Electromagnetic instability of a beam of charged particles in a dense plasma

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We investigate magnetic-field generation due to filamentation of a beam of charged particles propagating in a dense plasma under conditions of strong current neutralization. The filamentation mechanism is determined by inductive or dissipative magnetic-field accumulation which leads to an inertialess restructuring of the equilibrium of the charged-particle beam. The characteristic generation times of a magnetic field that leads to a substantial increase of the angular spread of the particles are indicated for typical beam and laser experiments.

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The subject of the present article is a dense plasma in which a beam of high-energy particles moves. The plasma is described in the two-component magnetic-gasdynamics approximation. This approach is possible if the spatial scale of interest to us is less than the stagnation length of the high-energy particles and the characteristic frequency of the process is lower than the frequency of the electron-ion collisions in the plasma. We shall be interested in processes that lead to breakup of the uniform beam of charged particles into jets and to generation of a magnetic field in the plasma. This question arose in connection with research into controlled thermonuclear fusion using strong-current charged-particle beams, where it is necessary to transport mega-ampere beams over distances up to 10 m through a dense plasma. We call attention to the fact that the beam filamentation and field generation, which are considered in detail in the present article, can take place and play an important role not only in propagation of high-current beams, but also in media so dissimilar in their parameters as cosmic plasma and laser plasma. In the case of the latter, experimental evidence of filamentation exists.[1]

Local pinching of an ion beam as a result of development of instability in a low-conductivity plasma was investigated in Ref. 2. This tendency exists also in a high-conductivity plasma, and the instability can set in even much faster on the helicon branch of nonpotential oscillations.[3]

We shall analyze here such an instability in the presence of an extraneous beam current substantially stronger than the total current in the plasma

$$j=|\mathbf{j}_b+\mathbf{j}_p|=(c/4\pi)\,|\operatorname{rot}\mathbf{H}|\ll j_b. \tag{1}$$

This typical situation arises when the beam transport time τ through the plasma is shorter than the skin-effect time for the plasma:

$$\tau\ll\tau_{\text{ск}}=4\pi\sigma r_b^2/c^2. \tag{2}$$

Here $\mathbf{j}_b$ and $\mathbf{j}_p$ are respectively the beam-current and plasma-electron densities, σ is the plasma conductivity, and r_b is the beam radius.

The instability considered hereafter extends over the following frequency and wave-vector ranges

$$\omega_{He}\gg\omega\gg\omega_{Hi},\qquad k\ll\omega_{pe}/c. \tag{3}$$

We shall describe the instability using the hydrodynamic equation for the ions

$$m_i\frac{d\mathbf{V}}{dt}=z_ie\mathbf{E}-\frac{z_ie^2n}{\sigma}(\mathbf{V}-\mathbf{v}) \tag{4}$$

and an equation for the electrons

$$\mathbf{E}+\frac{1}{c}[\mathbf{v}\times\mathbf{H}]+\frac{en}{\sigma}(\mathbf{v}-\mathbf{V})=-\frac{\nabla p}{en}, \tag{5}$$

which has the meaning of Ohm's law.

Ohm's law takes into account besides the Hall term also the term with the plasma-electron pressure; this term is generally speaking not potential and can influence the helicon electron perturbations. For local perturbations with $kr_b\gg1$, however, we can use the adiabat $pn^{-\alpha}=\text{const}$, and the term with the pressure balances the potential component of the electromagnetic force.

The beam-particle motion is described by the "hydrodynamic" equation

$$n_bm_b\gamma_b\frac{d\mathbf{v}_b}{dt}=z_ben_b\mathbf{E}+\frac{1}{c}[\mathbf{j}_b\times\mathbf{H}]-T_b\nabla n_b, \tag{6}$$

which is valid if the magnetic field H is strong enough and the beam particles manage to execute several radial oscillations in the magnetic channel.

