Electron diffusion in a quantizing magnetic field

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The transverse diffusion coefficient of a gas of electrons scattered by charged centers in a magnetic field quantized to the limit is calculated without the use of perturbation theory (the slow-electron scattering is either quasiclassical or resonant). The value of D is shown to depend nonmonotonically on the electron energy. The complicated character of this dependence in the case of slow collisions with attracting centers is due to the decisive role of the resonant scattering. It is found that slow electrons diffuse much more rapidly when scattered by attracting centers than by repelling ones.

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This paper deals with electron-gas diffusion, due to scattering by potential centers, in a plane perpendicular to a magnetic field. The magnetic field is assumed to be so strong that during the time between the scatterings the electrons manage to complete many revolutions around its force line. In a classical (nonquantizing) field the corresponding diffusion coefficient is the mean square of the transverse displacement of the Larmor-orbit centers of the electrons per unit time (Ref. 1, p. 295). If, for example, the Larmor radius is small compared with the effective radius of the action of the scattering centers, it is convenient to regard the orbit displacement as a drift in the electric field of these centers (Ref. 1, p. 308; Ref. 2). In a quantizing field, however, the classical concepts of Larmor motion are not applicable. Nonetheless, the character of the diffusion remains qualitatively the same as before[3,4] (see also Ref. 5). The meaning of the diffusion coefficient becomes in this case very clear if the Landau gauge $\mathbf{A} = H(0, x, 0)$ is used (the z axis is along the magnetic field $\mathbf{H}$ and the x axis is along the diffusion direction)[4]:

$$D = {}^1/_2 \left\langle \sum_{N', k_y', k'} \lambda^4 (k_y - k_y')^2 W(N, k_y, k \to N', k_y', k') \right\rangle . \quad (1)$$

Here, N, k_y, and k are Landau numbers, W is the probability of the indicated transition, and λ is the magnetic length. The angle brackets denote averaging over the initial states of the electrons. Inasmuch $-\lambda^2 k_y$ is the mean value of the x coordinate of the electron in the state (k_y), Eq. (1) can be interpreted as the mean square of the displacement of the electrons across the magnetic field per unit time.

The value of D was calculated for scattering in a quantizing magnetic field by many workers, in connection with finding the transverse magnetoconductance σ_{xx} (with which it is directly connected by the Einstein relation). It is well known that the use of perturbation theory leads in this case to a logarithmic divergence of σ_{xx} because of too fast a growth of the diffusion coefficient when the electron energy is decreased (see the review by Kubo *et al.*[4]). It was possible to go outside the framework of perturbation theory, however, only for the case of short-range centers with a characteristic dimension much smaller than λ.[6–8] (See Ref. 9 concerning the limit of applicability of Refs. 6–8 for slow collisions). In the case of long-range centers, however, such as charged ones whose screening radius is large compared λ, the scattering was calculated in the Born approximation (see, e.g., Ref. 10), which cannot be used for slow collisions. An exception is Ref. 11, where scattering of slow electrons by attracting charged centers is considered. The authors of that reference, however, confine themselves to citing some estimates, and have made a number of errors.

The present paper is devoted to calculation of the diffusion coefficient for scattering of electrons of arbitrary energy by charged centers in a magnetic field that is quantizing to the limit. It is assumed that the processes of scattering by different centers are independent of one another.[1)] To calculate the value of D [Eq. (1)] it suffices then to consider electron scattering by one center and to multiply the result by the total number of centers.

The Landau gauge is convenient for the derivation of the general equations. Yet it is not convenient for a concrete calculation of a collision with one center, since it does not take into account the symmetry of this single scattering act. Indeed, not one of the Landau numbers, N, k_y, and k is conserved in this case. However, scattering by a center whose potential is axisymmetric relative to the $\mathbf{H}$ direction conserves the projection of the electron angular momentum $(-M)$ on this direction. It is therefore natural to use a gauge in which M is a quantum number, namely $\mathbf{A} = \frac{1}{2}\mathbf{H} \times \mathbf{r}$ (Ref. 12). Its convenience was pointed out already in Refs. 4 and 5. In Ref. 4 was obtained a general expression for the diffusion coefficient in the representation of the M numbers in an extremely quantizing magnetic field, when all the electrons are concentrated on the lower Landau level. In the same paper, as well as in Ref. 5, it was shown that if the scattering by the long-range centers is quasiclassical then, even for an extremely quantizing magnetic field, the classical interpretation of diffusion is applicable within the framework of the treatment[1,2] of the drift of a wave packet in crossed electric and magnetic fields. The cited papers contain, however, no concrete results. These will be obtained below, using calculations for arbitrary electron energies, including those for which the quasiclassical approach to the determination of D is not applicable.

