Radiative corrections to the atomic levels in a strong electromagnetic field

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The influence of an electromagnetic field on the radiative corrections to the atomic energy levels is studied for the case of "strong fields" for which the interaction between the atom and the field is of the order of or greater than the radiative effects. The analysis is carried out on the basis of the Schwinger-Dirac equation for the propagation function of a bound electron in the field and on the basis of the density matrix apparatus in the Furry representation. It is shown that in the strong-field approximation the radiative shifts and widths are manifest as radiative corrections to the quasi-energies. Intensity effects in the radiative corrections to the atomic levels are obtained in ultrahigh resolution experiments in the case of single-photon resonance. Some multiphoton processes are considered by taking into account the effect of the field on the radiative structure of the levels.

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1. INTRODUCTION

Recent developments of methods of ultrahigh resolution laser spectroscopy have uncovered new possibilities for new precision measurements of the radiative shifts (RS) in the region of radio and optical frequencies of atomic transitions in ordinary[1–4] and muonic[5] atoms.

To account for the recently attained high experimental accuricies (for example, 10^{-5} for the $2s_{1/2}-2p_{1/2}$ state of the Lamb shift[1] and 2×10^{-3} for the RS of the ground state $1s_{1/2}$ in hydrogen and deuterium,[2] 2×10^{-7} for the anomalous magnetic moment of the electron in radio-frequency resonance experiments[6]), a need has therefore arisen for a more thorough theoretical study of the interaction of an atom with a strong electromagnetic field, with account taken of the radiative corrections.

The RS δ_i due to the interaction of the electron with the radiation field are usually neglected in the study of the behavior of the atomic levels in an electromagnetic field within the framework of the customary approaches (the density-matrix formalism,[7,8] methods based on the equations for the amplitudes[9] or propagation functions). According to one point view, to take the RS into account it suffices to add them to the energies of the atomic levels that are not perturbed by an external electric field. This procedure, which is valid for week external fields, is not correct in the case of "strong fields" whose intensity E, while smaller than the characteristic atomic energy $E_{\rm at}=p_{\rm at}^{\;3}/em\,[\,p_{\rm at}=(2mI)^{1/2}$ is the characteristic momentum of the electron or muon, and $I=(Ze^2)^2m$ is the ionization energy of the atomic system] corresponds, however, to an atom-field interaction energy much larger than the RS and the widths γ_i of the atomic levels. In the latter case it is necessary to take into account the influence of the strong field on the RS and on the level widths, and the radiative corrections must be calculated already for atomic energy levels perturbed by the external field.

The situation is further simplified in the indicated approaches by the nonrelativistic treatment of the interaction with the radiation field, so that in principle it is necessary to take into account only the Bethe parts of the RS.

The present paper is an attempt to fill these gaps. We use below a consistent approach based on the $S(t)$-matrix formulation of quantum electrodynamics in an external field in the Furry representation, and the influence of the field on the radiative effects is taken into account in a natural fashion.

It is known that for a time dependent external field the self-energy operator, which is a function of two four-dimensional points x and x', depends separately on the times t and t' and not on their difference as in the case of stationary fields. It is shown in this paper that treatment of this quantity in the representation of the quasi-energy states (QES) simplifies substantially the investigation of radiative effects in a field that is periodic in time, and reduces it in fact to the stationary case. The convenience of this representation lies also in the fact that the use of the QES makes it possible, as shown by Ritus,[10] to take easily into account the shifts and the splittings of the atomic levels in the electromagnetic field.

Within the framework of such an approach, we obtain in Sec. 2 from the Schwinger-Dirac equation, for the propagation function of a bound electron in an electromagnetic field, equations that are generalizations, to include the case of the presence of a periodic external field, of the Low equations[11] for the RS and the widths of the atomic levels.

Section 3 deals with the resonant case. To be specific, we consider an atomic transition between two atomic states with nondegenerate energy levels ω_a and ω_b, to which a resonant external field of intensity $E\ll E_{\rm at}$ and of frequency $\omega\approx\omega_{ba}=\omega_b-\omega_a$ corresponds. The spectrum of such a system is obtained with account taken of the effects of the intensity and radiative structure of the levels, which depend on the parameter

$$\eta=|V_{ba}/\varepsilon|\approx|E\omega_{ba}/E_{\rm at}\varepsilon|\gg1,$$

which characterizes the interaction of the field with the atom in the resonant case, and with accuracy up to nonresonant effects of second order to the quantities δ_i and γ_i $(i=a,b)$.

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In the strong field limit $2|V_{ab}|\gg|\delta_b-\delta_a|$, $4|V_{ab}|\gg|\gamma_b-\gamma_a|$, for small detunings from resonance $\varepsilon=\omega_{ba}-\omega \ll|V_{ab}|$, where V_{ab} is the matrix element of the interaction of the field with the atom; the RS and the widths of the atomic levels in the field appear as radiative corrections to the quasi-energies (QE), and are expressed in the resonant approximation in simple fashion in terms of their zero-field values and the populations of the resonant states [see Eqs. (25) and (25a) below]. A similar situation takes place also when account is taken of the degeneracy of levels of arbitrary multiplicity [see formula (25b) below].

Included among the results of general character, besides those of Sec. 2, are also the equations obtained in Sec. 5 for the density matrix in the QES. These equations describe completely the intensity effect in the RS and in the level widths, in the e^2 approximation in the radiation field. This formalism is most convenient for the analysis of multiphoton atomic transitions for the case of "strong fields."

Another purpose of the present paper is to obtain the probabilities of certain known multiphoton processes with allowance for the RS. We consider processes of single-photon decay of atomic states as they are mixed by the resonant field (Sec. 4), stimulated transitions in a two-level system, and resonance fluorescence (Sec. 5).

