{p}) - \hbar\omega_0], \tag{A.1}$$

and in view of the small penetration of the electrons into the active region we can assume $\tau^+$ to be constant; $g(\varepsilon)$ is the state density of the electron.

The initial kinetic equation is of the form

$$\frac{\partial f}{\partial t} + \mathbf{F}\frac{\partial f}{\partial \mathbf{p}} = \int d^3\mathbf{p}' W(\mathbf{p}', \mathbf{p}) f^+(\mathbf{p}', t), \tag{A.2}$$

$$\frac{\partial f^+}{\partial t} + \mathbf{F}\frac{\partial f^+}{\partial \mathbf{p}} + G(\mathbf{p}) f^+ = 0. \tag{A.3}$$

The plus sign indicates here that the distribution function pertains to the active region ($\varepsilon(\mathbf{p}) > \hbar\omega_0$) of momentum space, $\mathbf{F}$ is the force due to the electric and magnetic fields, and

$$G(\mathbf{p}) = \int d^3\mathbf{p}' W(\mathbf{p}, \mathbf{p}') = \frac{4\pi g(\varepsilon(\mathbf{p}) - \hbar\omega_0)}{\tau^+ g(\hbar\omega_0)}. \tag{A.4}$$

The kinetic equation can be simplifed using variables connected with the electron trajectories in momentum space. Let

$$\mathbf{p} = \mathbf{p}(\mathbf{q}_\perp, q_\parallel + t) \tag{A.5}$$

be the solutions of the equations of motion $\dot{\mathbf{p}} = \mathbf{F}$. The two integration constants $\mathbf{q}_\perp$ characterize the electron trajectory (we assume that $\mathbf{q}_\perp = 0$ corresponds to the principal trajectory), and the third constant $q_\parallel$ is chosen as an additive time constant.

We use now expression (A.5) as a transformation to new variables $\mathbf{q}_\perp$ and $q_\parallel$. In terms of these variables, Eq. (A.3) can be easily solved and the solution can be represented in the form

$$f^+(\mathbf{p}(\mathbf{q}_\perp, \mathbf{q}_\parallel + t), t) = \Phi(\mathbf{q}_\perp, \mathbf{q}_\parallel)\exp\left\{-\int_0^{t+q_\parallel - \hat{\tau}(q_\perp)} d\tau\, G(\mathbf{p}(\mathbf{q}_\perp, \tau + \hat{\tau}(\mathbf{q}_\perp)))\right\}, \tag{A.6}$$

where we have used the time $\hat{\tau}(\mathbf{q}_\perp)$ of flight of the electron over the $\mathbf{q}_\perp$-th trajectory, defined in accord with the equation

$$\varepsilon(\mathbf{p}(\mathbf{q}_\perp, \hat{\tau}(\mathbf{q}_\perp))) = \hbar\omega_0. \tag{A.7}$$

Substituting next (A.6) in the right-hand side of (A.3), integrating it, and matching the obtained distribution function to expression (A.6) on the boundary of the active region, we obtain an integral equation for the function $\Phi(\mathbf{q}_\perp, q_\parallel)$. This function defines the electron distribution function on the boundary of the active medium. Indeed, it can be seen from (A.6) that

$$\Phi(\mathbf{q}_\perp, q_\parallel) = f^+(\mathbf{p}(\mathbf{q}_\perp, \hat{\tau}(\mathbf{q}_\perp)), -q_\parallel + \hat{\tau}(\mathbf{q}_\perp)) \equiv f^+(\mathbf{p}, -q_\parallel + \hat{\tau}(\hat{\mathbf{q}}_\perp))\,|_{\varepsilon(\mathbf{p}) = \hbar\omega_0}. \tag{A.8}$$

Consequently, the integral equation for $\Phi(\mathbf{q}_\perp, q_\parallel)$ is an integral equation for the function $f(\mathbf{p}, t)|_{\varepsilon(\mathbf{p}) = \hbar\omega_0}$, that determines the transverse profile of the principal trajectory. It can be represented in the form

$$f(\mathbf{p}(\mathbf{q}_\perp, \hat{\tau}(\mathbf{q}_\perp)), t) = \int dq_\parallel \int dt' \int d^2\mathbf{q}_\perp' D(\mathbf{q}_\perp') W(\mathbf{p}(\mathbf{q}_\perp', \hat{\tau}(\mathbf{q}_\perp') + t'), \mathbf{p}(\mathbf{q}_\perp, q_\parallel))\exp\left\{-\int_0^{t'} d\tau G(\mathbf{p}(\mathbf{q}_\perp', \tau + \hat{\tau}(\mathbf{q}_\perp')))\right\} f(\mathbf{p}(\mathbf{q}_\perp', \hat{\tau}(\mathbf{q}_\perp')), t - \hat{\tau}(\mathbf{q}_\perp) + q_\parallel - t'). \tag{A.9}$$

Here $D(\mathbf{q}_\perp)$ is the Jacobian of the transformation from the variables $\mathbf{p}$ to the variables $\mathbf{q}_\perp$ and $q_\parallel$. We have left out the limits of integration in (A.9), since they are determined in fact by an exponential that decreases rapidly with increasing $t'$ (it is assumed that $t' > 0$) and by the $\delta$ function contained in the definition of $W(\mathbf{p}, \mathbf{p}')$.

