Damping of crystallization waves in He^3 at low temperatures

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Crystallization waves that can propagate in He^3 at $T < \text{mK}$ are considered. It is shown that the principal mechanism of residual damping of such oscillations is the emission of spin waves into the volume of solid antiferromagnetic He^3. The value of the damping is calculated with account taken of the previously discussed [S. V. Iordanskiĭ and A. V. Smirnov, JETP Lett. **32**, 398 (1980); A. É. Meĭerovich and B. Z. Spivak, *ibid.* **34**, 551 (1981)] ferromagnetic ordering on the phase interface. It is shown that the existence of surface magnetization can lead to a nonmonotonic dependence of the residual damping on the wave number k. Experimental observation of this dependence would be convincing proof in favor of surface ferromagnetism.

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1. INTRODUCTION

One of the most interesting recent events in the physics of quantum crystals was the experimental observation[1] of the crystallization waves, predicted in Ref. 2, on the interface of solid and liquid He^4. These waves can propagate if the interface is atomically rough and the liquid is in the superfluid state. There is therefore every reason to assume that at $T < 1$ mK, when the liquid He^3 is in the superfluid B phase, such oscillations take place for this helium isotope, too. Since the hydrodynamics equations of He^3-B coincide with those of He II, the spectrum of the crystallization waves in He^3 will coincide with the spectrum obtained in Ref. 2 for the case of He^4:

$$\omega^2 = Sk^3 \equiv \frac{\rho_l}{(\rho_l - \rho_s)^2}\tilde{\alpha}k^3, \qquad \tilde{\alpha} = \alpha + \frac{\partial^2 \alpha}{\partial \vartheta^2}. \tag{1}$$

Here ρ_s and ρ_l are densities of the solid and the liquid, while α is the surface tension; a small term linear in k and accounting for the gravitational field has been left out.

For He^4 we have $\tilde{\alpha} \sim 0.2$–0.3 erg/cm^2 (Ref. 1). In the case of He^3 this quantity should be of the same order. Such an estimate is obtained if it is assumed that the surface energy per surface atom is of the order of the Fermi energy. Then $S \sim 3 \times 102$ cm^3/sec^2.

The principal mechanism of the damping of crystallization waves in He^4 at $T = 0$ is the decay into analogous excitations.[2] To determine the residual damping of such oscillations in He^3 it is necessary in turn to consider the possibility of excitation of spin degrees of freedom.

Below the Néel temperature $T_N \approx 1.03$ mK (Ref. 3) the solid bcc He^3 is a two-sublattice antiferromagnetic with easy-plane anisotropy due to the dipole interaction.[3] Therefore one of the branches of the magnon spectrum is gapless: $\omega(\mathbf{k}) = ck$ ($c \approx 8$ cm/sec, Ref. 4), and the second has a gap equal to the antiferromagnetic-resonance frequency shift $\Omega_{AF}/2\pi \approx 8.25 \times 10^5$ sec^{-1} (Ref. 3).

The velocities c_l and c_t of the longitudinal and transverse spin waves in He^3-B, connected by the relation $c_t^2 = 2c_l^2$ (Ref. 5), exceed c considerably ($c_l \sim 10^3$ cm/sec according to experimental data[6]).

It is easily seen that propagation of crystallization waves with a wave vector $\mathbf{k}$ in the interval

$$0.3 \text{ cm}^{-1} \sim c^2/S < k < c_t^2/S \sim 3 \cdot 10^3 \text{ cm}^{-1}, \tag{2}$$

should be accompanied by emission of a spin wave corresponding to the zero-gap branch of the solid-He^3 spectrum. Neither the B-phase spin wave nor the wave corresponding to the solid-phase spectrum gap branch is emitted.

The experiments[1] on the crystallization waves in He^4 were carried out at $k \sim 10$–10^2 cm^{-1}. Being of interest also in connection with possible experiments on He^3, this wave-vector region lies within the interval (2).

Emission of a spin wave leads to damping of the crystallization wave. To determine the extent of this damping it is necessary to solve a system of equations that describe the boundary oscillations and the excitation of the spin degrees of freedom.

There is at present theoretical proof of the existence of surface ferromagnetism on the interface between solid and liquid He^3 (Refs. 7, 8). Experiments[9] performed on liquid He^3 filling the space between Grafoyl particles reveal a surface ferromagnetic contribution to the susceptibility. It will be shown in this paper that the existence of surface magnetization can lead to a nonmonotonic k dependence of the crystallization-wave damping.