Even if total curent neutralizaton was obtained at the initial instant and there is no magnetic field, the finite plasma resistance causes the reverse current to attenuate partially and a magnetic field to appear. Under condition (1) this field is equal to

$$H_\varphi=c\int_0^t\frac{d}{dr}\frac{j_b(r,t)}{\sigma(r,t)}dt, \tag{7}$$

from which follows a useful estimate for the magnetization parameter

$$\omega_{He}/\nu=(n_b/n)\,(L_b/r_b), \tag{8}$$

where $L_b=v_b\tau$ is the "length" of the beam. Actually ω_{He}/ν can be larger if the beam is injected in a previously produced magnetic transport channel.

Before we proceed to derive the equations for the local pinching of the beam, let us explain qualitatively the nature of this instability for the case when the ion-motion perturbation can be neglected and the charged-particle beam-current density is uniform over the cross section.

Let an electron-velocity perturbation v_φ be produced along the azimuthal angle φ, and let it lead to pinching of the

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magnetic field because the magnetic field is frozen into the electrons. Since the electron liquid is incompressible, a radial velocity v_r sets in and produced, by virtue of the freezing-in equation

$$\frac{d}{dt}\frac{\mathbf{H}}{n}=\left(\frac{\mathbf{H}}{n}\nabla\right)\mathbf{v}$$

a perturbation H_r.

It follows from (5) that $c^{-1}v_{z0}H_r$ generates an azimuthal electric field. The magnetic field H_z induced thereby fixes a new quasiequilibrium and increases the azimuthal displacement of the electron liquid:

$$\partial H_z/\partial r=(4\pi e n_0/c)\,v_\varphi.$$

Flute-like perturbations of the magnetic field are then produced on the beam surface and are oriented along the beam propagation direction. From the freezing-in equation it follows that the growth rate of the magnetic field H_z, and hence also the increment of the instability for a current j_{z0} that is uniform over the cross section, is proportional to the plasma density gradient.

We note now the main premises that will be made in this article to obtain a sufficiently simply physical picture of the phenomenon. Equation (4) is written under the assumption that the ions are not magnetized ($\omega\gg\omega_{Hi}$) and the ion pressure can be neglected.

No account is taken in (5) of the plasma-electron inertia; this is equivalent to magnetization of the electrons ($\omega\ll\omega_{He}$) in the indicated wavelength range.

The main effect of the action of the magnetic field on the beam is the bending of the beam trajectories by the perturbed magnetic field. In this case we can neglect the perturbation of the particle velocity in the beam, this being due to the presence of a large parameter I_b/I, where I_b is the charged-particle-beam current and I is the total current. The electric term in (6) can be neglected compared with the magnetic relative to the parameter $n_b/n\ll 1$, as follows from Eqs (1) and (5). When account is taken of the beam continuity equation

$$\partial n_b/\partial t+\operatorname{div}\,(n_b\mathbf{v}_b)=0$$

this is in essence equivalent to the quasistatism condition $\omega/kv_b\ll 1$, a result obtained also, when account is taken of the foregoing, from the induction equation

$$-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}=\operatorname{rot}\mathbf{E}. \tag{9}$$

In the "hydrodynamic" formula (6) the temperature T_b is assumed constant and equal to

$$T_b=\gamma_b m_b v_T{}^2\equiv\gamma_b m_b v_b{}^2\theta^2, \tag{10}$$

where v_b is the longitudinal velocity in the beam and θ is a small angle that characterizes the scatter of the beam-particle velocities. The beam particles move almost in a straight line because the total current is much less than the Alfven current for the beam particles:

$$I_b>I_A\gg I. \tag{11}$$

In addition, it will be assumed hereafter that the wavelength of the pertubations along the beam exceeds greatly not only the transverse wavelength but also the period of the betatron oscillations in the beam:

$$k_z{}^2/k^2\ll\theta^2=I/I_A\ll 1. \tag{12}$$

To obtain the fundamental equations that describe the considered nonpotential perturbation, we apply the curl operation to Eq. (5). With account taken of the induction equation (9) and of the adiabatic equation $p=Kn^\alpha$ we obtain

$$\frac{\partial\mathbf{H}}{\partial t}=\operatorname{rot}[\mathbf{v}\times\mathbf{H}]+\frac{ec}{\sigma}\operatorname{rot}n(\mathbf{v}-\mathbf{V}) \tag{13}$$

assuming that σ is constant. Taking into account the quasineutrality, $n=z_iN$, where n is the electron density, Maxwell's equation for the determination of the magnetic field takes the form

$$\operatorname{rot}\mathbf{H}=\frac{4\pi en}{c}(\mathbf{V}-\mathbf{v})+\frac{4\pi e}{c}z_b n_b\mathbf{v}_b. \tag{14}$$

