Theory of anomalous acoustical skin effect in metals

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The distribution of the displacement field $u(z)$ in a metallic halfspace $z>0$ is calculated for the case in which a transverse sound wave of frequency ω is excited at the boundary; the frequency satisfies the condition $\omega\tau \gg 1$, where τ is the conduction electron mean free path time. It is shown that the electrons whose velocity is parallel to the wave vector "drag" the displacement wave over a distance $z\sim l = v_F\tau$ (v_F is the Fermi velocity). In the collisionless limit ($l\to\infty$) the sound wave decays nonexponentially.

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1. It is well known[1–4] that in the propagation of high-frequency sound in metals at low temperatures, a spatial dispersion is more significant that temporal dispersion. This is connected with the fact that the Fermi velocity of the electrons v_F is much greater than the speed of sound s ($v_F/s \sim 10^3$), thanks to which the condition $kl=\omega\tau v_F/s \gg 1$ ($l=v_F\tau$ is the free path length of the electrons, $k=\omega/s$ is the wave vector and ω is the sound frequency) is satisfied at a comparatively low frequency (even at $\omega\tau\ll 1$). The condition $kl\gg 1$ in most cases leads to the result that l, and hence τ, drops out of the formulas everywhere and further increase in the frequency (from $\omega\tau\ll 1$ to $\omega\tau\gg 1$) does not change the character of the sound propagation. Recently, in connection with the availability of sources of hypersound, and also with the development of pulse methods, interest has arisen in the properties of solids at superhigh sound frequencies—in the case of metals, at frequencies that are high compared with the relaxation frequency $\nu = 1/\tau(\omega\gg\nu)$. Attention has been called in theoretical papers to such aspects of sound propagation in metals, in which the temporal dispersion (in terms of the parameter $\omega\tau$) is quite appreciable. Thus in Refs. 5–7, it is shown that the existence of parabolic points and points of flattening on the Fermi surface of a metal leads to angular anomalies in the sound velocity and the sound absorption coefficient; these anomalies are sensitive to the value of $\omega\tau$ (even in the case $kl\gg 1$). Moreover, it was noted in Ref. 7 that at $\omega\tau\gg 1$ the asymptote (as $z\to\infty$) of the displacement field $u(z)$ in a metallic halfspace $z>0$, on which a longitudinal sound wave of frequency ω is incident, should differ significantly from the asymptote in the intermediate case

$$s/v_F \ll \omega\tau \ll 1. \tag{1}$$

When the inequalities (1) are satisfied, the damping distance of the sound wave in a metal, d, is determined by the electron absorption coefficient Γ (at low temperatures the electrons are the principal cause of dissipation of the sound energy):

$$d = s/\Gamma. \tag{2}$$

For longitudinal sound wave,[1)] in accord with Refs. 2 and 3, $\Gamma\sim\omega s/v_F$ and $d\sim v_F/\omega$, i.e., of the order of the path over which the electron moves during one period of the field. For transverse sound, the case is complicated by the partial transformation of the sound energy into electromagnetic, thanks to which Γ depends on the relation between the wavelength s/ω of the sound and the skin depth $\delta=\delta(\omega)$ at the sound frequency.[8] It should be emphasized that the expression for Γ, when condition (1) is satisfied, does not contain the relaxation time τ (although the coefficients Γ are different at $\omega\tau\ll 1$ and $\omega\tau\gg 1$), i.e., there exists a collisionless limit for Γ, $\Gamma\to\Gamma_\infty$ as $l\to\infty$. This shows that at $l>d_\infty = s/\Gamma_\infty$ a mechanism of "pulling" of the disturbance over a distance $\sim l$ should exist. It was made clear in Ref. 7 that the pulling is determined by electrons whose velocity is parallel to the wave vector of the sound; they play no role in the formation of the absorption coefficient. Therefore, the investigation of the pulling can be the source of additional information on the conduction electrons in comparison with the other acoustical effects.

