Negative magnetoresistance in n-type germanium and its analysis on the basis of the theory of quantum corrections to the conductivity

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It is shown that the experimental results on negative magnetoresistance of germanium doped with antimony at densities $(2.7–52)\times 10^{17}$ cm^{-3} can be described by a new theory of this phenomenon, involving the influence of the magnetic field on the quantum corrections to the conductivity. Three types of corrections are considered, connected with the following: localization of the noninteracting electrons, b) Coulomb repulsion of the electrons; c) allowance for spin splitting. The relaxation time of the phase of the electron wave function and the dependence of this time on the electron density and temperature are estimated. It is found that this time is determined by the damping time of the single-electron excitations at short mean free paths and by the frequency of the electron-phonon collisions. The diffusion anisotropy coefficient determined from the angular dependences of the magnetic conductivity in a weak magnetic field exceeds by two or three times the value for single-valley germanium at the same antimony density, and is close in value to the effective-mass anisotropy coefficient.

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In connection with a new theory[1–5] that takes into account the quantum corrections to the kinetic coefficients, it became possible to explain exhaustively a number of experimental facts concerning, in particular, the dependence of the galvanomagnetic coefficients on the temperature, magnetic field, and electron density in strongly doped semiconductors at low temperatures. One of the most noticeable phenomena for which no satisfactory explanation was found for a long time is the anomalous negative magnetoresistance in semiconductors. In Refs. 2–4 is proposed a new theoretical explanation of the negative magnetoresistance, involving allowance for the influence of the magnetic field on the quantum corrections to the conduction.

In Ref. 6 are cited experimental data (see Figs. 1 and 3 of Ref. 6) on negative magnetoresistance of germanium doped with antimony, with electron density $n = (3.7–52)\times 10^{17}$ cm^{-3}, as well as preliminary results of their analysis on the basis of the theory of quantum corrections (*ibid.*, Figs. 2a and 2b). In the present paper we discuss in greater detail the analysis procedure used in Ref. 6, and present results of an investigation of the anisotropy of the negative magnetoresistance and new conclusions concerning the contribution made to the magnetoconductance by the interaction. The figures of Ref. 6 will hereafter be accompanied by the letter A (Fig. A1, etc.). The basic characteristics of the samples are listed in the table. The planes of the samples were perpendicular to the $\langle 111\rangle$ crystallographic direction, and the current direction is indicated in the table. The experimental change $\Delta\sigma^E(H)$ of the conductivity in a magnetic field was calculated from measurements of the magnetoresistance and of the Hall *emf* in the following manner:

$$\mathbf{H}\perp\mathbf{J},\quad \Delta\sigma_{xx}{}^E(H)=\frac{\rho_{xx}{}^H}{(\rho_{xx}{}^H)^2+(\rho_{xy}{}^H)^2}-\frac{1}{\rho_0}$$

$$\mathbf{H}\parallel\mathbf{J},\quad \Delta\sigma_{xx}{}^E(H)=1/\rho_{xx}{}^H-1/\rho_0, \tag{1}$$

$$\rho_{xx}{}^H=E_x(H)/J_x,\qquad \rho_{xy}{}^H=E_y(H)/J_x,\qquad \rho_0=\rho_{xx}{}^H(H=0),$$

where J_x, E_y, and E_x are the components of the vectors of the current density $\mathbf{J}$ and of the measured electric field $\mathbf{E}$ along the laboratory frame. Relations of the type (1) between the conductivity tensor σ_{rp} and the resistivity tensor ρ_{rp} are valid in the general case only for a spherical equal-energy surface in cubic crystals. In the particular cases $\mathbf{H}\perp\mathbf{J}$ and $\mathbf{H}\parallel\mathbf{J}$ for the orientations of $\mathbf{J}$ indicated in the table, however, the relations (1) are also valid.

