Mobility of carriers in inversion layers in semiconductors

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A theoretical investigation is made of the influence of the surface roughness on the carrier mobility in inversion layers in semiconductors. A relationship is obtained between this mobility and the main parameters of the roughness: the average height of the projections and the characteristic scale in the tangential direction. The dependence of the mobility on an electric field normal to the surface is found. The temperature dependence of the mobility is studied: in contrast to the well-known Schrieffer theory, the dependence agrees well with the available experimental results.

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1. INTRODUCTION

Schrieffer[1] in his well-known treatment studied the surface mobility of carriers in semiconductors under conditions of their totally diffuse reflection from the surface. Numerous subsequent experimental investigations (the fullest bibliography can be found, for example, in Greene's papers[2]) have shown that, although the surfaces always scatter strongly, such scattering cannot be regarded as totally diffuse. Introduction of the Fuchs boundary condition with a phenomenological diffuseness coefficient $\hat{p}$ (which represents the fraction of the carriers which are scattered diffusely) does not account for the experimentally observed temperature dependence of the surface mobility. This is due to the fact that this approach ignores the dependence of the scattering efficiency on the angle of incidence of carriers and on their energy. Moreover, the diffuseness coefficient can only be introduced in certain cases (we shall discuss this aspect later).

We shall investigate the surface mobility of carriers in semiconductors allowing for their scattering by the surface rouhness. We shall describe the surface scattering employing a boundary condition obtained by Fal'kovskiĭ.[3] It relates explicitly the distribution function $f^>$ of carriers reflected from the surface to the distribution function $f^>$ of the incident carriers, and can be written in the form[1]

$$f^>(\mathbf{p})=f^<(\mathbf{p})+p_z\int q_z W(\mathbf{q}-\mathbf{p})[f^<(\mathbf{q})-f^<(\mathbf{p})]\frac{d^2\mathbf{q}}{\pi^2}. \quad (1)$$

Here, p_z and q_z are the moduli of the components of the momentum normal to the surface, which can be expressed in a self-evident manner in terms of the tangential components p and q and in terms of the energy $\varepsilon\equiv p_0^2/2m$:

$$p_z=(p_0^2-p^2)^{1/2},\quad q_z=(p_0^2-q^2)^{1/2},$$

valid in the case of elastic scattering by the surface projections.[2] the function $W(\mathbf{q}-\mathbf{p})\equiv W(\rho)$ is the Fourier transform of a binary correlation function of the surface of the surface projections. It is essentially different from zero and is of the order of $(ad)^2$ in the range $\rho<d^{-1}$, where a and d represent, respectively, the average height of the surface projections and the characteristic scale of the projections in the tangential direction. For example, in the case of a Gaussian distribution of the surface projections, the function in question is

$$W(\rho)=8\pi(ad)^2\exp(-\rho^2d^2/2).$$

Equation (1) is derived on the assumption that $p_z, a\ll 1$. For this reason we shall assume that the average height of the surface projections is much less than the thermal electron wavelength, i.e., we shall assume that $a\ll\lambda_T/2\pi$. This assumption is quite reasonable because, for example, in the case of electrons at room temperature in silicon we have $\lambda_T\sim 100$ Å.

We shall consider the specific case of a model p-type semiconductor with an inversion region near its surface (Fig. 1). The z axis is directed into the semiconductor along the normal to its surface; the x axis is in the plane of the surface and it is parallel to a longitudinal electric field E_x. Electrons are localized near the surface in a layer of thickness $z\approx z_T=kT/eE_s$. We shall assume that in this layer the field E_s is independent of z. Under steady-state conditions this assumption is obeyed only if the surface electron density Γ_n is much lower than the surface concentration of donors Γ_{sc} in the space charge region. We shall use the constancy of the field to simplify the calculations; we can show that if the opposite inequality is obeyed, the surface mobility differs from that obtained by us only by a numerical factor. There is another situaton of interest in practice, when E_s is constant: this is the case of transient depletion, which appears (for example) in charge-couple devices.[4]

The boundary condition (1) was used earlier by Fal'kovskiĭ to analyze the influence of the surface roughness on the anomalous skin effect in the absence[5] and in the presence[6] of a magnetic field, and also on the conductivity of thin

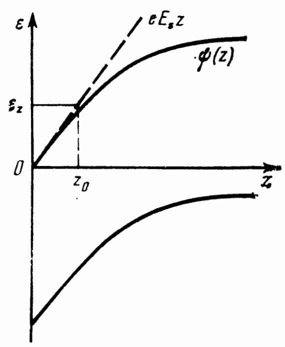

FIG. 1.