2. In the present work, we have calculated the displacement field distribution $u(z)$ at large distances from the boundary $z=0$ of a metallic halfspace $z>0$, on which a transverse sound wave of frequency is excited. For simplicity, the Fermi surface of the metal is assumed to be spherical.[2)] The pulling of the transverse wave differs from the pulling of the longitudinal, not only in the role of the co-moving electromagnetic fields, but also in the structure of the coefficients that determine the effect (in particular, the deformation potential). Since the electrons interact chiefly with the transverse sound, the velocity of these electrons being perpendicular to the wave vector, the transverse sound is pulled more weakly than the longitudinal; however, as will be shown below, this weakening is not significant.

For calculation of the displacement field $u_x \equiv u(z)$, it is necessary to solve the boundary-value problem, for which one must add to the system of elasticity and Maxwell equations which are connected with the kinetic equation for the conduction-electron distribution function χ (see Ref. 9), boundary condition for $u(z)$, for the electric field $E_x\equiv E(z)$ and for $\chi=\chi(\mathbf{p}, z)$ ($\mathbf{p}$ is the quasimomentum of the electron). If the conditions for $u(z)$ and $E(z)$ are determined by the experimental conditions (see below), the condition for $\chi(\mathbf{p}, z)$ at $z=0$ should describe the interaction of the electrons with the boundaries of the metal. In order to simplify the analysis,

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we limit ourselves to specular scattering

$$\chi(p_z<0;\ z=0)=\chi(p_z>0;\ z=0).$$

Since the pulling is determined by the electrons traveling normal to the boundary, the choice of specular reflection cannot be justified by the anomalous role of the glancing electrons with $p_z \ll p_F$ (p_F is the Fermi momentum), as in the case of the calculation of the impedance of a metal under the conditions of the anomalous skin effect (see, for example, Ref. 10). The diffuseness of the reflection (or multichanneling[11]) on the pulling phenomenon requires special consideration.

The specularity of the reflection of electrons permits a significant simplification of the problem, allowing us to proceed from consideration of the halfspace ($z>0$) to the entire space ($z \gtrless 0$),[12] by continuing the functions $E(z)$ and $u(z)$ in even fashion:

$$E(-z)=E(z),\quad u(-z)=u(z). \tag{3}$$

Then the necessary equations are valid over the entire space:

$$\begin{gathered} d^2u/dz^2+(\omega^2/s^2)u(z)=-F(z),\\ F(z)=f(z)+(im_0\omega/e\rho s^2)j(z),\\ d^2E/dz^2=-(4\pi i\omega/c^2)j(z), \end{gathered} \tag{4}$$

$j(z)\equiv j_x(z)$ is the current density, $f(z)\equiv f_x(z)$ is the force density. Such a setup of the problem allows us to use the Fourier method in which the expressions for the kinetic coefficients (in the τ approximation) do not differ from those obtained for infinite space. We shall denote the Fourier components of all the functions by the same letters as the initial functions:

$$E(k)=\frac{1}{2\pi}\int_{-\infty}^{\infty}E(z)e^{ikz}\,dz,\qquad u(k)=\frac{1}{2\pi}\int_{-\infty}^{\infty}u(z)e^{ikz}\,dz \tag{5}$$

and so on. Then, according to Ref. 9, the Fourier components of the current density are

$$\begin{gathered} j(k)=e^2\langle v_xRv_x\rangle\mathscr{E}(k)+ek\omega\langle v_xR\Lambda_x\rangle u(k),\\ \mathscr{E}(k)=E(k)+(m_0\omega^2/e)u(k), \end{gathered} \tag{6}$$

(ρ is the density of the metal, e and m_0 are the charge and the "heavy" mass of the electron) and those of the force density are

$$f(k)=(iek/\rho s^2)\langle\Lambda_xRv_x\rangle\mathscr{E}(k)+(i\omega k^2/\rho s^2)\langle\Lambda_xR\Lambda_x\rangle u(k). \tag{7}$$

