# Nonlinear magnetization waves in ferrites

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Effective equations of motion that describe the nonlinear dynamics of magnetization in an anisotropic ferrite with two magnetic sublattices are derived and investigated. Solutions that describe localized dynamic solitons (including two- and three-dimensional ones) as well as plane topological solitons (domain walls) are obtained and analyzed.

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Nonlinear magnetization waves are being intensively investigated theoretically at present and are widely used for the description of experiments on the motion of domain walls and of magnetic domains. The dynamics of nonlinear waves was investigated in greatest detail for two types of magnets: one-sublattice ferromagnets (see, e.g., the review[1] and compensated magnets with two equivalent sublattices—antiferromagnets and weak ferromagnets.[2–5] The investigations have shown that the laws governing the dynamics of the nonlinear waves differ fundamentally for these two classes of magnet. These differences are most pronounced in the maximum steady-state velocities $V_c$ of the domain wall; whereas in ferromagnets the maximum wall velocity is small to the extent that the relativistic-interaction constants are small, in antiferromagnets the value of $V_c$ is expressed only in terms of the exchange-interaction constant and can reach tens of kilometers per second.[6]

There is one more important class of magnets, viz., ferrimagnets or ferrites, which includes magnets with several nonequivalent sublattices. Practically all nonmetallic magnets with spontaneous magnetic moment of pure exchange origin are ferrites, by virtue of which the study of ferrites is timely from the practical viewpoint. In addition, the study of nonlinear dynamics of ferrites is of considerable interest also for purely theoretical reasons: as shown in Ref. 7, the equations that describe the dynamics of an isotropic ferrite in the exchange approximation belong to the class of those exactly integrable by the method of the inverse problem of scattering theory.

We investigate in this paper magnetic solitons in an anisotropic ferrite having two sublattices. We study various types of magnetic solitons whose existence calls magnetic anisotropy. These include plane topological solitons that describe the dynamics of domain walls, as well as localized dynamic solitons, among them two- and three-dimensional ones.

It will be shown that the effective-ferromagnet model customarily used to describe experiments on the dynamics of nonlinear waves in ferrites is adequate only when the lengths of the sublattice magnetization vectors $M_1$ and $M_2$ are not too close to each other, namely, when the following inequality holds:

$$|M_1-M_2|/M_{1,2}\gg(\beta/\delta)^{1/2}, \tag{1}$$

where $\beta$ is the anisotropy constant $\delta$ is the exchange constant. For Heisenberg magnets, $\beta/\delta$ is a small parameter, $\beta/\delta\lesssim 10^{-2}$–$10^{-3}$. In many known ferrites, however, particularly in those widely used to study the motion of domain walls of epitaxial iron-garnet films, the magnetization lengths of the sublattices are close enough to each other, and inequality (1) may not hold. In addition, this inequality is known to be violated near the ferrite compensation point, near which $M_1\to M_2$.

## 1. EFFECTIVE EQUATIONS FOR FERRITE MAGNETIZATION

We consider a model of a two-sublattice anisotropic ferrite whose state is determined by two sublattice-magnetization vectors $\mathbf{M}_1(\mathbf{r},t)$, $\mathbf{M}_2(\mathbf{r},t)$; $\mathbf{M}_{1,2}^2=M_{1,2}^2$, $M_{1,2}=\text{const}$. We write the energy of the magnet in the form

$$W=\int d\mathbf{r}\left\{\delta\mathbf{M}_1\mathbf{M}_2+\frac{\alpha_1}{2}(\nabla\mathbf{M}_1)^2+\frac{\alpha_2}{2}(\nabla\mathbf{M}_2)^2+\alpha_3(\nabla\mathbf{M}_1)(\nabla\mathbf{M}_2)+W_a(\mathbf{M}_1,\mathbf{M}_2)\right\}. \tag{2}$$

Here $\delta>0$ is the constant of homogeneous exchange between the sublattices, $\alpha_i$ are the inhomogeneous-exchange constants, and $W_a$ is the magnetic-anisotropy energy. The ground state of the ferrite corresponds to antiparallel orientation of the sublattice magnetization, and the total ferrite magnetization is $|M_1-M_2|$.