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metal films.[7,8] The specific nature of the semiconductor problem is that the motion of carriers occurs in the presence of an electric field which drives them toward the surface. These carriers collide periodically with the surface and the frequency of such collisions increases on reduction in p_z (the period is $T_z = 2p_z/eE_s$). This has the effect that glancing electrons are scattered strongly by the surface. In the case of thin films, the situation is reversed: glancing films collide rarely with the surface ($T_z \propto p_a^{-1}$), and their collision time is governed entirely by the bulk mechanisms. Therefore, as the bulk scattering time τ tends to infinity, the mobility of carriers in thin films rises without limit, which is not true of an inversion layer in a semiconductor.

Strictly speaking, the boundary condition (1) is applicable only when the localization length z_0 of electrons is much greater than their wavelength, i.e., when the quantization effects are unimportant near the surface. For this reason all the results obtained will apply to this specific case. Moreover, we shall show that is possible to go to the quantum situation in the limit.

2. GENERAL SOLUTION

As in Schrieffer's work,[1] we shall solve the transport equation

$$\frac{p_z}{m}\frac{\partial f_1}{\partial z}+eE_s\frac{\partial f_1}{\partial p_z}+\frac{f_1}{\tau}=\frac{ep_xE_xf_0}{mkT}. \quad (2)$$

Here, $f_1 = f - f_0$ is the correction to the equilibrium distribution function f_0. We shall consider the specific case when the electron gas is nondegenerate, i.e., when

$$f_0 \propto \exp\{-[p^2/2m+eE_sz]/kT\}$$

(we shall show later how this result can be obtained in the degenerate case). We shall also assume that the bulk scattering time τ is independent of the electron energy. This assumption does not affect significantly the results obtained, because in fields in which the surface mobility becomes much less than the bulk value (and is, consequently, governed by the surface scattering mechanisms), it ceases to depend on τ.

We shall solve Eq. (2) by introducing an energy parameter

$$\varepsilon_z=p_z^2/2m+eE_sz \quad (3)$$

and we shall assume that f_1 is a function of ε_z and z. Consequently, Eq. (2) reduces to

$$\frac{\partial f_1(\varepsilon_z,z)}{\partial z}+\frac{mf_1(\varepsilon_z,z)}{\tau p_z(\varepsilon_z,z)}=\frac{ep_xE_xf_0}{kT}\frac{1}{p_z(\varepsilon_z,z)}. \quad (4)$$

It has the following solutions:

$$f_1^> = \frac{e\tau}{m}\frac{p_xE_xf_0}{kT}\left\{1-\alpha_1(\varepsilon_z)\exp\left[\frac{p_z(\varepsilon_z,z)}{p_\tau}\right]\right\},$$
$$f_1^< = \frac{e\tau}{m}\frac{p_xE_xf_0}{kT}\left\{1-\alpha_2(\varepsilon_z)\exp\left[-\frac{p_z(\varepsilon_z,z)}{p_\tau}\right]\right\}, \quad (5)$$

where $p_\tau = eE_s\tau$; $\alpha_1(\varepsilon_z)$ and $\alpha_2(\varepsilon_z)$ are integration constants dependent on ε_z; here, p_z is understood to be the positive branch of the root $p_z = [2m(\varepsilon_z - eE_zz)]^{1/2}$. At the turning point of carriers $z_0 = \varepsilon_z/eE_s$ (Fig. 1) the condition $f^> = f^<$ should be satified; therefore,

$$\alpha_1(\varepsilon_z)=\alpha_2(\varepsilon_z)=\alpha(\varepsilon_z).$$

The function $\alpha(\varepsilon_z)$ is found from the boundary condition (2) and we should seek $\alpha = \alpha(p_2^2)$, because when $z = 0$ we have $\varepsilon_z = p_z^2/2m$.