Using (14) we can rewrite Eq. (13) in the form

$$\frac{\partial\mathbf{H}}{\partial t}=\operatorname{rot}[\mathbf{v}\times\mathbf{H}]+\frac{z_b ec}{\sigma}\operatorname{rot}n_b\mathbf{v}_b-\frac{c^2}{4\pi\sigma}\operatorname{rot}\operatorname{rot}\mathbf{H}. \tag{15}$$

Equations (14) and (15) are the basic ones for the investigation of the nonpotential perturbations. It is now necessary to substitute in them the perturbed quantities that describe the beam and the plasma ions. We use hereafter for the ions an equation obtained by eliminating the electric field from Eqs. (4) and (5)

$$m_i\frac{d\mathbf{V}}{dt}=-z_i\left(\frac{e}{c}[\mathbf{v}\times\mathbf{H}]+\frac{\nabla p}{n}\right). \tag{16}$$

Adding here the continuity equation for the ions, we can obtain the following equations for the velocity perturbation and the ion density

$$i\omega m_i\mathbf{V}=\frac{z_ie}{c}[\mathbf{v}_0\times\mathbf{H}]+\frac{z_ie}{c}[\mathbf{v}\times\mathbf{H}_0]+m_i\nabla\left(\frac{n}{n_0}c_s{}^2\right), \tag{17}$$

$$-n(\omega^2-k^2c_s{}^2)=(z_ie/m_ic)\operatorname{div}n_0([\mathbf{v}_0\times\mathbf{H}]+[\mathbf{v}\times\mathbf{H}_0]), \tag{18}$$

where $c_s^2=(z_i/m_i)\partial p/\partial n$. The expressions that follow from (17) and (18) for the perturbations of the ion velocities and of the density simplify greatly subsequently when the quasiclassical approximation $k^2r_b^2\gg 1$ is used.

From (6) and from the continuity equation for the beam particles we can obtain, using the quasiclassical approximation, the following expression for the current-density perturbation:

$$n_b=\frac{1}{(\omega-k_zv_{b0})^2-k^2v_T{}^2}\times\left\{i\frac{k_\varphi j_{b0}}{\gamma_b m_b c}H_r-\frac{\omega-k_zv_{b0}}{\gamma_b m_b c}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{j_{b0}}{\omega-k_zv_{b0}}H_\varphi\right)\right\}, \tag{19}$$

where $j_{b0}=z_b e n_{b0}v_{b0}$. Using Eq. (6) for the beam-particle velocity within the framework of the employed quasiclassical approximation, we can show that the principal role is played by the perturbation of the longitudinal current j_z in the beam, while the perturbations of j_r and j_φ, which are connected with the bending of the beam, are small and can be neglected.

Before we substitute the expressions obtained in the basic equations (14) and (15) we note that, just as in Ref. 3, it is convenient to use not the three magnetic-field components connected by the condition $\operatorname{div}\mathbf{H}=0$, but the components H_r and $\operatorname{curl}_r\mathbf{H}$. It is then convenient to use in place of Eq. (15) its r-component and the result of the application of the operation curl_r to this equation, as well as the continuity equation for the plasma electron density

$$\partial n/\partial t+\operatorname{div}(n\mathbf{v})=0. \qquad (20)$$

Within the framework of our approximations, the perturbations of the beam density and of the plasma electrons are

$$n_b=\frac{1}{(\omega-k_z v_{b0})^2-k^2 v_T^2}\frac{ik_\varphi j_{b0}}{\gamma_b m_b c}H_r, \qquad (21)$$

$$n=-\frac{1}{\omega^2-k^2 c_s^2}\frac{z_i e}{m_i c}ik_\varphi n_0 v_0 H_r. \qquad (22)$$

The expressions for the velocity perturbations are simplified correspondingly.