The angular brackets denote integration over the Fermi surface:

$$\langle\varphi\rangle=\frac{2}{(2\pi\hbar)^3}\oint_{(F)}\varphi\frac{dS}{v}, \tag{8}$$

where dS is an element of area of the Fermi surface (for a spherical Fermi surface, $dS/v=(p_F^2/v_F)dO$, $dO=\sin\theta d\theta d\varphi$), $\mathbf{v}$ is the velocity of the electron, R is the Green's function of the kinetic equation in the τ approximation:

$$R=\frac{1}{i(\mathbf{kv}-\omega)+\nu}=\frac{1}{i(kv_z-\omega)+\nu}. \tag{9}$$

The vector $\mathbf{\Lambda}$ has the components $\lambda_{ik}n_k$, where $\mathbf{n}=\mathbf{k}/k$ (in our case, $n_z=1$, $n_x=n_y=0$), λ_{ik} is the renormalized neutrality condition[3] for the deformation potential. For a spherical Fermi surface, we can set

$$\Lambda_x\equiv\lambda_{xz}=\tilde{m}v_xv_z. \tag{10}$$

This expression takes into account correctly the symmetry of the deformation potential (we note that $\langle\Lambda_x\rangle=0$), and the angle-independent factor $\tilde{m}$ has the dimensionality of mass. It can be called the "effective mass of the electron-phonon interaction," $\tilde{m}\sim m_0$, and also, $m^*=p_F/v_F$.

The existence of the boundary ($z=0$) manifests itself in the fact that the derivatives du/dz and dE/dz at $z=0$ have discontinuities

$$\begin{gathered} \Delta(du/dz)_0=2(du/dz)_{z\to+0}\equiv2u'(0),\\ \Delta(dE/dz)_0=2(dE/dz)_{z\to+0}\equiv2E'(0), \end{gathered}$$

in terms of which we can express the Fourier components $u(k)$ and $E(k)$, through (4):

$$\begin{aligned} u(k)=&-\frac{1}{\pi}\frac{k^2-(4\pi i\omega e^2/c^2)\langle v_x^2R\rangle}{D(\omega,k)}u'(0)\\ &-\frac{iek}{\pi\rho s^2}\frac{\langle\Lambda_xRv_x\rangle+(m_0\omega/k)\langle v_x^2R\rangle}{D(\omega,k)}E'(0); \end{aligned} \tag{11}$$

$$\begin{aligned} E(k)=&-\frac{1}{\pi}\frac{k^2-\omega^2/s^2-(i\omega k^2/\rho s^2)(\langle\Lambda_x^2R\rangle}{D(\omega,k)}E'(0)\\ &+\frac{1}{\pi}\frac{(2m_0\omega/k)\langle v_xR\Lambda_x\rangle+(m_0^2\omega^2/k^2)\langle v_x^2R\rangle)}{D(\omega,k)}E'(0)\\ &-\frac{4\pi i\omega ek}{\pi c^2}\frac{\langle v_xR\Lambda_x\rangle+(m_0\omega/k)\langle v_x^2R\rangle}{D(\omega,k)}u'(0), \end{aligned} \tag{12}$$

where the dispersion function is

$$\begin{gathered} D(\omega,k)=D_0(\omega,k)-V(\omega,k),\\ D_0(\omega,k)=(k^2-\omega^2/s^2)\left(k^2-\frac{4\pi i\omega e^2}{c^2}\langle v_x^2R\rangle\right), \end{gathered} \tag{13}$$

and $V(\omega,k)$ includes all the "cross" terms:

$$\begin{aligned} V(\omega,k)=(i\omega k^2/\rho s^2)\{&(4\pi i\omega e^2/c^2)\\ &\times(\langle\Lambda_xRv_x\rangle^2-\langle\Lambda_x^2R\rangle\langle v_x^2R\rangle)+k^2\langle\Lambda_x^2R\rangle\\ &+2m_0k\omega\langle\Lambda_xRv_x\rangle+m_0^2\omega^2\langle v_x^2R\rangle\}. \end{aligned} \tag{14}$$