The volume interaction between the layers causes the surface magnetization to be connected by definite relations with the antiferromagnetic order parameter of solid He^3, as well as with the order parameter of the B phase and the vector normal to the interface. Before we proceed to derive a system of equations we must establish these relations.

2. CONNECTION BETWEEN THE ORDER PARAMETERS ON THE INTERFACE

It is now firmly established that the B phase is a Balian-Werthamer state[10] with an order parameter $d_{ij} = 3^{-1/2}\Delta R_{ij}$, where Δ is the energy gap and R_{ij} is an arbitrary (neglecting dipole forces) matrix of the spin-system rotation relative to the orbital system. The problem of finding the connection between R_{ij}, the normal $\mathbf{n}$ to the interface,

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and the surface magnetization **M**, is closely connected with the question of the behavior of d_{ij} near a solid wall, which was considered in Ref. 11. It was shown within the framework of the Ginzburg-Landau theory that the boundary condition on a specularly reflecting wall, in the case of system with p pairing, takes the form $n_i d_{ij}|_{z=0} = 0$. It was established in Ref. 12 that a solution satisfying such a condition is sought for the Ginzburg-Landau equations in the form

$$d_{ij}(z) = 3^{-1/2}\Delta\{(\delta_{il} - n_i n_l)\psi(z) + [1 - f(z)]n_i n_l\}R_{lj}. \quad (3)$$

The function $f(z)$ in (3) decreases from unity at $z = 0$ to zero over a distance of the order of the coherence length $\xi(T)$. To determine the influence of the wall on the order-parameter orientation in the volume, a simpler form of $d_{ij}(z)$ with $\psi(z) = 1$ was used.

Ferromagnetic ordering on the interface between the phases increases substantially the difference between the densities of particles with different spin projections on the vector **M** in the Fermi-liquid boundary layer, hindering thereby the pairing of the quasiparticles in states with zero spin projection of the pair on this vector. We note that for $d_{ij}(z)$ in the form (3) there are no pairs near the wall with zero spin projection on the $n_k R_{kj}$ direction, as can be easily verified by checking that the contraction of the right-hand side of (3) with $n_k R_{kj}$ vanishes at $f(z) = 1$. Therefore the ferromagnetic ordering on the interface should not lead to additional depairing if **M** and $n_k R_{kj}$ are parallel or antiparallel. Since, however, the amplitudes of pairing into states with spin projections $+1$ and -1 are now different on the vector **M**, we have in place of (3)

$$d_{ij}(z) = 3^{-1/2}\Delta\{{}^1/_2(\delta_{il} - n_i n_l)[\psi_{\uparrow\uparrow}(z) + \psi_{\downarrow\downarrow}(z)] + {}^1/_2 i[\psi_{\uparrow\uparrow}(z) - \psi_{\downarrow\downarrow}(z)]\varepsilon_{ilk}n_k + [1 - f(z)]n_i n_l\}R_{lj}, \quad (4)$$

and the connection between **M**, **n**, and R_{ij} is given by

$$n_i R_{ij} M_j = \pm|\mathbf{M}|. \quad (5)$$

The amplitudes for pairing into states with spin projections $+1$ and -1 on $n_k R_{kj}$ are proportional to the functions $\psi_{\uparrow\uparrow}(z)$ and $\psi_{\downarrow\downarrow}(z)$ contained in (4), inasmuch as in the case $R_{ij} = \delta_{ij}$ the order parameter (4) corresponds to a pair wave function

$$\Psi_{\mu\nu}(\hat{\mathbf{k}}; z) \sim i d_{ij}(z)\hat{k}_i(\sigma_j\sigma_2)_{\mu\nu} \sim \begin{pmatrix} \psi_{\uparrow\uparrow}(z)(i\hat{k}_y - \hat{k}_x) & [1 - f(z)]\hat{k}_z \\ [1 - f(z)]\hat{k}_z & \psi_{\downarrow\downarrow}(z)(i\hat{k}_y + \hat{k}_x) \end{pmatrix}.$$

The influence of surface ferromagnetism on the reflection of volume quasiparticles from the boundary should reduce effectively to moving apart of the "walls" that reflect the quasiparticles with different spin projections on **M**, to a distance on the order of atomic. Therefore at $a \ll z < \xi(T)$ the difference

$$[\psi_{\uparrow\uparrow}(z) - \psi_{\downarrow\downarrow}(z)] \sim a/\xi(T).$$

By virtue of this relation, the difference between (4) and (3) is small. At atomic distances, apparently, $[\psi_{\uparrow\uparrow}(z) - \psi_{\downarrow\downarrow}(z)] \sim 1$.

