Rossby solitons: Stability, collisions, asymmetry and generation by flows with velocity shear

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(Submitted 1 September 1982)
Zh. Eksp. Teor. Fiz. **84**, 1357–1372 (April 1983)

In continuation of previous work [S. V. Antipov *et al.*, Sov. Phys. JETP **55**, 85 (1982)] the principal properties of Rossby solitons in shallow rotating fluids of constant depth have been investigated. It was shown for the first time that: 1) of all geostrophic vortices (with proper rotation frequency less than that of the system) only Rossby solitons are stable, and the other vortices are rapidly transformed into zonal flows; 2) the drift velocity of Rossby solitons increases with their amplitude; 3) collisions of Rossby solitons of different amplitudes (which invariably drift in the same direction) are inelastic; at sufficiently high velocities of approach the vortices merge into one Rossby soliton, the lifetime of which is determined by the viscosity of the medium; 4) as predicted theoretically [V. I. Petviashvili, JETP Lett. **32**, 619 (1980)], Rossby solitons are observed only in the form of solitary anticyclones, i.e., solitary fluid elevations (on the other hand, cyclones are fluid depressions which decay rapidly); consequently it is not possible to create a Rossby soliton in the form of a solitary cyclone-anticyclone vortex pair [V. D. Larichev and G. M. Reznik, Doklady Akad. Nauk SSSR **231**, 1077 (1976)] (for vortex sizes exceeding the Rossby radius): in place of the expected vortex pair one obtains a solitary anticyclone of larger size; 5) countercurrents in the geometry under consideration exhibit an instability (a variation on the classical Kelvin-Helmholtz instability). The nonlinear regime of this instability is characterized by a marked asymmetry: the excitation thresholds of vortices of opposite vorticity are approximately the same, whereas anticyclones (where the vorticity is antiparallel to the angular velocity vector of the global rotation of the system) have significantly larger amplitudes and sizes than cyclones; anticyclones are Rossby solitons. The probable relation between the observed asymmetry to the nature of the vortex in the Big Red Spot of Jupiter is considered.

PACS numbers: 47.30. + s, 47.20. + m

I. INTRODUCTION

The work reported in the present paper had two purposes. First of all, after the experiments[1] which first obtained and investigated Rossby solitons in a rotating shallow fluid of constant depth, which were predicted by theory,[2] it became necessary to clarify a number of questions of principle regarding the most general properties of Rossby solitons. The answers to these questions (briefly enumerated as items 1) through 4) in the Abstract) comprise the first part of the paper. Second, both because of the intrinsic logic of the investigation, and because of the obvious applications to the physics of planetary atmospheres, it became necessary to study properties of Rossby solitons which are not isolated (as in Refs. 1 and 2), but exist against the background of zonal flows with velocity shear. The experiments that were carried out, in which a Rossby soliton appears in a completely different situation, namely as a consequence of a Kelvin-Helmholtz instability, are described in the second half of this paper. This part of the experiment was influenced by the quite effective experiments of Obukhov and collaboratores,[3–5] in which the instability of zonal countercurrent flows was investigated in a different geometry, and where beautiful vortices were obtained, reminiscent of the cyclones of Antarctica. The geometry of our experiments has a very specific nature, allowing one to compare the results obtained here with the properties of the famous vortex in the Big Red Spot of Jupiter, and make some assertions on its putative nature.

The paper also considers some methodological questions, to wit: why are such "genuine" vortices, which effectivly drag with them fluid particles, called not simply (and not only) vortices but also solitary waves or solitons; why are solitary waves which are not conserved in mutual collisions called solitons; why do vortices of both signs (both cyclones and anticyclones) drift around the rotation axis of the system in the same direction; and other similar questions.

2. THE EXPERIMENTAL SETUPS AND CONDITIONS

Two experimental installations have been used in this work. The *first* (Fig. 1) had already been used in the previous experiments.[1] It was based on a small paraboloid of 28 cm maximum diameter rotating around the vertical symmetry axis with an angular velocity (the rotation period was $T_0 = 580$ ms) at which the fluid spreads along the surface of the paraboloid in an even layer of approximately constant depth. The thickness H_0 of the fluid layer varied between 3 and 10 mm. As a working fluid we have used either water or a solution of nickel sulfate in water (which had a viscosity approximately three times larger than water). In order to visualize the flows of the fluid we have used white test particles floating on the surface of the fluid on the background of

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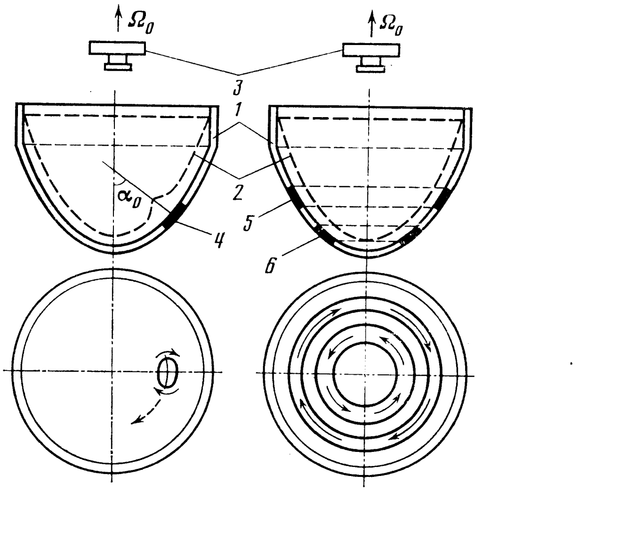

FIG. 1. Schematic drawing of the experimental setups[1,12] for the excitation of single Rossby solitons (left) and for the generation of zonal flows of the fluid (right): 1—vessel with parabolic profile of the bottom, 2—surface of the fluid (water), which spreads along the parabolic bottom when rotating, 3—photographic camera rotating along with the vessel, 4—rotating "priming disk," 5 and 6—rotating rings producing countercurrent flows with velocity shear. The paraboloid rotates around a vertical axis counterclockwise with angular velocity Ω_0. In the top views the solid arrows represent the anticyclonal direction of rotation of the priming disk and the direction of flow of the fluid for anticyclonal shear, and the dotted arrow shows the drift direction of the Rossby soliton (the soliton fall behind the global rotation of the fluid). The angle α_0 is the angle between the rotation axis of the vessel and the normal to the fluid surface at the active point.

the black bottom of the vessel, and a photographic camera rotating along with the paraboloid. To excite a solitary Rossby vortex at some working point (at a distance of 10 cm from the rotation axis) a thin metallic "priming disk," the diameter of which could be varied between 1.5 and 5 cm, was situated near the bottom of the fluid. The disk was brought for 2 to 3 seconds into rotation around the normal to the paraboloid at the working point and it gradually entrained the fluid on top of it and in its neighborhood into a local rotation. In the soliton scattering experiments the rotation velocity of the "priming disk" was varied in time, and it was thus possible to generate different Rossby vortices moving with different velocities one behind the other. In the experiments where bound pairs of vortices (cyclone-anticyclone pairs) were created, two priming disks situated next to each other at a distance of 5 mm along the same "meridian" symmetrically with respect to the point with $r = 10$ cm were used in place of the single disk. The rotation velocity, the duration of the rotation and the instant of switching on were separately adjusted for each disk. The directions of rotation could be pairwise changed: for instance in one experiment the interior disk rotated in the anticyclonal direction and the exterior disk (the one farther from the axis) in the cyclonal direction; in a second experiment the interior disk rotated in the cyclonal direction and the exterior disk in the anticyclonal direction.