We consider scattering within the zeroth Landau band, neglecting all the higher bands. To this end it suffices to have

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the cyclotron energy much higher than the Bohr energy, i.e., $\lambda \ll a$, where a is the Bohr radius. These conditions are realized in semiconductors with narrow band gaps and in a number of astrophysical objects. We assume that the potential energy of the electron in the field of the center is of the form

$$V(r) = \pm (e^2/r) \exp(-r/r_s)$$

(the long-wave approximation of screening theory). The screening radius r_s is assumed to be much larger than λ. In addition, it is assumed to be isotropic. The last approximation ceases to hold for the case of degenerate electrons in a quantizing magnetic field, when the Fermi wave vector $k_F \lesssim 1/r_s$ (Refs. 13 and 14) (or, equivalently, $\lambda/a \lesssim (na^3)^{-3/8}$, where n is the electron density). We shall not consider this case.

Neglect of the virtual transitions to higher Landau bands makes the electron scattering by the center one-dimensional. This is most clearly seen in an axisymmetric gauge, in which the transverse motion of the electrons is then entirely determined by the magnetic field, and the scattering alters only the longitudinal motion. We denote by $F_M(z)$ the longitudinal wave function of an electron scattered with a definite value of M. This function describes scattering by the one-dimensional potential.

$$V_M(z) = \langle M | V(r) | M \rangle, \tag{2}$$

where the matrix elements is taken on the functions of the transverse motion (it can be said that the longitudinal motion is adiabatically slow compared with the transverse). In one-dimensional scattering the electron has only two possibilities—to be scattered forward and to be reflected. The corresponding scattering amplitudes $f_M^{\pm}(k)$, where k is the wave vector, are determined by the asymptotic behavior of the wave function:

$$F_M(z \to \pm\infty) = e^{ikz} + f_M^{\pm}(k) e^{\pm ikz}.$$

In terms of these amplitudes it is possible to express the diffusion coefficient $D(k)$ of electrons with longitudinal-motion energy $\hbar^2 k^2/2m$. This is easiest to do by carrying out a gauge transformation in Eq. (1). Expanding the wave functions contained in this equation in the Landau gauge in terms of the wave function in the axisymmetric gauge, we obtain after straightforward but cumbersome transformations the diffusion coefficient

$$D(k) = (\pi\hbar |k| \lambda^4 n_c/m)\,(d^+(k) + d^-(k)), \tag{3}$$

where n_c is the density of the scattering centers and

$$d^{\pm}(k) = \sum_{M=0}^{\infty} (M+1) |f_{M+1}^{\pm}(k) - f_M^{\pm}(k)|^2,$$

$$f_M^{\pm}(k) = -\frac{im}{\hbar^2 |k|} \int_{-\infty}^{\infty} dz\, e^{\mp ikz} V_M(z) F_M(z). \tag{4}$$

The summation is over the values $M = 0, 1, 2, \ldots$, corresponding to the zeroth Landau band. In Ref. 4 was obtained an analogous expression in terms of the phase shifts of scattered waves of different parity. Equations (3) and (4), however, are somewhat more lucid and convenient for calculations.

We note that from Eq. (1) it is possible to obtain also an expression for the diffusion coefficient of electrons scattered from an arbitrary state (Nk):

$$D = (\pi\hbar\lambda^4 n_c/m) \sum_{N'k'} \sum_{M=-\min(N,N')}^{\infty} |k'| \{ |f_{M+1} - f_M|^2 (N+M+1)^{1/2} (N'+M+1)^{1/2}$$

$$+ {}^1/_2 |f_M|^2 [((N+M)^{1/2} - (N'+M)^{1/2})^2$$

$$+ ((N+M+1)^{1/2} - (N'+M+1)^{1/2})^2] \},$$

where f_M are the amplitudes of the scattering $k \to k'$, with change $N \to N'$ of the number of the Landau band, of electrons with angular momentum projection $(-M)$. At $N = N' = 0$ we obtain Eq. (3).