2. PROPAGATION FUNCTION OF A BOUND ELECTRON IN THE PRESENCE OF A PLANE-WAVE FIELD

The Feynman propagation function of a bound electron interacting with the field of a plane wave $\mathbf{A}(\mathbf{r},t)=\mathrm{Re}[\mathbf{A}_0\exp(-it)]$, which takes in the Furry representation the form

$$G(x,\ y)=\langle 0|T(\psi(x)\bar{\psi}(y)S)|0\rangle/S_0,$$

satisfies the Dirac-Schwinger equation

$$(i\gamma\Pi+m)G(x,y)+\int M(x,x')G(x',y)d^4x'=-i\delta^{(4)}(x-y).\ (1)$$

Here $\Pi_\alpha=-i\partial/\partial x_\alpha-eA_\alpha$, where the field $A_\alpha=(\mathbf{A},0)+(0,U)$ is the sum of the Coulomb potential $U(r)$ of the nucleus and of the vector potential $\mathbf{A}$ of the field, while $S=S(\infty,-\infty)$ is the scattering matrix, with $S_0=\langle 0|S|0\rangle$, and the operator $\psi(x)$ satisfies the Dirac equation in the field A_α. The renormalized self-energy operator

$$M(x,\ x')=e\gamma_\alpha A_{\text{eff}\ \alpha}(x)\delta(x-x')+\Sigma(x,\ x'),\qquad(2)$$

[where the effective potential $A_{\text{eff}\,\alpha}(x)$ contains the corrections that must be added to A_α to allow for the polarization of the vacuum and Σ is the mass operator] describes the radiative effects in the presence of an external field. In the e^2 approximation in the radiation field we have for the nonrenormalized values of the quantities

$$\gamma A_{\text{eff}}^{(0)}(x)=ie\int d^3x'\,dt'\gamma_\mu\,\mathrm{Sp}[\gamma_\mu S_F(\mathbf{x}',t';\mathbf{x}',t'+\varepsilon)]\times|_{\varepsilon\to+0}D(x-x'),$$

$$\Sigma_{(0)}(x,\ x')=ie^2D(x-x')\gamma_\mu S_F(x,\ x')\gamma_\mu,$$

where S_F is the propagation function of the bound electron in the field $\mathbf{A}$, and D is the photon propagation function.[1)]

It is known that for a time-dependent electromagnetic field the quantity $M(x,x')$, which is a function of two four-dimensional points, depends separately on t and t', and not on the difference $t-t'$ as in the case of stationary fields. However, both this quantity and Eq. (1) become substantially simpler if we use the solutions of the Dirac equation with a definite quasi-energy E_n (Refs. 10 and 14):

$$\psi_n(\mathbf{r},t)=e^{-iE_nt}\Phi_n(\mathbf{r},t)=e^{-iE_nt}\sum_\alpha\sum_{q=-\infty}^{\infty}a_{n\alpha}(q)e^{iq\omega t}\varphi_\alpha(\mathbf{r}),\quad(3)$$

$$\left(\alpha\Pi+\beta m+eU(r)-i\frac{\partial}{\partial t}\right)\Phi_n=E_n\Phi_n.\qquad(4)$$

The functions Φ_n make up a complete set

$$\sum_n\Phi_n(\mathbf{r},t)\Phi_n^+(\mathbf{r}',t)=\delta(\mathbf{r}-\mathbf{r}')\qquad(5)$$

and are chosen together with QE by letting $E_n\to\omega_n$ and $\Phi_n\to\varphi_n$ as $t\to-\infty$ when $\mathbf{A}=0$, where ω_n are the atomic energy levels and φ_n are the corresponding atomic wave functions.

We shall use the expansion of the operators $\psi(x)$ in terms of the system of solutions ψ_n. In this case the propagation function

$$S_F=\langle 0|T(\psi(x)\bar{\psi}(x'))|0\rangle$$

takes the form

$$S_F(x,x')=\frac{1}{2\pi i}\int_{-\infty}^{\infty}d\sigma e^{i\sigma(t'-t)}\sum_n\frac{\Phi_n(x)\bar{\Phi}_n(x')}{E_n(1-i0)-\sigma},\qquad(6)$$

where the summation is over all the QES corresponding to atomic states with positive and negative frequencies.

The use of solutions (3), which hold for the "atom + periodic field" system with neglect of relaxation phenomena, greatly simplifies the analysis of the radiative effects in a time-periodic field. The reason is that the matrix element of the operator M in the energy representation between the functions Φ_n

$$M_{nm}(\sigma,\sigma')=\frac{1}{2\pi}\int d^4x\,d^4x'e^{i(\sigma t-\sigma't')}\bar{\Phi}_n(x)M(x,x')\Phi_m(x')\quad(7)$$

can be represented by virtue of the periodicity conditions

$$\Phi_n(\mathbf{r},\ t+2\pi/\omega)=\Phi_n(\mathbf{r},\ t)$$

in the form of an expansion in harmonics:

$$M_{nm}(\sigma,\sigma')=\sum_{q=-\infty}^{\infty}\delta(\sigma-\sigma'-q\omega)M_{nm}^{(q)}(\sigma'),\qquad(8)$$

where the quantities $M_{nm}^{(q)}$ are of the form

$$M_{nm}^{(q)}(\sigma)=\langle\langle\Phi_n|e^{iq\omega t}M(\sigma)\Phi_m\rangle\rangle.\qquad(9)$$

In these expressions the double angle brackets denote the usual integration with respect to the coordinates and averaging over the period $2\pi/\omega$, while the operator $M(\sigma)$ is the renormalized value of the operator

$$M_{(0)}(\sigma)=\gamma A_{\text{eff}}^{(0)}+\Sigma_{(0)}(\sigma),$$

$$\Sigma_{(0)}(\sigma)=\frac{ie^2}{(2\pi)^4}\int\frac{d^4k}{k^2}\gamma_\mu\frac{1}{i\gamma(\Pi-k)-\beta\sigma+m-i0}\gamma_\mu.\qquad(10)$$

The operator $\Sigma_{(0)}$ is renormalized in standard fashion,[12] while the quantities $A_{\rm eff}^{(0)}$ are renormalized with the aid of the condition $A_{\rm eff}(A_\alpha = 0) = 0$.

Let us clarify the method of obtaining the expansion (8). Its validity can be easily verified for the matrix element of the first term in (2), if we use the $\tilde{\delta}$ function in the space of periodic functions with period $2\pi/\omega$:

$$\tilde{\delta}(\tau)=\sum_{q=-\infty}^{\infty} e^{iq\omega\tau},\quad \frac{\omega}{2\pi}\int_{-\pi/\omega}^{\pi/\omega} d\tau\tilde{\delta}(\tau-\tau')f(\tau)=f(\tau'). \quad (11)$$

To transform the contribution of the mass operator it is necessary to use expression (6) and the representation for the photon propagation function

$$D(x)=-\frac{i}{(2\pi)^4}\int\frac{d^4k}{k^2-i0}e^{ikx},$$

as well as Eq. (4) and relations (5) and (11), with the aid of which integration is carried out with respect to the coordinates and the time in the matrix element (7).