Since we are interested in high frequencies ($\omega\gg\nu$), the relaxation frequency ν should be left only in the resonant denominator of the expression (9), which, together with (10), allows us to express all the kinetic coefficients in (12) in terms of $\langle v_x^2\rangle$ and $\langle v_x^2R\rangle$:

$$\begin{gathered} \langle\Lambda_xRv_x\rangle=\frac{\tilde{m}}{k}(-i\langle v_x^2\rangle+\omega\langle v_x^2R\rangle),\\ \langle\Lambda_x^2R\rangle=\frac{\tilde{m}^2\omega}{k}(-i\langle v_x^2\rangle+\omega\langle v_x^2R\rangle), \end{gathered} \tag{15}$$

and also to simplify formula (14) significantly:

$$\begin{gathered} V(\omega,k)=\frac{\omega^2\tilde{m}^2}{\rho s^2}\left\{i\omega\left[k_0^2+\left(1+\frac{m_0}{\tilde{m}}\right)^2k^2\right]\langle v_x^2R\rangle\right.\\ \left.+\left[k_0^2+\left(1+\frac{2m_0}{\tilde{m}}\right)k^2\right]\langle v_x^2\rangle\right\},\\ k_0^2=\frac{4\pi e^2}{c^2}\langle v_x^2\rangle. \end{gathered} \tag{16}$$

3. Formulas (12)–(15) demonstrate an important property of the solution of Eqs. (4). Because of the resonant denominator of R, the kinetic coefficients can have singularities as $\nu\to0$ (in the collisionless limit). The condition for the

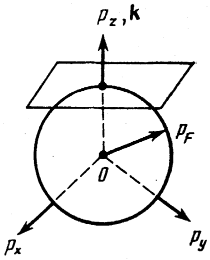

FIG. 1. The plane perpendicular to the sound wave vector **k** (the z axis) is tangent to the Fermi surface at the limiting point. The electrons at the point of tangency determine the pulling effect.

existence of the singularity is multiplicity of the zero of the denominator of R.[7] As a rule, this takes place at a definite value of the wave vector at an isolated point on the Fermi surface (we call it the critical point). If the Fermi surface is a sphere, the kinetic coefficients have a singularity at $k=\omega/v_F$ because $d(kv_F\cos\theta-\omega)/d\theta$ vanishes at $\theta=0$; the angle θ is measured from the z axis, the critical point coincides with the limiting point (see Fig. 1). If the numerator of the integrand for the kinetic coefficients differs from zero at the critical point, the coefficient itself goes to infinity (as a rule, logarithmically); however, according to (11) and (12), the diverging terms cancel one another if the divergence is due to the isolated point on the Fermi surface: the coefficients of $u'(0)$ and $E'(0)$ tend to a finite limit, since the quadratic terms in $V(\omega,k)$

$$\langle\Lambda_x R v_x\rangle^2-\langle\Lambda_x^2 R\rangle\langle v_x^2 R\rangle$$

cancel one another here.

In our case (transverse sound isotropic dispersion law, see Ref. 15), the kinetic coefficients do not diverge at $\omega=k/v_F$, since $v_x=0$ at the limiting point. However, what has been pointed out must be kept in mind when considering the pulling effect in the general case.

We now give the expressions obtained for $\langle v_x^2 R\rangle$ and $\langle v_x^2\rangle$ after integration over the spherical Fermi surface. According to (8) and the definition of k_0^2,

$$\langle v_x^2\rangle=n/m^*,\quad k_0=\omega_L/c,\quad \omega_L^2=4\pi ne^2/m^*, \tag{17}$$

n is the number of electrons per unit volume and

$$\langle v_x^2 R\rangle=(in/m^*\omega)\psi(\xi),\quad \xi=(\omega+i\nu)/kv_F,$$
$$\psi(\xi)=\frac{2}{3}\xi\left[\xi+\frac{1}{2}(\xi^2-1)\ln\frac{\xi-1}{\xi+1}\right]. \tag{18}$$

4. With the help of (11), the displacement field $u(z)$ is determined by the sum of two integrals:

$$u(z)=-\frac{u'(0)}{\pi}\int_{-\infty}^{\infty}\frac{k^2+k_0^2\psi(\xi)}{D(\omega,k)}e^{-ikz}\,dk$$
$$-\frac{eE'(0)}{\pi}\frac{\tilde{m}n}{\rho s^2m^*}\int_{-\infty}^{\infty}\frac{1-\psi(\xi)}{D(\omega,k)}e^{-ikz}\,dk. \tag{19}$$

