Muon transfer from hydrogen to helium

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It is found that μ^- mesons stopped in a gas mixture of hydrogen, helium, and xenon (hydrogen pressure about 20 atmospheres, helium and xenon densities relative to hydrogen 0.05–2 and $\sim 10^{-4}$ respectively) are transferred from the $p\mu$ atoms in the ground state to helium atoms at a rate $\lambda_{\rm He} = (3.6 \pm 1.0)\cdot 10^8\ \rm sec^{-1}$. The result is in good agreement with the calculations in which a novel mesic-molecular mechanism of $p\mu$-atom charge exchange with helium nuclei is taken into account. The dependence of the probability for $p\mu$-atom formation in the ground state on the helium density is measured. An analysis of this dependence and a comparison of it with the corresponding data for π^- mesons indicate that muons can also be transferred from excited levels of $p\mu$ atoms at a rate higher than in the case of $p\pi$ atoms (transfer constant $\Lambda_\mu = 3.8 \pm 0.3$ compared with $\Lambda_\pi = 1.84 \pm 0.09$).

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Experimental investigations of the transfer of μ^- mesons from hydrogen to atoms of other elements

$$p\mu + Z \rightarrow Z\mu + p \tag{1}$$

have shown that in the impurity-density region $c_Z \sim 10^{-5}$–10^{-3} transfer is possible to atoms of all the investigated elements, with the exception of helium, for which it is suppressed by several orders of magnitude.[1] This is attributed to the fact[2,3] that if the $p\mu$ atom colliding with the Z atom is in the ground state, the intermolecular terms of the systems $p\mu + Z$ and $Z\mu + p$ have no crossings of pseudocrossings for $Z = 2$ and 3. This suppression mechanism, however, does not apply to collisions of excited mesic atoms:

$$p\mu^* + \mathrm{He} \rightarrow \mathrm{He}\,\mu + p. \tag{2a}$$

In fact, experiments[4] on the transfer of pions from $p\pi$ atoms that exist only in the excited state to atoms Z have shown the existence of transfer at $c_Z > 0.1$ to all atoms $Z > 1$, and helium is no exception. By analogy with π^- mesons, one can expect in the mixture to be able to observe also transfer of muons at suitable helium densities.

We have undertaken a search for the transfer (2a) of muons from excited levels of $p\mu$ atoms to helium. The preliminary result obtained for part of the data by analyzing the integral yield of the particles was published earlier.[5] As for transfer of muons from the ground state of $p\mu$ atoms:

$$p\mu + \mathrm{He} \rightarrow \mathrm{He}\,\mu + p, \tag{2b}$$

the previously existing theoretical estimate[2,3] of its rate $\lambda_{\rm He} \sim 10^6\ \rm sec^{-1}$ gave no grouinds for taking this process into account under our conditions. However, a reduction of the entire obtained material without allowance for the process (2b) did not yield self-consistent results. It follows therefore that a correct result can be obtained only through a reduction that includes an analysis of the temporal distributions of the particles and is not subject to limiting assumptions. Another factor that stimulated this analysis was the conclusion reached in Ref. 6, where they considered a new transfer mechanism (2b) connected with intermediate formation of a mesic molecule $p\mu$ He, and a higher value of the rate of the process was obtained: $\lambda_{\rm He}^{\rm theor} = 4.4\cdot 10^7\ \rm sec^{-1}$. In the present paper we present results of a detailed reduction of data obtained by us in the experiment.[5]

ORGANIZATION OF EXPERIMENT

We consider processes that take place in a mixture of hydrogen and helium gases after a muon is stopped in it (Fig. 1). The probability $W_{\rm H}$ of formation of the mesic-hydrogen atom can be expressed in terms of the ratio A of the rates of atomic capture of a meson by helium and by hydrogen and in terms of the helium concentration $c = n_{\rm He}/n_{\rm H}$ (where $n_{\rm He}$ and $n_{\rm H}$ are the atomic densities of the mixture components):

$$W_{\rm H} = (1 + Ac)^{-1}. \tag{3}$$

The produced excited $p\mu$ atom, owing mainly to collisions with the mixture atoms, goes over gradually to the ground state. If it collides with an impurity atom in the course of de-