The experimental data were analyzed under the assumption that

$$\Delta\sigma^E(H)=\Delta\sigma^C(H)+\sum_q \Delta\sigma^q(H),$$

where the first term is the classical magnetoconductivity, and the second is the sum of the different quantum corrections (the symbol q designates the type of correction). The theoretical functions $\Delta\sigma^C(H)$ and $\Delta\sigma^q(H)$ were calculated using the experimental values of ρ_0 and of the Hall mobility

$$\mu_H=(\rho_{xy}{}^H/H\rho_{xx}{}^H)_{H\to 0}.$$

The condition for the applicability of the theory is

$$k_F l>1, \tag{2}$$

TABLE I. Basic characteristics of the investigated samples at $T = 4.2$ K.

Sample	Orientation of x and $\mathbf{J}$	$n=1/eR_H$, 10^{17} cm^{-3}	σ_0, $\Omega^{-1}\cdot$cm^{-1}	ε_F, K	$k_F l$	λ	C
1	$11\bar{2}$	3.7	58.5	32.4	5.5	2	1.5
2	$1\bar{1}0$	4.3	74.1	38.4	6.6	–	–
3	$11\bar{2}$	5.5	87.9	45.6	7.3	1.8	1.8
4	$1\bar{1}0$	11	200	72.9	13	1	2.3
5	$11\bar{2}$	52	954	203	37	0.5	3.3

Note: R_H is the Hall coefficient

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where l is the mean free path and k_F is the wave vector of an electron with a Fermi energy ε_F. The parameter $k_F l$ can be expressed in terms of the diffusion coefficient D or the conductivity σ as follows:

$$k_F l = 3Dm^*/\hbar = {}^3/_2\sigma/G_0 k_F,$$

where $G_0 = e^2/2\pi^2\hbar = 1.23\cdot10^{-5}\ \Omega^{-1}$. Here and elsewhere the condition $\varepsilon_F \gg kT$ is assumed satisfied. It can be seen from the table that the criterion (2) is satisfied for all the samples.

We took into account the following types of quantum correction: the conductivity change $\Delta\sigma^L > 0$ due to the weak localization of the electrons[2,3]; the contribution $\Delta\sigma^I$ connected with the electron interaction[3]; the magnetoconductivity $\Delta\sigma^S < 0$ due to the spin interaction of the electrons.[5]

Using a common symbol q in place of the indices L, I and S that identify the type of correction, we can write down the theoretical dependences of the corresponding magnetoconductivities in the general form[2,3,5]:

$$\Delta\sigma^q(H) = a_q G_0 l_H^{-1} F_q(b_q H), \tag{3}$$

where $l_H = (\hbar c/eH)^{1/2}$ is the magnetic length, a_q, b_q and F_q are respectively coefficients and a function that depend on the type of quantum correction:

$$a_L = 1-\beta(\lambda), \quad F_L(x_L) \equiv F(1/\delta) \equiv f_3(x),$$
$$b_L H = x_L = 4l_L^2/l_H^2, \tag{4}$$
$$a_I = -\lambda, \quad F_I(x_I) \equiv \varphi_3(x_I)/2\pi,$$
$$b_I H = x_I = 4l_I^2/l_H^2, \tag{4a}$$
$$a_S = -R(z), \quad F_S(x_S) = g_3(x_S)/2(\pi x_I)^{1/2},$$
$$b_S H = x_S = g\mu_B H/kT. \tag{4b}$$

The coefficients a_L and a_I are connected via the electron interaction constant λ. For Coulomb repulsion of the electrons $\lambda > 0$ and $\Delta\sigma^I < 0$. The functions $F(1/\delta)$, $\varphi_3(x)$, $\beta(\lambda)$ are cited respectively in Refs. 2, 3, and 4; $1/\delta \equiv x_L$, $g_3(x) = G(x) - G(0)$; the function $G(x)$ was introduced in Ref. 5;

$$R(z) = z^{-1}\ln(1+z), \tag{5}$$

where $z = 4k_F^2 r_0^2 = \pi a_B k_F$, r_0 is the screening radius, a_B is the electron Bohr radius, and l_L and l_I are the diffusion lengths:

$$l_L = (D\tau_\varphi)^{1/2}, \quad l_I = (D\hbar/2\pi kT)^{1/2}, \tag{6}$$

τ_φ is the relaxation time of the electron wave-function phase[3]; μ_B is the Bohr magneton, and g is the spin splitting factor for the electrons. The coefficient 2π in (4a) accounts for the errata in Ref. 3.[1)]