The dynamics of the vectors $\mathbf{M}_1$ and $\mathbf{M}_2$ is determined by the Landau-Lifshitz equations[8,9]

$$\partial\mathbf{M}_\alpha/\partial t=-g[\mathbf{M}_\alpha\times\mathbf{H}_\alpha^{\text{eff}}],\quad \mathbf{H}_\alpha^{\text{eff}}=-\delta W/\delta\mathbf{M}_\alpha,\quad \alpha=1,2, \tag{3}$$

where $g$ is the gyromagnetic ratio, assumed to be the same for both sublattices, $g=2\mu_0/\hbar$, where $\mu_0$ is the modulus of the Bohr magneton.

The ferrite magnetization $\mathbf{M}$ is determined by the sum of the sublattice magnetizations. It is convenient also to introduce the vector $\mathbf{L}$,

$$\mathbf{L}=\mathbf{M}_1-\mathbf{M}_2,\qquad \mathbf{M}=\mathbf{M}_1+\mathbf{M}_2 \tag{4}$$

and express the equations of motion (3) in terms of the vectors $\mathbf{M}$ and $\mathbf{L}$. We note that by virture of the constancy of the sublattice-magnetization lengths, the vectors $\mathbf{M}$ and $\mathbf{L}$ are connected by two identities:

$$\mathbf{ML}=M_1^2-M_2^2,\qquad \mathbf{M}^2+\mathbf{L}^2=2(M_1^2+M_2^2). \tag{5}$$

In place of Eqs. (3) it is convenient to consider their linear combinations, sum and difference. Using (2) and (3) it

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is easy to verify that the exchange constant enters only in the difference of these equations. Within the framework of the long-wave approximation whose applicability we shall discuss below, it must be assumed that the characteristic dimension $x_0$ of the inhomogeneity in the distribution of the magnetization is large compared with the lattice constant $a$, $x_0 \gg a$. It is then obvious that the term proportional to the exchange constant is patently larger than the terms proportional to the spatial derivatives of the vectors $\mathbf{M}_1$ and $\mathbf{M}_2$, whose order of magnitude is $M\alpha/x_0^2 \sim \delta M (a/x_0)^2 \ll \delta M$, and is larger than the terms that stem from the anisotropy energy. It follows therefore that the difference between Eqs. (3) can be approximately written in the form

$$\partial \mathbf{L}/\partial t = g\delta [\mathbf{L} \times \mathbf{M}]. \quad (6)$$

Solving this equation for the vector $M$ with allowance for relations (5) we obtain

$$\mathbf{M} = \frac{\mathbf{L}}{L^2}(M_1^2 - M_2^2) + \frac{1}{g\delta L^2}\left[\frac{\partial \mathbf{L}}{\partial t} \times \mathbf{L}\right]. \quad (7)$$

We see from this formula that the magnetization of the ferrite consists of two parts: the first term has the same form as in the equilibrium state and is due to the inequality of the sublattice magnetization lengths. The second term, however, is purely of dynamic origin and is determined by the ferrite sublattice noncollinearity that takes arises in the dynamics. Allowance for the second term is not an exaggeration of the accuracy only if $|M_1 - M_2| \ll M_{1,2}$. In the equilibrium state of the ferrite we have here $|\mathbf{M}| \ll |\mathbf{L}| \sim M_{1,2}$. It is just this case, in which the ferrite dynamics does not reduce to the one-sublattice model, that we shall be interested hereafter. We introduce the notation

$$M_0^2 = {}^1/_2 (M_1^2 + M_2^2), \quad M_s = (M_1^2 - M_2^2)/|\mathbf{L}| \approx M_1 - M_2,$$
$$\mathbf{l} = \mathbf{L}/|\mathbf{L}|$$

and rewrite (7) in terms of the unit vector $\mathbf{l}$:

$$\mathbf{M} = M_s \mathbf{l} + \frac{1}{g\delta}\left[\frac{\partial \mathbf{l}}{\partial t}\mathbf{l}\right]. \quad (8)$$

Using the relation (8) we can eliminate the magnetization $\mathbf{M}$ from the system of equations for the vectors $\mathbf{M}$ and $\mathbf{L}$ (3) and obtain for $\mathbf{l}$ an expression that is advantageously written in the form

$$-\frac{M_s}{2gM_0^2}\frac{\partial \mathbf{l}}{\partial t} = \alpha \mathbf{l} \times \left[\Delta \mathbf{l} - \frac{1}{c^2}\frac{\partial^2 \mathbf{l}}{\partial t^2}\right] - \frac{1}{2M_0^2}\left[\mathbf{l} \times \frac{\partial W_a}{\partial \mathbf{l}}\right], \quad (9)$$

where $c = gM_0(2\alpha\delta)^{1/2}$; the quantity $\alpha$ has the meaning of the effective inhomogeneous exchange constant, $\alpha = (\alpha_1 + \alpha_2 - 2\alpha_3)/2$; $W_a$ is the magnetic-anisotropy energy density expressed in terms of the vector $\mathbf{l}$.