As for the connection between **M** and the unit vector **l** parallel to the magnetization of the sublattice in solid He^3, we assume

$$\mathbf{Ml} = 0. \quad (6)$$

This relation is obtained if it is assumed that several atomic layers of a solid antiferromagnet are acted upon by a "molecular" field directed along **M** and produced by the ferromagnetic layer.

The dipole energy in the volume of He^3-B is reached when the angle of the rotation effected by the matrix R_{ij} is equal to $\arccos(-\frac{1}{4})$ (see, e.g., Ref. 5). The rotation axis here remains arbitrary. The dipole forces in the volume of solid and liquid He^3 do not lead to a complete lifting of the degeneracy with respect to rotation in spin space. In fact, by placing the vector **l** in the easy plane at arbitrary orientation and then choosing **M** to satisfy Eq. (6), we can always place the rotation axis $\boldsymbol{\eta}$ in such a way that the matrix $R_{ij}(\boldsymbol{\eta}, \arccos(-\frac{1}{4}))$ satisfies the relation (5).

Final lifting of the degeneracy is by dipole forces of surface origin. The primary contribution to the surface dipole energy $\mathcal{F}_s^d$ is made by the term connected with the distortion of the B-phase order parameter near the boundary. It was shown in Ref. 13 that in the case of an extraneous wall the rotation axis η^0 is perpendicular to this wall at equilibrium. In the case of a ferromagnetic boundary the rotation axis also tends to be perpendicular, inasmuch as at distances larger than atomic, as already noted, $(\psi_{\uparrow\uparrow} - \psi_{\downarrow\downarrow}) \ll 1$. At $T = 0$ the corresponding surface constant is $g_s^{(1)} \sim g_B\xi_0 \sim 10^{-9}$ erg/cm^2, where $g_B \sim 10^{-3}$ erg/cm^3 is the volume energy density of the dipole interaction in He^3-B and ξ_0 is the pair dimension. In the situation considered $\mathcal{F}_s^d$ depends on the orientation of the mutually perpendicular vectors **M** and **l**. The order of magnitude of the constants corresponding to such terms can be easily determined from an estimate of the interaction energy of two nuclear moments separated by atomic distance: $g_s^{(2)} \sim \mu^2/a^5 \sim 10^{-10}$ erg/cm^2. We see that $g_s^{(1)}$ can exceed $g_s^{(2)}$ by one order, and we shall therefore assume that in the case of an interphase boundary at equilibrium

$$\boldsymbol{\eta}^0 = \pm\mathbf{n}^0. \quad (7)$$

We assume for the sake of argument that the normal is directed towards the solid, and the upper signs are chosen in (5) and (7). At equilibrium we have then $\mathbf{M}^0 = M\mathbf{n}^0$. The vector $\mathbf{l}^0$ is then directed along the intersection of the separation boundary and the easy plane.

3. SYSTEM OF EQUATIONS

It is convenient to derive the equations on the basis of the Lagrangian formalism. We shall not include the surface dipole energy $\mathcal{F}_s^d$ in the system Lagrangian. The condition for the validity of this approximation will be stipulated below.

We represent the Lagrangian of the system as a sum of four terms:

$$\mathcal{L} = \mathcal{L}_B + \mathcal{L}_{AF} + \mathcal{L}_F + \mathcal{L}_{CR}. \quad (8)$$

The first three terms of the sum pertain to the spin dynamics, which we shall describe, as is frequently done, with the aid of

small rotation angles φ_i. To keep the matrix $R_{ij}^0 = R_{ij}(\boldsymbol{\eta}^0, \arccos(-\frac{1}{4}))$ that corresponds to equilibrium from entering explicitly in the Lagrangian and in the equations, we make first the following substitutions for all the spin vectors, as well as for the unit vector $\boldsymbol{\nu}$ perpendicular to the easy plane in the antiferromagnetic bcc He^3:

$$l_i \to R_{ki}^0 l_k, \quad M_i \to R_{ki}^0 M_k, \quad \varphi_i \to R_{ki}^0 \varphi_k, \quad \nu_i \to R_{ki}^0 \nu_k. \tag{9}$$