It is shown in Refs. 2 and 3 that $\Delta\sigma^L(H)$ and $\Delta\sigma^I(H)$ have an asymptotic value $\sim H^{1/2}$ in the strong fields. For a comparison of theory with experiment it turned out to be quite useful to know also the second term of the expansion in $H \to \infty$, as is evidenced, e.g., by analysis of the measured negative magnetoresistance in the inversion channel of a silicon field-effect transistor.[7]

$$f_3(x) \to 0.605 - 2/x^{1/2}, \quad \varphi_3(x) \to \pi f_3(x). \tag{7}$$

The asymptotic relations (7) are valid at values $x \gtrsim 100$ (accurate to 6%), whereas the first term of the expansions (7), corresponding to $\Delta\sigma \sim H^{1/2}$, is valid only at $x \gtrsim 3\times10^3$ (with the same accuracy). Thus, for all three types of correction we have as H

$$\Delta\sigma^q \to a_q G_0 (c_q'/l_H - c_q''/l_q),$$

where l_L and l_I are determined by (6), $l_S = l_I$, the coefficients are

$$c_L' = 0.605, \quad c_I' = 0.302, \quad c_S' = 0.25(g\hbar/Dm_0)^{1/2},$$
$$c_L'' = 1, \quad c_I'' = {}^1/_2, \quad c_S'' = \theta/4\pi, \quad \theta = 1.3^{\,2)},$$

and m_0 is the mass of the free electrons.

Consequently, at $x > 100$ the functions $\Delta\sigma^q(H^{1/2})$ are described by lines whose slopes in the case of the corrections $\Delta\sigma^L$ and $\Delta\sigma^I$ are determined by the coefficients a_L and a_I, i.e., by the interaction constant λ (4), (4a). The intercepts of the lines with the ordinate axis are the conductivities, which differ only by a numerical coefficient from the corresponding temperature correction

$$\Delta\sigma^q(T) = \sigma(T) - \sigma(0).$$

Actually,[5,8]

$$-a_L G_0/l_L = [1-\beta(\lambda)]G_0/l_L = \Delta\sigma^L(T),$$
$$-a_I G_0/2l_I = {}^4/_3\lambda G_0\theta(kT/2D\hbar)^{1/2} = 2\Delta\sigma^I(T)^{\,3)},$$
$$(-a_S G_0/l_I)(\theta/4\sqrt{\pi}) = {}^1/_2 R(z) G_0\theta(kT/2D\hbar)^{1/2} = -{}^1/_2\Delta\sigma^S(T).$$

In multivalley semiconductors (if intervalley hopping of electrons can be neglected) it is necessary to sum the magnetoconductivity for the different valleys, with allowance for the orientation of the axes of the ellipsoid axes of the equal-energy surface $\varepsilon(\mathbf{k})$.[2,3] Let the unit vectors $\mathbf{v}$, $\mathbf{r}$, $\mathbf{p}$, and $\mathbf{h}$ denote the following directions in the crystal: $\mathbf{v}$-along the singled-out axis of the ellipsoid $\varepsilon(\mathbf{k})$, $\mathbf{r}$—the direction of the current $\mathbf{J}$; $\mathbf{p}$—the direction of the measured electric field $\mathbf{E}$; $\mathbf{h}$—the direction of the magnetic field $\mathbf{H}$, and let r_p, h_ν, r_ν, h_r, etc. be the cosines of the angles between the corresponding vectors. We can then write for one of the valleys labeled by the index (ν):

$$\alpha_J^{(\nu)} \equiv D_{rp}^{(\nu)}/D_t = r_p - (1-1/K)r_\nu p_\nu,$$
$$\alpha_H^{(\nu)} \equiv (D_c^{(\nu)}/D_t)^2 = 1/K + (1-1/K)h_\nu^2, \tag{8}$$
$$\alpha_s^{(\nu)} \equiv (g^{(\nu)}/g_t)^2 = \varkappa^2 + (1-\varkappa)h_\nu^2,$$