Therefore the asymptotic behavior of $u(z)$ depends on the zeros of $D(\omega,k)$ and on the singularities of the integrands. Since the function $\psi(\xi)$ contains a logarithmic singularity at the points $k=\pm(\omega/v_F+i\nu/v_F)$, the displacement field splits into a sum of two terms:

$$u(z)=u_{\text{ord}}(z)+u_{\text{anom}}(z). \tag{20}$$

The first (ordinary) term is due to the zeros of the dispersion function $D(\omega,k)$; the second (anomalous) term—to the logarithmic singularity.

The zeros of $D(\omega,k)$ are close to their unperturbed values (at $V\equiv0$): to the acoustic $k=\pm\omega/s$ and to the electromagnetic—the roots of the equation

$$k^2=-k_0^2\psi(\xi),\quad \xi=(\omega+i\nu)/kv_F. \tag{21}$$

The roots of interest to us are

$$k^2=-\varkappa^2,\quad \varkappa\approx\begin{cases}(3\pi/4)^{1/3}(\omega k_0^2/v_F)^{1/3}, & \nu\ll\omega\ll k_0v_F\\ k_0, & \omega\gg k_0v_F\end{cases}. \tag{22}$$

Calculation of the integrals entering into formula (19) is carried out in Appendix I. According to (I1)–(I6), by taking it into account that $\omega\ll k_0v_F\sim10^{13}\ \text{s}^{-1}$, we have

$$u(z)\approx-u'(0)\{(is/\omega)\exp(i\omega z/s-z/d)+(\mu\omega^2k_0^2/Ms^2(\varkappa^2+\omega^2/s^2)\varkappa)\exp(-\varkappa z)+(3s^2\mu/M\omega v_F)(v_F/\omega z)^2\exp(-i\omega z/v_F-z/l)\}$$
$$-eE'(0)(n\tilde{m}/\rho s^2m^*)\{(i\omega^3/s^3)\exp(i\omega z/s-z/d)+\varkappa^{-1}(\varkappa^2+\omega^2/s^2)^{-1}\exp(-\varkappa z)-(2s^2/\omega k_0^2v_F)(v_F/\omega z)^2\exp(i\omega z/v_F-z/l)\}. \tag{23}$$

Here $\mu/M=\tilde{m}^2n/\rho m^*$ (M is the mass of the ion). This relation is the definition of the quantity μ, which is of the order of the mass of the free electron, d is the damping distance of the sound wave and is determined by the imaginary part of the "acoustical" root of the equation $D(\omega,k)=0$ (various limiting cases are analyzed in Ref. 13). It must be kept in mind that the assumption that the Fermi surface is totally isotropic leads to a significant increase in the value of d in comparison with the general case. The quantity $\varkappa$ is determined by formula (22), ω/s and $\varkappa$ become equal at $\omega=sk_0(s/v_F)^{1/2}\sim3\cdot10^7\ \text{s}^{-1}$ (see Ref. 22). The obtained formulas show that in the collisionless limit ($\varkappa l$, $l/d\gg1$), at sufficiently large distances, a quasiwave due to the electrons of the limiting point on the Fermi surface "survives." The electrons move with velocity v_F along the normal from the surface of the metal, pulling both the electromagnetic and the acoustic fields. Because of the fact that the transverse component of the velocity of the electrons at the limiting point is equal to zero, $\psi(\xi)$ has a singularity of the type $\Delta\ln\Delta$ ($\Delta=k-\omega/v_F$), which leads to a quadratic dependence of $v_F/\omega z$ of the amplitude of the quasiwave (see Eq. 23), i.e., it increases its damping somewhat (in comparison with the longitudinal case[7]).