Substituting the expressons for the density perturbations and for the beam and plasma ion velocities in the basic equations and eliminating from them the velocity components v_φ and v_z of the plasma electrons, we obtain a system of three equations for H_r, $\operatorname{curl}_r\mathbf{H}$, and v_r. Using the foregoing simplifying assumptions, this system of equation takes the form

$$\left(1+\frac{\omega_{pb}^2}{k^2c^2}+\frac{\omega_{pi}^2}{k^2c^2}\frac{k_z v_0}{\omega}\right)\operatorname{rot}_r\mathbf{H}=-\frac{4\pi e}{c}n_0 v_r, \qquad (23)$$

$$\left[-i\tilde\omega+\frac{k^2c^2}{4\pi\sigma}\left(1+\frac{\omega_{pb}^2}{c^2}\frac{v_{b0}^2}{(\omega-k_z v_{b0})^2-k^2v_T^2}+\frac{\omega_{pb}^2}{k^2c^2}\right)\right]\times H_r=ik_\varphi H_{\varphi 0}v_r, \qquad (24)$$

$$\left[-i\tilde\omega+\frac{k^2c^2}{4\pi\sigma}\left(1+\frac{\omega_{pb}^2}{k^2c^2}\right)\right]\operatorname{rot}_r\mathbf{H}$$
$$=-i\frac{k_\varphi c H_{\varphi 0}}{4\pi e n_0}\left(k^2+\frac{\omega_{pb}^2}{c^2}\frac{k_\varphi^2 v_{b0}^2}{(\omega-k_z v_{b0})^2-k^2v_T^2}+\frac{\omega_{pb}^2}{c^2}\right.$$
$$\left.+\frac{\omega_{pi}^2}{k^2c^2}\frac{k_z v_0}{\omega}+\frac{\omega_{pi}^2}{c^2}\frac{k_\varphi^2 v_0^2}{\omega^2-k^2c_s^2}\right)H_r+ik_\varphi\frac{\partial v_0}{\partial r}H_r. \qquad (25)$$

Here $\tilde\omega=\omega-k_z v_0$ and $v_0=j_{b0}/en_0$. In the derivation of the system (23)–(25) we assumed satisfaction of the condition

$$\tilde\omega\omega\gg v_A^2(k_\varphi^2+\omega_{pb}^2/c^2),\qquad v_A^2=z_iH_{\varphi 0}^2/4\pi m_i n_0, \qquad (26)$$

which means that the perturbations in questions are rapid and they can be considered with the Alfven perturbations neglected. It follows from (34) that the inequality (26) holds at $k^2c^2/\omega_{pi}^2\gtrsim 1$ when account is taken of the other inequalities.

From the system (23)–(25) at $k^2\gg(\omega_{pi}^2/c^2)(k_z v_0/\omega)$ we obtain the following final dispersion equation for the nonpotential perturbations:

$$\left(\frac{4\pi e n_0}{k_\varphi c H_{\varphi 0}}\right)^2\left[\tilde\omega+i\frac{k^2c^2}{4\pi\sigma}\left(1+\frac{\omega_{pb}^2}{k^2c^2}\right)\right]$$
$$\times\left[\tilde\omega+i\frac{k^2c^2}{4\pi\sigma}\left(1+\frac{\omega_{pb}^2}{c^2}\frac{v_{b0}^2}{(\omega-k_z v_{b0})^2-k^2v_T^2}+\frac{\omega_{pb}^2}{k^2c^2}\right)\right]$$
$$=\left(k^2+\frac{\omega_{pb}^2}{c^2}\frac{k_\varphi^2 v_{b0}^2}{(\omega-k_z v_{b0})^2-k^2v_T^2}\right.$$
$$\left.+\frac{\omega_{pb}^2}{c^2}+\frac{\omega_{pi}^2}{c^2}\frac{k_\varphi^2 v_0^2}{\omega^2-k^2c_s^2}-\frac{4\pi e n_0}{cH_{\varphi 0}}\frac{\partial v_0}{\partial r}\right)$$
$$\times\left(1+\frac{\omega_{pb}^2}{k^2c^2}\right). \qquad (27)$$

In the absence of a beam, Eq. (27) leads to an expression for the frequency of helicons that are weakly damped on account of the finite value of the conductivity.