Following Refs. 14 and 15, we write down the parts of the Lagrangian that correspond to the B phase and to the solid, with allowance for the concrete form of the dipole anisotropy:

$$\mathcal{L}_B = \frac{1}{2}\frac{\chi}{\gamma^2}\int_{z<0} d^3r \left\{ (\dot{\boldsymbol{\varphi}})^2 - c_t^2 \left(\frac{\partial \boldsymbol{\varphi}}{\partial x_i}\right)^2 - \frac{1}{2}(c_l^2 - c_t^2)\left[(\operatorname{div}\boldsymbol{\varphi})^2 + \frac{\partial \varphi_i}{\partial x_j}\frac{\partial \varphi_j}{\partial x_i}\right] - \Omega_B^2 (\boldsymbol{\varphi}, \boldsymbol{\eta}^0)^2 \right\}. \tag{10}$$

$$\mathcal{L}_{AF} = \frac{1}{2}\frac{\chi_\perp}{\gamma^2}\int_{z>0} d^3r \left\{ (\dot{\boldsymbol{\varphi}})^2 - (\mathbf{l}^0\dot{\boldsymbol{\varphi}})^2 - c^2\left[\left(\frac{\partial \boldsymbol{\varphi}}{\partial x_i}\right)^2 - \left(\mathbf{l}^0 \frac{\partial \boldsymbol{\varphi}}{\partial x_i}\right)^2 \right] - \Omega_{AF}(\boldsymbol{\varphi}[\mathbf{l}^0 \times \boldsymbol{\nu}])^2 \right\}. \tag{11}$$

Here χ and $\chi_\perp$ are respectively the susceptibilities of He^3-B and of the bcc He^3, Ω_B is the frequency of the longitudinal resonance in the B phase, and γ is the gyromagnetic ratio for the He^3 nucleus. We note that the integrand terms in square brackets in (10) differ from each other by a divergence of a certain vector. This means that when the variables in the volume of the liquid are varied they correspond to equal terms in the Lagrange equations. The fact that (10) contains their arithmetic mean rather than some other combination cannot follow from phenomenological considerations but is based on the need for obtaining, when varying the variables on the boundary, a correct expression for the normal component of the spin current, which is known from the microscopic theory (see, e.g., Ref. 5). Neglect of the difference between these terms led to incorrect boundary conditions in Ref. 15.

If we confine ourselves to the part of the Lagrangian corresponding to the ferromagnetic transition layer[14,15] to the term of lowest order in $\boldsymbol{\varphi}$ and $\dot{\boldsymbol{\varphi}}$, which is not a total derivative with respect to time, we obtain

$$\mathcal{L}_F = \frac{1}{2}\frac{M}{\gamma}\int_{S^0} d^2r_\parallel (\mathbf{n}^0[\dot{\boldsymbol{\varphi}} \times \boldsymbol{\varphi}])|_{z=+0}. \tag{12}$$

In expression (12) the ferromagnetic layer if formally referred to the solid.

The last term in (8) corresponds to crystallization waves. It can be written in the form

$$\mathcal{L}_{CR} = \frac{1}{2}\frac{(\rho_l - \rho_s)^2}{\rho_l}\int_{S^0}\int_{S^0} d^2r_\parallel\, d^2r_\parallel' \frac{\dot{\zeta}(\mathbf{r}_\parallel)\dot{\zeta}(\mathbf{r}_\parallel')}{2\pi|\mathbf{r}_\parallel - \mathbf{r}_\parallel'|} - \frac{1}{2}\tilde{\alpha}_{\alpha\beta}\int_{S^0} d^2r_\parallel \frac{\partial \zeta}{\partial x_\alpha}\frac{\partial \zeta}{\partial x_\beta}$$

$$\tilde{\alpha}_{\alpha\beta} = \alpha\delta_{\alpha\beta} + (\partial^2\alpha/\partial\vartheta_\mu\partial\vartheta_\nu)\,\varepsilon_{\alpha\mu i}\varepsilon_{\beta\nu j} n_i^0 n_j^0. \tag{13}$$

Here ζ is the displacement of the phase interface, ζ_μ is the angle of rotation of the normal to the boundary (the Greek subscripts correspond to the unperturbed plane surface S^0 and run through the values 1 and 2). The first term in the right-hand side of (13) is the energy, expressed in terms of $\dot{\zeta}$ of the potential motion of an incompressible liquid whose velocity satisfies the boundary conditions

$$\mathbf{v}|_{z=-\infty} = 0, \quad (\mathbf{v}\mathbf{n}^0)|_{z=-0} = [(\rho_l - \rho_s)/\rho_l]\dot{\zeta}.$$

The second term is the increment, taken with opposite sign, of the surface energy.