where $D_{rp}^{(\nu)}$ are the components of the diffusion current, $D_c^{(\nu)}$ is the effective value of the diffusion coefficient in the magnetic field $\mathbf{H}$ (Ref. 3) and depends on the direction of $\mathbf{H}$ and on the ratio $K = D_t/D_l = \mu_t/\mu_l$ in the same manner as the cyclotron frequency $\omega_c^{(\nu)}$, with K replaced by $K_m = m_l^*/m_t^*$:

$$(\omega_c^{(\nu)})^2 = \omega_t^2[1/K_m + (1-1/K_m)h_\nu^2].$$

The indices l and t denote the principal components of the tensors along (r_ν, $h_\nu = 1$) and across (r_ν, $h_\nu = -0$) the singled-out axis $\mathbf{v}$ of the ellipsoid; $\varkappa = g_t/g_l$; μ_t, μ_l are the drift mobilities; m_t^* and m_l^* are the effective masses. Using these symbols and summing the magnetoconductivities of N valleys, we obtain for the classical magnetoconductivity

$$\Delta\sigma_{rp}^{c}=-\frac{\sigma_t}{N}\sum_{(\nu)=1}^{N}\left(\alpha_J^{(\nu)}-\frac{h_r h_p}{K\alpha_H^{(\nu)}}\right)Q\left(x_C\sqrt{\alpha_H^{(\nu)}}\right), \tag{9}$$

where

$$Q(x)=x^2/(1+x^2),\qquad x_C=\mu_t H,\qquad \mu_t=\mu_H(2K+1)/(K+2).$$

For the quantum corrections we obtain on the basis of Refs. 2, 3, and 4 and expressions (4)–(4b)

$$\Delta\sigma_{rp}{}^{q}=a_q G_0 l_H{}^{-1}F_{rp}{}^{q}, \tag{10}$$

$$F_{rp}^{q}(x_q)=K^{1/2}\sum_{(\nu)=1}^{N}\alpha_J^{(\nu)}\left(\alpha_H^{(\nu)}\right)^{1/4}F_q\left(x_q\sqrt{\alpha_H^{(\nu)}}\right) \tag{10a}$$

for the corrections $\Delta\sigma^L$ and $\Delta\sigma^i$ and

$$F_{rp}^{S}(x_S)=K^{1/2}\sum_{(\nu)=1}^{N}\alpha_J^{(\nu)}F_S\left(x_S\sqrt{\alpha_S^{(\nu)}}\right) \tag{10b}$$

for the correction $\Delta\sigma^S$. The quantities a_q, x_q, and F_q in (10)–(10b) are the same as in (4)–(6), but with $D=D_t$ and $g=g_l$.

In antimony-doped germanium, the relaxation time of the interalley scattering τ_v is large ($\tau_v\approx 4\cdot 10^{-11}$ sec, Ref. 9) and, as shown by estimates of the ratio $D_c^{(\nu)}\tau_v/l_H^2$, expressions (10)–(10b) can be used for the entire magnetic-field range used in the experiments. In a weak magnetic field

$$\Delta\sigma_{rp}^{q}|_{H\to 0}=c_q{}^{0}H^2 a_q G_0 K^{1/2}\left[\sum_{(\nu)=1}\alpha_J^{(\nu)}\alpha_{H,S}^{(\nu)}\right], \tag{11}$$

where the numerical coefficient c_q^0 is determined by the asymptotic form of the functions $F_q(x_q)$ as $x_q\to 0$.[2,3,5] The last factor in (11) determines the anisotropy of the magnetoconductivity in a weak magnetic field. Since in the case of repulsion between electrons ($\lambda>0$) we have $\Delta\sigma^i<0$, just as $\Delta\sigma^S$, and both types of correction have similar dependences on the magnetic field, it is precisely the investigation of the anisotropy which could identify the type of correction that appears in each particular case. For example, for one of the valleys in a multivalley semiconductor we have as $H\to 0$

$$\Delta\sigma^{I}(H_t)/\Delta\sigma^{I}(H_l)=1/K<1$$

in the case when the magnetoconductivity is determined by the Coulomb interaction, and

$$\Delta\sigma^{S}(H_t)/\Delta\sigma^{S}(H_l)=\varkappa^2>1$$

for the spin interaction.