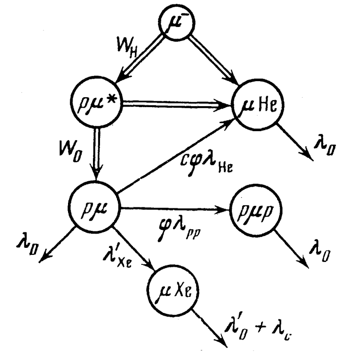

FIG. 1. Scheme of the process: next to the broad arrows are indicated the transition probabilities, and next to the narrow ones the rates of the processes.

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excitation, the transfer (2a) can take place. We denote by W_0 the probability that the $p\mu$ atom will retain the muon and reach the ground state. Then, if N_0 mesons were stopped in the mixture, by the time the mesic hydrogen atoms reach the ground state the number of $p\mu$ and He μ atoms will be respectively

$$N_{10}=N_0W_HW_0,$$
$$N_{20}=N_0(1-W_HW_0). \tag{4}$$

The time of de-excitation of the $p\mu$ atom in hydrogen at a pressure of several tens of atmospheres is according to Ref. 7 certainly much less than 10^{-9} sec. The processes that proceed during this stage are marked in Fig. 1 by broad arrows.

Next, the $p\mu$ atom, being already in the ground state, can give up a muon to the helium atom [if the process (2b) takes place], produce a $p\mu p$ mesic molecule, or vanish via muon decay. We assume also that the mixture contains a small addition of a "testing" gas—xenon, such that the rate of transfer of a muon from the xenon is comparable with the rates of its decay: $\lambda'_{Xe} \sim \lambda_0 = 0.455\cdot10^6$ sec^{-1}. For the conditions considered it corresponds to a density $c_{Xe} \sim 10^{-4}$, i.e., the influence of the xenon on the atomic capture of a meson and on the cascade in the mesic hydrogen can be neglected. The produced Xe μ atoms vanish rapidly because of capture of the muon by the xenon nucleus (the rate is $\lambda_C \sim 10^{-7}$ sec^{-1}) or as a result of the meson decay (the rate is $\lambda'_0 \approx \lambda_0$ in the field of the nucleus). As for the He μ-atoms and $p\mu p$ molecules, they vanish only as a result of muon decays. Thus, during the second stage the characteristic times of the processes are of the order of microseconds, and consequently the first stage can be considered for them to be instantaneous, and the number of the mesic atoms, determined in (4) and pertaining to the end of the first stage, can be referred to the zeroth instant of time. Now the dependence of the number of considered objects on the time can be described in the following fashion

$$\begin{aligned}
p\mu&:\ N_1(t)=N_{10}e^{-\lambda_S t},\\
\text{He}\,\mu&:\ N_2(t)=N_{20}e^{-\lambda_0 t}+N_{10}B_2c\,(e^{-\lambda_0 t}-e^{-\lambda_S t}),\\
p\mu p&:\ N_3(t)=N_{10}B_3(e^{-\lambda_0 t}-e^{\lambda_S t}),\\
\text{Xe}\mu&:\ N_4(t)=N_{10}B_4[e^{-\lambda_S t}-e^{(\lambda_C+\lambda_0')t}]\\
&\approx N_{10}B_4e^{-\lambda_S t}\quad(\text{at}\quad t>0.4\,\text{sec}),\\
\lambda_S&=\lambda_0+\lambda_{Xe}'+\varphi\lambda_{pp}+c\varphi\lambda_{He}=\lambda_1+c\varphi\lambda_{He},\\
B_2&=\varphi\lambda_{He}/(\lambda_S-\lambda_0),\quad B_3=\varphi\lambda_{pp}/(\lambda_S-\lambda_0),\\
B_4&=\lambda_{Xe}'/(\lambda_C+\lambda_0'-\lambda_S).
\end{aligned} \tag{5}$$