Equation (9) generalizes the equations used before the describes the nonlinear dynamics of magnets. If $M_1 = M_2$, i.e., $M_s = 0$, this equation goes over directly into the one that describes the dynamics of antiferromagnets or weak ferromagnets.[2,4] If, however, the noncollinearity of the sublattices is neglected (formally—if we take the limit as $\delta \to \infty$ or $c^2 \to \infty$, Eq. (9) goes over into the Landau-Lifshitz equation for a one-sublattice ferromagnet with magnetization $\mathbf{M}_s = M_s \mathbf{l}$. One can therefore speak, when investigating various solutions of (9), of "antiferromagnetic" ($M_s \to 0$) and "ferromagnetic" ($\delta \to \infty$) limits.

It is convenient to introduce the angle variables for the vector $\mathbf{l}$:

$$l_x = \sin\theta\cos\varphi, \quad l_y = \sin\theta\sin\varphi, \quad l_z = \cos\theta, \quad (10)$$

in terms of which (9) takes the form

$$-\frac{\nu}{gM_0}\frac{\partial\varphi}{\partial t}\sin\theta = \alpha\left(\Delta\theta - \frac{1}{c^2}\frac{\partial^2\theta}{\partial t^2}\right)$$
$$-\alpha\sin\theta\cos\theta\left[(\nabla\varphi)^2 - \frac{1}{c^2}\left(\frac{\partial\varphi}{\partial t}\right)^2\right] - \frac{1}{2M_0^2}\frac{\partial W_a}{\partial\theta}, \quad (11)$$

$$\frac{\nu}{gM_0}\frac{\partial\theta}{\partial t}\sin\theta = \alpha\nabla(\sin^2\theta\nabla\varphi) - \frac{\alpha}{c^2}\frac{\partial}{\partial t}\left(\sin^2\theta\frac{\partial\varphi}{\partial t}\right)$$
$$-\frac{1}{2M_0^2}\frac{\partial W_a}{\partial\varphi}. \quad (12)$$

Here $W_a$ is the magnetic-anisotropy energy density expressed in terms of the angle variables $\theta$ and $\varphi$, and $\nu = M_s/2M_0$. Without loss of generality, we can assume $\nu \geqslant 0$.

## 2. ONE-DIMENSIONAL SOLITONS IN A UNIAXIAL FERRITE

Consider a ferrite with purely uniaxial anisotropy. Assuming that $|\mathbf{M}| \ll |\mathbf{L}|$, we can write the magnetic-anisotropy energy in the form

$$W_a = {}^1/_4\beta(L^2 - L_z^2) = \beta M_0^2 \sin^2\theta, \quad (13)$$

where $\beta$ is the effective anisotropy constant and the preferred crystal axis coincides with the axis.

The ground state of the ferrite corresponds at $\beta > 0$, as can be readily seen, to $L_z^2 = L^2$ or to $\theta = 0, \pi$. We consider a one-dimensional soliton solution that describes a nonlinear solitary wave propagating along a certain axis $x$ with velocity $V$. We can seek this solution in the form

$$\theta = \theta(\xi), \quad \varphi = \omega t + \psi(\xi), \quad \xi = x - Vt. \quad (14)$$

This two-parameter soliton (the parameters are $V$ and $\omega$) corresponds to magnetization precession with frequency $\omega$ in a reference frame moving at the soliton velocity $V$. Such solitons were investigated both in ferromagnets[10] and in antiferromagnets.[2]

Corresponding to the localized solution of the equations of motion (11) and (12) are the natural boundary conditions

$$\theta \to 0, \quad \partial\theta/\partial\xi \to 0, \quad \partial\psi/\partial\xi < \infty \quad \text{for} \quad \xi \to \pm\infty. \quad (15)$$

We proceed to investigate the soliton structure. Since the anisotropy energy (13) is independent of the angle $\varphi$, Eq. (12) with allowance for (14) and (15), is integrated in elementary fashion:

$$\frac{d\psi}{d\xi} = -\frac{V\omega}{c^2(1 - V^2/c^2)} - \frac{\nu V}{2gM_0\alpha(1 - V^2/c^2)\cos^2(\theta/2)}. \quad (16)$$