The second installation (Fig. 1) had at its basis the same paraboloid as the first one, but now two troughs were machined into the central part of the paraboloid in which two rings of width 3 cm each could slide, rotating around the vertical symmetry axis. The distances between the rings and the walls of the troughs were 1 mm. The distance between the rings was also 3 cm, and its middle was at $r = 10$ cm from the rotation axis. The rings rotated relative to the paraboloid in opposite directions with the same angular velocity and created in the reference frame of the paraboloid counterflowing flows with velocity shear, the direction of which could be reversed: for cyclonal shear the outer ring was rotating faster than the paraboloid, and the inner one was rotating slower; for anticyclonal shear the outer ring lagged behind, whereas the interior one advanced relative to the rotation of the paraboloid. The magnitude of the shear angular velocity Ω_{shear} could be smoothly varied from zero to a value equal to the angular velocity of the main rotation $\Omega_0 = 2\pi/T_0$. The depth of the fluid in these experiments ranged between $H_0 = 2$–20 mm. Observations have shown that as long as the magnitude of the velocity shear did not exceed a certain threshold, a laminar flow established itself in the fluid with two sharp jumps in the velocity in the region of the gaps between the rings and the adjacent portions of the wall of the vessel. The width of these jumps which were separated by a distance of 3 cm was approximately equal to the depth of the fluid H_0, i.e., was of the order of several millimeters. Accordingly, the photographs of the traces of the test particles represent arcs of concentric circles around the rotation axis. In this regime the layers of fluid which are adjacent to the rings move approximately with the velocity of the appropriate ring. When the magnitude of the velocity shear exceeded a certain value, the character of the motion of the fluid changed radically, and quite interesting phenomena come into play, which we describe below.

3. EXPERIMENTAL DATA

A. Previous results

Before reporting the experimental data obtained in the present work we would like to remind the reader that Rossby solitons, which are solitary "shallow water" gravity waves in the model of a rotating planet, are characterized by the following characteristic traits[1]:

1) A Rossby soliton is a solitary anticyclonal vortex, rotating around its proper vertical axis opposite to the global rotation of the medium. There is approximate equilibrium between the Coriolis force directed towards the center of the vortex and creating an elevation of the fluid, and the gradient of the hydrostatic pressure (this is the so-called geostrophic equilibrium). The frequency of the proper rotation of the vortex is small compared to the frequency of the global rotation, and therefore the centrifugal force from the proper rotation is much smaller than the Coriolis force.

2) The characteristic size (diameter) of the soliton, $2a$, defined as the distance between the opposite points of its profile at which the linear velocity of the proper rotation is maximal (see Fig. 2), is connected to the Rossby radius r_R by the relation

$$2a \gtrsim 2.5 r_R, \tag{1}$$

$$r_R = (g^* H_0)^{1/2} / f_0, \tag{2}$$

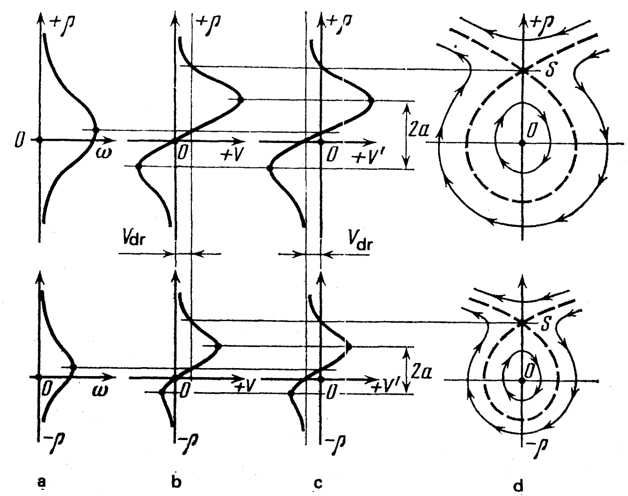

FIG. 2. The spatial profiles of the angular velocity ω of the rotation of a vortex (a), the linear velocity V of the particles in the reference frame of the fluid (b), the linear velocity V' of particles in the reference frame of the vortex (c), and the flow lines in the reference frame of the vortex (d) for a single Rossby soliton. In all pictures the vertical axis represents the distance ρ from the center of the vortex, the signs plus and minus correspond to displacements to the periphery of the paraboloid and the rotation axis, respectively. The linear velocity is chosen with a plus sign for motions in the direction of the drift of the vortex (left to right) and with a minus sign for motion in the opposite direction. The lower series of illustrations corresponds to the regime with a smaller region of captured particles, the region having contracted on account of viscosity. The capture region is bounded by the loop of the separatrix, denoted by the dotted line passing through the point S.

where H_0 is the depth of the fluid, $g^* = g/\cos\alpha$, g is the acceleration of gravity, $f_0 = 2\Omega_0\cos\alpha$ is the Coriolis parameter, and α is the angle between the vector Ω_0 of the angular velocity of the global rotation and the normal vector to the surface of the fluid at the given point. In the conditions of our experiment, $r_R = 2.1$ cm for $H_0 = 5$ mm.

3) A Rossby soliton drifts around the rotation axis of the system against the direction of the global motion of the fluid with a velocity V_{dr} which is close to the Rossby velocity V_R (see below for details). In the parabolic model (Fig. 1) the Rossby velocity is defined by

$$V_R = H_0\Omega_0 \sin\alpha_0. \tag{3}$$

For example, in the typical conditions of our experiments $H_0 = 5$ mm, $\Omega_0 = 11\ s^{-1}$, $\sin\alpha_0 = 0.8$, and $V_R = 4.5$ cm/s.

4) If the amplitude of the soliton is sufficiently large, so that the linear velocity of the proper rotation (which is maximal on the velocity profile of the vortex) exceeds its drift velocity, the soliton will entrain all fluid particles except those which are at the periphery of the vortex and have a velocity of rotation smaller than V_{dr}. In the conditions of our experiments entrainment of particles occurs definitely if the rise ΔH of the vortex above the level of the surrounding fluid satisfies the (sufficient) condition $\Delta H \gtrsim 0.2\,H_0$.

In order to illustrate what was said, the lower row of Fig. 2 shows the same items as the upper row, but some time later. Figure 2 shows the profiles of the angular velocity of the proper rotation of the vortex (position a), of the linear velocity of rotation of the vortex at a section of the meridional plane perpendicular to its drift direction (position b), and the profile of the velocity of the motion of the fluid in the reference frame comoving with the vortex itself (position c). A positive sign of the velocity at the profiles b and c corresponds to motion in the direction of the drift of the vortex, a negative sign corresponds to motion in the opposite direction. Figure 2d shows the streamlines in the reference frame of the vortex. The curve with the point S where the velocity vanishes in the frame of the vortex is the separatrix; inside this curve is the region of particles captured by the vortex and outside it is the region of passer-by particles; at the point S the velocity of rotation is equal to the drift velocity of the vortex. As the vortex moves and its rotation slows down on account of viscosity, the diameter of the region of captured particles decreases, as shown by the comparison of the lower row in Fig. 2 with the upper row.