We turn now to the propagation function G, which is transformed, neglecting radiative effects, into the propagation function (6). Taking the expansion (8) into account, we represent G in the form

$$G(x,x')=\frac{1}{2\pi i}\sum_{n,m}\sum_{q=-\infty}^{\infty}\Phi_n(x)\bar{\Phi}_m(x')e^{-iq\omega t}\times\int_{-\infty}^{\infty}d\sigma e^{i\sigma(t'-t)}G_{nm}^{(q)}(\sigma).$$

Equation (1) reduces in this case to a system of equations for the coefficient propagation functions:

$$G_{nm}^{(q)}(\sigma)=[E_n(1-i0)-\sigma-q\omega]^{-1}\{\delta_{mn}\delta_{q0}+\sum_p\sum_{q_1=-\infty}^{\infty}M_{np}^{(q-q_1)}(\sigma+q_1\omega)G_{pm}^{(q_1)}(\sigma)\}, \quad (12)$$

which generalize the Low equations for the shifts and the natural widths of atomic levels to include the case when a monochromatic field periodic in time is present.

One of the methods of investigating this system of equations is to reduce it to a form in which are subdivided explicitly the terms containing the ratios $M_{ij}^{(q)}/(E_i - E_j + q\omega)$ of quantities of the order of the radiative effects to the difference of the QE.

Introducing new quantities, defined by the relation $R_{nm}^{(q)}(\sigma) = G_{nm}^{(q)}(\sigma)/G_{mm}^{(0)}(\sigma)$ for two groups of values of the indices ($n\neq m$ with q arbitrary and $n = m$ with $q\neq 0$) and $R_{nm}^{(q)}$ at $n = m$ and $q = 0$, we transform the equations in the following manner:

$$G_{nn}^{(0)}(\sigma)=1/(E_n+W_n(\sigma)-\sigma), \quad (13)$$

$$[\sigma+q\omega-E_n-M_{nn}^{(0)}(\sigma+q\omega)]R_{nm}^{(q)}(\sigma)=M_{nm}^{(q)}(\sigma)+\sum_{q_1\neq 0}M_{nm}^{(q-q_1)}(\sigma+q_1\omega)R_{mm}^{(q_1)}(\sigma)+\sum_{q_1\neq q}M_{nn}^{(q-q_1)}(\sigma+q_1\omega)R_{nm}^{(q_1)}(\sigma)+\sum_{p\neq m,n}\sum_{q_1}M_{np}^{(q-q_1)}(\sigma+q_1\omega)R_{pm}^{(q_1)}(\sigma). \quad (14)$$

The propagation functions are then represented in the form

$$G(x,x')=\frac{1}{2\pi i}\sum_{n,m}\Phi_n(x)\bar{\Phi}_m(x')\times\int_{-\infty}^{\infty}\frac{d\sigma\exp[i\sigma(t'-t)]}{E_m+W_m(\sigma)-\sigma}\sum_{q=-\infty}^{\infty}e^{-iq\omega t}R_{nm}^{(q)}(\sigma), \quad (15)$$

in which are explicitly separated the terms $R_{mn}^{(q)}$ with contain high orders of e^2 and describe multistep nonradiative transitions between the QES with allowance for the RS and the level widths of the intermediate states, in terms of which is expressed also the quantity

$$W_n(\sigma)=M_{nn}^{(0)}(\sigma)+\sum_{q\neq 0}M_{nn}^{(-q)}(\sigma+q\omega)R_{nn}^{(q)}(\sigma)+\sum_{p\neq n}\sum_q M_{np}^{(-q)}(\sigma+q\omega)R_{pn}^{(q)}(\sigma). \quad (16)$$

When the effects of the plane-wave field are neglected, as a result of averaging over the period, the only nonvanishing terms in expression (9) are the matrix elements with zeroth harmonic $M_{ij}^{(q)}(\sigma, \mathbf{A} = 0) = \delta_{q0}m_{ij}(\sigma)$. The quantities m_{ij} are here the matrix elements of the self-energy operator at $\mathbf{A} = 0$ between the atomic wave functions, and its diagonal elements determine the shifts and widths of the atomic levels:

$$\delta_i=\mathrm{Re}\, m_{ii}(\omega_i),\quad \gamma_i=-2\,\mathrm{Im}\, m_{ii}(\omega_i).$$

As seen from (14), in this limit $R_{nm}^{(q)}\to\delta_{q0}R_{nm}^{(0)}(\mathbf{A} = 0)$, and Eq. (15) goes over into the known expression for the atomic propagation function.[11]

At low values of the ratio

$$|M_{nm}^{(q)}(E_n)/(E_m+M_{mm}^{(0)}-E_n-M_{nn}^{(0)}+q\omega)|\ll 1 \quad (17)$$

for the permissible values of the indices ($n\neq m$ with q arbitrary; $n = m$ with $q\neq 0$) the functions R and W, as follows from (14) and (16), can be represented in the form of an iteration series in terms of this quantity. In the lowest-order approximation we obtain the expression

$$G(x,x')=\frac{1}{2\pi i}\int_{-\infty}^{\infty}d\sigma e^{i\sigma(t'-t)}\sum_n\frac{\Phi_n(x)\bar{\Phi}_n(x')}{E_n+M_{nn}^{(0)}(\sigma)-\sigma}, \quad (18)$$

in which the radiative shifts δE_n and the widths Γ_n of the atomic levels in the field appear as radiative corrections to the quasi-energies E_n:

$$\delta E_n-\frac{i}{2}\Gamma_n=M_{nn}^{(0)}(E_n)=\frac{\omega}{2\pi}\int_{-\pi/\omega}^{\pi/\omega}dt\langle\Phi_n|M(E_n)|\Phi_n\rangle. \quad (19)$$

For fields $E\ll E_{\rm at}$ and in the absence of resonances, the field effects in the functions (3) are given by the terms of order $\xi = e|\mathbf{A}_0|/p_{\rm at} = (2IE/\omega E_{\rm at})\ll 1$ (Ref. 10), and it is these which determine the difference between the quantity (17) at $q = 0$ and the usual parameters δ_i/ω_{ij}, γ_i/ω_{ij}, and $|m_{ij}|/\omega_{ij}$ of perturbation theory in the zero-field case. Contributions with nonzero harmonics are suppressed by the additional factor $\xi^{|q|}$. However, the conditions (17), and consequently also the expressions (18) and (19), are valid also in the strong-field limit, when the field shifts of the atomic levels exceed in order of magnitude the values of the radiative effects in the field. An illustration of this case in the presence of resonances is given in the next section.