At sufficiently large M, at which f_M changes little when M changes by unity, the summation in (3) can be replaced by integration. Introducing[4,9,15] at $M \gg 1$ a quasiclassical impact parameter $\rho = (2M)^{1/2}\lambda$ (which has the meaning of the distance, in a plane perpendicular to the field direction, within which electron density is mainly concentrated), we obtain for the contribution to $d^{\pm}$ from such M:

$${}^1/_2 \int |\partial f_M^{\pm}/\partial\rho|^2 \rho\, d\rho. \tag{5}$$

The one-dimensional potentials $V_M(z)$ take then at $M \gg 1$ the form

$$V_M(z) = \pm [e^2/(\rho^2+z^2)^{1/2}] \exp[-(\rho^2+z^2)^{1/2}/r_s], \quad M \gg 1. \tag{6}$$

We shall consider separately the cases of weak ($r_s \gg a$) and strong ($r_s \ll a$) screening. We shall also distinguish between attracting and repelling centers.

Weak screening, $r_s \gg a$

A. Attracting centers

We calculate first the quantity d^+ that describes the contribution made to the diffusion coefficient by the forward-scattering processes.

1) At sufficiently large k (here and elsewhere $k > 0$), the Born approximation is valid

$$f_M^+ = -(im/\hbar^2 k) \int_{-\infty}^{\infty} V_M(z)\, dz. \tag{7}$$

Substitution of (6) in (7) shows that at $\rho \gg r_s$ the amplitude f_M^+ decreases exponentially with increasing ρ. Therefore the main contribution to the sum d^+ are made by $\rho \lesssim r_s$. For these we have the estimate

$$f_M^+ \sim (1/ka) \ln(r_s/\rho),$$

from which it follows that the integral (5) diverges logarithmically at small ρ. This means that it is necessary to use in place of Eqs. (5) and (6), which are valid for large M, the more accurate expressions (2) and (4). It then becomes possible to carry out the summation (4) exactly, after which we get

$$d^+=2(ka)^{-2}\ln(2^{1/2}r_s/\lambda), \quad k\gg\ln(a/\lambda)/a. \tag{8}$$

This result agrees with that obtained in Ref. 10. Its validity is restricted to values of k at which the electron energy $\hbar^2k^2/2m$ is much larger than its binding energy in the potential $V_M(z)$ for the determining ρ (perturbation theory). At $\rho\sim\lambda$, according to Ref. 16, the characteristic binding energy is a maximum, $\sim(\hbar/ma^2)\ln^2(a/\lambda)$, from which follows in fact the presented inequality.

2) If $\ln(a/\lambda)\gg 1$, there exists a region $1/a\ll k\lesssim\ln(a/\lambda)/a$, in which the Born formula (7) is not applicable only for $\rho\ll a$ (such that $\ln(a/\rho)\gtrsim ka\gg 1$). For such ρ, however, Eqs. (7) can be modified in simple manner and corresponds to scattering by a δ-like potential (see, e.g., Ref. 15, Appendix I):

$$f_M^+=ik_{bM}/(k-ik_{bM}). \tag{9}$$

The quantity $\hbar^2k_{bM}^2/2m$ is the binding energy of an electron with angular-momentum projection $(-M)$. With logarithmic accuracy,[16]

$$k_{bM}=2\ln(a/\rho)/a, \quad \ln(a/\rho)\gg 1.$$

Substituting the last equality in (9) and (5) we easily find the contribution to d^+ from $\rho\lesssim a$ (it is necessary in this case to integrate in (5) from $\rho\sim\lambda$ to $\rho\sim a$). We thus obtain

$$d^+=\frac{1}{2ka}\left[\frac{k\nu a}{(k\nu a)^2+1}+\operatorname{arctg}\frac{1}{k\nu a}\right]+\frac{2}{(ka)^2}\ln\frac{2^{1/2}r_s}{a},$$
$$k\gg 1/a, \quad \nu=1/2\ln(a/\lambda)\ll 1. \tag{10}$$

The last term describes the contribution from $\rho\gtrsim a$.