B. The stability of Rossby solitons

After the detection of Rossby solitons[1] there naturally arose the question: are they just one of the "equally valid" particular solutions of the corresponding nonlinear equation, or are they singled-out physically by something and should be observed in preference over other geostrophic vortices? To answer this question we have carried out the following experiment. By means of priming disks of different diameters we have excited anticyclonal geostrophic vortices of different sizes, and have compared their parameters (size, drift velocity, lifetime) with the corresponding characteristics of the Rossby solitons. (Insofar as cyclonal vortices are concerned, it was already established in Ref. 1 that they are unstable and decay rapidly, see Subsection E below.) The lifetime τ of the vortex was defined as the time interval from the formation of the vortex (its separation from the priming disk) to the time when the vortex lines open up and the tracks of the test particles stop forming closed trajectories around the axis of the vortex. One of the mechanisms of this phenomenon is illustrated by Fig. 2: if the vortex is stable, the slowing down of the rotation of the vortex to a state in which the maximal rotation velocity falls below the drift velocity, $V_{rot} \leqslant V_{dr}$, leads to the disappearance of the capture region (Fig. 2). We call this mechanism laminar (for an explanation of the definition of the lifetime of a vortex for this case see Fig. 3). However, the experiments have shown that there exists another mechanism which limits the lifetime of the vortex even for $V_{rot} > V_{dr}$. This mechanism is related to the decay of the vortex into zonal flow: in other words, there occurs a change of the spatial structure of the vortex for which the round "hillock" transforms into a drawn-out "crest." This second mechanism is thus related to the instability of the vortex against decay.

The results of the experiments in which the working fluid was water are represented in Fig. 3. (We remark right away that these results do not depend on the viscosity of the medium; see e.g., Fig. 3d as well as Ref. 6a.) Figures 3a and 3b show the dependence of the lifetime of a vortex on its diameter, defined similarly to the diameters of the Rossby vortices, at the instant when the vortex may be considered as formed; $H_0 = 5$–6 mm in the case 3a and $H_0 = 3$ mm in the

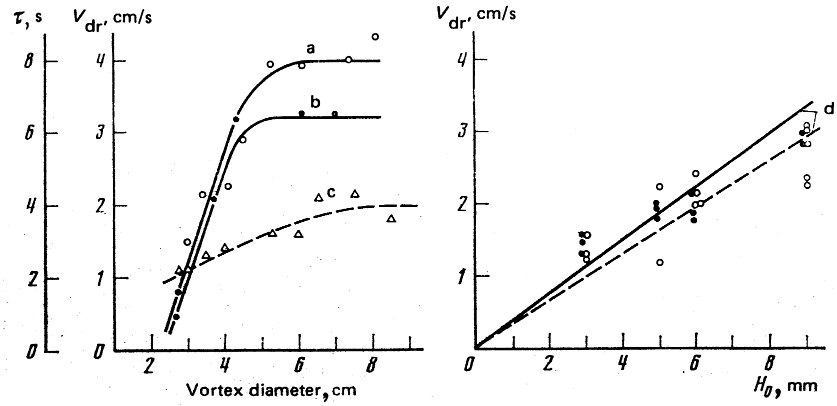

FIG. 3. The dependence of the lifetime of vortices in water on their diameter: a) $H_0 = 5$ mm and 6 mm; b) $H_0 = 3$ mm. The dependence of the drift velocity on the diameter of the vortices (c) for $H_0 = 5$–6 mm and on the depth of the fluid (d); in the case d the diameter of the vortex satisfies the condition (4), and the prime denotes the case when the working fluid was a nickel sulfate solution. As diameter of the vortex we have taken the distance 2a between the points on the profile where the linear velocity is maximal (see Fig. 2).

case 3b. It is clear that if the diameter of the vortex exceeds a certain magnitude

$$2a \gtrsim (2.5-3)\, r_R, \tag{4}$$

which increases with the growth of the depth of the fluid, the lifetime of the vortex reaches saturation, τ_{max}; its magnitude is obviously determined by the time of laminar opening-up of the vortex trajectories. If the diameter of the vortex is substantially lower than the threshold (4), the lifetime of the vortex turns out relatively short; it is determined by the decay of the vortex into a zonal motion[6a] and is smaller the smaller the diameter of the vortex becomes. For instance, if the diameter is only half as large as the threshold (4), the decay of the vortex into a zonal flow occurs over a time of the order of one rotation of the paraboloid, i. e., considerably faster than one rotation of the vortex around its own axis. Vortices of different diameter also have different drift velocities (for constant amplitude ΔH. Figure 3c shows that as the diameter of the vortex increases so does the drift velocity, which reaches saturation. It is clear from Fig. 3d that the maximal drift velocity, just like the Rossby velocity, is approximately proportional to the depth H_0 of the fluid, but numerically $V_{dr} < V_R$. Thus, $V_{dr} \approx V_R/2.5$ under the conditions of Fig. 3d. If one takes into account the fact that according to the theory[2] the drift velocity of the soliton must be somewhat larger than V_R (or, what amounts to the same, if one takes into account the dependence of V_{dr} on the amplitude of the soliton), one can see that the experimental value of V_{dr} is smaller than the theoretical value[2] by approximately a factor of three. There are reasons to believe that this circumstance does not contradict the principles of the theory,[2] and is partly due to the geometry of the paraboloid used in the experiments.

A comparison of the experimental data with the enumerated properties of Rossby solitons leads to the conclusion that the vortices which are observed in the range of conditions (4) are Rossby solitons which are the only ones among the geostrophic vortices which are stable. This stability also manifests itself in the fact that an extended anticyclonal disturbance of a more or less arbitrary form decays into a "chain" of Rossby solitons (see Ref. 1) or reorganizes itself into a single Rossby soliton. A typical example of this type is given in Fig. 4, which shows the time evolution of an anticyclonal vortex which initially had an irregular form. It can be seen that over a time of the order of several rotations of the paraboloid the vortex takes on the shape, size, and drift velocity which are typical of a Rossby soliton, and continues to exist stably over the "viscosity" time, in other words, becomes a regular Rossby soliton. This regularity expresses the fact (to which V. V. Yan'kov called our attention) that a Rossby soliton is an "attraction" type solution of the nonlinear equation.[2]

C. The drift velocity of a Rossby soliton as a function of its amplitude

An investigation of the dependence of the drift velocity V_{dr} of the vortices on their amplitude ΔH was carried out by means of a camera which with one winding-up could take a series of approximately 15 pictures with adjustable exposures and intervals between exposures. Each such series corresponded to one pulse of the priming disk and yielded both a graph of the motion of the vortex relative to the fluid (from which V_{dr} was determined), and a total picture of the distribution of velocities of the particles in the vortex as a function of time. An example of such a series was given in Fig. 4. (Similar series of pictures illustrate, among other things, the laminar opening-up of the particle trajectories in vortices of sufficiently large size, satisfying the condition (4).) The velocities of the particles in the vortex were determined directly from photographs of the type of those shown in Fig. 4, and the amplitude ΔH was estimated from the maximal velocity V_{rot} in the vortex from the equation of geostrophic equilibrium (the Coriolis force is equated to the pressure gradient in the fluid)

$$2V_{rot}\Omega_0 \cos\alpha \approx g^* \Delta H/a, \tag{5}$$

where a is the radius of the vortex corresponding to maximal velocity. The rotation velocity in the vortex and its amplitude ΔH decrease with time on account of the viscosity over the duration of each series of photographs (Fig. 5) and this allowed us to construct the dependence of V_{dr} on ΔH. A typical example of such a dependence is shown in Fig. 5. It is clear that V_{dr} increases with ΔH—in agreement with the theory[2]—and reaches saturation for large amplitudes (the latter phenomenon surpasses the framework of the theoretical predictions). It should be noted that the investigated