The dispersion equation (27) does not admit of a direct transition to the dispersion equations considered in Refs. 3 and 4. The reason is that it was derived without including terms proportional to the magnetic field, compared with j_{b0} relative to the parameter $I/I_b\ll 1$.

If we neglect in (17) dissipation and the perturbation of the ions, and if the density gradient is strong enough and the extraneous current is uniform enough over the cross section, we can obtain the instability indicated above. Retaining in the right-hand side of (27), besides k^2, only the term with the gradient of the velocity v_0, and using the unperturbed equation of motion for the plasma ions, we obtain

$$(4\pi e n_0/k_\varphi c)^2\tilde\omega^2=k^2H_{\varphi 0}^2+4\pi(m_i/z_i)a_r\partial n_0/\partial r, \qquad (28)$$

where $\mathbf{a}$ is the acceleration acting on the plasma. If we neglect in the right-hand side of (28) the term $k^2H_{\varphi 0}^2$, which corresponds to helicon perturbations, instability is present at

$$a_r\frac{1}{n_0}\frac{\partial n_0}{\partial r}<0,$$

i.e., when the acceleration direction is antiparallel to the density gradient. This criterion coincides formally with the Rayleigh-Taylor criterion for the analog of the free-fall acceleration in the absence of acceleration of the medium. The growth rate is increased in this case by $k^2c^2\omega_{pi}^2$ times compared with the usual Rayleigh-Taylor instability.

To estimate the growth rates we shall use hereafter relations that connect, in order of magnitude, the local values of n_b, v_{b0}, and v_T with the beam current I_b, with the Alfven current $I_A=\gamma_b m_b v_{b0}c^2/e$ and with the total current I:

$$\frac{\omega_{pb}^2}{c^2}=\frac{1}{r_b^2}\frac{I_b}{I_A},\qquad \frac{\omega_{pb}^2 v_{b0}^2}{c^2v_T^2}=\frac{1}{r_b^2}\frac{I_b}{I}.$$

We neglect now in the right-hand side of the dispersion equation (27) the terms with the gradient of the velocity v_0 and with the plasma-density perturbation. In this case the instability is due to the anisotropy of the beam-particle function

$$\left(1+\frac{\omega_{pb}^2}{k^2c^2}\right)^{-1}\left(\frac{4\pi e n_0}{k_\varphi c H_{\varphi 0}}\right)^2\tilde\omega^2=k^2-\frac{1}{r_b^2}\left(\frac{I_b}{I}-\frac{I_b}{I_A}\right). \qquad (29)$$

For a total current $I<I_A$, when the longitudinal beam velocity greatly exceeds its transverse velocity so that the beam

particles have an appreciable velocity anisotropy, a change takes place in the dispersion of the helicon perturbations at $k^2r_b^2 < I_b/I$ and instability sets in. We note that an instability due to the change of the oscillaton dispersion in a strong longitudinal magnetic field, was noted in Ref. 5 on the helicon branch of the oscillations.

We assess now the role of the ion term in (27). We use for this purpose the equation for the plasma-electron temperature

$$n_0\partial T/\partial t = (\alpha-1)j_{b0}^2/\sigma. \tag{30}$$

Using the expression for the magnetic field from (7) and the connection between the equilibrium quantities in the beam

$$c^{-1}j_{b0}H_{\varphi 0}+T_b\partial n_{b0}/\partial r=0, \tag{31}$$

we obtain the following estimate for the plasma temperature T:

$$n_0T\approx\tau j_{b0}^2/\sigma\approx n_{b0}T_b. \tag{32}$$

It follows therefore that in the dispersion equation (27) at $\omega^2 \ll k^2c_s^2$ the ion term is of the order of the beam term. Thus, Eq. (29) for the growth rate of the unstable helicon perturbation is of the correct order of magnitude also when the ions are taken into account.