3) Let now $1/r_s\ll k\ll 1/a$. It can be easily shown that the contribution to d^+ from $\rho\lesssim a$ is then of the order of 1. It will be shown below that the contribution from $\rho\gg a$ is much larger. It was established in Ref. 9 that for these large ρ the scattering is quasiclassical at $k\gg 1/r_s$. This means that the coefficient of passage through the potential $V_M(z)$ is equal to $|1+f_M^+|\approx 1$, so that

$$f_M^+\approx -1+\exp(i\theta_M), \tag{11}$$

$$\theta_M=\int_{-\infty}^{\infty}(k_M(z)-k)\,dz. \tag{12}$$

Here θ_M is the change of the phase of the electron wave function due to its acceleration on passing through the potential well, and

$$k_M(z)=[k^2-2mV_M(z)/\hbar^2]^{1/2} \tag{13}$$

is the quasiclassical wave vector. Substituting Eqs. (11)–(13) in (5) we find that

$$d^+=\frac{1}{2}\int_0^{\infty}\left(\frac{\partial\theta_M}{\partial\rho}\right)^2\rho\,d\rho=\frac{1}{2}\int_0^{\infty}\left[\int_{-\infty}^{\infty}\frac{\partial k_M(z)}{\partial\rho}dz\right]^2\rho\,d\rho. \tag{14}$$

We then have for the diffusion coefficient

$$D=\frac{\pi pn_ec^2}{2me^2H^2}\int_0^{\infty}\left[\int_{-\infty}^{\infty}\frac{\partial p(z)}{\partial\rho}dz\right]^2\rho\,d\rho \tag{15}$$

(the contribution from the backward scattering is negligibly small, see below). Here $p(z)$ is the classical momentum and $p=p(\pm\infty)$. This expression agrees fully with the classical drift-approximation formula[1,2] for the case when the diffusion is determined by the scattering of electrons with ρ much larger than the Larmor radius r_L. In this case ρ should be taken to mean the impact parameter of the Larmor "circle." This correspondence was indicated in Refs. 4 and 5. It is, of course, not accidental. Indeed, even though the magnetic field is quantizing, the tranverse motion of the electron at $M\gg 1$ is in a certain sense quasiclassical. Therefore in the considered range of energy variation both the longitudinal and transverse motions are quasiclassical, and the parameter ρ/λ plays the role of the parameter ρ/r_L is the theory of Refs. 1 and 2. We note, however, that actual calculations[1,2] were made only for the case $r_s\gg r_L\gg 1/k^2a$, when a substantial contribution to D is made also by $\rho\lesssim r_L$. In our case, however $\lambda\ll 1/k^2a$ and the decisive $\rho\gg\lambda$, so that the results of Refs. 1 and 2 are not directly applicable.

The formulas cited are valid for $1/r_s\ll k\ll 1/a$. It is convenient to calculate d^+ separately in two regions of the indicated interval. We consider first the region $1/(ar_s)^{1/2}\ll k\ll 1/a$. The values of ρ that determine in d^+ in this case lie in the interval $1/k^2a\lesssim\rho\lesssim r_s$. In fact, at $\rho\ll 1/k^2a$ it is possible to neglect k in Eqs. (12) and (13), and for θ_M we have then

$$\theta_M(k\to 0)=\int_{-\infty}^{\infty}[-2mV_M(z)/\hbar^2]^{1/2}dz=4(\pi r_s/a)^{1/2} \quad \rho\ll r_s, \tag{16}$$

i.e., in first-order approximation θ_M does not depend at all on ρ. This means in fact that the contribution to d^+ from small ρ is negligibly small. In the region $\rho\gg 1/k^2a$ perturbation theory is applicable and

$$\theta_M=-(m/\hbar^2k)\int_{-\infty}^{\infty}V_M(z)\,dz.$$

For $\rho\gg r_s$, the phase θ_M is exponentially small and such large ρ likewise make no contribution to d^+. If, however $1/k^2a\ll\rho\ll r_s$, it follows from the last formula that

$$\partial\theta_M/\partial\rho=-2/ka\rho \tag{17}$$

meaning that the integral d^+ diverges logarithmically at the end points of this interval. Numerical integration makes it possible to find the $d^+(k)$ dependence with accuracy higher than logarithmic:

$$d^+=\frac{2}{(ka)^2}\ln(0.9k^2ar_s), \quad 1/(ar_s)^{1/2}\ll k\ll 1/a. \tag{18}$$

We consider now the region $1/r_s\ll k\ll 1/(ar_s)^{1/2}$. The decisive value for it is $\rho\sim r_s$, since at smaller ρ the value of θ_M (16) is independent of ρ, and at larger ones it is exponentially small. Neglecting k in (14) and integrating numerically, we obtain

$$d^+=\tfrac{1}{2}\int_0^{\infty}[\partial\theta_M(k\to 0)/\partial\rho]^2\rho\,d\rho=0.35r_s/a,$$
$$1/r_s\ll k\ll 1/(ar_s)^{1/2}. \tag{19}$$

The lower limit of the indicated inequality is connected with the fact that at smaller k the condition of quasiclassical scattering is violated for $\rho\sim r_s$.