Substituting expression (16) in (11), we obtain an ordinary differential equation for the polar angle $\theta$:

$$\theta'' + A\sin\theta - B\sin\theta\cos\theta + D\frac{\sin(\theta/2)}{\cos^3(\theta/2)} = 0, \quad (17)$$

where

$$A=\frac{\nu\omega}{\alpha g M_0(1-V^2/c^2)^2},\quad B=\frac{\beta c^2(1-V^2/c^2)-\alpha\omega^2}{\alpha c^2(1-V^2/c^2)^2}$$
$$D=\nu^2V^2/2\alpha^2(gM_0)^2(1-V^2/c^2)^2. \tag{18}$$

Integrating (17) with allowance for the boundary conditions (15) we obtain the explicit form of the soliton solution:

$$\mathrm{tg}^2\frac{\theta}{2}=\frac{\varkappa^2/2}{(A^2+2BD)^{1/2}\,\mathrm{ch}\,\varkappa\xi+(A+D)}, \tag{19}$$

where the quantity $\varkappa=\varkappa(V,\omega)$ is determined by the relation

$$\varkappa^2=4(B-A-D/2). \tag{20}$$

It is easily seen that the solution (19) exists subject to satisfaction of the condition $\varkappa^2\geqslant 0$, which can be written in the form

$$\frac{(\omega/gM_0+\nu\delta)^2}{(2\beta\delta+\nu^2\delta^2)}+\left(\frac{V}{c}\right)^2\leqslant 1. \tag{21}$$

The region of admissible parameters of a soliton on the plane $(V,\omega)$ is thus located inside an ellipse (see Fig. 1).

We discuss now the structure of a nonlinear wave of the form (19). The singularities of this solution are, first, points located at the boundary of the region where the solution exists ($\varkappa^2=0$), and second, those values of the parameters $V$, $\omega$, and $\nu$ for which $A=D=0$. Everywhere except at these singular points the solution (19) describes a localized soliton, i.e., a nonlinear wave to which corresponds one and the same magnetization value ($\theta=0$) as $\xi\to-\infty$ and $\xi\to+\infty$.

With approach of the soliton parameters $(V,\omega)$ to the boundary of the region of exsitence of the localized solutions (21), the soliton amplitude decreases, and the region of its localization $\Delta\xi\sim 1/\varkappa$ increases. The behavior of the solution on the boundary itself, however, depends essentially on the sign of $A+D$. If $A+D>0$ (corresponding to this condition are the points of the upper half of the ellipse (22)), the soliton amplitude tends to zero as $\varkappa^2\to 0$ and the soliton is completely delocalized and vanishes at $\varkappa^2=0$. In the lower half of the ellipse ($A+D<0$), the behavior is in principle different: at $\varkappa^2=0$ the soliton amplitude remains constant, but the dependence of the magnetization on the coordinate $\xi$ becomes algebraic:

$$\mathrm{tg}^2\frac{\theta}{2}=\frac{|A+D|}{(A+D)^2\xi^2+D/2}. \tag{22}$$

the solution (22) describes the so-called algebraic soliton.

Far from the compensation point, when the inequality

$$\nu\gg(\beta/\delta)^{1/2} \tag{23}$$

is satisfied, the soliton solution (19) goes over into the analogous solution obtained in Ref. 10 for a ferromagnet. The boundary of the soliton states is then transformed as follows: as $\delta\to\infty$ the center ellipse drops down to infinity away from the origin, so that the upper part of the ellipse becomes a parabola and the lower part does not manifest itself at all.[1] In the ferromagnetic limit the soliton frequency is therefore bounded only from above by the parabola

$$\omega<g\beta M_0/\nu-\nu^2V^2/4\alpha\beta(gM_0)^2. \tag{24}$$

Naturally, no algebraic solitons of the type (22) exist in the ferromagnetic limit.

We consider now the case $A=D=0$. Equation (17) has then a solution of the form

$$\mathrm{tg}(\theta/2)=\exp(B^{1/2}\xi), \tag{25}$$

which differs fundamentally from the localized solution (19). It corresponds to different values of the magnetization as $\xi\to-\infty$ and $\xi\to+\infty$:

$$\theta(-\infty)=0,\qquad \theta(+\infty)=\pi.$$

This solution describes a 180-degree domain wall constituting a plane topological soliton.