Expressions (19) can be rewritten in terms of the matrix elements over the quasi-energy wave functions, by using Eq. (4) and formulas (5) and (11):

$$M_{nn}^{(0)}(E_n)=\frac{\alpha}{4\pi^2}\int\frac{d^3k}{|\mathbf{k}|}$$

$$\times\sum_{q=-\infty}^{\infty}\Big[\sum_m\frac{Q_{\mu,nm}^{(q)}(\mathbf{k})Q_{\mu,mn}^{(-q)}(-\mathbf{k})}{E_n-E_m(1-i0)-q\omega-|\mathbf{k}|E_m/|E_m|}$$

$$+\frac{|\mathbf{k}|}{\pi}\frac{Q_{\mu,nn}^{(q)}(\mathbf{k})J_\mu^{(q)}(\mathbf{k})}{\mathbf{k}^2-(q\omega)^2-i0}\Big]+\delta m\langle\langle\Phi_n\Phi_n\rangle\rangle. \quad (20)$$

Here

$$Q_{\mu,nm}^{(q)}(\mathbf{k})=\langle\langle\Phi_n|\gamma_\mu\exp[i(\mathbf{kr}+q\omega t)]|\Phi_m\rangle\rangle,$$

the quantity

$$J_\mu^{(q)}(\mathbf{k})=\frac{\omega}{2\pi}\int_{-\pi/\omega}^{\pi/\omega}d^3r\,dt\,e^{-i(\mathbf{kr}+q\omega t)}\,\mathrm{Sp}[\gamma_\mu S_F(\mathbf{r},t;\mathbf{r},t+0)] \quad (21)$$

is the Fourier component of the current induced in vacuum by the external field A_μ, δm is the mass renormalization constant, and $\alpha=e^2/4\pi$.

3. RADIATIVE CORRECTIONS IN THE RESONANT CASE

The purpose of the present section is the calculation of the RS and of the widths of the nondegenerate levels ω_a and ω_b of the atom in a periodic field in the presence of one-photon resonance between the levels and the field: $\varepsilon=\omega_{ba}-\omega\ll\omega_{ba}$.

In the approximation

$$|V_{ba}|\ll\omega_{ba},\quad V_{ba}=-e\langle\varphi_b|\alpha\mathbf{A}_0|\varphi_a\rangle,$$

taking the nonresonant states $\varphi_i(i\neq a,b)$ into account by perturbation theory, we have for the QE spectra (see, e.g., Refs. 15 and 16)

$$E_{a,b}=\omega_{a,b}+\Delta_{a,b}\pm\frac{\varepsilon'}{2}\left(1-\frac{2}{|\varepsilon'|}\Omega'\right),$$

$$E_i=\omega_i+\Delta_i\quad(i\neq a,\ b). \quad (22)$$

Here $2\Omega'=[(\varepsilon')^2+|V_{ab}|^2]^{1/2}$, $\varepsilon'=\varepsilon+\Delta_b-\Delta_a$, and the quantities E_a and E_b are jointly represented by one expression with a comma between the subscripts. These expressions take into account the nonresonant Stark shifts of the levels Δ_j (Ref. 10), which are of the order of $\approx\xi^2 I$ at $\omega\approx I$ and which can be comparable with the RS in typical laser experiments.

The populations n_{ja} of the atomic states φ_a in the QES Φ_j, with allowance for the nonresonant corrections, can be calculated by using the following formula[17]:

$$n_{ja}=\sum_{q=-\infty}^{\infty}|a_{ja}(q)|^2=\partial E_j/\partial\omega_a.$$

Neglecting the quantities $\partial\Delta_i/\partial\omega_j$, which are second-order nonresonant corrections, we obtain

$$n_a\equiv n_{aa}=n_{bb}=\frac{1}{2}\left(1+\frac{|\varepsilon'|}{2\Omega'}\right)+\frac{\varepsilon'|V_{ab}|^2}{4|\varepsilon'|\Omega'\omega_{ba}},$$

$$n_b\equiv n_{ab}=n_{ba}=\frac{1}{2}\left(1-\frac{|\varepsilon'|}{2\Omega'}\right)-\frac{\varepsilon'|V_{ab}|^2}{4|\varepsilon'|\Omega'\omega_{ba}}. \quad (23)$$

The resonant wave functions $\Phi_{a,b}$ corresponding to adiabatic turning-on of the field as $t\to-\infty$, are well known. They are given in Sec. 4 [see Eqs. (30) at $L=0$, $\sigma_a=E_a$, $\sigma_b=E_b$, and also Refs. 15 and 16]. Using them together with Eqs. (25) and (27) we can easily verify that the conditions (17) for the resonant states $\varphi_{a,b}$ lead to new strong-field conditions

$$|\delta_b-\delta_a|\ll2|V_{ab}|,\quad {}^1/_2|\gamma_b-\gamma_a|\ll2|V_{ab}| \quad (24)$$

for small detunings $\varepsilon\ll|V_{ab}|$, $\varepsilon\lesssim\delta,\gamma$, together with the presented criteria for the validity of perturbation theory with respect to the radiation field. Accordingly, expression (18) also is accurate to terms e^2, $\delta_i/2\Omega$, and $\gamma_i/2\Omega$ $(i=a,b)$. The conditions (24) mean that the radiative effects are small compared with the field-induced shifts and splittings of the atomic levels; the quasi-energy spectra of the atom in an external periodic field manifests itself when they are satisfied. It must be noted that there exists a wide range $\alpha^3\ll(E/E_{at})\ll1$ of E for which the conditions $E\ll E_{at}$ and (24) are compatible.