The transverse magnetoconductivity σ_{xx} was calculated in Ref. 11 for $r_s \gg a$. It is stated there that at a characteristic electron energy $\bar{\varepsilon}$ much lower than the Bohr energy one can regard the phase θ_M as independent of k. It follows from the exposition above that this holds true only for much lower energies ($m\bar{\varepsilon}/\hbar^2 \ll 1/ar_s$). In the region $1/ar_s \ll m\bar{\varepsilon}/\hbar^2 \ll 1/a^2$, however, it is necessary to use Eq. (18). In the case of nondegenerate electrons it follows then from the Einstein relation that

$$\sigma_{xx} \propto H^{-2}\varepsilon^{-1/2} \ln (m\varepsilon a r_s/\hbar^2).$$

For a temperature higher than the Bohr energy, the authors of Ref. 11 assume that the contribution to the conductivity from the scattering of electrons of energy $\hbar^2 k^2/2m$ is determined by $\rho \gtrsim 1/k^2 a$. It was shown above that this is so only for electron energy lower than the Bohr value ($k \lesssim 1/a$). Therefore the expression obtained in Ref. 11 for σ_{xx} is incorrect, nor is the criticism there of the known formula for σ_{xx} from Ref. 10. It is clear that at sufficiently high temperature (or in a sufficiently strong magnetic field) σ_{xx} is determined by the values of k given in (8), and the equation of Ref. 10 is still valid: $\sigma_{xx} \propto H^{-2} \ln H$.

4) We investigate now the scattering of slow electrons with $k \ll 1/r_s$. In this case it is convenient to divide the range of variation of ρ in two parts. It can be easily seen that in the first of them, when $\rho \gg r_s \ln(r_s/a)$, the potential well $V_M(z)$ contains only one shallow level. As will be established below, such large ρ contribute to the diffusion coefficient only at very small k. Therefore let first $\rho \ll r_s \ln(r_s/a)$, for which the wells are deep and contain many levels. The characteristic energy spacing between the highest levels is of the order of $\hbar^2/mr_s^2$. The electrons, on the other hand, have a much lower energy. We shall show that in this case they are strongly reflected ($f_M^+ \approx -1$), with the exception of the resonance case, when the well contains a level with zero energy. Indeed, at resonant interaction with a level whose energy is $\hbar^2 k_{bM}^2/2m$ (with $|k_{bM}| \ll 1/r_s$), the scattering amplitude takes the form (9) (Ref. 12, p. 626). It can be seen that $f_M^+ \to 0$ as $k_{bM} \to 0$, i.e., at exact resonance (at the threshold of creation of the level) the slow electrons pass through the well with unity probability, but at a small detuning from resonance they are completely reflected. Thus, the transmission coefficient $|1 + f_M^+|^2$, depending on ρ, has periodically repeating abrupt and narrow peaks that repeat whenever a level with zero energy appears in the well. The value of d^+ is determined by the derivative $\partial f_M^+/\partial\rho$, therefore the main contribution to it are made by just these peaks. Expression (9) is substantially different from -1 when $|k_{bM}| \lesssim k \ll 1/r_s$. Using the standard quasiclassical method of matching in the vicinity of turning points, we obtain for such small k_{bM}

$$k_{bM} = [\theta_M(k\to 0) - \pi(n+1/2)]/2\pi r_s, \qquad (20)$$

where n is an integer. From this we have for the number of peaks in a unit interval of variation of ρ:

$$|\partial n/\partial\rho|_{k_{bM}=0} = \pi^{-1}|\partial\theta_M(k\to 0)/\partial\rho|$$

and for the contribution to d^+ from each such peak

$$\int |\partial f_M^+/\partial\rho|^2 d\rho = (4kr_s)^{-1}|\partial\theta_M(k\to 0)/\partial\rho|.$$

Multiplying the last two expressions and integrating over all the ρ, we find that

$$d^+ = (4\pi k r_s)^{-1} \int_0^\infty [\partial\theta_M(k\to 0)/\partial\rho]^2 \rho\, d\rho = 0.06/ka, \qquad (21)$$
$$(1/r_s)(r_s/a)^{1/3}(\lambda/r_s)^2 \ll k \ll 1/r_s.$$