Before substituting the solutions of Eq. (5) in Eq. (1), it sould be pointed out that the surface mobility may differ from the bulk value only in fields such that $p_\tau \gg p_T$, which is equivalent to the condition $l_s \ll l_T$ [$p_T = (2mkT)^{1/2}$ is the thermal momentum, $l_s = kT/eE_s$ is the localization length of carriers near the surface, and l_T is the mean free path in the bulk]. This condition occurs in Schrieffer's theory precisely as the condition of a considerable difference between the surface and bulk mobilities, and it reflects the circumstance that carriers should collide more frequently with the surface than be scattered in the bulk. In our problem the condition is necessary but insufficient, because the scattering probability differs from unity for collisions with the boundary. Therefore, we cannot regard this condition as always satisfied. Then, in the expressions of Eq. (5) the difference between the exponential function and unity is important only when the difference $f^> - f^<$ is obtained. Retaining in this difference only the first term of the expansion and introducing a new function $\xi(\varepsilon_z) = 1 - \alpha(\varepsilon_z)$, we obtain from Eq. (1)

$$-2p_x\frac{p_z}{p_\tau}[1-\xi(p_z^2)]=p_z\int\frac{d^2\mathbf{q}}{\pi^2}q_zW(\rho)[q_x\xi(q_z^2)-p_x\xi(p_z^2)]. \quad (6)$$

An analytic solution of this equation can be found in two limiting cases: 1) $p_Td \ll 1$; 2) $p_Td \gg 1$.

We shall have to calculate the surface mobility

$$\mu_s=\int[p_x(f_1^<+f_1^>)d^3\mathbf{p}\,dz]/2mE_x\int f_0\,d^3\mathbf{p}\,dz. \quad (7)$$

Under the same assumptions as those used to derive Eq. (6), we find that

$$f_1^>+f_1^<\approx 2(e\tau/m)(E_xf_0/kT)p_x\xi(p_z^2). \quad (8)$$

Substituting this function into f_0 of Eq. (7) and introducing new variables $s = \varepsilon_z/\varepsilon$ and $c = \varepsilon/kT$, where ε_z is given by Eq. (3) and $\varepsilon = p^2/2m + \varepsilon_z$ (the Jacobian of the transformation is proportional to t), we obtain

$$\mu_s=\frac{2}{\sqrt{\pi}}\frac{e\tau}{m}\int_0^1 ds\int_0^\infty dt\,t^{3/2}s^{1/2}(1-s)e^{-t}\xi(s,t). \quad (9)$$

3. SOLUTION IN THE $p_Td \ll 1$ CASE

If $d \ll p_T^{-1}$, the function $W(\mathbf{p})$ can be regarded as constant and can be taken outside the integral in Eq. (6). Then, the integral with $q_x\xi(q_z^2)$ vanishes because the integrand is odd. As a result, we obtain

$$\frac{p_z}{p_\tau}(1-\xi)=\frac{1}{\pi}W_0\xi p_z\int q_z\,dq^2, \quad (10)$$

and hence

$$\xi=[1+W_0p_0^3p_\tau/3\pi]^{-1}. \quad (11)$$

It should be noted that although the scattering probability in Eq. (10) does depend on the angle of incidence of carriers (it is proportional to p_z), this dependence disappears from the function ξ which is used as a measure of the effectiveness of the surface scattering. This is due to the fact mentioned earlier: glancing electrons collide more frequently with the surface (the collision frequency is $T_z^{-1} \propto p_z^{-1}$).

Equation (9) allows us to find the surface mobility

$$\mu_s=\frac{8}{15\sqrt{\pi}}\frac{e\tau}{m}\int_0^\infty dt\left[1+\frac{W_0}{3\pi}p_T^3p_\tau t^{3/2}\right]^{-1}t^{5/2}e^{-t}. \qquad (12)$$

If $W_0p_T^3r_\tau/3\pi \ll 1$, we can ignore the second term in the denominator of the integrand and we then obtain $\mu_s = e\tau/m$, i.e., the surface mobility is identical with the bulk value. When the opposite inequality is obeyed, we can ignore unity in the denominator and we then obtain

$$\mu_s=\frac{8\sqrt{\pi}}{5}\frac{v_T}{E_s}\frac{1}{p_T^4W_0}, \qquad (13)$$

where $v_T = p_T/m$ is the average thermal velocity of carriers. In fact, the condition of validity of Eq. (13) is $\mu_s \ll e\tau/m$.

For a Gaussian distribution of projections the function W_0 is $W_0 = 8\pi(ad)^2$. In this case we obtain from Eq. (13)

$$\mu_s=\frac{1}{5\sqrt{\pi}}\frac{v_T}{E_s}\frac{1}{(p_T^2ad)^2}=\frac{1}{5\sqrt{\pi}}\frac{v_T}{E_s}\left[\frac{(\lambda_T/2\pi)^2}{ad}\right]^2, \qquad (14)$$

where $\lambda_T = 2\pi/p_T$. If we regard defects as spherical craters of radius r with the concentration N, we find that $W_0 \propto r^6N$.