Here φ is the ratio of the density of the hydrogen gas to that of liquid hydrogen, and the meaning of the remaining symbols is clear from Fig. 1. It can be seen that the temporal distribution of the Heμ atoms is described mainly by the exponential $\exp(-\lambda_0 t)$, and the distributions for the $p\mu$ and Xe μ atoms can be described by the exponential $\exp(-\lambda_S t)$. We distinguish accordingly also between the temporal spectra of the secondary particles produced in processes initiated by the $p\mu$ or He μ atoms: decay electrons, cascade γ rays, and products of nuclear capture of muons by xenon. We note that He μ atoms produce only decay electrons, while the remaining particles, including an appreciable number of neutral ones, come only from $p\mu$ atoms and their derivative products.

The method used by us consists of determining, from an analysis of the temporal spectra of the neutral and charged particles, measured at a certain helium density, the contributions made to them by the processes caused initially by the produced $p\mu$ and He μ atoms. Such a separation is possible because the components of the spectrum differ both in shape and in their ratio in the spectra of the neutral and charged particles. A measure of the sought contributions can be the numbers of the $p\mu$ and He μ atoms produced towards the end of the de-excitation state (N_{10} and N_{20}). From (4), with allowance for the fact that the total number of stoppings is $N_{10}+N_{20}$, it follows that

$$W=W_HW_0=N_{10}/(N_{10}+N_{20}). \tag{6}$$

The value of W determined in this manner is the probability that a muon stopped in a mixture of hydrogen and helium will be captured just by the hydrogen, and the produced $p\mu$ atom will reach the ground state without giving up a muon to the impurity atom. This value, obtained from measurements at the different helium concentrations, yields a function $W(c)$ that contains information on the atomic capture of muons and their transfer (2a) from the excited $p\mu$ atoms. We note that to determine W it sufficies to know only the relative values of the numbers of the mesic atoms.

Another consequence of the analysis of the measured distributions is the answer to the question whether the rate λ_S depends on the helium density. Since, as determined in (5), $\lambda_1 = \text{const}(c)$, the presence of such a dependence would be a direct indication of the existence of the transfer (2b) from the ground state: $\lambda_{He} \neq 0$.

Let us see how the spectra $E(t)$ of the charged particles and $G(t)$ of the neutral particles are formed, with account taken of the inaccuracy in their separation. We assume that the charged and neutral particles are identified using a counter with a thin scintillator that encloses a working volume of the target in such a way that the particles landing in the principal detectors pass through the scintillator. Whether or not an identifying counter operates is an indication of the type of particle. Errors in the determination of the particle type are due to two main causes: incomplete efficiency of registration of charged particles by this counter and (if the main detector registers a neutral particle) the landing of an accompanying charged particle on the identifying counter. The He μ atoms and $p\mu p$ molecules produce only decay electrons:

$$\begin{aligned}
G'(t)&=\varepsilon_2(1-\varepsilon_4)\lambda_0[N_2(t)+N_3(t)],\\
E'(t)&=\varepsilon_2\varepsilon_4\lambda_0[N_2(t)+N_3(t)],
\end{aligned} \tag{7a}$$

where ε_e is the probability of registration of the electron by the detector, and ε_4 is the probability of registration of an electron by an identifying counter. The $p\mu$ atoms produce decay electrons; after transfer of the muon to the xenon, several cascade γ rays are emitted; this is followed by intense capture of muons by the xenon nuclei with emission of several particles, both charged and neutral. This complicated picture becomes simpler because at $t > 0.4\,\mu$sec the time distributions for the $p\mu$ and $Xe\mu$ atoms become similar (5). This makes it possible to represent all the proceses in the $Xe\,\mu$ atom as occuring at one instant—at the time of annihilation of the $p\mu$ atom. It will be made clear by the exposition that follows that the ensuing error, by a constant factor, will be automatically taken into account in the normalization. We can now introduce the following parameters: the total efficiency ε_1 of registration of the vanishing of a $p\mu$ atom (including also as a result of muon decay) and the probability ε_3 that some particle will simultaneously be registered by the identifying counter. We then obtain for the spectra for the $p\mu$ and $Xe\,\mu$ atoms

$$G''(t)=\varepsilon_1(1-\varepsilon_3)(\lambda_0+\lambda_{Xe}')N_1(t),\quad E''(t)=\varepsilon_1\varepsilon_3(\lambda_0+\lambda_{Xe}')N_1(t). \tag{7b}$$