The dispersion equation (27) yields the transition from the dissipative mode considered in Ref. 2 to the helicon instability. The dissipative instability considered in Ref. 2 is obtained from (27) as $\sigma\to 0$ for a cold beam ($T_b = 0$)

$$-\omega^2\equiv\gamma^2=\omega_{pb}^2v_{b0}^2/c^2. \tag{33}$$

Taking into account the thermal spread of the beam as $\sigma\to 0$, the growth rate of the dissipative perturbation is

$$\gamma=\frac{I_b}{I_A}\frac{c^2}{4\pi\sigma r_b^2}\left(\frac{1}{\theta^2}-1\right). \tag{34}$$

If the magnetic field is not zero and the beam particles move on equilibrium orbits, then $\theta^2 = I/I_A$. In the case of a zero magnetic field the angle $\theta = v_\perp/v$ characterizes the anisotropy of the particle distribution in the beam. In this case instability exists at all σ.

It is easily seen from a comparison of (7) with (34) that the evolution time of such an instability coincides with time of generation of the main magnetic field, and at first glance this result is not applicable anywhere. Yet this may not be the case. Consider the propagation of an electron beam of width $2a$ in the direction of the z axis in the half-space $z\geqslant 0$. If at $|x|\leqslant a$ and in the region $z>0$ the beam is uniform the main magnetic field H_y is generated only near the planes $z=0$ and $|x|=a$, and the field penetrates into the beam region only for a certain finite time. It is clear that for a sufficiently broad beam the generation of a magnetic field inside the beam turns out to be substantial as a result of instability.

Obviously, the foregoing is valid in the case when there is no noticeable entry of a magnetic field on account of the motion of the electronic component of the plasma in which the magnetic field is frozen.[6]

We estimate now the growth rate of the dissipative instability for typical laser-plasma parameters, when the power flux is 10^{13} W/cm^2 and higher, the hot-component temperature is $T_e\approx 1$ keV, and $z_{\text{eff}}\approx 5$. The critical Alfven current corresponding to this flux is only of the order of 1 kA.

If the indicated streams are incident on a spherical target with radius $R\approx 10^{-2}$ cm, the total flux of the hot particles reaches 10^5 Alfven currents. For the parameters indicated, the growth rate of the considered perturbations is $\gamma\approx 10^{-8}n_b$ and is equal to 10^9 sec^{-1} at $n_b\gtrsim 10^{17}$. The characteristic scale of these perturbations can be estimated from the condition of the stabilization of the instability by the diffusion spreading of the magnetic field in the plasma:

$$r\gtrsim(c^2/4\pi\sigma\gamma)^{1/2}\approx 10^{-3}\text{ cm}.$$

Papers dealing with the discussed dissipative instability have recently been published. Haines[7] considered, as applied to a laser plasma, overheat instability, when the change of the conductivity of the medium is significant. The very existence of the instability was found formally to be connected with some form of energy balance in the plasma. This does not reflect its universal electrodynamic origin, and the concrete results of Ref. 6 are not applicable to a laser plasma where there is no noticeable difference between the electron and ion temperatures. Hughes *et al.*[8] analyzed numerically the nonlinear stage of the dissipative instability.[2]

It must be emphasized that as the magnetic-field generation proceeds a situation inevitably arises in which the magnetization of the plasma electrons becomes substantial and the magnetic field begins to influence the dispersion properties of the perturbations. Instability exists in this case even on the helicon branch.

The helicon-instability growth rate that follows from (29) with allowance for $k^2r_b^2 < I_b/I$, is

$$\gamma\approx\frac{cH_{\varphi 0}}{2\pi en}\frac{I_b}{I}\frac{1}{r_b^2}. \tag{35}$$

This expression does not depend at all on the magnetic field, but is determined only by the beam current, by the plasma density, and by the characteristic radius of the beam:

$$\gamma\approx I_b/\pi en_0r_b^3. \tag{36}$$

For the typical beam parameters discussed in the program for inertial fusion with the aid of light ions (current $I_b = 3.5$ MA, plasma density $n_0 = 10^{18}$ cm^{-3}, $r_b = 1$ cm, pulse duration $\tau = 50$ nsec we get from (36) a growth rate $\gamma\approx 10^7$ sec^{-1}, which is insufficient for instability development. However, at typical parameters of experimental studies of the passage of high-current electron beams through a plasma ($I_b = 30$–100 kA, $r_b = 1$–2 cm, $n_0\approx 10^{14}$) the instability considered is significant.