5. The jumps in the derivatives $u'(0)$ and $E'(0)$ are determined by the specific setup of the problem. In contactless excitation (i.e., excitation by an electromagnetic field), since the transformation of the electromagnetic energy into acoustic is very small, we can express $E'(0)$ in terms of the value of the magnetic field at the surface, the amplitude of which is approximately twice the amplitude of the wave incident on the metal $[E'(0)=(i\omega/c)H(0)]$, and set $u'(0)$ equal to zero:

$$u(z)\approx-(ie\omega/c)(n\tilde{m}/\rho s^2m^*)H(0) \times\{(is^3/\omega^3)\exp(i\omega z/s-z/d)+\varkappa^{-1}(\varkappa^2+\omega^2/s^2)^{-1}\exp(-\varkappa z) -(2s^2/\omega v_Fk_0^2)(v_F/\omega z)^2\exp(i\omega z/v_F-z/l)\}. \tag{24}$$

In contact excitation of sound, because of the smallness of the impedance of the metal, the electric field at the boundary can be considered to be zero. This makes it possible to express $E'(0)$ in terms of $u'(0)$ with the help of (12) (see Appendix II, in which the small parameter $\mu/M\ll1$ is used in the calculations):

$$eE'(0)/u'(0)=k_0^2s^2\tilde{m}\beta(\alpha), \tag{25}$$

$$\beta(\alpha)\approx\begin{cases}1, & \alpha\ll1\\ \dfrac{\sqrt{3}(1-i\sqrt{3})}{4\pi}\dfrac{\ln\alpha}{\alpha^{2/3}}, & \alpha\gg1,\end{cases}$$

where $\alpha=(3\pi/4)(k_0^2s^3/v_F\omega^2)$. The parameter α can be larger or smaller than unity [see the discussion in connection with formula (22)].

Substituting the value of $eE'(0)$ in (23), we obtain

$$u(z)\approx-(is/\omega)u'(0)\{[1+(\mu/M)(k_0^2s^2\beta/\omega^2)]\exp(i\omega z/s-z/d) +(3i\mu s/Mv_F)(1-\beta/3)(v_F/\omega z)^2\exp(i\omega z/v_F-z/l)+\ldots\}. \tag{26}$$

The dots denote the omitted electromagnetic wave which is damped over the skin depth. The obtained expression shows that the role of the electric field is not small even in contact excitation of sound: terms containing the factor β can be of the same order as the terms due to direct (contact) excitation (they do not contain β).

6. The foregoing analysis allows us to conclude how the formulas given here change for an aribitrary dispersion law. First, not only the electrons of the limiting points take part in the pulling (for an aribtrary Fermi surface, there can be several such points), but also those for which v_z reaches a maximal value (Fig. 2). This means that a spectrum of quasiwaves with velocities $\sim v_F$ should be excited in the metal. Second, in the case of a complicated Fermi surface, Eqs. (10) and (15) do not hold, and the kinetic coefficients should have singularities at $k=\pm\omega v_z^{\max}$. In turn, this means that the quasiwaves are damped more slowly in the general case than according to the l/z^2 law [see (24) and (26)].

In conclusion, we emphasize that it is simplest to observe the pulling effect by exciting a sound pulse in a plate of thickness L and measuring the signal reflected from the rear wall, which arrives after a time interval $\sim 2L/v_F\ll 2L/s$. Furthermore, the electron pulling of the sound has been observed experimentally[14] and the correct theoretical interpretation is given in Ref. 15. However, in the works cited,[14] the propagation of sound in a metal in a comparatively strong magnetic field was observed, which is also reflected in the constructed theory of the pulling effect.[15] As is seen from the present paper, the pulling of the excitation (electromagnetic or acoustic) by the electrons is a universal phenomenon which should be observed under the simplest conditions.

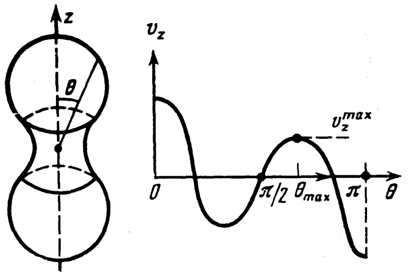

FIG. 2. Fermi surface of the dumbbell type. The sound propagates along the axis. The pulling effect is determined not only by the electrons at the turning point, but also by the electrons on the line of parabolic points (indicated). At the right is the dependence of v_z on the angle θ.