For our purposes it suffices to determine only the relative values of four factors made up of the parameters ε_1–ε_4. This can be done with the aid of normalization measurements with pure hydrogen and with a mixture of hydrogen and xenon. Assume that N_0 muons are stopped in the hydrogen. The decay electrons will be registered and form spectra

$$G(H_2)=\varepsilon_2(1-\varepsilon_4)\lambda_0N_0e^{-\lambda_0 t},\quad E(H_2)=\varepsilon_2\varepsilon_4\lambda_0N_0e^{-\lambda_0 t}.$$

Addition of xenon does not change the number of stoppings, but now the principal role is played by transferred to the xenon:

$$G(H_2+Xe)=\varepsilon_1(1-\varepsilon_3)(\lambda_0+\lambda_{Xe}')N_0e^{-\lambda_1 t}+\varepsilon_2(1-\varepsilon_4)\lambda_0B_3N_0(e^{-\lambda_0 t}-e^{-\lambda_1 t}),$$

$$E(H_2+Xe)=\varepsilon_1\varepsilon_3(\lambda_0+\lambda_{Xe}')N_0e^{-\lambda_1 t}+\varepsilon_2\varepsilon_4\lambda_0N_0B_3(e^{-\lambda_0 t}-e^{-\lambda_1 t}).$$

From these data we determine the factors of the exponentials. We redesignate them as follows:

$$\varepsilon_1(1-\varepsilon_3)(\lambda_0+\lambda_{Xe}')N_0=G_0,\quad \varepsilon_2(1-\varepsilon_4)\lambda_0N_0=\gamma E_0,$$

$$\varepsilon_1\varepsilon_3(\lambda_0+\lambda_{Xe}')N_0=\varepsilon G_0,\quad \varepsilon_2\varepsilon_4\lambda_0N_0=E_0.$$

The meaning of this notation is that G_0 and E_0 are the normalizations of the basic components in the spectra of the neutral and charged atoms, while the factors γ and ε determine the admixture of the other component in each spectrum. Now the factors of the exponential (7) can be replaced by G_0/N_0, $\varepsilon G_0/N_0$, E_0/N_0, and $\gamma E_0/N_0$; then, substituting for $N_i(t)$ their expression from (5), we can unify (7a) and (7b):

$$G(t)=\frac{N_{10}}{N_0}[(G_0-\gamma E_0B_S)e^{-\lambda_S t}+\gamma E_0B_Se^{-\lambda_0 t}]+\frac{N_{20}}{N_0}\gamma E_0e^{-\lambda_0 t},$$

$$E(t)=\frac{N_{10}}{N_0}[(\varepsilon G_0-E_0B_S)e^{-\lambda_S t}+E_0B_Se^{-\lambda_0 t}]+\frac{N_{20}}{N_0}E_0e^{-\lambda_0 t}, \tag{8}$$

$$B_S=B_2+B_3.$$

Approximation of the spectra of the neutral and charged particles by these expressions makes it possible to find the values of N_{10}/N_0, N_{20}/N_0, and λ_{He} [the last quantity enters in λ_S and via the latter in B_i—see Eq. (5)], and from the relative values of N_{10} and N_{20}, according to (6), follows the value of W for the given density.