We turn now to the calculation of δE_n and Γ_n. From (19) and (20), taking full account of the resonance effects of the intensity with the parameter $\eta=|V_{ab}|/\varepsilon$, and with accuracy that includes first-order nonresonant corrections, we obtain

$$\delta E_{a,b}-\frac{i}{2}\Gamma_{a,b}=n_{a,b}\left(\delta_a-\frac{i}{2}\gamma_a\right)+n_{b,a}\left(\delta_b-\frac{i}{2}\gamma_b\right)$$

$$\mp\frac{\varepsilon|V_{ab}|}{2|\varepsilon|\Omega}\left(\delta_F-\frac{i}{2}\gamma_F\right), \quad (25)$$

$$\delta_F=\frac{\alpha}{6\pi}|V_{ab}|\Big[\sum_{i\neq a}|\mathbf{P}_{ai}|^2\left(\ln\frac{m}{|\omega_a-\omega_i|}-1\right)$$

$$+\sum_{i\neq b}|\mathbf{P}_{bi}|^2\left(\ln\frac{m}{|\omega_b-\omega_i|}-1\right)\Big]$$

$$\gamma_F=\frac{\alpha}{3}|V_{ab}|\left(\sum_{\omega_i<\omega_a}|\mathbf{P}_{ai}|^2+\sum_{\omega_i<\omega_b}|\mathbf{P}_{bi}|^2\right), \quad (26)$$

where the populations n_a and n_b are given by (23), $\mathbf{P}_{ij}=\langle\varphi_i|\mathbf{v}|\varphi_j\rangle$, and $\mathbf{v}$ is the electron-velocity operator.

Let us clarify the method of obtaining this result. Calculating first the resonant contribution by means of Eq. (19), we note that it suffices to retain the effects of the plane-wave field only in the functions $\Phi_{a,b}$, and they can be neglected in the operator $M(E)$. Indeed, in the resonance approximation there remains in (20) the contribution of the harmonics with $q=0,\ \pm1,\ \pm2$. If the field shifts of the atomic levels are neglected in them compared with the frequencies of the unperturbed atomic transitions, we note that the summation over the functions (3) in the first term of (20) reduces to summation over the atomic wave functions, while in (21) the only nonzero current component is $J_\mu^{(0)}(\mathbf{k})|_{\mathbf{A}=0}$, which is induced by the Coulomb field.[2)] These arguments lead to the first two terms of (25) from which the nonresonant corrections to the level populations were left out.

Similar reasoning and the use of the resonant wave functions lead to the following results:

$$M_{ba}^{(1)}(\omega_a)=\frac{\varepsilon V_{ab}}{4|\varepsilon|\Omega}(m_{aa}-m_{bb}),$$
$$M_{ab}^{(-1)}(\omega_b)=\frac{\varepsilon V_{ab}^*}{4|\varepsilon|\Omega}(m_{aa}-m_{bb}), \quad (27)$$

which are used later on.

To calculate the contributions δ_F and γ_F, i.e., to take into account the previously neglected field corrections to the atomic-transition frequencies, it suffices to start from the part $M_{nn}^{<}$, which is of low frequency in terms of the energy of the virtual photons and which is obtained from (20) after renormalization of the mass. In particular, we have (see Ref. 18)

$$\delta E_n^{<}=\frac{2\alpha}{3\pi}\sum_{q=-\infty}^{\infty}\sum_{j\neq n}|\mathbf{P}_{nj}^{(q)}|^2(E_n-E_j-q\omega)\ln\frac{m}{2|E_n-E_j-q\omega|},$$

$$\mathbf{P}_{nj}^{(q)}=\langle\langle\Phi_n|e^{iq\omega t}\mathbf{v}|\Phi_j\rangle\rangle.$$

Direct calculation of the low-frequency part of $\delta E_n^{<}$ with the aid of the QE spectrum (22) and the corresponding wave functions, and with account taken of the corrections of first order in Ω/ω_{nj} and ε/ω_{nj} to the difference of the levels ω_{nj}, leads to an expression similar to (25) with the quantities[3)] (26). In this approximation,

$$\delta_F\approx\xi\max(\delta_a,\delta_b),\quad \gamma_F\approx\xi\max(\gamma_a,\gamma_b),$$

i.e., they are of the order of the nonresonant corrections to the quantities $\max(\delta_i)$, $\max(\gamma_i)$, $i=a, b$. They become comparable with the zero-field values $\max(\delta_a,\delta_b)$ and $\max(\gamma_a,\gamma_b)$ under the condition $|V_{ba}|\ll\omega_{ba}$, which are realized for fields with $E\lesssim E_{\rm at}$, but also at $E\ll E_{\rm at}$ for a system with close levels $\omega_{ba}\ll I$. These cases, however, call for a special analysis that includes also the calculation of the QES outside the frameworks of the resonant approximation and of perturbation theory.

In the resonant approximation, the RS of the quasi-energies take the following form

$$\delta E_{a,b}=\frac{1}{2}\left(1\pm\frac{|\varepsilon|}{2\Omega}\right)\delta_a+\frac{1}{2}\left(1\mp\frac{|\varepsilon|}{2\Omega}\right)\delta_b. \quad (25a)$$

At $|V_{ab}|\gg\varepsilon$, owing to the equality of the populations $n_a\approx n_b$, the shifts δE_a and δE_b become of the same order even if δ_a and δ_b differ substantially. In the limiting case of the fields $|V_{ab}|\ll\varepsilon$ we obtain $\delta E_i\to\delta_i$.

We make a few remarks concerning allowance for the level degeneracy in the calculation of the radiative corrections to the nonresonant field. It is known[16] that the presence of degeneracy does not change the number of the QE compared with the nondegenerate case, but leads to expressions for the populations that are different from (23). This modifies Eqs. (25) somewhat. In the particular case when the lower level ω_a is not degenerate, we obtain in the resonant approximation with the aid of the corresponding adiabatic quasi-energy wave functions

$$\delta E_a=\frac{\frac{1}{2}\left(1+\frac{|\varepsilon|}{2\Omega_W}\right)\delta_a+\frac{1}{2}\left(1-\frac{|\varepsilon|}{2\Omega_W}\right)\sum_i^N|V_{ab_i}|^2\delta_{b_i}}{\left(\sum_j^N|V_{ab_j}|^2\right)}. \quad (25b)$$

Here

$$\Omega_W=\tfrac{1}{2}\left(\varepsilon^2+\sum_i|V_{ab_i}|^2\right)^{1/2}$$

and the indices b_i pertain to the states φ_{b_i} of the N-fold degenerate level ω_b. Without writing out the expressions for δE_{b_i}, we note only that they lead to a splitting of the quasi-energy E_b into components in accordance with the different RS values δ_{b_i} (it is known that values of δ_{b_i} that differ from one another are obtained for states that are degenerate in the orbital momenta $l=j-1/2$ and $l=j+1/2$, but not in the magnetic quantum number).