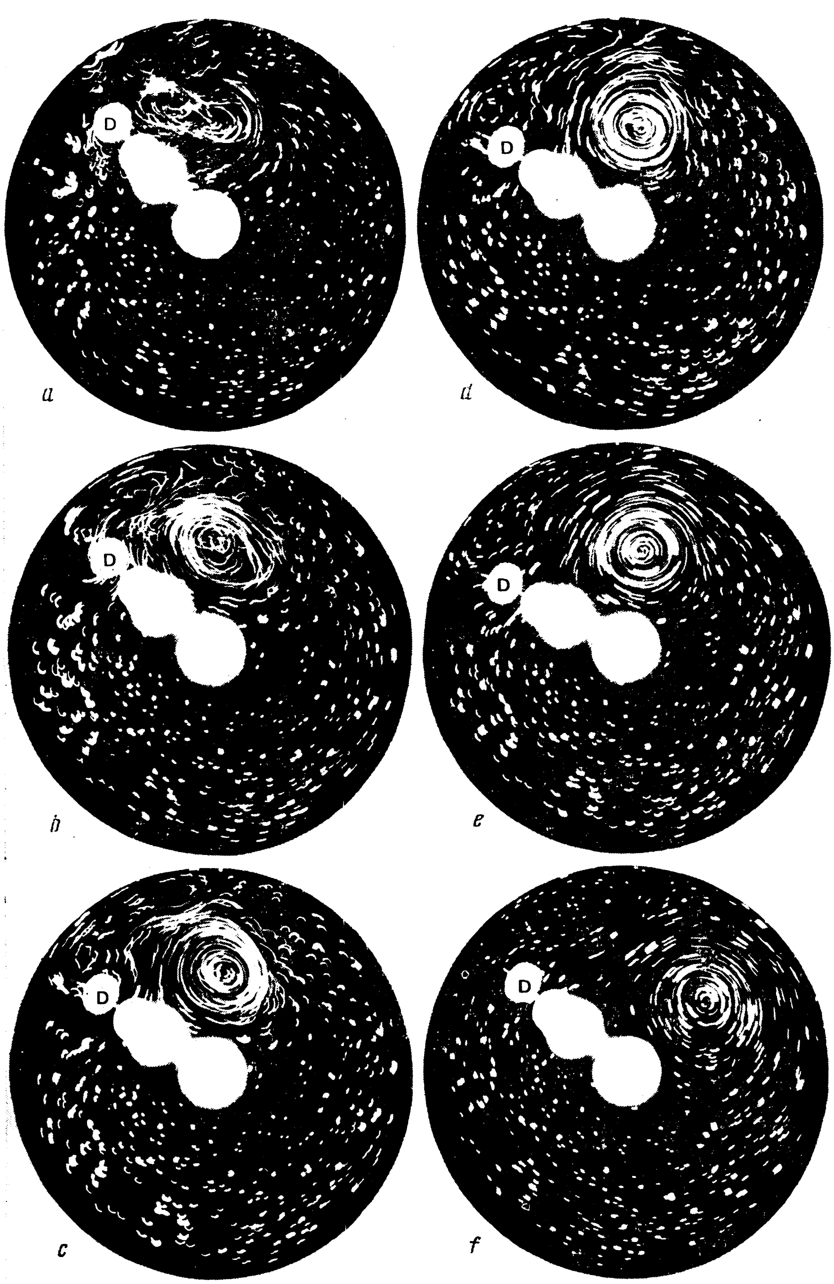

FIG. 4. A series of photographs showing the evolution of an anticyclonal Rossby soliton from an irregular initial disturbance, photographed by a camera rotating along with the paraboloid. A top view of the trajectories of white test particles floating on the surface of the water, with a black background, is shown. The white spot is the priming disk and its driver. The intervals between frames a–e were 0.6 s (one revolution of the vessel). The last interval was about 2.4 s (four revolutions of the paraboloid). The depth of the water was $H_0 = 3$ mm, the exposure time of the camera is 0.3 s.

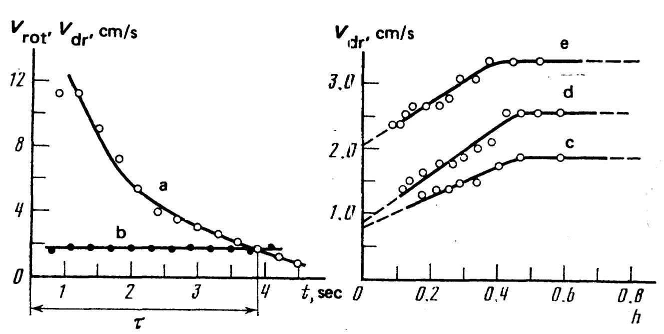

FIG. 5. The velocity of rotation of the particles in the vortex, which is maximal on its profile (a) and the drift velocity (b), as functions of time. The vortex diameter satisfies the condition (4), $H_0 = 5$ mm. The quantity τ is the lifetime of the vortex. The dependence of the drift velocity of the Rossby soliton on its relative amplitude $h = \Delta H / H_0$ (c,d,e). $H_0 = 5$ mm (c), 6 mm (d), 9 mm (e), the diameter of the priming disk was $D = 5$ cm (c), 5 cm (d), 4 cm (e).

range of variation of the amplitude of vortices is sufficiently wide: the relative amplitude $h = \Delta H / H_0$ may reach values close to unity (whereas the theory[2] assumes that the magnitude of h is considerably smaller than unity). The data represented in Fig. 5 may be considered as a basis for experiments on the collisions of Rossby solitons. Since, as follows from what was said above (and Ref. 1), Rossby solitons always drift in the same direction, one can only produce collisions between solitons of different amplitudes which propagate one behind the other.

D. Collisions of Rossby solitons

Before discussing the experimental data on collisions of Rossby vortices we make one methodological remark. According to an early definition (see, e.g., Ref. 7) a soliton was defined only as a solitary wave which remained unchanged in collisions with similar waves. However, the subsequent development of soliton physics has shown that this definition may not quite serve its purpose. Indeed, it is now well known (see, e.g., Ref. 8) that the result of the interaction of two-dimensional solitary waves described by the Kadomtsev-Petviashvili equation[9] (a two-dimensional generalization of the Korteweg–de Vries equation), or of three-dimensional solitary waves depends in principle on the angle under which the collision takes place; namely, in a definite range of collision angles there appears an irreversible effect: a third solitary wave is created. It hardly makes sense to call the same solitary waves solitons for some collision angles and not to call them solitons for other angles. We will designate as a soliton any solitary wave, whether or not it has passed the collision test.

The result of experiments on the (most typical) collisions of Rossby vortices (Figs. 6 and 7; see also Ref. 6b) show that such collisions occur according to the following scenarios.

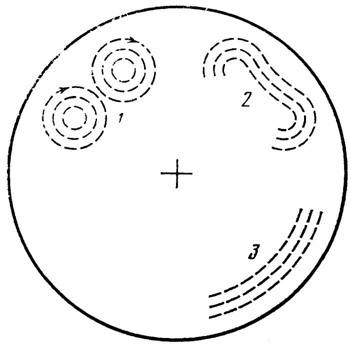

FIG. 7. The restructuring of colliding anticyclones into a zonal flow (schematic). The digits 1, 2, 3 denote successive stages of the process.