We linearize, finally, the boundary condition (5):

$$(\delta_{ij} - n_i^0 n_j^0)(\varphi_j|_{z=+0} - \varphi_j|_{z=-0}) - \varepsilon_{i\alpha j}(\partial\zeta/\partial x_\alpha) n_j^0 = 0. \tag{14}$$

The action $W = \int dt$ corresponding to the Lagrangian (8) should be varied subject to the additional condition (14). The condition (6) is automatically satisfied, since $\boldsymbol{\varphi}|_{z=+0}$ describes rotation of a pair of mutually perpendicular vectors $\mathbf{M}$ and $\mathbf{l}$.

We write out the resultant expression with allowance for (7). We omit hereafter the superscript 0, which corresponds to equilibrium.

Variation of W with respect to the angle variables yield first of all the equations of motion in the volume:

$$(\partial^2/\partial t^2 - c^2\Delta)(\boldsymbol{\varphi} - \mathbf{l}(\boldsymbol{\varphi}\mathbf{l})) + \Omega_{AF}^2[\mathbf{l}\times\boldsymbol{\nu}](\boldsymbol{\varphi}[\mathbf{l}\times\boldsymbol{\nu}]) = 0, \quad z>0, \tag{15}$$

$$(\partial^2/\partial t^2 - c_t^2\Delta)\boldsymbol{\varphi} - (c_l^2 - c_t^2)\nabla \operatorname{div}\boldsymbol{\varphi} + \Omega_B^2 \mathbf{n}(\boldsymbol{\varphi}\mathbf{n}) = 0, \quad z<0, \tag{16}$$

as well as the boundary conditions

$$\{(M/\gamma)\varepsilon_{ijk}\dot{\varphi}_j n_k + J_{ik}^{AF} n_k\}|_{z=+0} = J_{ik}^B n_k|_{z=-0}, \tag{17}$$

$$J_{ik}^B n_i n_k|_{z=-0} = 0, \tag{18}$$

where the spin-current tensors in the bcc He^3 and in He^3-B are respectively

$$J_{ik}^{AF} = -\frac{\chi_\perp}{\gamma^2}c^2\frac{\partial}{\partial x_k}(\varphi_i - l_i(\boldsymbol{\varphi}\mathbf{l})), \tag{19}$$

$$J_{ik}^B = -\frac{\nu}{\gamma^2}\left\{ c_t^2\frac{\partial \varphi_i}{\partial x_k} + \frac{1}{2}(c_l^2 - c_t^2)\left[\delta_{ik}\operatorname{div}\boldsymbol{\varphi} + \frac{\partial \varphi_k}{\partial x_i}\right]\right\}. \tag{20}$$

We note that equations fully analogous to (16) were first obtained in the Hamiltonian formalism in Ref. 16.

Variation of the action with respect to the variable ζ leads to the equation

$$-\frac{(\rho_l - \rho_s)^2}{\rho_l}\int d^2r_\parallel' \frac{\ddot{\zeta}(\mathbf{r}_\parallel')}{2\pi|\mathbf{r}_\parallel - \mathbf{r}_\parallel'|} + \tilde{\alpha}_{\alpha\beta}\frac{\partial^2\zeta}{\partial x_\alpha \partial x_\beta} + \varepsilon_{\alpha\beta j} n_j n_k \frac{\partial J_{\alpha k}^B}{\partial x_\beta}\Big|_{z=-0} = 0. \tag{21}$$

The system (14)–(21) is a system of equations and of effective boundary conditions needed to determine the damping of the crystallization wave.