For fields that are not strong, when the conditions (24) are not satisfied, it is necessary to take into account in (16) also quantities that contain all the powers of the ratio M/Ω. Neglecting in this case the terms of order δ_i^2/I and γ_i^2/I compared with the RS and with the level widths, we obtain in the vicinity of the values $\sigma\approx E_a$ and $\sigma\approx E_b$, respectively,

$$R_{ba}^{(1)}(\sigma)\approx M_{ba}^{(1)}(\sigma)/(\sigma+\omega-E_b-M_{bb}^{(0)}(\sigma+\omega)),$$
$$R_{ab}^{(-1)}(\sigma)\approx M_{ab}^{(-1)}(\sigma)/(\sigma-\omega-E_a-M_{aa}^{(0)}(\sigma-\omega)), \quad (28)$$

This leads with the aid of (23) and (25)–(28) to the following two solutions for the poles of the propagation function (15), corresponding to the levels ω_a and ω_b in a resonant field

$$\sigma_{a;1,2}=\omega_{aR}+\frac{\varepsilon_R}{2}-\frac{i}{4}(\gamma_a+\gamma_b)\pm\frac{\varepsilon_R}{|\varepsilon_R|}\tilde{\Omega},$$
$$\sigma_{b;1,2}=\omega_{bR}-\frac{\varepsilon_R}{2}-\frac{i}{4}(\gamma_a+\gamma_b)\pm\frac{\varepsilon_R}{|\varepsilon_R|}\tilde{\Omega}. \quad (29)$$

Here $\omega_{iR}=\omega_i+\Delta_i+\delta_i$, $\varepsilon_R=\omega_{bR}-\omega_{aR}-\omega$, and the quantity

$$\tilde{\Omega}=\Omega_R+\frac{V_{ab}}{2\Omega}\left(\delta_F-\frac{i}{2}\gamma_F\right)+\frac{|V_{ab}|^2}{8\Omega\omega_{ba}}\left[\delta_a-\delta_b-\frac{i}{2}(\gamma_a-\gamma_b)\right]$$

consists of a part

$$\Omega_R=\tfrac{1}{2}\left\{\left[\varepsilon_R+\frac{i}{2}(\gamma_a-\gamma_b)\right]^2+|V_{ab}|^2\right\}^{1/2}$$

and corrections on the order of the nonresonant intensity effects to the RS and to the level width in the lowest approximation. When these corrections are neglected, expressions (29) can be obtained also within the framework of the formal method of the "effective non-Hermitian Hamiltonian." The noted quantities, however, are of the order of $(E/E_{\rm at})\delta_i$ ($i=a,b$), and for fields $E>cE_{\rm at}$ they should be taken into account together with the radiative effects of higher order in e^2, if the latter are calculated with accuracy determined by the value of c.

4. AMPLITUDES OF SINGLE-PARTICLE QES

We turn to the amplitudes

$$f_n(\mathbf{r}, t) = \langle 0 | T(\psi(x)S) | n \rangle / S_0$$

of the single-particle states $|n\rangle$ of an atomic system in an external field; these states are connected with the propagation function by the equation

$$f_n(r,t) = \lim_{t' \to -\infty} \int d^3r' G(x,x') \gamma_0 e^{-iE_n t'} \Phi_n(\mathbf{r}', t')$$

and the condition that the radiation field had been turned off at $t' \to -\infty$. It leads to an expression of the type (3) for f_n, with a complex QE. The existence of such quasistationary quasi-energy solutions follows directly from the corresponding Schwinger-Dirac equation for the single-particle amplitudes and is connected with the following periodicity property

$$M(\mathbf{r}, t+2\pi/\omega;\ \mathbf{r}', t'+2\pi/\omega) = M(\mathbf{r}, t; \mathbf{r}', t')$$

of the self-energy operator.

We calculate now the amplitudes f_a and f_b for the states $|a\rangle$ and $|b\rangle$, which are connected with the resonant field. With the aid of expressions (25)–(28) we obtain (at $\varepsilon > 0$) for the quantities normalized at $t = 0$, accurate to the admixture of nonresonant states,

$$f_a = \frac{B}{\sqrt{2}} e^{-i\sigma_a t} \left\{ \left(1+\frac{\varepsilon}{2\Omega}\right)^{1/2} (1+L)\varphi_a - \frac{V_{ab}}{|V_{ab}|}\left(1-\frac{\varepsilon}{2\Omega}\right)^{1/2} \times (1-L) e^{-i\omega t}\varphi_b \right\},$$

$$f_b = \frac{B}{\sqrt{2}} e^{-i\sigma_b t} \left\{ \frac{V_{ab}^*}{|V_{ab}|}\left(1-\frac{\varepsilon}{2\Omega}\right)^{1/2} (1-L) e^{i\omega t}\varphi_a + \left(1+\frac{\varepsilon}{2\Omega}\right)^{1/2} \times (1+L)\varphi_b \right\}, \quad (30)$$

where

$$B = \frac{1+L^*}{|1+L|}\left(1+|L|^2+\frac{\varepsilon}{\Omega}\operatorname{Re} L\right)^{-1/2}; \quad \sigma_a \equiv \sigma_{a;2}; \quad \sigma_b \equiv \sigma_{b;1}$$

and the quantity

$$L = (m_{aa} - m_{bb}) / [2(\sigma_a - E_a - 2\Omega) - m_{aa} - m_{bb}]$$

describes the radiative correction to the effect of mixing of the resonant states by the external field and leads to the onset of nonorthogonality of the amplitudes f_a and f_b of the order of the quantity Im L.[4)]

When the interaction with the radiation field is turned off, the amplitudes (30) go over into the known resonant quasi-energy wave functions; in the limiting case of weak field they go over into the quasistationary amplitudes of an atom unperturbed by the external field.