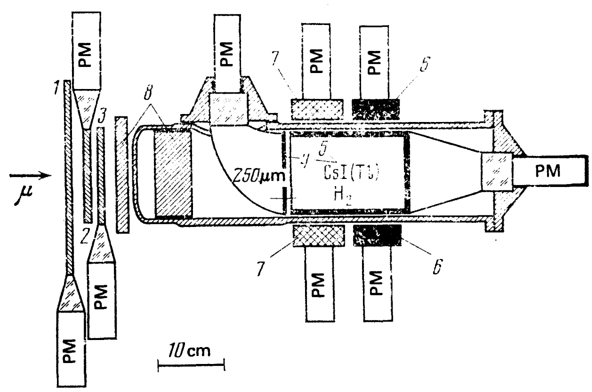

FIG. 2. Diagram of experimental setup: 1) counter that monitors the passage of the second meson; 2, 3, 4) monitoring counters; 5) vessel counter enclosing the working volume of the target; 6, 7) secondary-particle detectors with NaI and stilbene scintillators, respectively; 8) decelerating filter; PM) photomultiplier.

MEASUREMENTS AND DATA REDUCTION

The experiments were performed with the muon beam of the JINR synchrocylotron. A diagram of the setup with the gas target is shown in Fig. 2 and is described in detail in Ref. 8, while the system for selection and registration of the events is described in Ref. 9. Altogether three measurement runs were made, the principal difference between which was in the amount of xenon in the mixture at the measured helium densities. The procedure, however, also differed substantially: while the configuration of the apparatus was the same, targets of two different constructions were in fact used, with plastic scintillators in counters 4 and 5 (runs I and II) and with CsI crystals (III). As a result, the three groups of our data were obtained under substantially different conditions with respect to the background level and the efficiencies of counters 4 and 5. The fact that the results obtained from these data are in agreement indicates that the indicated factors do not introduce noticeable systematic errors. In each run, the measurements were made with an empty target, with pure hydrogen, with a mixture H_e + Xe, and with several batches of helium added (H_2 + Xe + He_i). In addition, for control purposes, we carried out measurements with pure helium. The data on the compositions of the mixtures are given in Table I. A measure of the xenon density is the rate λ'_{Xe}, whose ratio to the rate λ_0 is the value listed in the table.

Each detector was used to record both neutral secondary particles and charged particles. To reduce the data, we plotted respectively the "γ" and "e" spectra (depending on

TABLE I. Properties of the composition of the gas mixture in the experiment.

Run	Hydrogen pressure, atm	λ'_{Xe}/λ_0	Values of helium density
I	20.0	⩾8	0.46; 0.89
II	16.5	3	0.19; 0.38; 0.70; 1.03; 2.14
III	24.6	1	0.05; 0.11; 0.29

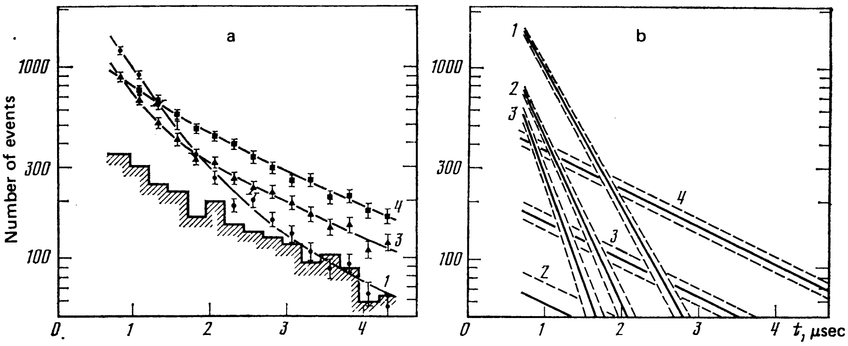

FIG. 3. a) Examples of temporal γ spectra (measured at the following helium densities: 1) 0; 2) 0.19; 3) 0.38; 4) 1.03) and b) their resolution into the components $\exp(-\lambda_S t)$ (steep straight lines) and $\exp(-\lambda_0 t)$ (straight lines with smaller slope). Dashed histogram—spectrum obtained with empty target. The spectra 2 and 3 are similar, therefore only one of them is shown in the left-hand side of the figure.

whether the operation of the counter 5 coincided in time) in the time interval 0.7–8.2 μsec in steps of 0.25 μsec.