It can be seen that the decisive $\rho \sim r_s$. The restriction from below on the values of k follows from the fact that at smaller k it is impossible for these ρ to replace the summation (4) by the integration (5). Indeed, such a replacement is valid if

$$|\partial f_M^+/\partial M| \sim |\partial f_M^+/\partial\rho|(\lambda^2/\rho) \ll 1.$$

The width of the peak, however, which determines the characteristic value of the derivative, decreases with decreasing k, and can be shown that it is of the order of $kr_s(ar_s)^{1/2}$ for $\rho \sim r_s$. From this follows inequality (21). We note that decisive role of resonant scattering at $k \ll 1/r_s$ was pointed out in Ref. 11. The estimate given there is of the same order as (21), but the region of its applicability cited there is incorrect.

With further decrease of k most peaks make no contribution to d^+ (in other words, the greater part of the terms in the sum (4) pertains to the intervals between the peaks). The probability that some peak makes a contribution is proportional to its width. The latter is proportional to k (see above), from which it follows that $d^+ \propto k$. We confine ourselves to the estimate

$$d^+ \sim kr_s(r_s/\lambda)^4,$$
$$(1/r_s)(\lambda/r_s)^4 \ln(r_s^5/\lambda^4 a) \ll k \ll (1/r_s)(\lambda/r_s)^2(r_s/a)^{1/3}. \qquad (22)$$

Thus, $d^+(k)$ is a nonmonotonic function: it increases with decreasing k in region (21), and decreases in region (22). However, at still smaller k the value of d^+ again increases and tends to infinity as $k \to 0$. This divergence is determined by the contribution of the electrons with impact parameters $\rho \gg r_s \ln(r_s/a)$. Scattering by shallow wells corresponding to such large ρ is described by Eq. (9) with

$$k_{bM} = -\frac{m}{\hbar^2}\int_{-\infty}^{\infty} V_M(z)\,dz = [(2\pi)^{1/2}/a](r_s/\rho)^{1/2}\exp(-\rho/r_s), \qquad (23)$$

where $\hbar^2 k_{bM}^2/2m$ is the binding energy of a single discrete level in the well $V_M(z)$. Substitution (9) and (23) in (5) shows that, with logarithmic accuracy, the contribution to d^+ from $\rho \gg r_s \ln(r_s/a)$ is equal to $\frac{1}{4}\ln(1/ka)$. Comparing this value with (22), we find the region where d^+ is determined by scattering by shallow wells:

$$d^+ = {}^1/_4 \ln(1/ka), \quad k \ll (1/r_s)(\lambda/r_s)^4 \ln(r_s^5/\lambda^4 a), \quad \ln(1/ka) \gg \ln(r_s^5/\lambda^4 a). \qquad (24)$$

The divergence of d^+ as $k \to 0$ is easily understood: the decisive ρ are such that $k_{bM} \sim k$. Therefore the smaller k the larger the decisive $\rho \sim r_s \ln(1/ka)$ and the larger d^+. We note that a similar divergence for the scattering cross section in a strong magnetic field was discussed in Refs. 9 and 15.

We turn now to the contribution made to $D(k)$ by backward scattering. In the Born approximation

$$f_M^- = -(im/\hbar^2 k)\int_{-\infty}^{\infty} \exp(-2ikz)\, V_M(z)\, dz,$$

and in complete analogy with the calculation of d^+ we obtain for d^-

$$d^- = 2(ka)^{-2} \ln (2^{1/2} k\lambda)^{-1}, \quad k \gg \ln(a/\lambda)/a,$$

which coincides with the result of Ref. 10. The decisive impact parameters are in this case $\lambda \lesssim \rho \lesssim 1/k$.

If $\ln(a/\lambda) \gg 1$, by repeating the entire reasoning that led to Eq. (10), we find that d^- is determined in the interval $1/a \ll k \lesssim \ln(a/\lambda)/a$ by the same formula without the last term.

For the values of k in the region $1/r_s \ll k \ll 1/a$ the contribution to $D(k)$ from the reflection processes is small (i.e., $d^- \ll d^+$). Indeed, in this region the value of d^+ is determined by large $\rho \gg a$ for which the scattering is quasiclassical. In the quasiclassical approach, however, the reflection from a one-dimensional well is weak and can be neglected. Finally, scattering of slow electrons whose wavelength is much larger than the characteristic dimension of the center is isotropic and $f_M^- = f_M^+$. Therefore $d^- = d^+$ at $k \ll 1/r_s$.