4. RESIDUAL DAMPING OF CRYSTALLIZATION WAVE

We confine ourselves hereafter to the wave-vector region in which the following inequalities hold:

$$10\ \text{cm}^{-1} \sim g_s^{(1)}\gamma^2/\chi c_l^2 \ll k \ll \Omega_B/c_l \sim 10^3\ \text{cm}^{-1} \tag{22}$$

(according to Ref. 17, $\Omega_B^2/(2\pi)^2 \approx 5\cdot 10^{10}\ \text{sec}^{-2}$). The left-

hand inequality in (22) means in essence that the characteristic value of the B-phase spin current on the boundary greatly exceeds the surface dipole forces. It is just this circumstance which makes it possible to leave the energy $\mathcal{F}_s^d$ out of the Lagrangian. Allowance for this energy would lead in (17), (18), and (21) to additional terms that can be neglected by virtue of the smallness of the ratio $g_x^{(1)}\gamma^2/(\chi c_t^2 k)$.

In the propagation of a crystallization wave with wave vector $\mathbf{k}$ and frequency ω the dependence of the variables on $\mathbf{r}_\parallel$ and on the time reduces to the factor $\exp(i\mathbf{k}\cdot\mathbf{r}_\parallel - i\omega t)$:

$$\varphi(\mathbf{r},\ t)=\Phi(z)\ \exp\ (i\mathbf{k}r_\parallel-i\omega t),\quad \zeta(\mathbf{r}_\parallel,\ t)=Z\exp\ (i\mathbf{k}\mathbf{r}_\parallel-i\omega t). \tag{23}$$

It will be clear from the calculation results that the corrections to the spectrum (1) to allow for the excitation of the spin degrees of freedom will be found to be small. Therefore if the inequalities (22) are satisfied it is easy to show that the following chain of inequalities holds:

$$c^2k^2\ll|\omega|^2\ll c_t^2k^2\ll\Omega_B^2<\Omega_{AF}^2. \tag{24}$$

The inequalities (24) make it possible to simplify significantly the calculation of the dependence of the angle variables on z.

The solution we seek for Eq. (15) consists of two modes: one is a traveling wave propagating into the interior of the solid He^3, and the other decreases exponentially as $z\to\infty$. It follows from (24) that for the first mode we can neglect c^2k^2 compared with ω^2, and for the second we can neglect c^2k^2 and ω^2 compared with Ω_{AF}^2. In this approximation we obtain

$$\Phi(z)=\mathbf{v}(\mathbf{v}\Phi(+0))\exp\left(i\frac{\omega}{c}z\right)$$
$$+[\mathbf{l}\mathbf{v}]([\mathbf{l}\mathbf{v}]\Phi(+0))\exp\left(-\frac{\Omega_{AF}}{c}z\right),\quad z>0. \tag{25}$$

The exponentially decreasing solution of Eq. (16) consists of three modes. Two of them decrease over lengths of the order of k^{-1} with increasing penetration into the He^3-B, and the third decreases over the dipole length $\xi_d\sim c_t/\Omega_B$. In this situation one of the three spin-wave in He^3-B dipole gap. The inequalities (24) make it possible, when all three modes are determined, to solve the static problem for Eq. (16), i.e., neglect the frequency. In addition, when determining the mode that decreases on ξ_d we can neglect $c_t^2k^2$ compared with Ω_B^2. At the accuracy indicated we obtain

$$\Phi(z)=[\hat{\mathbf{k}}\times\mathbf{n}]([\hat{\mathbf{k}}\times\mathbf{n}]\Phi(-0))\exp(kz)+\hat{\mathbf{k}}(\hat{\mathbf{k}}\Phi(-0))\exp\left(\frac{c_t}{c_l}kz\right)$$
$$+\mathbf{n}(\mathbf{n}\Phi(-0))\exp\left(\frac{\Omega_B}{c_t}z\right),\quad z<0,\quad \hat{\mathbf{k}}=\frac{\mathbf{k}}{|\mathbf{k}|}. \tag{26}$$

The dispersion equation can now be obtained as the condition for the solvability of the system of linear equations (14), (17), (18), and (21), in which all the quantities should be expressed in terms of $\Phi(-0)$, $\Phi(+0)$, and Z with the aid of (23), (29), and (26). But since the determination of the dispersion equation by this method calls for writing down the determinant of a matrix, of seventh order it is preferable to eliminate in succession $\Phi(-0)$ and $\Phi(-0)$ from the equations.