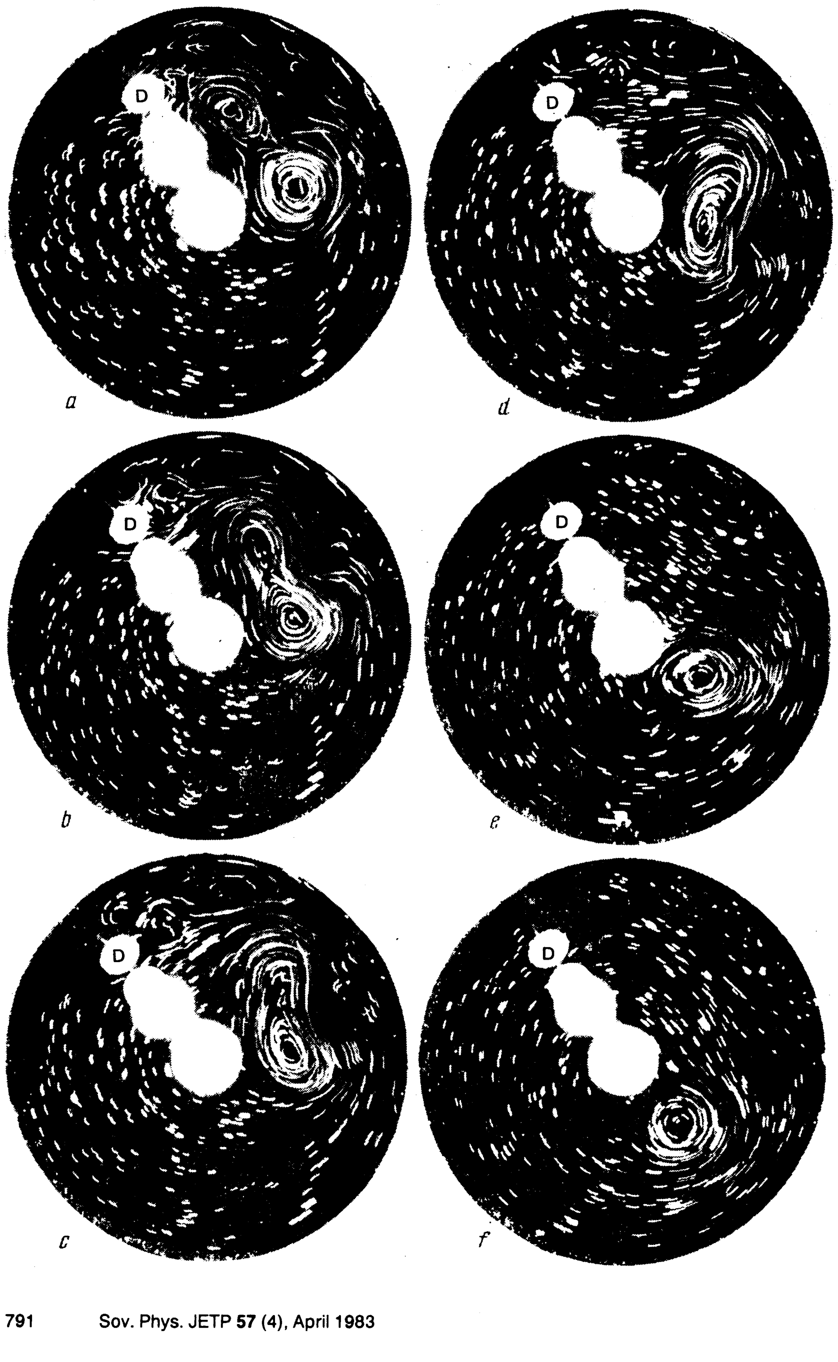

FIG. 6. Different stages of approach and fusion of initially separated Rossby anticyclones produced by the priming disk D. The intervals between the frames a–f (in seconds): 0.6, 0.6, 1.8, 1.8. The white spots are the priming disk D and its drive.

Scenario 1 (Fig. 6). At a sufficiently large approach velocity of the vortices they fuse irreversibly into a single vortex, which continues to drift stably in the fluid and has a lifetime determined by the viscosity of the medium; the new vortex is also a Rossby soliton. In the conditions of Fig. 6 both vortices are formed approximately in the same manner. The experiments show that under different conditions, when one of the vortices is formed much better than the other, it will "survive," swallowing its partner, independently of their mutual position.

Scenario 2 (see Ref. 6b). At a small velocity of approach the solitons exist practically without interaction. The examples of vortex structures ("chains of vortices") given in Ref. 1 belong to the same category.

Scenario 3 (Fig. 7). For an intermediate velocity of approach the solitons (which have approximately equal amplitudes) mutually destroy each other and reorganize themselves into a zonal flow.

Finally, we stress the fact that in the experiments described here we never observed the phenomenon where one vortex passes through the other, as is proper for some types of soliton.[7] Thus, the interaction of Rossby solitons is generally inelastic: it seems that it is energetically more convenient for the solitons to fuse, rather than remain separate. But in order for such a fusion to be possible, if one can judge by the data presented here, the velocity of approach must exceed a certain threshold.

E. Asymmetry

It was discovered in Ref. 1 that in the experimental geometry which was investigated there do not exist long-lived (stable) solitary cyclonal Rossby solitons: such objects are unstable and decay rapidly, at least for sizes larger than or of the order of the Rossby radius (2). Cyclones with radii smaller than the Rossby radius have not been investigated, since in this parameter range (in our geometry) capillary phenomena may have a substantial influence. Anticipating a frequently asked question, we call attention to the fact that the sizes of cyclones which exist stably in the Earth's atmosphere seem to be determined by scales smaller than the Rossby radius (2), which for the terrestrial atmosphere is about 3000 km (the corresponding diameter is already 6000 km), i.e., turns out to be "too large." (The size of a terrestrial cyclone can be estimated from the example given in Ref. 4: only along the periphery of Antarctica is there room for a chain of six cyclones, and consequently their radii are smaller than the Rossby radius (2).) In this sense terrestrial cyclones fall into the range of parameters which was not investigated in the present work (as well as in Ref. 1). It seems most likely that the atmospheric vortices (but the geostrophic ones, with a proper rotation period smaller than the global rotation period of the planet) are, from the point of view of wave motion, three-dimensional, rather than two-dimensional formations, and may therefore have radii smaller than the Rossby radius (2) (in this connection, see, e.g., Ref. 10). The data on the sizes of cyclones in a system with zonal flows can be found in subsection F.

After the discovery of the instability of solitary cyclones there remained the question of principle: could such objects not exist within the solitary cyclone-anticyclone pair discovered in Larichev and Reznik's theoretical paper[11]? According to them such a Rossby pair soliton in which the vortices are situated on the same meridian could move either against the global motion of the fluid with a velocity exceeding the Rossby velocity (3) (same as a single soliton), or in the direction of the global motion with arbitrary velocity. In the geometry of our experiments this would mean that in the first of the indicated cases the anticyclone should situate itself closer to the system axis, whereas in the second case the cyclone should be closer to the axis. (We recall that cyclone-anticyclone structures have been observed in the previous experiments,[1] where it was stressed that the cyclones entering into these structures have a manifestly secondary character: they are "unwound" by the neighboring (primary) anticyclones on account of viscosity; accordingly, the particle velocities in them are considerably lower than in anticyclones. Therefore the observation of vortex structures indicatd in Ref. 1 more likely refers to the coexistence of almost noninteracting anticyclones.)

The investigation of the possibility of production of a Rossby soliton pair was carried out by means of the described installation with two independent priming disks of diameter 3 cm situated next to each other (on the same meridian at neighboring latitudes) rotating in opposite directions. The experiment was carried out in two variants. In the first variant the priming disk closer to the system axis rotated anticyclonally and the neighboring disk rotated cyclonally. Three series of pictures were taken in this variant: in the first series only the interior disk was switched on, in the second series only the second disk was switched on, and in the third series both disks were active.