For the current orientations indicated in the table, the coefficients $\alpha_J^{(\nu)}$ and $\alpha_H^{(\nu)}$ in (9) take on the values

$$\mathbf{H}\|\langle 111\rangle\perp\mathbf{J}\left\{\begin{array}{ll}\alpha_H^{(1)}=1, & \alpha_H^{(2-4)}={}^1/_9(1+8/K),\\ \alpha_J^{(1)}=1, & \sum_{(\nu)=2}^{4}\alpha_J^{(\nu)}={}^1/_3(5+4/K),\end{array}\right. \tag{12a}$$

$$\mathbf{H}\|\langle 110\rangle\|\mathbf{J}\left\{\begin{array}{ll}\alpha_H^{(1,2)}=1/K, & \alpha_H^{(3,4)}={}^1/_3(2+1/K),\\ \alpha_J^{(1,2)}=1, & \alpha_J^{(3,4)}={}^1/_3(1+2/K).\end{array}\right. \tag{12b}$$

Figure A1 shows plots of $\Delta\sigma^E(H)-\Delta\sigma^C(H)$ for several samples with different antimony densities ($\mathbf{H}\|\langle 111\rangle$, $T=4.2$ K). At $H\lesssim 30$ kG we have $\Delta\rho/\rho_0<0$, and consequently the correction due to the weak localization of the electrons ($\Delta\sigma^L>0$) makes the main contribution to the change of the conductivity, i.e., in a weak magnetic field we have

$$\Delta\sigma^{E}-\Delta\sigma^{C}=\Delta\sigma^{L}. \tag{13}$$

The functions $\Delta\sigma^L(H)$ in (10), (10a) and (14a), calculated for different samples, are shown in the same figure. The values of τ_φ obtained from the condition (13) are shown in Fig. A2a.

It can be seen from (10) that the values of the magnetoconductivity for different samples, normalized to the value $G_0 l_H^{-1}(\Omega^{-1}\cdot\mathrm{cm}^{-1})=1.52(H\,[\mathrm{kG}])^{1/2}$ and plotted as functions of $\Delta\sigma^E-\Delta\sigma^C$ should form, if (13) is satisfied and the values of τ_φ are correctly determined, a family of parallel curves shifted along the ordinate axis by the value of the coefficient $a_L=1-\beta(\lambda)$. Figure 1 shows plots of $\Delta\sigma_E(H)-\Delta\sigma^C(H)$, reduced by this method. The solid line in the same figure represents the function $F_{xx}^{L}(x_L)$, [Eq. (10a)] multiplied by 0.5, at values $\alpha_J^{(\nu)}$ and $\alpha_H^{(\nu)}$ corresponding to (12a). As seen from the figure, the function (10a) with a coefficient $a_L=0.5$ describes well the experimental results for all the samples with exception of the region of strong