Vanishing of the "arrival" term in Eq. (6) reflects the fact that a wave scattered by a defect of size $r \ll \lambda_T$ is cylindrically symmetric. It is precisely in this case that we can introduce the diffuseness coefficient which depends naturally both on the angle of incidence of carriers and on their energy. A similar result is obtained also in the case of scattering by small defects in the bulk, but then the scattering is $(r/\lambda)^4$ times weaker because of the smallness of the wave function near the surface on which it vanishes.

4. SOLUTION IN THE $p_Td \gg 1$ CASE

In this case we find that in the interval defined by Eq. (1) the function $W(\mathbf{\rho})$ differs from zero only in the region $\rho < d^{-1} \ll p_T$. Following Fal'kovskiĭ,[6] we shall expand the function $q_z[f(\mathbf{q}) - f(\mathbf{p})]$ in powers of $(\mathbf{q}-\mathbf{p})_i$ and reatin only terms to the differential equation

$$-2(p_z/p_\tau)(1-\xi)=Q_1p_z^2\nabla f^<+Q_2\nabla(p_z^2\nabla f^<), \qquad (15)$$

where

$$Q_1=\int\frac{d^2\mathbf{q}}{\pi^2}W(\rho)\rho,\quad Q_2=\frac{1}{2}\int\frac{d^2\mathbf{q}}{\pi^2}W(\rho)\rho^2. \qquad (16)$$

On the left-hand side of Eq. (15) we have already substituted the value for the difference $f^> - f^<$.

If $p_0 - p > d^{-1}$, then we have $Q_1 = 0$ because the integrand is an odd function, and Eq. (15) for $\xi(s)$ becomes

$$s(s-1)\xi''+(3s-1)\xi'+{}^1/_2\xi=(p_T/2Q_2p_\tau)s^{1/2}(1-\xi). \qquad (17)$$

Here, as in Eq. (9), we have $s = p_z^2/p_0^2$. Since in the limit $\xi \to 1$ the surface mobility reaches the bulk value, it would be interesting to consider the case when $\xi \ll 1$. Then, on the right-hand side of Eq. (17), we can ignore ξ compared with unity and the solution of this equation is of the form

$$\xi=c_1F_1+c_2F_2+\frac{p_0}{2Q_2p_\tau}\frac{\sqrt{2}}{\pi}\sin\frac{\pi}{\sqrt{2}}$$
$$\times\left[F_2\int_0^s x^{1/2}(1-x)F_1(x)\,dx-F_1\int_1^s x^{1/2}(1-x)F_2(x)\,dx\right]. \qquad (18)$$

Here

$$F_1=F(1+2^{-1/2},\ 1-2^{-1/2},\ 1,\ s),$$
$$F_2=F(1+2^{-1/2},\ 1-2^{-1/2},\ 2,\ 1-s) \qquad (19)$$

are hypergeometric functions. A particular solution of an inhomogeneous equation is selected so that it remains finite throughout the interval [0,1], whereas the solution F_1 has the divergence $(1-s)^{-1}$ in the limit as $s \to 1$ ans the solution F_2 diverges logarithmically as $s \to 0$.

We shall find the constant c_1 by rewriting Eq. (6) in the form

$$\left[2\frac{p_z}{p_\tau}+p_z\int\frac{d^2\mathbf{q}}{\pi^2}q_zW(\rho)\right]p_x\xi(p)=2p_x\frac{p_z}{p_\tau}$$
$$+p_z\int\frac{d^2\mathbf{q}}{\pi^2}q_zW(\rho)q_x\xi(q). \qquad (20)$$