Figure 3a shows by way of example the γ spectra obtained in run II at several helium densities, and Fig. 3b shows the results of a resolution of these spectra (after subtraction of the background) into the constituent exponentials $\exp(-\lambda_S t)$ and $\exp(-\lambda_0 t)$. It can be seen that with increasing amount of helium the contribution of the first exponential decreases and its slope increases; this is evidence of transfer of the muons to the helium from the ground state of the $p\mu$ atoms. At a density $c \approx 1$, this exponential is no longer observed. The contribution of the second exponential is a measure of the number of atoms produced Heμ, and becomes dominant at $c > 0.5$. Figure 4 shows the values of λ_S, as functions of the helium density, obtained for exposures $0 \leqslant c < 1$ in runs II and III. Despite the fact that each point is statistically close to the possible relation $\lambda_S = \text{const}$ $(c) = \lambda_1$, taken together they demonstrate a systematic increase of λ_S with increasing helium density.

The final reduction of the runs II and III consisted of simultaneously approximating all the spectra by expressions (8), in which the following substitutions were made

$$N_{10}{}^i = N^i_{st} W_i \quad \text{and} \quad N_{20}{}^i = N^i_{st}(1 - W_i)$$

and the following parameters were determined: λ_{He}—common for all the data; G_0, E_0, γ, and ε and the characteristics

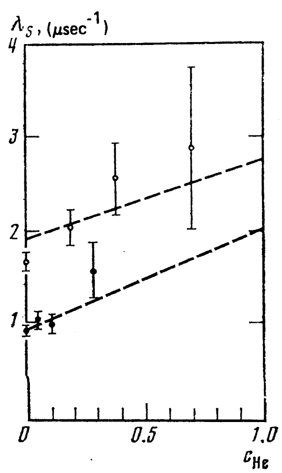

FIG. 4. Preliminary estimates of λ_S (O—run II, ●—run III) and the $\lambda_S(c)$ dependences obtained for these runs after the complete data reduction.

of the background—common for each run; the factors W_i—individual for each point with density c_i. As for the set of parameters N^i_{st} (the number of muons stopped in the measurement i), they were determined after preliminary analysis, and the ensuing $N_{st}(c)$ dependence was described by the formula

$$N_{st.}(c) = N_0(1 + Sc).$$

In the case when the stopping thickness of the target was much less than the range scatter of the beam, the factor S in this relation should be a constant equal to the ratio of the stopping abilities of helium and hydrogen: $S = S_0 = \text{const}$. Under our conditions it turned out that S depends, albeit weakly, on the density and changes from $S(0) = 1.15\, S_0$ to $S(c > 1) = S_0$. This may be due to the dependence of the efficiency of registration of the stoppings of the muons on the amount of matter in the target (this need not be clarified in detail, since we are interested in the ratio of the numbers of the produced mesic atoms, and not their total number). In the final reduction, the number of stoppings is described by a function of c and $S(c)$, with the $S(c)$ dependence obtained empirically.

The reduction of the data of run I has anomalies connected with the overwhelming role, at a xenon density $\sim 10^{-3}$, of muon transfer from the $p\mu$ atoms to the xenon, compared with the other competing processes of the second stage (the influence of the xenon on the de-excitation at this density can be neglected as before). In this case, the temporal spectrum of the γ rays is a steep exponential $\exp(-\gamma_S t)$ with $\lambda_S \gg \lambda_0$, whose shape remains practically unchanged when the helium density is changed. Some of the γ rays enter in the registration interval and produce at its starting section a sharply pronounced peak. The substantial difference between the shapes of the γ-ray spectra and the decay electrons from Heμ atoms makes it possible to separate them reliably, and for the particle yields we have in analogy with (8)

$$G = G_0(N_{st}/N_0)W, \quad E = E_0(N_{st}/N_0)(1 - W),$$

from which we obtain the value of W for the given concentration.