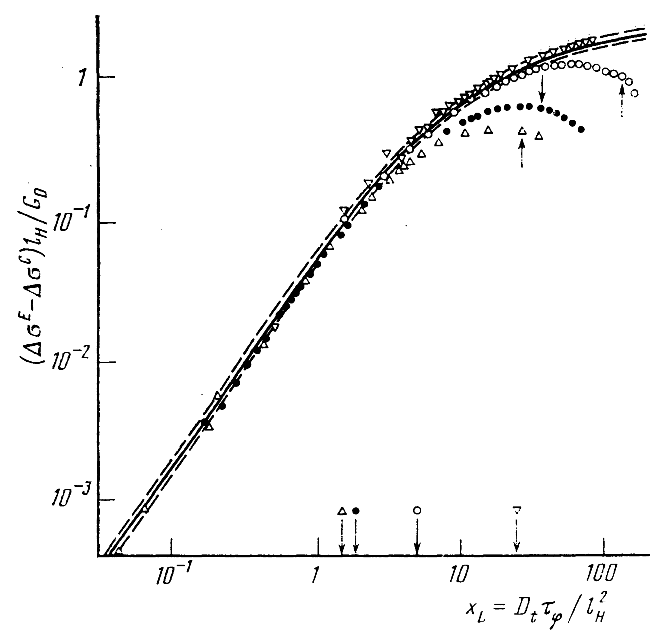

FIG. 1. Plots of $(\Delta\sigma^E-\Delta\sigma^C)l_H/G_0$ vs x_L at $T=4.2$ K, $\mathbf{H}\|\langle 111\rangle$ for the following samples: Δ—1, ●—3, ○—4, ∇—5, Solid and dashed lines—calculation of the function $(0.5\pm 0.05)F_{xx}{}^{L}(x_L)$ (10a). The values of x_L at $H=1$ kG are shown by the arrows with the symbols. The arrows at the curves correspond to the magnetoconductivity when condition (14) is satisfied.

magnetic fields. We are unable to estimate the electron interaction constant λ on the basis of the value $\beta(\lambda) = 1 - a_L = 0.5$, since there is no calculation of the function $\beta(\lambda)$ for positive values of λ and $\beta > 0.017$ (Ref. 4).

As seen from Fig. 1, the experimentally observable magnetoconductivity in strong magnetic fields deviates from the theoretical $\Delta\sigma^L(H)$ dependence. We have proposed[6] that this discrepancy may be due to the need to take into account the contribution $\Delta\sigma^I(H) < 0$ (Ref. 3) connected with the Coulomb repulsion of the electrons. Indeed, the difference between the experimental and calculated curves in Fig. 1 is well described by the ratio $\Delta\sigma^I(H)/G_0 l_H^{-1}$(4a), (10), (10a), (12a) with the interaction-constant values listed in the table (these values of λ are 2π times larger than assumed in Ref. 7 in view of the errata in Ref. 3). The decrease of λ from 2 to 0.5 with increasing density is in qualitative agreement with the relation[3]:

$$1/\lambda = 1/\bar{\lambda} + C, \qquad C = \ln(\gamma \varepsilon_F/\pi kT),$$

where $\bar{\lambda}$ is the nonrenormalized interaction constant and $\ln\gamma = 0.577$. However, the quantitative estimates of the constant $\bar{\lambda}^{-1} = \lambda^{-1} - C$ at the values $0.5 < \lambda < 2$ and at the values of C indicated in the table lead to negative values of $\bar{\lambda}$, and this contradicts the initial electron-repulsion assumption. We assume as a result that the contribution $\Delta\sigma^I(H)$ must be taken into account in a strong magnetic field, but it is insufficient to explain all the discrepancies between the calculated and experimental relations in Fig. 1.

The decrease of the magnetoconductivity in strong magnetic fields may be connected also with the correction $\Delta\sigma^S$ (Ref. 5). Indeed, when the arguments of the functions $F_L(x_L)$ [Eqs. (4), (6)] and $F_S(x_S)$ [Eq. (4b)] are compared, it can be seen that the contribution of $\Delta\sigma^S$ increases in comparison with $\Delta\sigma^L$ with decreasing conductivity $\sigma_0(H=0)$, in qualitative agreement with the results shown in Fig. 1. To estimate $\Delta\sigma^S$ we used the values $g_l = 0.9$, $g_t = 1.9$, $a_B = 42$ Å, $z = \pi a_B k_F/N$ and the approximation $g_3(x_s) \approx 0.053 x^{S2}$ (Ref. 5), which undoubtedly overestimates $g_3(x_S)$ at $x_S > 0.5$. The calculations made for sample 1 on the basis of relations (4b), (5), and (11) have shown that $\Delta\sigma^S$ does not exceed 10% of the contribution $\Delta\sigma^I$ in the entire magnetic-field range (if $\lambda \approx 1$). Consequently, in our case the correction $\Delta\sigma^S$ can be disregarded, since the relative contribution of $\Delta\sigma^S$ for the remaining samples is even smaller.

The discrepancy between theory and experiment in strong magnetic fields is possibly due to the fact that in samples 1–4 at $H \approx 20$–30 kG the condition

$$\hbar\omega_c(\nu) \approx \varepsilon_F \tag{14}$$

is satisfied for one of the valleys ($\mathbf{v}\langle 111\rangle \| \mathbf{H}$), and the theory developed in Refs. 2, 3, and 5 is not applicable.