The right-hand side of the above equation is finite for any values of c_1 and c_2. Therefore, $\xi(p)$ on the left-hand side cannot have a divergence p^{-2} in the limit as $p \to 0$ and, consequently, we have $c_1 = 0$. The appearance of the solution F_1 is associated with the fact that the expansion (15) is not valid at $p = 0$ if $f(\mathbf{p})$ diverges. This is not strictly true in the case of the logarithmically diverging solution F_2, because it is valid only in the range $p_0 - p > d^{-1}$. We can find the constant c_2 by allowing for the first term on the right-hand side of Eq. (15) because near the boundary of the region p_0 it differs from zero. As $p_z \to 0$, the left-hand side of Eq. (15) tends to zero. The term with Q_2 also vanishes because it is a solution of the equation with $Q_1 = 0$. As a result, we obtain the boundary condition

$$p_z^2\nabla f=0 \quad \text{for} \quad p_z=0, \qquad (21)$$

which in the case of Eq. (17) becomes

$$sd\xi/ds=0. \qquad (22)$$

Consequently, the constant c_2 vanishes. In fact, Eq. (15) can be regarded as the diffusion equation for particles whose concentration is f in a plane p, and the left-hand side of this equation describes their drift because of acceleration during the motion in a fied E_x in the bulk, whereas the right-hand side describes diffusion in the momentum space due to the surface scattering. Then, the boundary condition of Eq. (21) is simply the condition for the absence of a particle flux across the boundary p_0.

When Fal'kovskiĭ calculated the carrier mobilities in a thin metal film,[8] he obtained formulas similar to Eqs. (15) and (17), but the boundary condition for his equations differed considerably from that obtained by us. This was due to the fact that, as pointed out earlier, in the case of a thin film the free time between collisions with the surface is $T_z \propto p_z^{-1}$, i.e., glancing electrons collide rarely with the surface, and their scattering time is governed by the bulk scattering time.

In this case the boundary condition specifies that the mobility of glancing electrons should attain the bulk value. However, in our case we have a different situation: glancing electrons collide more frequently with the surface ($T_z \propto p_z$); they are scattered strongly by the surface and the bulk time τ drops out from the final results. This is reflected in Eq. (15) by the presence on the left-hand side of a factor p_z as a result of which in the limit as $p_z \to 0$ this side tends to zero and we obtain the boundary condition (21). In the case of a thin film we find that p_z is replaced with p_z^{-1}, which gives rise to a divergence in the limit as $p_z \to 0$, and Eq. (21) cannot be used, but we then have a chance to achieve matching with the bulk mobility directly.

Substituting the solution (18) into Eq. (9), we obtain

$$\mu_s = \frac{12\sqrt{2}}{\pi^{3/2}} \sin\frac{\pi}{\sqrt{2}} \frac{v_T}{E_s} \frac{1}{Q_2} \int_0^1 ds\, s^{1/2}(1-s) F_2(s) \times \int_0^s dx\, x^{1/2}(1-x) F_1(x) \approx \frac{0.132}{Q_2} \frac{v_T}{E_s}. \tag{23}$$

The value of the integral was obtained by numerical methods. In the case of a Gaussian distribution of the surface projections, we have $Q_2 = 16(a/d)^2$ and

$$\mu_s \approx 8.4 \cdot 10^{-3} (v_T/E_s)(d/a)^2. \tag{24}$$

5. DISCUSSION

We shall begin an analysis of the results from the case $p_T d \ll 1$, i.e., we shall assume $d \ll \lambda_T/2\pi$, which is usually realized in semiconductors (in contrast to metals, when the opposite inequality $\lambda_F \ll d$ is more likely because the Fermi wavelength obeys $\lambda_F \ll \lambda_T$). Comparing the mobility given by Eq. (14) with the Scjrieffer value, when $\mu_s \propto v_T/E_s$, we find that the former is higher than the latter because $(\lambda_T^2/ad)^2 \gg 1$. This is due to the fact that the vanishing of the carrier wave function on the surface and the smallness of the tangential scale of the scattering centers compared with the thermal wavelength reduce the scattering cross section because of $(ad/\lambda_t^2)^2 \ll 1$.

The dependences of the mobility on the normal field given by Eqs. (13) and (14) are the same as in the diffuse scattering case, i.e., when $\mu_s \propto E_s^{-1}$, but the temperature dependence is considerably different. The Schrieffer mobility rises with temperature as $T^{1/2}$. This is due to the fact that on increase in temperature the carriers move away from the surface, collide less frequently with the surface, and are scattered less. On the other hand, it follows from Eqs. (13) and (14) that the mobility decreases as $T^{-3/2}$, which is a consequence of a reduction in the carrier wavelength on increase in temperature: $\lambda_T \propto T^{-1/2}$. Electrons can then approach the surface more closely (because of λ_T^{-2}) and this increases the effectiveness of the surface scattering. Such a temperature dependence is reported in many investigations, for example in Refs. 9 and 10, and it supports the conclusion that the scattering by projections plays an important role at high temperatures.