We have thus established the function $d^+(k) + d^-(k)$ for the case of attraction centers at arbitrary k. This function is shown schematically in Fig. 1 (curve 1).

B. Repelling centers

Obviously, the results for attraction and repulsion centers coincide in the Born approximation when $k \gg \ln(a/\lambda)/a$. At $\ln(a/\lambda) \gg 1$, however, the agreement region is wider, $k \gg 1/a$, inasmuch as in the interval $1/a \ll k \lesssim \ln(a/\lambda)/a$ the $d^{\pm}(k)$ dependence is determined in fact by scattering from δ-like potentials (see above), the reversal of the sign of which reduces only to the inessential reversal of the sign of k_{bM} in (9).

For smaller k, i.e., at

$$1/r_s \ln^{1/2}(r_s/a) \ll k \ll 1/a,$$

as can be easily shown in the case of an attraction center, $D(k)$ is determined by the quasiclassical forward scattering of electrons with $\rho \gg a$. For these ρ the coefficient of passage through the potential barrier $V_M(z)$ changes abruptly from a small value to unity when the electron energy exceeds the top of the barrier (this means also that the barrier is quasiclassically high). In other words, for small ρ such that the top of the barrier exceeds $\hbar^2 k^2/2m$, the amplitude f_M^+ is equal to -1 with exponential accuracy, but in the opposite case it is described by Eqs. (11)–(13). Consequently d^+ is given by expression (14) if we replace in it the lower integration limit by the value of ρ defined from the condition

$$\hbar^2 k^2/2m = V_M(0) = (e^2/\rho) e^{-\rho/r_s}. \tag{25}$$

Let $1/(ar_s)^{1/2} \ll k \ll 1/a$. It is easily seen that in this case the decisive impact parameters are in the range $2/ka^2 < \rho \lesssim r_s$. If the values of ρ are far from the end points of this interval then, as follows from the derivation of (17), the value of $\partial\theta_M/\partial\rho$ for repelling centers differs from this expression only in sign. Therefore the differences between the integrals d^+ for attraction and repulsion centers reduces only to replacement of the numerical factor under the logarithm sign in (18). Numerical integration yields for it a value 6.5 instead of 0.9.

We consider now the interval

$$1/r_s \ln^{1/2}(r_s/a) \ll k \ll 1/(ar_s)^{1/2}.$$

In this case the cutoff of the lower integration limit in (14) influence the result substantially. Indeed, condition (25) corresponds then to a value $\rho \gg r_s$ for which the phase θ_M is exponentially small. Since however, in this case the dependence (25) of ρ on k is only logarithmic, namely $\rho \sim r_s \ln(1/k^2 a r_s)$, it follows that d^+ decreases with decreasing k in power-law fashion:

$$d^+ \sim (kr_s)^2 \ln^2(1/k^2 a r_s), \quad 1/r_s \ln^{1/2}(r_s/a) \ll k \ll 1/(ar_s)^{1/2}.$$

The lower limit on the applicability region is due to the fact that at small k the quasiclassical treatment for the decisive ρ is no longer correct, and the main contribution to d^+ is made by scattering from shallow and narrow barriers with large $\rho \gg r_s \ln(r_s/a)$. For these small k the $d^+(k)$ dependence takes the form (24) with logarithmic accuracy; we note that $\ln(1/ka) \gg \ln(r_s/a)$.

Finally, in analogy with the case of the attraction potential, the reflection processes make no significant contribution to $D(k)$ in the region $1/r_s \ln^{1/2}(r_s/a) \ll k \ll 1/a$ where the scattering is quasiclassical, and make a contribution equal to that for forward scattering at smaller k.

The general form of the function $d^+(k) + d^-(k)$ for weakly screened repelling centers is shown in Fig. 1 (curve 2). It is remarkable that slow electrons diffuse much faster when scattered by attracting centers than by repelling ones, even though the scattering cross section in the former case is much smaller than in the latter.[9]

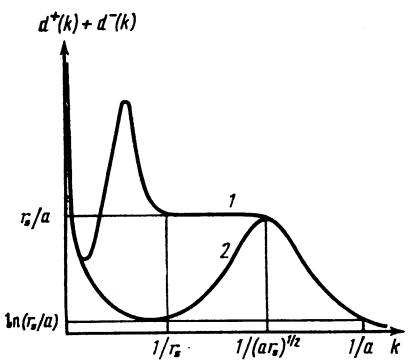

FIG. 1. Plot of $d^+(k) + d^-(k)$ (see Eq. (3)). 1—Scattering by attracting centers, 2—by repelling centers. Screening radius r_s much larger than the Bohr radius a.