The solutions obtained from (30) for the instantaneous application of the external field can be used to express, for example, the probability $dW(\nu)$ of relaxation of a two-level system with states $|a\rangle$ and $|b\rangle$ at the initial instant $t = 0$ to another ground state $|i\rangle$ unperturbed by the field. At $t \gg (\gamma_a + \gamma_b)^{-1}$, assuming the transition $|a\rangle \to |i\rangle$ not to be forbidden, we obtain in the resonant and "single-photon" approximations

$$dW(\nu) = \gamma_{ai} \left| \frac{R}{\nu+\omega_i-\sigma_a} + \frac{1-R}{\nu+\omega_i+\omega-\sigma_b} \right|^2 \frac{d\nu}{2\pi}, \quad (31)$$

where

$$R = \frac{1}{2}\left(1+\frac{|\varepsilon|}{2\Omega}\right)(1+L)^2 \Big/ \left(1+\frac{|\varepsilon|}{2\Omega}+L^2\right)$$

and the nonresonant contributions should be left out of the quantities σ_a and σ_b [here as well as in Eqs. (30)]. The "single-photon approximation" is known[9] to be valid in the presence of a resonance under the condition $\gamma_{ba} \ll \gamma_a + \gamma_b$, where γ_{ba} is a partial width. When this condition is satisfied we can neglect the multiphoton spontaneous radiation in a transition where a resonant field is present; this corresponds to the first-order approximation in the parameter $\gamma_{ba}/(\gamma_a + \gamma_b)$.

The question of radiative effects with allowance for multiphoton transitions will be investigated in the next section. Such a formulation of the problem within the framework of the propagation-function method entails a laborious technique of summation over the number of radiated photons. A more adequate approach to this group of problems is known to be the density-matrix formalism.

5. RADIATIVE EFFECTS DESCRIBED BY A DENSITY MATRIX

The customarily employed density-matrix method is based on equations of the Markov type (equations that do not take into account memory effects in the relaxation), which contain only the nonrelativistic part of the widths of the atomic levels without RS.[7,8] As applied to ultrahigh resolution experiments, we present equations free of this approximation, which take account of the intensity effects in the RS and in the width.

It is convenient to introduce the density matrix in the following manner:

$$\rho_{\alpha\beta}(t) = S^+(t, -\infty) |\alpha\rangle\langle\beta| S(t, -\infty),$$

where $|\alpha\rangle$ and $|\beta\rangle$ are multiparticle states that contain no photons, with positive and negative frequencies of the atomic system in an external classical field. A factor equal to the unit operator in the space of the photon states is implied in the right-hand side. This matrix leads to the expansion

$$j_{\Gamma\mu}(x) = S^+(t, -\infty) j_\mu(x) S(t, -\infty) = \sum_{\alpha,\beta} \rho_{\alpha\beta}(t) \langle\alpha| j_\mu(x) |\beta\rangle \quad (32)$$

for the current operator

$$j_\mu(x) = -\frac{e}{2}[\bar{\psi}\gamma_\mu, \psi]$$

in the Furry representation, $x = (x, t)$.

The equations for the density matrix follow from the equation for the $S(t)$ matrix. Taking the radiative effects into account in the e^2 approximation with the aid of Eqs. (7), (8), and (11) and with the aid of expansion (32), we obtain

$$i\frac{d}{dt}(\rho_{nm}-\rho_{nm}^{(0)})=\frac{1}{2\pi}\int_{-\infty}^{t}dt'\int_{-\infty}^{\infty}d\sigma\exp[i\sigma(t'-t)]$$

$$\times\Big\{\sum_{k,q}\{\rho_{nk}(t')M_{mk}^{(q)}(\sigma+E_k)$$

$$\times\exp[i(E_{mk}-q\omega)t]-\rho_{km}(t')M_{nk}^{(q)*}(-\sigma+E_k)$$

$$\times\exp[-i(E_{nk}-q\omega)t]\}+i\sum_{p,k,q}\rho_{pk}(t')M_{pn,mk}^{(q)}(\sigma)\exp[i(E_{mn}-E_{kp}$$

$$-q\omega)t]\Big\}+\sum_{k,q}\{\rho_{nk}(t)\Lambda_{mk}^{(q)}e^{i(E_{mk}-q\omega)t}-\rho_{km}(t)\Lambda_{nk}^{(q)*}$$

$$\times\exp[-i(E_{nk}-q\omega)t]\}. \quad (33)$$

Here

$$i\frac{d}{dt}\rho_{nm}^{(0)}=\int d^3x\{A_\mu^{(-)}(x)[\rho_{nm}(t),j_{\Gamma\mu}(x)]$$

$$+[\rho_{nm}(t),j_{\Gamma\mu}(x)]A_\mu^{(+)}(x)\}$$

are terms that vanish upon averaging over the photon vacuum, where $A_\mu^{(\pm)}(x)$ are the positive- and negative-frequency parts of the radiation field;

$$M_{pn,mk}^{(q)}(\sigma)=-\frac{\alpha}{4\pi}\sum_{q_1}\int\frac{d^3k}{|\mathbf{k}|}Q_{\mu,pn}^{(q_1)}(\mathbf{k})Q_{\mu,mk}^{(q-q_1)}(-\mathbf{k})$$

$$\times\{\delta(E_{pn}-\sigma-|\mathbf{k}|-q_1\omega)+\delta[E_{km}+\sigma-|\mathbf{k}|+(q-q_1)\omega)]\}; \quad (34)$$

$$eQ_{\mu,nm}^{(q)}(\mathbf{k})=-\frac{\omega}{2\pi}\int_{-\pi/\omega}^{\pi/\omega}dt\int d^3x\langle n|j_\mu(x)|m\rangle$$

$$\times\exp\{i[\mathbf{kx}+(q\omega-E_{nm})t]\}$$

and the counterterms of the mass renormalization

$$\Lambda_{nk}^{(q)}=\delta m\langle\langle\Phi_n e^{iq\omega t}\Phi_k\rangle\rangle$$

are explicitly separated.