In Figs. A2a, b the experimental values of τ_φ are compared with the theoretical phase-relaxation time values calculated in Refs. 10 and 3:

1) with the damping time $\tau_{ee}^{(2)}$ of the quasistatic excitations[10] in the presence of a large number of defects that limit the mean free path:

$$\frac{\hbar}{\tau_{ee}^{(2)}} = \frac{\sqrt{6}}{4}\frac{\varepsilon^{3/2}}{(k_F l)^2}\left(\frac{\tau}{\hbar}\right)^{1/2}, \tag{15}$$

where $\varepsilon \approx kT$ is the electron energy reckoned from F and is the momentum relaxation time;

2) with the relaxation time τ_φ'' of the phase of the wave function under conditions of quasi-elastic electron-phonon interaction.[3] In our case ($s\varepsilon_F\tau_{\rm ph}/\hbar v_F > 1$ (s is the speed of sound, v_F is the Fermi velocity, $\tau_{\rm ph}^{-1}$ is the frequency of the electron-phonon collisions), therefore $\tau_\varphi'' \sim \tau_{\rm ph}$ (Ref. 3). The values of $\tau_{\rm ph}$ were estimated in the following manner:

$$\hbar/\tau_{\rm ph} = \hbar(\tau_{\rm ph}^0)^{-1}(\varepsilon_F/kT)^{1/2}, \tag{16}$$

where $(\tau_{\rm ph}^0)^{-1}$ is the frequency of the electron-phonon collisions in a nondegenerate semiconductor. (The measured $\tau_{\rm ph}^0$ in germanium at a carrier density $n < 10^{12}$ cm^{-3} are given in Ref. 11 for the temperature range of interest to us.)

Figure A2,a shows the values of $\hbar\tau_{ee}^{(2)}$(15) and $\hbar/\tau_{\rm ph}$(16) calculated for the investigated samples. It can be seen that the experimental values of τ_φ at low electron densities are close to the values of $\tau_{ee}^{(2)}$, and at a density $n \approx 5\times 10^{18}$ cm^{-3} they are close to the value of $\tau_{\rm ph}$. The dashed line in Fig. A2a is the sum $\hbar/\tau_{ee}^{(2)} + \hbar/\tau_{\rm ph}$, which describes well the experimental $\hbar/\tau_\varphi(\varepsilon_F)$ dependence. The theoretical plot of $\hbar/\tau_{ee} \times (T)$ [Eq. (15] shown in Fig. A2b and calculated for sample 3 is also in good agreement with the experimental $\hbar\tau_\varphi(T)$ obtained for this sample on the basis of the $\Delta\rho/\rho_0(H^2)$ dependence as $H \to 0$ (Fig. A3).

It must be noted that in all the calculations we used an anisotropy coefficient $K = K_m = m_l^*/m_t^*$($K_m = 19.3$ for n-Ge), whereas it is known that the ratio $K = D_t/D_l = \mu_t/\mu_l$ is much less than K_m in strongly doped semiconductors, owing to the anisotropy of the relaxation time τ_l/τ_t (Ref. 12). This value $K = 19.3$ was assumed by us on the basis of an analysis of the experimental results of an investigation of the anisotropy of the negative magnetoresistance. In a weak magnetic field, the ratio of the transverse $\Delta\sigma_\perp^L(\mathbf{H}\perp\mathbf{J})$ and longitudinal $\Delta\sigma_\|^L(\mathbf{H}\|\mathbf{J})$ magnetoresistances depends only on the anisotropy coefficient K. In our case we have in accord with (11) and (12a, b)