The integral equation (A.9) is an exact consequence of the initial kinetic equation (A.2)–(A.3). It can be greatly simplified because of the presence of the small parameter $\tau^+/\hat{\tau}$ that enters, according to (A.4), in the integral expression in the exponential. Indeed, owing to this parameter the exponential decreases rapidly with increasing $t'$, i.e., the main contribution to the integral is made by the region of small $t'$. But then it follows from the presence of the $\delta$ function in $W(\mathbf{p}, \mathbf{p}')$ that the kernel of the equation differs significantly from zero only in the region of small $\mathbf{p}(\mathbf{q}_\perp, q_\parallel)$. As a result the function $f(\mathbf{p}(\mathbf{q}_\perp, \hat{\tau}(\mathbf{q}_\perp)), t)$ has a sharply pronounced maximum in the vicinity of $\mathbf{q}_\perp = 0$, and it is therefore convenient to solve (A.9) in the moment approximation. The larger the moment $\mathbf{q}_\perp$ of the function $f(\mathbf{p}, t)$, the higher the power of the small parameter $\tau^+/\hat{\tau}$ in it. Consequently, cutting off the system of moment equations at a definite moment, we can obtain a solution with arbitrary accuracy relative to the indicated parameter. However, since the initial equation (A.9) is homogeneous, this procedure does not yield a solution, but leads to a certain equation for the zeroth moment:

$$J(t) = \int d^2\mathbf{q}_\perp D(\mathbf{q}_\perp) f(\mathbf{p}(\mathbf{q}_\perp, \hat{\tau}(\mathbf{q}_\perp)), t) \equiv \int d\mathbf{S}_p \frac{d\mathbf{p}}{dt} f(\mathbf{p}, t), \tag{A.10}$$

which is the total flux of the electrons through the boundary of the active region at the given instant of time $t$.

We obtain such an equation in the lowest nonvanishing approximation in $\tau^+/\hat{\tau}$ (in the case of a parabolic bottom of the electron band, the leading term of the expansion is $(\tau^+/\hat{\tau})^{2/3}$). To this end we integrate both sides of (A.9) with respect to $D(\mathbf{q}_\perp)d^2\mathbf{q}_\perp$, make in the right-hand side the change of variables in accordance with the formula

$$u=\int_0^{t'} d\tau G(\mathbf{p}(\mathbf{q}_\perp',\tau+\hat{\tau}(\mathbf{q}_\perp'))) \tag{A.11}$$

and transform from the variables $\mathbf{q}_\perp$ and $q_\parallel$ to the variables $\mathbf{p}$ (in accord with (A.5 at $t=0$). We obtain then the equation

$$J(t)=\int d^2\mathbf{q}_\perp' D(\mathbf{q}_\perp') \int d^3\mathbf{p} \int_0^\infty du e^{-u} \frac{1}{4\pi g(\varepsilon(\mathbf{p}))} \delta\{\varepsilon(\mathbf{p}(\mathbf{q}_\perp',\hat{\tau}(\mathbf{q}_\perp')))-\hbar\omega_0-\varepsilon(\mathbf{p})\} f(\mathbf{p}(\mathbf{q}_\perp',\hat{\tau}(\mathbf{q}_\perp')),t-\hat{\tau}(\mathbf{q}_\perp)+q_\parallel-t'). \tag{A.12}$$

To obtain the approximation indicated above it suffices to retain in (A.11) the zeroth order of the expansion in $\mathbf{q}_\perp'$; this leads to

$$t'=\eta u^{2/3}, \quad \eta=\left[\frac{8\pi}{3\tau^+(\hbar\omega_0)^{1/2}}\left(\frac{d\varepsilon}{dt}\Big|_{\substack{t=\hat{\tau}\\ \mathbf{q}=0}}\right)^{1/2}\right]^{-2/3} \sim \hat{\tau}\left(\frac{\tau^+}{\tau}\right)^{2/3}. \tag{A.13}$$

The integration with respect to $d^2\mathbf{q}_\perp'$ in the right-hand side of (A.2) includes then only the function $f$ and leads to the quantity $J(T')$ whose argument (if we confine ourselves to expansion up to terms quaratic in $\mathbf{p}$ inclusive) can be represented in the form

$$t_1=t-\hat{\tau}+\mathbf{Ap}+\mathbf{p}B\mathbf{p}-\eta u^{2/3}. \tag{A.14}$$

Expanding now the integrand $J(t')$ in the vicinity of $t-\hat{\tau}$ in a Taylor series, we obtain the sought increment to the complete equation (3.4) for $J(t)$, where

$$a=\Gamma\left(\frac{5}{3}\right)\eta\left\{\frac{2m}{3}\mathrm{Sp}\,B\,\frac{d\varepsilon}{dt}\Big|_{\substack{t=\hat{\tau}\\ \mathbf{q}=0}}-1\right\}, \tag{A.15}$$

$$b=\Gamma\left(\frac{5}{3}\right)\eta\frac{m}{3}|\mathbf{A}|^2\frac{d\varepsilon}{dt}\Big|_{\substack{t=\hat{\tau}\\ \mathbf{q}=0}}, \tag{A.16}$$

with $\Gamma(5/3)\approx 0.903$.

In the case of crossed electric and magnetic fields of interest to us

$$a=-a_0\left(\frac{\tau^+}{\tau_0}\right)^{2/3}\tau_0\frac{1}{1-(p_0/2p_c)^2}, \tag{A.17}$$

$$b=a_0\left(\frac{\tau^+}{\tau_0}\right)^{2/3}\tau_0^2, \tag{A.18}$$

where

$$a_0=\frac{1}{12}\left(\frac{3}{\pi\sqrt{2}}\right)^{2/3}\Gamma\left(\frac{5}{3}\right)\approx 0.056; \tag{A.19}$$

$$\tau_0=\frac{p_0}{eE_0}\left(1-\left(\frac{p_0}{2p_c}\right)^2\right)^{-1/2} \tag{A.20}$$

is a characteristic time of the order of the time of flight $\hat{\tau}$ along the principal trajectory.

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