The coefficients $A$ and $D$ vanish simultaneously in two cases. First, at $\nu=0$ (strictly at the compensation point) we have $A=D=0$ for all values of the parameters $V$ and $\omega$. At this point in a ferrite, just as in an antiferromagnet, there are no localized solitons, and the two-parameter solutions describe moving domain walls.

At any nonzero (even arbitrarily small) value of $\nu$ the situation is different: $A=D=0$ only if $V=0$ and $\omega=0$. In this case the solution (25) describing a domain wall at rest. Thus, everywhere except at the compensation point itself, motion of the domain walls is impossible in the model of a purely uniaxial ferrite (13). The dynamics of the domain walls in a more general model will be considered in Sec. 4.

We dwell now on the conditions for the applicability of the soliton solutions obtained above. In the derivation of the effective equations of motion (11) and (12) we used in essence the approximation $|\mathbf{M}|\ll|\mathbf{L}|$. Starting from relations (8), (18), and (20) it is easy to show that this condition is satisfied if the following inequality holds at $\nu\ll 1$:

$$(c-V)/c\gg\max\{\beta/\delta;\ \nu\}. \tag{26}$$

The condition for the applicability of the long-wave approximation, i.e., that the soliton be macroscopic, namely $(a/x_0)\sim a\varkappa\ll 1$, also leads to the inequality (26).

Thus, at $\nu\ll 1$ the approximations used by us are thus

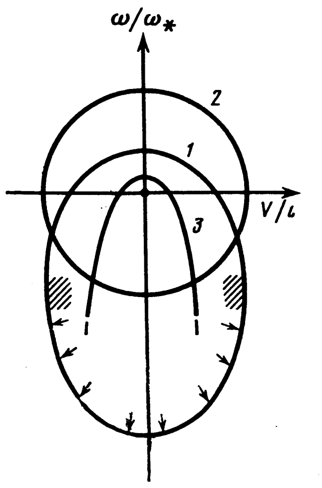


FIG. 1. Region of existence of localized solutions of Eq. (17). Here $\omega_*$ $gM_0(2\beta\delta)^{1/2}$, $c=gM_0(2\alpha\delta)^{1/2}$. 1 – $\nu\ll 1$; 2 – $\nu=0$; 3 – $\nu\approx 1$. The arrows mark the part of the ellipse in which algebraic soliton (22) exists.

valid in the entire region of existence of soliton solutions, with the exception of a narrow region of velocities close to $c$. This region is shown shaded in Fig. 1.

If $\nu \sim 1$, however, inequality (26) ceases to be significant. The condition for the applicability of the macroscopic description reduces in this case to a limitation of the precession frequency in the soliton:

$$|\omega| \ll g\delta M_0, \quad \text{or} \quad V \ll c. \tag{27}$$

### 3. THREE-DIMENSIONAL AND TWO-DIMENSIONAL SOLITONS

The one-dimensional solitons considered in the preceding section are dynamic. Their existence is due to the presence of integrals of motion of the system—the momentum **P** and the number of spin deviations (summary projection of the $z$-projection $I_z$ of the magnetization)

$$I_z = \int d\mathbf{r}\,(M_s - M_z(\mathbf{r},t)) = M_s \int d\mathbf{r}\left\{1-\cos\theta + \frac{\dot{\varphi}\sin^2\theta}{g\delta M_s}\right\}. \tag{28}$$

It can be assumed (in analogy with the case of ferromagnets[11] and antiferromagnets[2]) that in this system there can exist non-one-dimensional solitons.

We confine ourselves to an analysis of immobile solitons ($V=0$). At $V=0$ the one-dimensional solution determined by formulas (14), (16), and (19) become much simpler: the azimuthal angle $\varphi$ ceases to depend on the coordinate. It is easy to verify that in the three-dimensional case there also exist soliton solutions with uniform precession. Corresponding to these solutions are

$$\varphi = \omega t, \quad \theta = \theta(r), \tag{29}$$

where $r^2 = x^2 + y^2$ for the two-dimensional and $r^2 = x^2 + y^2 + z^2$ for the three-dimensional case.

Substituting (29) in (11) and (12) we find that the angle $\theta = \theta(r)$ satisfies the equation

$$l_0^2\left(\frac{d^2\theta}{dr^2} + \frac{d-1}{r}\frac{d\theta}{dr}\right) + \Omega\sin\theta - \sin\theta\cos\theta = 0, \tag{30}$$

where $d = 2$, or $w$ is the dimensionality of the solution;

$$l_0^2 = \frac{\alpha}{\beta - \alpha\omega^2/c^2}; \quad \Omega = \frac{\nu\omega}{gM_0(\beta - \alpha\omega^2/c^2)}. \tag{31}$$

Investigation and numerical integration of an equation similar to (30) were carried out in Refs. 11 for $d=3$ and 12 for $d=2$ in an analysis of solitons in a ferromagnet, and we can use these results directly.