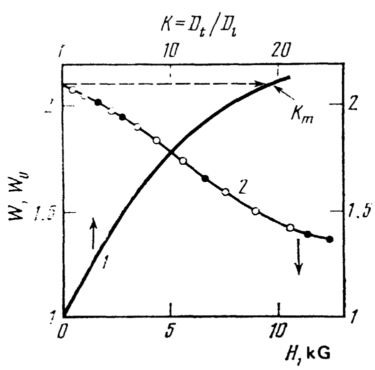

FIG. 2. Plots of: 1 — $W_0(K)$ (17), calculation; 2 — $W(H)$ (18), experiment, sample 2, $T = 4.2$ K, dark circles—from the angular dependences, H in the (112) plane, light circles—from the field dependences $(\Delta\sigma^E - \Delta\sigma^C)_{\perp,\|}(H)$. The dashed line joins the values of $W(H\to 0) = W_0(K)$.

$$W_0(K)=(\Delta\sigma_\perp{}^L/\Delta\sigma_\parallel{}^L)_{H\to 0}={}^1/_3(8K^2+11K+8)/(K^2+7K+1). \quad (17)$$

The $W_0(k)$ dependence is shown in Fig. 2, which shows also the experimental dependence

$$W(H)=(\Delta\sigma^E-\Delta\sigma^C)_\perp/(\Delta\sigma^E-\Delta\sigma^C)_\parallel \quad (18)$$

on the magnetic field for sample 2, obtained from measurements of the angular dependences of the magnetoresistance and of the Hall constant. It can be seen from the figure that as $H\to 0$ the value of $W(H)$ tends to the value $W_0(K)\approx 2.1$ corresponding to $K\approx 20$. Kawabata[2] also used $K=20$ for a comparison of the theory with experiment, and obtained good agreement. The question why the negative magneto resistance has a stronger anisotropy than proposed by the theory[3] remains unclear.

On the other hand, the ratios of the longitudinal $\Delta\rho_\parallel^{(1)}$ and transverse $\Delta\rho_\perp^{(1)}$ negative magnetoresistance at $H\approx 1$ kG, which follow from the experimental data of Refs. 13 and 14 for "single-valley" germanium (elastically deformed along the ⟨111⟩ axis) with antimony density $n\approx 4\times 10^{17}$ cm^{-3} are close to $K\approx 5.5$ (Ref. 13) and $K\approx 7.6$ (Ref. 14), and are thus closer to the universally assumed values of the anisotropy coefficient at this electron density.[12] If account is taken of the possible influence of the correction $\Delta\sigma^S$ on the anisotropy of the negative magnetoresistance in a weak magnetic field, then the values of K in single-valley germanium should be even lower. Indeed, taking into account the quantum corrections of all three types, we obtain

$$(\Delta\rho_\parallel^{(1)}/\Delta\rho_\perp^{(1)})_{H\to 0}\approx(\Delta\sigma_\parallel^{(1)}/\Delta\sigma_\perp^{(1)})_{H\to 0}=W_0^{(1)}=KA, \quad (19)$$

where for the case $\lambda>0$ (electron repulsion)

$$A=[1-u(1+v)]/[1-u(1+vK\varkappa^2)],$$

$$u=\left|\frac{\Delta\sigma_\parallel{}^I}{\Delta\sigma_\parallel{}^L}\right|_{H\to 0}=\left|\frac{a_I}{2\pi a_L}\left(\frac{\hbar/\tau_\varphi}{kT}\right)^{1/2}\right|\ll 1,\qquad v=\left.\frac{\Delta\sigma_\parallel{}^S}{\Delta\sigma_\parallel{}^I}\right|_{H\to 0}\geqslant 0.$$

If $v=0$, then $A=1$ and W_0^1 of Eq. (19) yields the values of K cited above. Since $v\approx 0.5/\lambda$ for the samples of Refs. 13 and 14, and $K\varkappa^2\gg 1$ for n-Ge, we have

$$A\approx 1+uvK\varkappa^2>1$$

and the values of K determined from (19) can be even smaller. Thus, when the spin interaction of the electrons is taken into account, the discrepancy between the values of K in single-valley and four-valley germanium at the same electron density becomes even larger.

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[1)] Private communication from B. L. Al'tshuler and A. G. Aronov.
[b)] $\theta\approx 2.5$ in the paper by Al'tshuler and Aronov.[8]
[3)] If $\Theta\approx 2.5$, there is no coefficient 2 in the last part of the equation.

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