APPENDIX I
Calculation of the integrals in formula (19)

$$J_1=\int_{-\infty}^{\infty}\frac{k^2+k_0^2\psi(\xi)}{D(\omega,k)}e^{-ikz}\,dk,\qquad \xi=\frac{\omega+i\nu}{kv_F};$$
$$J_2=\int_{-\infty}^{\infty}\frac{1-\psi(\xi)}{D(\omega,k)}e^{-ikz}\,dk. \tag{I1}$$

For the calculation of the integrals, it is convenient to use the contour shown in Fig. 3. We have

$$J_\alpha=-2\pi i\Sigma_{\rm Res}^{\alpha}+I_\alpha,\quad \alpha=1,2, \tag{I2}$$

where I_α are the integrals around the cut. Here k_s and k_e are the exact values of the zeros of $D(\omega,k)$. Then, taking into account the closeness of the exact values of the zeros to their unperturbed values [$k_s\approx-\omega/s$, $k_e\approx-i\varkappa$, see Eq. (22)] we have

$$-2\pi i\Sigma_{\rm Res}^{1}\approx(\pi is/\omega)\exp(i\omega z/s-z/d) +(\pi\mu/M)[\omega^2k_0^2/s^2(\varkappa^2+\omega^2/s^2)\varkappa]\exp(-\varkappa z),$$
$$\mu/M=\tilde{m}^2n/\rho m^*. \tag{I3}$$

We have made use of the fact that $\varkappa^2\ll k_0^2$ at $\omega\ll k_0v_F$. Similarly,

$$-2\pi i\Sigma_{\rm Res}^{2}\approx(i\pi s^3/\omega^3)\exp(i\omega z/s-z/d) +[\pi/\varkappa(\varkappa^2+\omega^2/s^2)]\exp(-\varkappa z). \tag{I4}$$

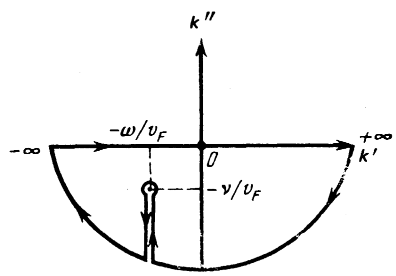

FIG. 3. Contour of integration for calculation of the integrals J_1 asnd J_2 [see (I1)].

In the calculation of the integrals I_1 and I_2 around the cut, we must make use of the smallness of the dimensionless parameter μ/M. In accord with (16)–(18)

$$V(\omega,k)=(\mu\omega^2/Ms^2)\times\{[k_0^2+(1+2m_0/\tilde{m})k^2]-[k_0^2+(1+m_0/\tilde{m})^2k^2]\psi(\xi)\},$$

$$I_1=\exp\left(i\frac{\omega}{v_F}z\right)\frac{\mu\omega^2k_0^2}{Ms^2}\times\int_{\nu/v_F}^{\infty}\frac{e^{-k''z}\,dk''[k_0^2+(1+2m_0/\tilde{m})k^2](\psi_1-\psi_2)}{(k^2-\omega^2/s^2)^2(k^2+k_0^2\psi_1)(k^2+k_0^2\psi_2)},$$

where $k=-\omega/v_F+ik''$, $\psi_{1,2}$ is the value of the function $\psi(18)$ on the different edges of the cuts. Transforming to the new variable of integration ζ according to the formula

$$k''=\zeta/z+\nu/v_F$$

and noting that as $z\to\infty$ the principal role is played by small k'', we obtain the asymptotic value of I_1:

$$I_1\approx(3\pi\mu s^2/Mv_F\omega)(v_F/\omega z)^2\exp(i\omega z/v_F-z/l).\qquad(\mathrm{I5})$$

We have made use of the fact that $\omega\ll k_0v_F$. In the calculation of the integral, I_2 we can omit $V(\omega,k)$ in the dispersion function because of the smallness of μ/M. In the same approximation as was used in the calculation of I_1.