The experiments yielded the following results. Activation of the "anticyclonal" disk produced a good anticyclone of the form which had been repeatedly demonstrated both in Ref. 1 and in the present work. The switching-on of the "cyclonal" disk led to the effect explained in Fig. 8 in place of the expected cyclone, an anticyclone invariably unwinds(!). The activation of both disks leads in any case to the formation of only an anticyclone, naturally one more powerful

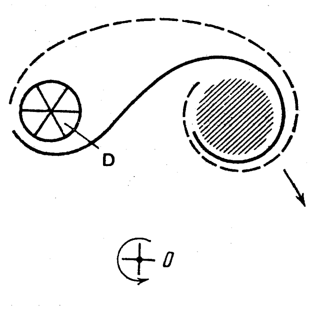

FIG. 8. The excitation of a Rossby anticyclone (shaded) for different directions of rotation of the priming disk D. The solid line represents the case when the disk rotates cyclonally, the dotted line depicts anticyclonal rotation of the disk. The arrows indicate the direction of rotation of the paraboloid around the point O and the drift direction of the Rossby soliton, respectively.

than in the two preceding series (this result should not cause any bewilderment). The resulting vortex is longlived and of a very regular shape—approximately like the one in Fig. 4 (starting from frame c). In the second variant of the experiment the interior disk rotated in the cyclonal direction and the neighboring one was given an anticyclonal rotation. Only anticyclones were obtained in these experiments. The experiments have proved that no matter what the disturbance of the fluid, it never propagated in the direction along the global rotation: the motion of the vortices is invariably in a direction opposite the revolution of the system, i.e., in the same direction as the drift of isolated Rossby solitons (anticyclones or the mentioned cyclone-anticyclone structures).

Thus, in the conditions of the geometry of our experiments only the nonlinearity responsible according to theory[2] for the predominance of anticyclones over cyclones manifests itself, but not that nonlinearity which according to the theory[11] finds its realization in isolated cyclone-anticyclone pairs. It is possible that for the observation of such paired solitons one must carry out experiments with vortices having radii a considerably smaller than the Rossby radius (2). In our device such experiments are impeded by two circumstances: the possible interference from capillary phenomena, and a violation of the shallow water condition ($a \gg H_0$).

F. The generation of Rossby solitons by fluid flows with velocity shear (the cyclone-anticyclone asymmetry of the nonlinear regime of the Kelvin-Helmholtz instability)

In this subsection, which is closely related to the preceding one, we discuss further manifestations of the asymmetry of the system with respect to the possible existence of vortices with opposite orientation of the vorticity (curl of the velocity) relative to the angular velocity vector of the system. One of the mechanisms for generation of vortices in a fluid (or plasma) is the Kelvin-Helmholtz instability of fluids with a velocity shear. Over the past few years, this instability was very effectively studied experimentally[3–5] in connection with the problems of the physics of planetary atmospheres. The characteristic of our experiments was that they were carried out on an installation which might be considered as a model for a homogeneous planetary atmosphere, the following conditions being simultaneously satisfied: 1) the fluid as a whole rotates around a vertical symmetry axis with angular velocity Ω_0; 2) the depth of the fluid is small compared to the Rossby radius (2); 3) the curvature of the surface of the rotating fluid is sufficiently large, so that the Coriolis parameter f_0 has a significant gradient in latitude (the so-called beta-effect); at the same time the curvature radius of the surface is much larger than the Rossby radius ($R \gg r_R$). Under these conditions the instability manifests itself in a new way—which is the subject of the present section.

The experimental setup was already described before (Fig. 1) and it was noted that for a certain threshold value of the magnitude of the velocity shear of the counterflows there appears a Kelvin-Helmholtz instability, whose character depends essentially on the sign of the vorticity (curl of the velocity) in the system of counterflows. For positive curl we shall say that the system exhibits cyclonal shear, and for negative vorticity (the curl is antiparallel to Ω_0) we call the shear anticyclonal. One of the most characteristic traits of the instability under consideration consists in the following Vortices which appear in cyclonal shear have quite small amplitudes and sizes (and these vortices do not become more intense upon arbitrary increase of the shear-up to its maximal value when the angular velocity of the flows relative to the paraboloid becomes almost equal to Ω_0). The depth H_0 of the fluid between the rings is kept under control and remains sufficiently large, for example, in the conditions of Ref. 12 $H_0 = 8$ mm for a ring velocity 91 cm/s. We note that the velocity of the flows satisfies the conditions $V_R \lesssim u < (g^* H_0)^{1/2}$. In this connection we indicate the depth H_0 of the water between the rings for various magnitudes and directions of the velocity (u) of the exterior ring: a) $H_0 = 17$ mm for $u = 0$, b) $H_0 = 22$ mm for $u = -43$ cm/s, and c) $H_0 = 12$ mm for $u = -43$ cm/s, under the conditions of Fig. 2 of Ref. 12. For anticyclonal shear even a small excess of the shear above the instability threshold leads to the formation of large-amplitude vortices (Fig. 9). These vortices exhibit the following properties: 1) they are anticyclones (rise of the fluid level); 2) their sizes are substantially larger than the Rossby radius (2); 3) they are solitary formations with a sharply defined velocity profile; 4) they are stable and are observed in a stationary manner; 5) they drift against the global fluid motion with a velocity close to the Rossby velocity (3). For example, they move relative to the fluid by approximately 15° of arc for one revolution of the paraboloid for $H_0 = 5$ mm. As the depth of the liquid increases by approximately a factor of two the drift velocity of the vortices increases to 24° per revolution of the paraboloid (i.e., to 7 cm/s). This velocity increases significantly also when the angular velocity Ω_0 increases (this is related to the latitudinal gradient of the depth of the liquid, see Ref. 1). As for the number of vortices, near the threshold of instability this number is equal to four, and for larger shear (up to the maximal value) it is equal to three.

We designate this instability as a Kelvin-Helmholtz instability, since in accord with the generally accepted definition it manifests itself through the generation and development of perturbations (in our case vortex perturbations), the characteristic scales of which are larger than the width of the layer in which the velocities of the flow vary.

Figure 9 is an illustration of the picture of observable anticyclones, corresponding to an anticyclonal shear with a velocity of the counterflows exceeding by a factor of about 1.5 the threshold of instability (for details we refer the reader to Ref. 12). As the experiments have shown, this picture is observed stationarily and revolves around the system axis; for instance, for $H_0 = 1$ cm it effects one revolution relative to the fluid in approximately 9 seconds.

Comparing the experimental data with the properties of Rossby solitons listed above (Sec. 3A), we come to the conclusion that anticyclonal vortices observed at negative shear are Rossby solitons. Regarding their oval shape, it is caused by the existence of velocity shear in the flow (the round soliton described above is observed in quiescent fluid, where there are no flows).