It is shown in Refs. 11 and 12 that an equation of the type (30) and at $l_0^2 > 0$ localized solutions with boundary conditions $\theta \to 0$ as $R \to \infty$ and $\theta' \to 0$ as $r \to 0$, provided that the inequality

$$0 < \Omega < 1, \tag{32}$$

is satisfied; this imposes a limit on the precession frequency $\omega$: a localized soliton exists at

$$0 < \omega < \omega_{(+)} = gM_0\delta\{-\nu + (\nu^2 + 2\beta/\delta)^{1/2}\}. \tag{33}$$

The quantity $\omega_{(+)}$ is the frequency of the linear ferrimagnetic resonance. It can be easily seen that $\nu \approx 1$ we have $\omega_{(+)} \approx g\beta M_0$, with decrease of the parameter $\nu$ the value of $\omega_{(+)}$ increases, and as $\nu \to 0$ the frequency $\omega_{(+)} \to gM_0(2\beta\delta)^{1/2}$. Without dwelling in detail on all the properties of the soliton solutions of Eq. (30), which were investigated in Refs. 11 and 12, we note only that the amplitude of the soliton (the quantity $\theta(0)$) and the region of its localization depend substantially on the parameter $\Omega$, i.e., on the precession frequency $\omega$. In particular, as $\Omega \to 1$ the soliton amplitude tends to zero ($\theta(0) \propto (1-\Omega)^{1/2}$), i.e., the soliton is degraded. More interesting is the case $\Omega \ll 1$. In this limit the soliton is a spherical region of radius $R \gg l_0$, in which the angle $\theta$ is close to $\pi$; this region is separated from the remainder of the magnet, in which $\theta \approx 0$, by a transition layer having a thickness of the order of $l_0$. The soliton radius $R$ is determined at $\Omega \ll 1$ by the formula $R = 2l_0/\Omega \gg l_0$ (Ref. 11), or

$$R = \frac{2(\alpha\beta)^{1/2} gM_0}{\nu\omega}\left(1 - \frac{\alpha\omega^2}{\beta c^2}\right)^{1/2}. \tag{34}$$

We discuss now the dependence of the soliton radius on the frequency $\omega$. It can be easily seen that at $\nu \approx 1$, just as in ferromagnets,[11] we have $R \gg l_0$ only at low precession frequencies: $\omega \ll \omega_{(+)} \approx g\beta M_0$. With decreasing $\nu$, the frequency interval to which $R \gg l_0$ corresponds increases and as $\nu \to 0$ it is practically equal to the entire soliton-existence interval $(0, \omega_{(+)})$. For example, at $\nu \ll (\beta/\delta)^{1/2}$ the soliton radius is much larger than $l_0$ even if $\omega \sim \omega_{(+)}$, but if the following inequality holds

$$(\omega_{(+)} - \omega)/\omega_{(+)} \gg \nu(\delta/\beta)^{1/2}. \tag{35}$$

The point $\nu = 0$ is singular for Eq. (39), since at $\nu = 0$ the parameter $\Omega = 0$ for all values of the precession frequency $\omega$. In this case the solitons can exist only when account is taken in the anisotropy energy of terms of the type $bL_z^4$ and only in the narrow frequency interval (of the order of $(b/\beta)\omega_{(+)}$) near the antiferromagnetic-resonance frequency $\omega_{(+)}$ (Ref. 2).

Thus, the conditions for the existence of on-one-dimensional solitons in ferrites at arbitrary nonzero values of $\nu$ (even small ones, $\nu \ll (\beta/\delta)^{1/2}$) differ in principle from the corresponding conditions in an antiferromagnetic. At $\nu \neq 0$ the soliton solutions, just as in a ferromagnet, exist in the entire interval $0 < \omega < \omega_{(+)}$. When the ferrite approaches the compensation point ($\nu \to 0$), first, this frequency interval expands on account of the increase in the frequency $\omega_{(+)}$ of the linear ferrimagnetic resonance. Second and more important, an increase takes place in the relative size of the frequency interval in which solitons with $R \gg l_0$ (magnon drops), which are known to be stable,[1] exist [see Eq. (35)].