Using (14) and (15) we can express $\Phi(-0)$ in terms of $\Phi(+0)$ and Z:

$$\Phi(-0)=[\hat{\mathbf{k}}\times\mathbf{n}]([\hat{\mathbf{k}}\times\mathbf{n}]\Phi(+0)-ikZ)+$$
$$+\hat{\mathbf{k}}(\hat{\mathbf{k}}\Phi(+0))+\frac{i}{2}\mathbf{n}(\hat{\mathbf{k}}\Phi(+0))\frac{c_lk}{\Omega_B}\left(\frac{c_t^2-c_l^2}{c_t^2+c_l^2}\right). \tag{27}$$

We express now the normal components of the spin current on both side of the boundary in terms of $\Phi(+0)$ and Z with the aid of (25)–(27):

$$J_{ij}^{AF}n_j|_{z=+0}=-\frac{\chi_\perp}{\gamma^2}c(i\omega v_iv_j-\Omega_{AF}[\mathbf{l}\times\mathbf{v}]_i[\mathbf{l}\times\mathbf{v}]_j\Phi_j(+0)$$
$$\times\exp(i\mathbf{k}\mathbf{r}_\parallel-i\omega t). \tag{28}$$

$$J_{ij}^{B}n_j|_{z=-0}=-\frac{\chi}{\gamma^2}c_t^2k\left\{\left([\hat{\mathbf{k}}\times\mathbf{n}]_i[\hat{\mathbf{k}}\times\mathbf{n}]_j+\frac{c_l}{c_t}\hat{k}_i\hat{k}_j\right)\Phi_j(+0)\right.$$
$$\left.-ikZ\varepsilon_{ijp}\hat{k}_jn_p\right\}\exp(i\mathbf{k}\mathbf{r}_\parallel-i\omega t). \tag{29}$$

We have left out of (29) the term containing the small factor c_lk/Ω_B.

The vector equation that connects $\Phi(+0)$ with the amplitude Z of the crystallization wave is obtained by substituting (28) and (29) in (17). Without dwelling on the calculations, we write directly the solution of this equation:

$$\Phi(+0)=\frac{ikZ}{D}\left\{[\hat{\mathbf{k}}\times[\mathbf{l}\times\mathbf{v}]](\mathbf{n}[\mathbf{l}\times\mathbf{v}])-i\mathbf{l}(\mathbf{l}[\hat{\mathbf{k}}\times\mathbf{n}])\frac{\omega}{c_lk}\frac{\chi_\perp c}{\chi c_t}\right.$$
$$\left.-i[[\hat{\mathbf{k}}\times\mathbf{n}][\mathbf{l}\times\mathbf{v}]]\frac{\omega}{c_lk}\frac{\gamma M}{\chi c_t}\right\}, \tag{30}$$

where

$$D=(\mathbf{n}[\mathbf{l}\times\mathbf{v}])^2\left[1-\left(\frac{\gamma M}{\chi c_t}\right)^2\frac{\omega^2}{c_lc_tk^2}\right]$$
$$-i\frac{\chi_\perp c}{\chi c_t}\frac{\omega}{c_lk}\left[1+\left(\frac{c_t}{c_l}-1\right)(\hat{\mathbf{k}}\mathbf{l})^2\right]. \tag{31}$$

In the numerator and the denominator of (30) we have neglected the terms that contain the small factors $\sim\omega/\Omega_{AF}$.

Substituting finally (29) and (30) in (21) and taking (23) into account we obtain the sought dispersion equation

$$\frac{(\rho_l-\rho_s)^2}{\rho_l}\frac{\omega^2}{k}-\bar{\alpha}_{\alpha\beta}k_\alpha k_\beta$$
$$+\frac{\chi}{\gamma^2}c_t^2k^3\left\{\frac{(\mathbf{n}[\mathbf{l}\times\mathbf{v}])^2}{D}-\frac{i\omega}{c_lk}\frac{\chi_\perp c}{\chi c_t}\frac{1-(\hat{\mathbf{k}}\mathbf{l})^2}{D}-1\right\}=0, \tag{32}$$

which can be solved by successive approximations, choosing as the zeroth approximation the unperturbed spectrum

$$\omega_0^2(\mathbf{k})=[\rho_l/(\rho_l-\rho_s)^2]\bar{\alpha}_{\alpha\beta}k_\alpha k_\beta k.$$