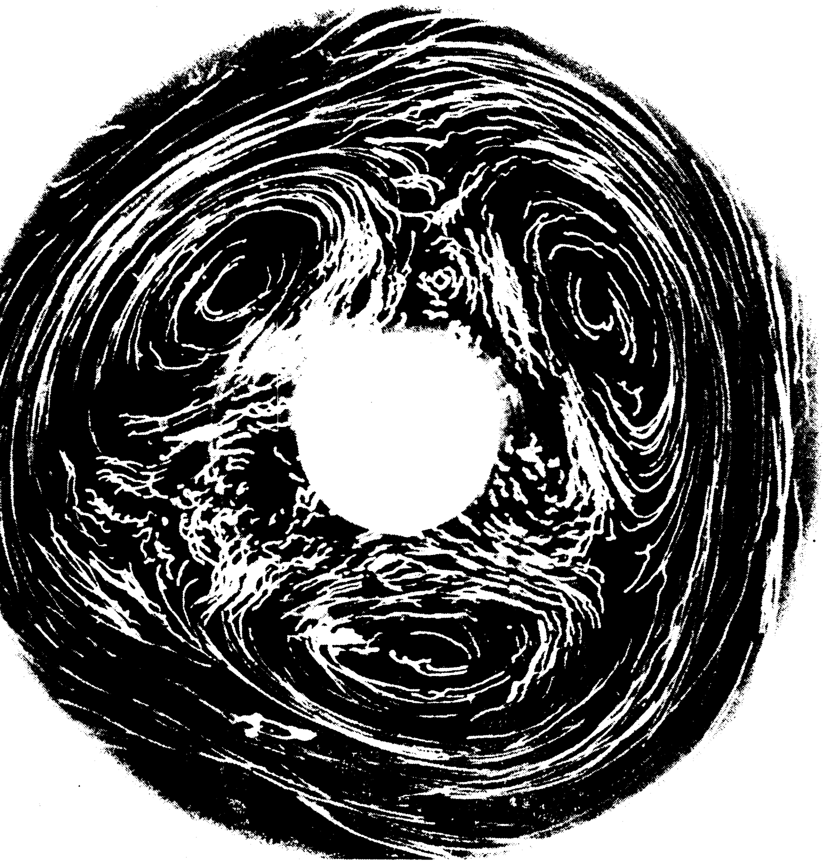

FIG. 9. The image of fluid flow for an anticyclonal shear of period $T_{\text{shear}} = 1.7$ s. The rotation period of the paraboloid was $T_0 = 0.58$ s. The figure shows a top view of the trajectory of white test particles floating on the surface of the water on the background of the black container bottom. The photograph was taken with a corotating camera with an exposure time of 0.25 s. The depth H_0 of the water was approximately 2 cm. The white spot in the center is part of the drive of the device. The exterior diameter of the paraboloid is 28 cm (see Fig. 1).

In the conditions of the experiments described here the threshold of observed instability corresponded to the rotation frequency Ω_{shear} of the counterflows relative to the paraboloid, which is approximately by one order of magnitude lower than the angular velocity Ω_0 of the global revolution. It is appropriate to point out here that, as was shown by the measurements, the flow velocity of the surface layer of the liquid (where the white test particles float) is smaller by a factor of 1.5 than the velocity of motion of the ring situated under the given layer of fluid. In this connection one should take into account the fact that in Ref. 12, when we mentioned the flow velocity we had in mind the velocity of the rings.

In the conditions of Fig. 9 the flow velocity at the surface of the fluid was approximately 20–30 cm/s, and the drift velocity of anticyclonal solitons (for $H_0 = 1$ cm) was approximately 7 cm/s.

The existence of a threshold for the Kelvin-Helmholtz instability in the presence of the β-effect follows from the known Rayleigh-Go criterion[13] (a generalization of the Rayleigh criterion to fluid flows in the presence of an inflection point in the velocity profile):

$$\beta - \frac{\partial^2 u}{\partial y^2} = 0, \tag{6}$$

where

$$\beta = -\frac{1}{r_R^2}\frac{\partial}{\partial y}(f_0 r_R^2)$$

(see Ref. 1), where y is the latitude (a coordinate measured along a meridian) and u is the fluid flow velocity ($u > 0$) if its direction coincides with that of the global revolution of the system; in the following estimates

$$|\partial^2 u/\partial y^2| \sim |u|/\delta^2,$$

where δ is the charactristic size of the shear). The fluid flow along the parallel leads to the appearance of a meridionally directed gradient of the fluid depth, the magnitude of which is determined by the condition of geostrophic equilibrium:

$$g^* \partial H/\partial y = 2\Omega_0 u \cos\alpha.$$

Making use of this equation together with Eqs. (2) and (3), we obtain an instability criterion (valid both for a paraboloid and for a system of the type of a planet)

$$V_R + u + r_R^2 \partial^2 u/\partial y^2 = 0. \tag{6'}$$

The validity of the criterion (6), (6') depends essentially on the sign of the flow velocity. Two cases are possible here: *First case*: $u > 0$. In this case the instability can develop only if $\delta < r_R$, in other words, it can generate only small vortices of sizes small compared to the Rossby radius (2). The geometry of this case is close to that of a tangential discontinuity. The threshold value of the flow velocity $|u| \sim V_R \delta^2 / r_R^2$, where the Rossby velocity V_R is defined by Eq. (3). *Second case*: $u < 0$. In this case the instability can be excited both on small scales (smaller than the Rossby radius) and on large scales (larger than the Rossby radius). The threshold value of the flow velocity at large scales is close in magnitude to V_R, i.e., to (approximately) that velocity with which the Rossby solitons propagate. This case is of greatest interest, since as a

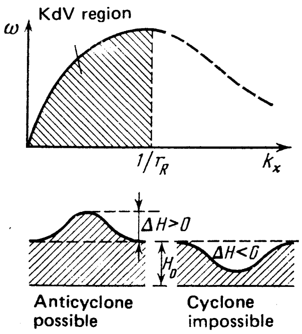

FIG. 10. A schematic drawing of the dispersion function $\omega(k_x)$ for Rossby waves. The shaded region is described by an equation close to the Korteveg–de Vries (KdV) equation (see, e.g., Refs. 7, 9) implying the possibility of solitary waves (solitons) of raised fluid level enhancements and the impossibility of solitary fluid level depression waves.

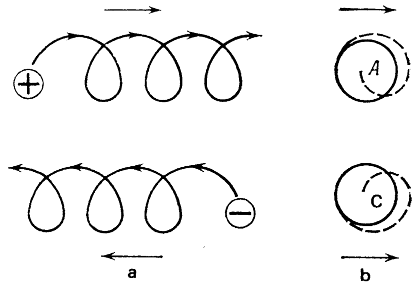

FIG. 11. A comparison of the drift of Rossby vortices with the drift of particles in an inhomogeneous magnetic field: a) the drift of positive ions and of electrons in a field perpendicular to the drawing, with a gradient in the picture plane directed from the top to the bottom; b) the drift of vortices of opposite signs in a rotating system, the angular momentum vector of which is perpendicular to the plane of the drawing, and with the gradient of the Coriolis parameter is the picture plane, from the top to the bottom. In the left half of the picture the straight arrow show the directions of drift of the ions and electrons, and in the right half of the picture they denote the common direction of drift of anticyclones and cyclones. The dotted lines show that the particle trajectories in the anticyclone become less steep in the upper part and steeper in the lower part, on account of the Coriolis force; conversely, in cyclone is less curved in the lower half, and more curved in the upper half; for this reason both vortices drift in the same direction.

consequence of the instability of the flow (which moves against the revolution of the system) the excitation of Rossby solitons is possible.

In the experiments described here, for any orientation of the vorticity of the counterflows one flow (the exterior one or the interior one) will always lag behind the revolution of the system (i.e., has $u<0$). According to our analysis it would seem to be able to excite large-scale ($\delta > r_R$) Rossby solitons. In other words, from this point of view practically identical conditions are set up for the excitation of large cyclonal and anticyclonal vortices. Nevertheless, the described experiments show that the excitation of large-scale *anticyclones* takes place, whereas that of large-scale *cyclones* does not. This agrees with the fact (see Subsec. E) that anticyclones are stable, and cyclones decay rapidly, apparently even faster than they are created (under the conditions of the geometry of our experiments. The cyclone-anticyclone asymmetry discussed in the present paper in its various manifestations corresponds to the intuitive notion that the direction of the Coriolis force is such that it facilitates the twisting of the particle trajectories into an anticyclone and impedes the formation of a cyclone (see also Fig. 11).