Higher growth rates are obtained when account is taken of the resonant character of the ion term in (27), when it plays the principal role in the right hand of this equation. With dissipation neglected and at $\omega_{pb}^2/k^2c^2 < 1$ the dispersion equation (27), with only the ion term taken into account, takes the form

$$\left(\frac{4\pi en_0}{k_\varphi cH_{\varphi 0}}\right)^2\tilde{\omega}^2=k^2+\frac{\omega_{pi}^2}{c^2}\frac{k_\varphi^2v_0^2}{\omega^2-k^2c_s^2}. \tag{37}$$

To investigate the resonant perturbation we put $\omega = kc_s + \Omega$, where $\Omega\ll kc_s$. We rewrite the dispersion equation (37) in the form

$$\left(\frac{4\pi e n_0}{k_\varphi c H_{\varphi 0}}\right)^2\left(\omega-k_z v_0+\frac{k_\varphi k c H_{\varphi 0}}{4\pi e n_0}\right)\left(\omega-k_z v_0-\frac{k_\varphi k c H_{\varphi 0}}{4\pi e n_0}\right)$$
$$=\frac{\omega_{pi}^2}{c^2}\frac{k_\varphi^2 v_0^2}{\omega^2-k^2c_s^2}$$

and substitute in it $\omega = kc_s + \Omega$. When the resonance condition

$$kc_s-k_z v_0=kk_\varphi c H_{\varphi 0}/4\pi e n_0 \tag{38}$$

is taken into account, the dispersion equation with respect to Ω at $k_z \ll k$ takes the form

$$\Omega^2/k_\varphi=\frac{cH_{\varphi 0}}{4\pi e n_0}\frac{\omega_{pi}^2}{c^2}\frac{v_0^2}{4c_s}. \tag{39}$$

Recognizing now that the perturbation growth rate should exceed the reciprocal skin-effect time, $\Omega > k_\varphi^2 c^2/4\pi\sigma$, we obtain an expression for the maximum growth rate

$$\Omega_{max}^3 \lesssim \frac{4\pi\sigma}{c^2}\left(\frac{cH_{\varphi 0}}{4\pi e n_0}\right)^2\frac{\omega_{pi}^4}{c^4}\frac{v_0^4}{16c_s^2} \tag{40}$$

or alternately

$$\Omega_{max}\lesssim\left(\frac{\sigma H_{\varphi 0}}{e n_0 c}\right)^{1/3}\frac{I_b}{\pi e n_0 r_b^3}\left(\frac{r_b\omega_{pi}}{c}\right)^{4/3}\left(\frac{v_0}{c_s}\right)^{2/3}\left(\frac{I}{32I_b}\right)^{1/3}.$$

Let us estimate the expression obtained for the growth rate under condition when beams of light ions are transported to a target, within a framework of the approach with inertial containment. The following characteristic parameters are indicated in Ref. 9: $I_b = 3$ MA, $r_b \approx 1$ cm, $n_0 = 10^{18}$, $v_0 = 10^7$ cm/sec, $c_s = 0.5\times 10^7$ cm/sec, $H = 6\times 10^3$ Oe, and $\sigma = 10^{15}$ sec^{-1}. In this case the resonant increment exceeds by more than one order the growth rate (36) and its order of magnitude is 10^8 sec^{-1}.

We have thus shown that when charged-particle beams propagate along high-density plasma channels there are excited rather fast perturbations that cause local pinching of the beam. Although we have not analyzed here the linear stage of the instability, it can be easily seen from the character of the phenomenon that the current and magnetic-field perturbations can grow to the values $I_A\theta^2$ and $H_{\varphi 0}$, respectively. The beam particles scattered by the magnetic-field fluctuation accumulate rapidly an average angle spread.

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