RESULTS OF EXPERIMENT

The results of a joint reduction of the data from the three runs are the following:

TABLE II. Measured values of W.

c_i	W_i	c_i	W_i
0.05	0.87±0.03	0.46	0.25±0.02
0.11	0.70±0.04	0.70	0.13±0.02
0.19	0.50±0.02	0.89	0.12±0.02
0.29	0.33±0.05	1.03	0.06±0.02
0.38	0.28±0.03	2.14	—

1. We obtained the rate of transfer of muons from $p\mu$ atoms in the ground state to helium (referred to the density of the liquid hydrogen):

$$\lambda_{He} = (3.6 \pm 1.0) \cdot 10^7 \text{ sec}^{-1}.$$

The value obtained agrees well with the rate calculated in Ref. 6 under the assumption that there exists a molecular mechanism of charge exchange of the $p\mu$ atoms with the helium atoms, $\lambda_{He}^{theor} = 4.4 \cdot 10^7 \text{ sec}^{-1}$. The dashed lines in Fig. 4 show plots of $\lambda_S(c)$ obtained by recalculating the obtained rate λ_{He} to the values of the hydrogen density in these runs. We recall that the points in these figures are the preliminary estimates of λ_S, determined individually for each helium density only from the argument of the corresponding exponential, without allowance for the constraints imposed on the spectra by expression (8). It can be seen that the final result agrees with this estimates, thus attesting to the correctness of the reduction.

2. The values of W were measured for different helium densities (the results are listed in Table II and shown in Fig. 5 in the form of points). It is of interest to compare them with the analogous relation for π^- mesons[4] (curve π on Fig. 5):

$$W_\pi(c) = W_H^\pi(c) W_{cap}^\pi(c),$$

where W_H^π is the probability of pion landing on the hydrogen [in analogy with $W_H(3)$ for the muon], while W_{cap}^π is the probability that the pion will be captured in the $p\mu$ atom by a proton and will not be recaptured by the helium. As can be seen from Fig. 5, the points for the muons lie much lower:

$$W(c) < W_\pi(c) \text{ or}$$

$$W_H(c) W_0(c) < W_H^\pi(c) W_{cap}^\pi(c) \quad (c > 0).$$

We assume that the probabilities of the atomic capture of π^- and μ^- mesons in the mixture of hydrogen with helium are described by the same relations, i.e., $W_H(c) = W_H^\pi(c)$. There are no direct experimental data on this question, but the calculations of the characteristics of the deceleration and capture of mesons at hydrogen and in hydrogen-helium mixtures[10] show that the total probabilities (i.e., for the entire energy interval of the slowing-down particle) of atomic capture for the π and μ mesons are practically equal. Thus, it follows from the preceding inequality that

$$W_0(c) < W_{cap}^\pi(c), \quad (c > 0),$$

which indicates the existence of a more intense transfer of the mesons from the excited levels of the $p\mu$ atoms than the pion transfer. The reason for the difference is apparently the nuclear capture of the pions from the nS state of the $p\pi$ mesic atom, which competes with the transfer.

For a quantitative description of the $W(c)$ dependence for muons, we used the parametrization of Ref. 4:

$$W_\mu(c) = W_H W_0 = -[(1 + Ac)(1 + \Lambda_\mu c^{4/3})]^{-1}. \quad (9)$$

Approximation of our data by this function, using the value $A = 1.84$ from Ref. 4, yielded for the muons a transfer constant

$$\Lambda_\mu = 3.8 \pm 0.3,$$

which is noticeably higher than the corresponding constant $\lambda_\pi = 1.84 \pm 0.09$ for pions.

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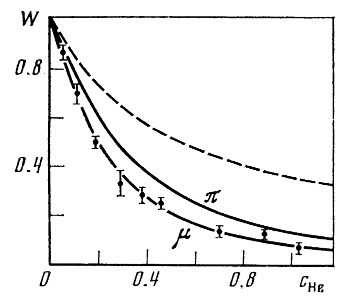

FIG. 5. Measured values of W_i and their approximation (μ) by Eq. (9); π) the function $W_\pi(c)$ from Ref. 4, dashed—the function $W_H^\pi(c)$ from Ref. 4.

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