The imaginary correction to the frequency, i.e., the damping, is obtained in first-order approximation. When writing down the result we invert the substitutions in (9):

$$\Gamma_0 = -\operatorname{Im}\omega = Ck^3 \sin^2\theta\{\lambda(\hat{\mathbf{k}}) + (\gamma M/\chi c_t)^2\Lambda(\mathbf{k})[1-\lambda(\hat{\mathbf{k}})]\}$$
$$\times\{\sin^4\theta[1-(c_l/c_t)(\gamma M/\chi c_t)^2\Lambda(\mathbf{k})]^2 + (\chi_\perp c/\chi c_t)^2\Lambda(\mathbf{k})[1+(c_l/c_t-1)\lambda(\hat{\mathbf{k}})]^2\}^{-1}, \quad (33)$$

where

$$C = \frac{1}{2}\frac{\rho_l}{(\rho_l-\rho_s)^2}\frac{\chi_\perp c}{\gamma^2},$$

$$\Lambda(\mathbf{k}) = \frac{\omega_0^2(\mathbf{k})}{c_t^2k^2} = \frac{\rho_l}{(\rho_l-\rho_s)^2}\frac{\bar{a}_{\alpha\beta}\hat{k}_\alpha\hat{k}_\beta k}{c_t^2}, \quad \lambda(\hat{\mathbf{k}}) = (\hat{k}_\alpha R_{\alpha i}{}^0 l_i^0)^2,$$

and θ is the angle between the phase interface and the easy plane in the antiferromagnetic bcc He^3. The coefficient C in (33) is estimated at $\sim 10^{-9}$ cm^3/sec. If we assume the thickness of the transition layer to be atomic, then $(\gamma M/\chi c_t)^2 \sim 10^2$. The ratio $(\chi_\perp d/\chi c_t)^2$. is of the same order or somewhat smaller.

It must be noted that the decay mechanism yields a negligibly small contribution to the damping compared with Γ_0 (Ref. 2).

$$\Gamma_{dec} \sim [\hbar\rho_l/(\rho_l-\rho_s)^2]\,k^5.$$

We recall that our result is valid in the wave-vector region for which the inequalities (22) hold. Expression (33) should provide a quantitative description of the dependence of the residual damping on k for $k = (0.5\text{–}2)\times 10^2$ cm^{-1}. At $k = 2\times 10^2$ cm^{-1} we have $\Gamma_0 \sim 10^2$ sec^{-1}, so that this damping can be measured provided, of course, that the possibility arises of observing crystallization waves in He^3 and of measuring their amplitude. It must be emphasized here that to measure Γ_0 we need also that the temperature part of the damping be not too large. At low temperatures the main contribution to Γ_T is made by collisions with thermal magnons from the solid He^3, since these excitations have the lowest velocity. Using the method of Ref. 2, we find

$$\Gamma_T \sim [\hbar\rho_l/(\rho_l-\rho_s)^2]\,(T/\hbar c)^4 k.$$

Estimates show that if $k = 2\times 10^2$ cm^{-1} the value of Γ_T becomes comparable with Γ_0 at $T \sim 10^{-23}$ mK.

The existence of surface magnetization can lead to a nonmonotonic dependence of Γ_0 on K. Differentiating (33) with respect to k at $\lambda(\hat{\mathbf{k}}) = 1$ we find that if

$$(2-\sqrt{3})\sin^4\theta(\gamma M/\chi_\perp c)^2 c_t/c_l > 1, \quad (34)$$

then at the points

$$k^{(1),(2)} = \frac{(\rho_l-\rho_s)^2}{\rho_l}\frac{c_t c_l}{\bar{a}_{\alpha\beta}\hat{k}_\alpha\hat{k}_\beta}\left(\frac{\chi c_t}{\gamma M}\right)^2$$
$$\times\left\{2-\frac{c_l}{c_t}\left(\frac{\chi_\perp c}{\gamma M}\right)^2\sin^{-4}\theta \mp \left(\left[2-\frac{c_l}{c_t}\left(\frac{\chi_\perp c}{\gamma M}\right)^2\sin^{-4}\theta\right]^2-3\right)^{1/2}\right\}$$

Γ_0 has a maximum and a minimum (the upper sign corresponds to the maximum). According to estimates, the condition (34) can be satisfied even at atomic thickness of the transition ferromagnetic layer.

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