We shall now consider the results reported in Refs. 9 and 10 in greater detail. These results were obtained by measuring the surface mobility in the case of a silicon MIS structure directly under an oxide formed by oxidation in dry oxygen[9] and then in the presence of chlorine.[10] In the latter case the surface mobility was much higher and its temperature dependence was in both cases described by the $T^{-3/2}$ law valid at high values of T. Obviously, such a reduction in the mobility could be very difficult to explain by an increase in the effectiveness of the scattering by phonons. Therefore, in this case the scattering by the surface projections was in our opinion the main scattering mechanism. One should point out that the surface roughness should be understood in a wide sense, because the only important feature is the vanishing of the wave function at the boundary; therefore, our treatment applies also to stacking faults near the surface, whose concentration decreases as a result of oxidation in the presence of chlorine.[10]

In the derivation of the expressions for the surface mobility we have assumed that the electron gas is nondegenerate near the surface. However, we can readily obtain the corresponding expressions also for the strong degeneracy case. We then have to substitute in Eq. (2) the expression $\partial f_0/\partial\varepsilon \propto \delta(\varepsilon - \varepsilon_F)$ instead of f_0/kT. The integration in Eq. (7) is then carried out in an elementary manner and, instead of Eq. (14), we obtain $\mu_s \propto (v_F/E_s)(\lambda_F^2/ad)^2$, where v_F and λ_F are, respectively, the electron velocity and electron wavelength at the Fermi level.

At low temperatures (high values of λ), exactly as for strong fields E_s, the nature of motion of carriers in an inversion layer near the semiconductor surface is influenced by the effects of quantization of such motion along the z axis. In the quantization limit, when carriers are in the first quantum band, the expression for the mobility was obtained by Entin[11] and Cheng[12]: for $\lambda_0 \gg a,d$, it has the form

$$\mu_s \propto (v_0/E_s)(\lambda_0^2/ad)^2,$$

where $v_0 \equiv \hbar/m\lambda_0$ and $\lambda_0 \sim (\hbar^2/meE_s)^{1/2}$ are, respectively, the characteristic velocity and localization length of electrons at the first level. Therefore, it is quite clear that in the limit we go over to a quantum situation involving the replacment of v_T and λ_T with v_0 and λ_0, respectively.

In the case of large amplitudes of surface projections it may happen that in the surface plane d the inequality $\lambda_T/2\pi \ll d$ is satisifed. This may occur also at high temperatures if λ_T is small. Then, the mobility is described by Eq. (24). The appearance of a factor $(d/a)^2 \gg 1$ is due to the fact that the probability that an electron is scattered by the surface is then $w_2 \sim p_z^2 a^2$ [in contrast to the case when $p_T d \ll 1$, when the probability is $w_1 \sim p_z p_3^0 (ad)^2$], but electrons are scattered into a solid angle $\Delta\Omega \sim (p_0 p_z d^2)^{-1} \ll$. The total number of collisions necessary for the scattering is

$$N_2 \sim (w_2 \Delta\Omega)^{-1} \sim p_0 d^2/p_z a^2,$$

so that after averaging over the angles we obtain $N_2 \sim d^2/a^2$, the effectiveness of the scattering in the case of collisions with the boundary is $\eta_2 = N_2^{-1} \sim a^2/d^2$. In the former case we have $\Delta\Omega = 2\pi$ and $\eta_0 \sim (ad/\lambda_T^2)^2$. Hence, it is clear that the effectiveness of electron scattering by a boundary, which in the first case rises with a temperature as T^2, reaches now saturation in the limit $kT \gg m/d^2$. The surface mobility will then increase with temperature as $T^{1/2}$ because of delocalization of carriers near the surface.

In fact, the validity of Eq. (24) is not limited to the condition $a \ll \lambda_T/2\pi$. It can easily be generalized to the case of rare surface projections of dimensions $r \gg \lambda_T$. The scattering cross section for such projections is $\sigma \gtrsim r^2$ [in contrast to the case when $r \ll \lambda_T/2\pi$, when — as shown above — we find that $\sigma \sim r^2(r/\lambda)^4$], and in the absence of scattering interference, i.e., when the condition $\sigma N \ll 1$ is obeyed, the efficiency of the scattering by the surface is characterized by r^2N. Clearly, in this case we should understand a and d in Eq. (24) to be r and $N^{-1/2}$, respectively.

1) We use $\hbar = 1$.

2) For simplicity, we assume that the dispersion law of carriers is quadratic.

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