### 4. DOMAIN-WALL MOTION IN A FERRITE

As noted in the preceding section, in a uniaxial ferrite at $V = \omega = 0$, there is no localized soliton state that satisfies the boundary conditions (15). In this case, however the equations (11), (12) have a solution (25) that differs in principle from the soliton solution in that it corresponds to different values of the magnetization as $x = -\infty$ and $x = +\infty$:

$$\theta(-\infty) = 0, \quad \theta(+\infty) = \pi. \tag{36}$$

In other words, the solution (25) describes a plane 180-degree domain wall (DW) that constitutes a one-dimensional topological soliton. When such an object moves, a change takes place in the summary $z$-projection of the ferrite magnetization (28), whereas motion of the soliton (19) leaves this quantity unchanged. The last circumstance, as shown in Ref. 13, makes impossible the motion of a domain wall in an uncompensated magnet, such as a ferrite at $\nu \neq 0$, in the case of uniaxial magnetic anisotropy. To obtain for the equations of motion a solution that describes a moving DW it is necessary to consider a more complicated form of the anisotropy energy $W_a$; for example, account must be taken of the anisotropy in the basal $XY$ plane of the magnet.

Consider a ferrite with biaxial (rhombic) anisotropy, whose magnetic-anisotropy energy is chosen in the form

$$W_a = {}^1/_4\beta(L^2 - L_z^2) + {}^1/_4\rho L_x^2 = M_0^2(\beta \sin^2\theta + \rho \sin^2\theta \cos^2\varphi), \quad (37)$$

where $\rho$ is the effective constant of the rhombic anisotropy. The equations of motion (11) and (12) admit then of a solution that satisfies the boundary conditions (36) and describes a moving 180-degree DW. Corresponding to this solution are

$$\varphi = \varphi(V) = \text{const}, \quad \cos\theta = -\text{th}[\varkappa(V)(x - Vt)]. \quad (38)$$

The azimuthal angle $\varphi(V)$ and the reciprocal thickness $\varkappa(V)$ of the domain wall are connected with each other and with the velocity $V$ by the relations

$$V\varkappa(V) = -(\rho g M_0/2\nu)\sin 2\varphi, \quad (39)$$

$$\varkappa^2(V) = (\beta + \rho\cos^2\varphi)/\alpha(1 - V^2/c^2). \quad (40)$$

An analysis of (39) and (40) shows that the solution (38) exists at velocities $V$ that do not exceed a certain limiting steady-state velocity $V_c$ given by

$$V_c = c[(\beta+\rho)^{1/2} - \beta^{1/2}]\{[(\beta+\rho)^{1/2} - \beta^{1/2}]^2 + 2\delta\nu^2\}^{-1/2}. \quad (41)$$

The relations obtained generalize the corresponding results for a ferromagnet (the so-called Walker solution, see Ref. 14) and for an antiferromagnet.[2–5] As already noted, in the description of DW dynamics and in the calculations of the velocity $V_c$ the model usually employed is that of an effective ferromagnet with fixed length of the summary magnetization vector $|\mathbf{M}| = |M_1 - M_2| = M_s$. It can be seen from (41) that this approximation is valid only if the inequality (23) is satisfied.

It must be emphasized that at arbitrary small but nonzero values of the parameter $\nu$ the limiting velocity $V_c$ vanishes at $\rho = 0$. Only at the compensation point, where $\nu = 0$, does the value of $V_c$ become equal to the minimum phase velocity $c$ of the spin waves, determined only by the exchange-interaction parameters. This result is typical of compensated magnets.[2–5]

It can be easily seen from (39) and (40) that to each value $V < V_c$ there correspond to values of the angle $\varphi(V)$ and of the reciprocal DW thickness $\varkappa(V)$, i.e., two types of DW. At low velocities ($V \ll V_c$) one of them corresponds to an angle close to $\pi/2$, and the other to zero. By analogy with a ferromagnet, these DW can be called respectively quasi-Bloch and quasi-Néel. Both types of DW coincide at $V = V_c$.