Strong screening, $\lambda \ll r_s \ll a$

It is easy to see that in the case of strong screening ($r_s \ll a$) all the potentials $V_M(z)$ are shallow (narrow). From this it follows directly that the diffusion coefficient does not depend on the sign of the charge of the centers. In the Born approximation (we assume that $\lambda \ll r_s$):

$$d^{\pm} = \frac{2}{(ka)^2} \ln \frac{2^{1/2} r_s}{\lambda[(k \mp k)^2 r_s^2 + 1]^{1/2}}, \quad k \gg \ln(r_s/\lambda)/a.$$

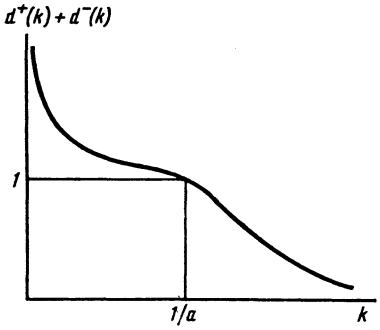

FIG. 2. Plot of $d^+(k)+d^-(k)$ vs k for the case of strong screening ($\lambda \ll r_s \ll a$).

The limit of applicability is imposed here by the condition that perturbation theory be valid for $\rho \sim \lambda$; this corresponds to a binding energy of the order of $(\hbar^2/ma^2)\ln^2(r_s/\lambda)$, $r_s \lesssim a$ in a potential $V_M(z) < 0$ [see the paragraph that follows Eq. (8)]. If in $(r_s/\lambda) \gg 1$, we find in analogy with the case (10) that $d^{\pm} = \pi/4ka$ in the region $1/a \ll k \ll \ln(r_s/\lambda)/a$. Finally, at very low electron energies $d^{\pm}$ depends logarithmically on k, exactly as the function (24):

$$d^{\pm} = {}^1/_4 \ln(1/ka), \quad k \ll 1/a, \quad \ln(1/ka) \gg 1.$$

The function $d^+(k)+d^-(k)$ for strong screening is shown in Fig. 2.

CONCLUSION

In conclusion, we describe qualitatively the physical causes of the behavior of $D(k)$. We were able to go outside the framework of the Born approximation because scattering of slow electrons is either quasiclassical or resonant. Thus if the electron wavelength is much less than the characteristic dimension r_s of the center, the quasiclassical approach is correct. In this case the diffusion is determined by forward scattering, which reduces only to acceleration or deceleration of the electrons, depending on the sign of the potential. The smaller the electron velocity the longer the time they stay in the range of action of the potential, and consequently the larger the diffusion coefficient. Therefore $D(k)$ continues to increase with decreasing k even outside the Born region (see Fig. 1). This dependence, however, continues only until the electron energy becomes of the order of the potential $V(r)$ at $r \sim r_s$. With further decrease of k the characteristic time of stay of the electrons in the field of the attracting center is determined not by the electron velocity but by the field of the center itself, and D/k ceases to depend on k. The factor $1/k$ is connected here with the electron flux incident on the center. (If the center is repelling, the electrons begin to be strongly reflected from the potential barrier, so that the time of their stay in the field of the center decreases, and with it D/k.)

At $k \sim 1/r_s$, resonant scattering begins and the time of interaction of the electron with the center increases (the electrons "stick" for some time to the resonant level). This leads to an increase of D/k. Further decrease of D/k with decreasing k is due to the narrowing of the parameter (ρ) regions in which the scattering is not resonant, the electrons are reflected and do not contribute to $D(k)$. Finally, at very small k the main contribution to $D(k)$ is made by large ρ. The wells for these are shallow and at sufficiently low energies the scattering is always resonant. In this region, D/k increases, since the effective cross section for resonant interaction increases. In the case of repelling centers this scattering mechanism determines the diffusion coefficient for all k smaller than those that admit of quasiclassical treatment. It is also responsible for the diffusion of slow electrons scattered from short-range centers (see Fig. 2), for which the one-dimensional potentials are shallow at all ρ.

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[1)]Although this approximation is customarily employed, its validity for the case of a strong magnetic field is far from obvious, since the motion of the electrons in such a field is one-dimensional to a greater degree.

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