$$I_2\approx-2\pi(v_F/\omega z)^2(s^2/\omega v_Fk_0^2)\exp(i\omega z/v_F-z/l).\qquad(\mathrm{I6})$$

APPENDIX II

Calculation of the factor $\beta=\beta(\alpha)$ in formula (25)

In accord with (12), using the smallness of μ/M and the Eqs. (15), we have

$$eE'(0)\int_{-\infty}^{\infty}\frac{dk}{k^2-(4\pi i\omega e^2/c^2)\langle v_x^2R\rangle}=-\omega^2k_0^2u'(0)\int_{-\infty}^{\infty}\frac{\tilde{m}+(i\omega/n)(m_0+\tilde{m})m^*\langle v_x^2R\rangle}{D_0(\omega,k)}dk.\qquad(\mathrm{II1})$$

In the limiting case of interest to us,

$$(4\pi i\omega e^2/c^2)\langle v_x^2R\rangle=(3\pi i/4)(k_0^2\omega/v_Fk)\operatorname{sgn}k,$$

$$eE'(0)\int_{-\infty}^{\infty}\frac{k\,dk}{k^3-(3\pi ik_0^2\omega/4v_F)\operatorname{sgn}k}=-\omega^2k_0^2u'(0)\int_{-\infty}^{\infty}\frac{[\tilde{m}+(3\pi i/4)(\omega/v_Fk)\operatorname{sgn}k(\tilde{m}+m_0)]dk}{(k^2-\omega^2/s^2-i0)[k^2-(3\pi i/4)(k_0^2\omega/v_Fk)\operatorname{sgn}k]}.\qquad(\mathrm{II2})$$

We have used the fact that k'' is much smaller than the reciprocal of the skin depth for the acoustical root of the dispersion equation, and have replaced k'' by $i0$. The characteristic wave vectors entering (II2) are ω/s and $(k_0^2\omega/v_F)^{1/2}$. In both cases, $\omega/v_Fk\ll1$, which allows us to omit the second term in the numerator in the integrand at the right. Taking into account the evenness of the integrands and transforming to the new integration variable we obtain

$$\frac{eE'(0)}{u'(0)}=-\tilde{m}k_0^2s^2\int_0^{\infty}\frac{x\,dx}{(x^2-1-i0)(x^3-i\alpha)}\Big/\int_0^{\infty}\frac{x\,dx}{x^3-i\alpha},\qquad(\mathrm{II3})$$

$$\alpha=(3\pi/4)(k_0^2s^3/v_F\omega^2).$$

The integral in the denominator is well known from the theory of the anomalous skin effect;[12]

$$\int_0^{\infty}\frac{x\,dx}{x^3-i\alpha}=\frac{\pi}{\alpha^{1/3}3\sqrt{3}}(\sqrt{3}+i).\qquad(\mathrm{II4})$$

We now calculate the integral in the numerator:

$$\int_0^{\infty}\frac{x\,dx}{(x^2-1-i0)(x^3-i\alpha)}=\frac{i\pi}{2(1-i\alpha)}+\fint_0^{\infty}\frac{x\,dx}{(x^2-1)(x^3-i\alpha)},$$

and the integral in the sense of the principal value has the simple estimate

$$\fint_0^{\infty}\frac{x\,dx}{(x^2-1)(x^3-i\alpha)}\approx\begin{cases}(i/3\alpha)\ln\alpha, & \alpha\gg1\\ -(\pi/\alpha^{1/3}3\sqrt{3})(\sqrt{3}+i), & \alpha\ll1\end{cases}.\qquad(\mathrm{II5})$$

Thus, comparing (II3) and (II4) with (25), we see that

$$\beta(\alpha)\approx\begin{cases}\dfrac{\sqrt{3}(1-i\sqrt{3})}{4\pi}\dfrac{\ln\alpha}{\alpha^{2/3}}, & \alpha\gg1\\ 1, & \alpha\ll1.\end{cases}$$

1) Only this case was considered in Ref. 7.

2) There are several observations at the end of this paper on the effect of pulling in metals with anisotropic Fermi surfaces.

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