G. Some considerations on the nature of the Big Red Spot of Jupiter

The obvious similarity between the properties of the vortices which appear for anticyclonal shear, described in the preceding section and the Big Red Spot of Jupiter, both in shape, physical parameters, and drift direction, as well as conditions of existence, calls attention to itself. It is known (see, e.g., Ref. 14), that the Big Red Spot is an anticyclonal vortex, probably a Rossby soliton,[2,10,15,16] with east-west zonal counterflows surrounding it; it drifts in a direction opposite to the revolution of the planet, and has dimensions exceeding the Rossby radius. The spot has an oval shape, prolate in the drift direction, in the same manner as the vortices in Fig. 9. In connection with the experimental results of the present paper the following fact seems important as a matter of principle: The Big Red Spot is "tied" to that latitude belt in the atmosphere of Jupiter where the shear of the zonal flows has anticyclonal direction and "ignores" a similar belt somewhat to the north where the shear, being smaller in magnitude, has a cyclonal direction. This fact is manifestly analogous to the cyclone-anticyclone asymmetry of the nonlinear regime of the Kelvin-Helmholtz instability discussed in the preceding section. The regime described there for the excitation of an anticyclone of size exceeding or of the order of the Rossby radius corresponds (as a whole) to the conditions in the Big Red Spot. It should be noted that according to the instability criterion derived in the preceding section, the maximal velocity of the zonal flow of the wind $|u|_{\max}$ must not be smaller than the Rossby velocity (3), whereas in the region of the Dot $|u|_{\max} = (50–50)$ m/s, and $V_R = 160$ m/s.[10] The escape from this difficulty could be seen, e.g., in the fact that from a wave point of view the Spot is not a two-dimensional formation, but rather a three-dimensional object, and in this case the wave motion along a vertical line reduces the drift velocity of the Spot, and it becomes significantly smaller than the Rossby velocity defined in Eq. (3) (in this connection see, e.g., Ref. 10). Regarding the fact that the Spot is only one (whereas along the periphery of the planet there is room for more than ten vortices of this scale), one might make the following conjecture: if the Spot appeared from a pre-threshold state of the system on account of some "local" process, then in the presence of hysteresis such an azymuthally inhomogeneous state could be maintained by the existing shear of the zonal flows even if the magnitude of the shear is not sufficient to excite a chain of vortices (in the conditions of Fig. 9 such a chain consists of three vortices); in each zonal belt there could exist one Spot under these conditions.

H. On the wave representation of Rossby vortices

The question has often been raised why we call the Rossby vortices not simply vortices but (solitary) waves? Our answer is that the wave approach turns out often to be more fruitful and informative than the simple vortex approach.

Thus, in the main section of this paper the wave approach allowed for an intuitive prediction of the cyclone-anticyclone asymmetry which is one of the most remarkable properties of the Rossby solitons. Let us dwell in more detail on this question. (For this purpose we shall, as before, use the two-dimensional theory, which suffices for the derivation of qualitative results.) The dispersion law of two-dimensional Rossby waves has the form

$$\omega=\frac{\beta k_x}{k_x^2+k_y^2+1/r_R^2}, \qquad (7)$$

where ω is the frequency of oscillations, and k_x and k_y are respectively the latitudinal and meridional wave numbers. The graph of the function $\omega(k_x)$ is shown in Fig. 10. It is important to note two circumstances: first, the phase velocity of the oscillations decreases with the increase of the wave number, and second, it increases as the depth of the fluid increases. Let us now consider some local enhancement of the fluid level. If there were no dispersion, the upper portions of the profile would catch up with the lower ones on account of the nonlinearity, which would lead to a steepening of the anterior wave front and a breaking of the wave. On the other hand, if there were no nonlinearity (the increase of the phase velocity with the depth of the liquid), then on account of dispersion the upper portions of the profile corresponding to its steep slopes (and to corresponding larger wave numbers of its spectral components) would fall behind the lower portions. In a soliton involving enhancement of the fluid level the two indicated effects of nonlinearity and dispersion have opposite signs and cancel mutually, and therefore a stable wave packet (the soliton) becomes possible. It is easy to see that for waves corresponding to a depression of the fluid level the effects of the nonlinearity and dispersion have the same signs, and a solitary wave becomes impossible. In other words, an anticyclonal Rossby soliton is possible, and a cyclonal one is not. (Of course, here we call attention only to a necessary condition; a proof that the soliton must exist has to be given separately,[2] but in the case of the cyclone the necessary condition for nonexistence is already satisfied.) As we have seen above, this expectation of the wave theory is confirmed by experiment. As regards the "usual" (or "simply vortex-type") approach it does not contain this essentially important information and has less predictive power than the wave approach.

Considering further the Rossby soliton from the viewpoint of balance between nonlinearity and dispersion it is easy to estimate its characteristic size. Indeed, if the wave numbers are too small ($k_x \ll + /r_R$) there is almost no dispersion and the nonlinearity dominates; for wave numbers which are too large ($k_x \gg 1/r_R$) dispersion dominates. Thus only for moderate wave numbers corresponding to a scale of the order of the Rossby radius r_R the balance between nonlinearity and dispersion is possible with the formation of a soliton. As we know the radius of the Rossby soliton is indeed close to the Rossby radius. And finally it can be seen that the drift velocity of the soliton relative to the medium must exceed somewhat (the more, the larger its amplitude) the maximal propagation speed of linear Rossby waves: $(\omega/k_x)_{\max} = V_R$. These conclusions are an additional illustration of the power of the wave approach. One may say simply: since a Rossby vortex has dispersion and moves relative to the medium it is convenient in some cases to consider it from the viewpoint of a wave.

I. On the drift direction of Rossby cyclones and anticyclones

This problem requires additional clarification. Since the Rossby vortices are wavelike objects they must drift relative to the fluid in the same direction as linear Rossby waves, i.e., opposite to the global revolution of the fluid, independently of their sign (cyclone or anticyclone). On the other hand, judging by the analogy between the motionof a fluid in the Coriolis force field and the motion of a plasma in the field of the Lorentz force,[1] at first sight there appears a difficulty: whereas plasma particles of opposite charges drift in opposite directions in an inhomogeneous magnetic field, vortices of opposite velocity curl (vorticity) must drift in the same direction. The essence of the difference is explained in Fig. 11, where the left half (a) refers to the motion of charged particles in an inhomogeneous magnetic field (perpendicular to the plane of the picture) and the right half (b) refers to the motion of vortices in the inhomogeneous Coriolis force field. The intensity of the magnetic field and of Coriolis force increase from the top of the bottom of the figure. It is clear that at the points of higher magnetic fields the trajectories of charged particles have larger curvature for both signs, and therefore particles of opposite signs drift in the opposite directions. But the particles in a vortex (Fig. 11,b) are deflected by the Coriolis force always in the same direction (e.g., in the Northern Hemisphere on Earth always to the right). This means that at the points with the larger Coriolis force the trajectories of particles in anticyclone are more curved, and the trajectories in a cyclone become straighter. As a result of this both cyclones and anticyclones drift in the same direction. This is exactly what happens in the experiment: both vortices drift in the same direction, against the rotation of the system as a whole (but the anticyclone is stable, whereas the cyclone decays rapidly).

The authors would like to express their gratitude to B. B. Kadomtsev, A. M. Obukhov, V. I. Petviashvili, V. V. Yan'kov, F. V. Dolzhanskiĭ, and Yu. L. Chernous'ko for stimulating discussions, and to A. N. Khvatov for help with this work.

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Translated by M. E. Mayer