The solution (38) describes DW motion "by inertia," i.e., without allowance for the driving force and for the dissipative processes. The driving force usually employed is an external magnetic field **H** applied in such a way that on account of the Zeeman energy

$$w_H = -(\mathbf{M}_1 + \mathbf{M}_2)\mathbf{H} = -\mathbf{M}\mathbf{H}$$

one of the separable DW of the homogeneous phase of the magnet (the wall in which $\mathbf{M}\cdot\mathbf{H} > 0$) becomes advantageous in energy over the other (in which $\mathbf{M}\cdot\mathbf{H} < 0$). The DW is then acted upon by a magnetic pressure $p_H$ directed towards the less advantageous phase. In addition, a moving DW is acted upon by a deceleration force $F(V)$ due to various dissipative processes and dependent on the DW velocity. At a definite value of $V$ equilibrium sets in, $p_H = F(V)$, and the DW motion becomes steady. The $V = V(H)$ dependence was obtained by Walker for ferromagnets (see Ref. 14), and in Refs. 3 and 4 for weak ferromagnets.

Let us consider the motion of a DW in a ferrite assuming that both the relaxational constant $\lambda$ and the driving field $H$ are small compared with the characteristic quantities of the problem (in particular, $H \ll \beta M_0$ and $\lambda \ll \beta; \delta$ ). In this case it can be assumed that the DW structure is the same as at $H = \lambda = 0$ and is described by Eqs. (38)–(41).

The magnetic pressure $p_H$ is determined by the equilibrium values of the vector **M** to the right and to the left of the DW. From (8) and (38) we get

$$p_H = H(M_z(-\infty) - M_z(+\infty))$$
$$= HM_0(l_z(-\infty) - l_z(+\infty)) = 4M_0H\nu. \quad (42)$$

To calculate the decelerating force we introduce into the equation of motion (3) the phenomenological relaxation terms in Gilbert's form. In this case we easily obtain for the deceleration force the expression

$$F(V) = (4\lambda M_0/g)V\varkappa(V). \quad (43)$$

Equating (42) and (43) and using the relations (39) and (40) we find that the steady-state velocity $V$ of the DW is connected with the field by the relation

$$V(H) = \frac{cH}{H_m}\left\{\left(\frac{H}{H_m}\right)^2 + \frac{8\delta\nu^2}{\rho^2}\left[\beta + \frac{\rho}{2}\left(1 \pm \left[1 - \left(\frac{H}{H_m}\right)^2\right]^{1/2}\right)\right]\right\}^{-1/2} \quad (44)$$

where

$$H_m = \lambda\rho M_0/2\nu^2, \quad (45)$$

the minus and plus signs in (44) pertain respectively to the quasi-Bloch and quasi-Néel DW. We see from (44) that steady-state motion of a DW is possible only at $H \leqslant H_m$. This result (the presence of a critical field) is typical of uncompensated magnets. For compensated magnets there is formally no such restriction.[3,4] From (45) we see also that $H_m \to \infty$ as $\nu \to 0$.

In weak fields ($H \ll H_m$) the DW velocity is linearly connected with the value of the field $H$, and the mobility of the quasi-Bloch DW wall ($\mu_B$) is larger than that of the quasi-

Néel wall ($\mu_N$):

$$\mu_B=\frac{gv}{\lambda}\left(\frac{\alpha}{\beta}\right)^{1/2}>\mu_N=\frac{gv}{\lambda}\left(\frac{\alpha}{\beta+\rho}\right)^{1/2}. \quad (46)$$

The limiting DW velocity $V_c$ is reached at a field value $H_c < H_m$:

$$H_c=\frac{2H_m}{\rho}[(\beta+\rho)^{1/2}-\beta^{1/2}][\beta(\beta+\rho)]^{1/4}<H_m. \quad (47)$$

At $H = H_m$ the velocities of both types of DW coincide and are equal to

$$V(H=H_m)=c\rho[8\delta v^2(\beta+\rho/2)+\rho^2]^{-1/4}<V_c, \quad (48)$$

while the differential mobility $\partial V/\partial H$ becomes infinite.

We discuss now the region of applicability of (44). It was derived under the assumption $H \ll \beta M_0$, i.e., it is meaningful to speak to DW motion in fields $H \sim H_m$ only if $H_m \ll \beta M_0$, i.e., at $v \gg \lambda^{1/2}$. In addition, the phenomenological allowance for the relaxation is approximate, and to calculate the decelerating force acting on a DW it is necessary to use a macroscopic approach that takes into account both the relaxation due to scattering and emission of magnons,[15] and on account of other subsystems of the crystal, particularly phonons.[16]

[1]The lower part of the ellipse corresponds at $v \sim 1$ to frequencies of the order of $g\delta M_0$, at which the conditions for the macroscopic description of the soliton are violated, since its characteristic dimension becomes of the order of the lattice constant $a$.


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