Only the density-matrix elements corresponding to single-particle atomic states with positive frequencies (designated by Latin letters) were retained in Eqs. (33); states with negative frequencies are taken into account only in the quantities M. This approximation is justified for problems in which there is no pair production and in which averaging of the density-matrix elements over the initial single-particle states of the system with positive frequencies is assumed.

The non-Markov character of Eqs. (33) is indicated by the integral, nonlocal in time, character of its terms. Equations in the Markov approximation obtained at $t\to\infty$ if the dependence on the variable σ in the matrix element of the operator M is neglected.

These equations, together with propagation function (15), are written in the QES basis.[5] As a result they are most convenient (both in the calculations and for the development of physical interpretations), as applied to the case of strong fields, when the field shifts and splitting of the atomic levels greatly exceed the radiative effects. We present them for the values

$$\bar\rho_{nm}(t)=\langle 0,\bar\psi_0|\rho_{nm}(t)|\psi_0,0\rangle$$

averaged over the initial QES of the atomic system and over the vacuum of the photons in the lowest order in M/Ω, when the connection between the diagonal and nondiagonal elements of the mean values $\bar\rho_{nm}(t)$ can be neglected, and when single-photon resonance is present.

As $t\to\infty$ we obtain in the resonant approximation, which is known to correspond to neglect of rapid oscillations with characteristic times ω_{ij}^{-1} compared with slow ones with times Ω^{-1},

$$i\frac{d}{dt}\bar\sigma_{ab}(t)=\Big[\bar E_b-\bar E_a-\omega-\frac{i}{2}(\Gamma_a+\Gamma_b-2M_{aa,bb}^{(0)}(0)\Big]\bar\sigma_{ab}(t), \quad (35)$$

$$\frac{d}{dt}\bar\rho_{bb}(t)=(-\Gamma_b+M_{bb,bb}^{(0)}(0))\bar\rho_{bb}+M_{ab,ba}^{(0)}(0)\bar\rho_{aa}, \quad (36)$$

$$\bar\rho_{aa}(t)+\bar\rho_{bb}(t)=1,\quad \bar\sigma_{ab}^*=\bar\sigma_{ba}.$$

We have used here the notation

$$\bar\sigma_{ab}(t)=e^{i(E_a+\omega-E_b)t}\bar\rho_{ab}(t),\quad \bar E_{a,b}=E_{a,b}+\delta E_{a,b}$$

and have left out from the equations also the non-Markov contributions of order $\alpha^3\gamma_i$ and $\alpha^3\delta_i$ to the RS and to the widths of the atomic levels.

It is important to note that the matrix element (34) are pure real and contribute only to the level width without changing the RS. The zeroth-harmonic values $M^{(0)}(0)$ coincide with the partial widths of the transitions between the QES, and the following relation holds for the positive-frequency part of the width

$$\Gamma_n=\sum_m M_{nm,mn}^{(0)}(0).$$

If the influence of the field is neglected in the operator M, Eqs. (35) and (36) without RS and with nonrelativistic parts of the widths coincide with the equations obtained in Ref. 20 from another viewpoint.

Following that reference, we can calculate in simple fashion the spectrum of the resonant fluorescence for the case of a strong field, by using an expansion of (32), Eq. (35) at $\gamma_a=0$ and $\gamma=\gamma_b$, as well as the expression

$$M_{aa,bb}^{(0)}(0)=M_{bb,aa}^{(0)}(0)=-\gamma|V_{ab}|^2/16\Omega^2.$$

The spectrum has the well-known three-peak structure[21] with the following form of the spectrum with respect to a frequency ν for the sidebands ("satellites") in the RS:

$$\frac{\gamma}{2}\Big(1+\frac{|V_{ab}|^2}{8\Omega^2}\Big)\Big/\Big\{\Big[\nu-\omega\pm\Big(\frac{2\varepsilon}{|\varepsilon|}\Omega+\delta E_a-\delta E_b\Big)\Big]^2+\frac{\gamma^2}{4}\Big(1+\frac{|V_{ab}|^2}{8\Omega^2}\Big)^2\Big\}. \quad (37)$$

The following remark is in order concerning the influence of the resonant field on the anomalous magnetic moment of the electron a_e in the simplified microwave-resonance scheme.[6] To this end we turn to the probability of a transition with spin flip from the state $|\varphi_1\rangle$ of the electron in the magnetic field $\mathbf{B}$ with spin antiparallel to the field direction, in the resonant approximation $\omega\approx\omega_0=eB/m$.

The probability of the transition from the initial state $|\varphi_i,0\rangle$ without photons, summed over all the final "atom in state $|\varphi_2\rangle$ plus an arbitrary number of photons with arbitrary polarizations and momenta" states, is of the form

$$W_{12}(t)=\sum_{n,m}\langle 0,\overline{\varphi}_1|\rho_{nm}(t)|\varphi_1,0\rangle\langle\varphi_2|m\rangle\langle n|\varphi_2\rangle,$$

and we obtain with the aid of Eqs. (33) of the Markov type[21]

$$W_{12}(t)|_{\gamma t\gg 1}=|V_{12}|^2/[2|V_{12}|^2+4(\omega-\omega_0-\delta)^2+\gamma^2]. \quad (38)$$

It is important to note that intensity effects in the value $\delta = a_e\omega_0$ of the radiative frequency shift of the magnetic transition ω_0, which determines the value of a_e, or of the order of the resonant contributions $(|V_{12}|/\omega_0)\delta$ and lead to similar effects $(|V_{12}|/\omega_0)a_e$ to the value of a_e.

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1) The mass operator for an unbound electron in an intense field that is constant in the coordinates and in time was calculated in Ref. 12; the mass radiative correction to the quasi-energy of a free electron in a plane-wave field was considered in Ref. 13.

2) This can be easily verified using as an example the propagation function from expression (6), in which the use of the resonant functions $\Phi_{a,b}$ and neglect of the field shifts of the levels actually lead to a pure Coulomb propagation function.

3) We note that nondiagonal contributions of the order of ξm_{ai} and ξm_{bi} $(i\neq a, b)$ from the admixture of nonresonant states φ_i to the resonant wave functions $\Phi_{a,b}$ have been left out from (25).

4) Non-orthogonal quasistationary states were investigated from the general point of view in Ref. 19.

5) The corresponding equations were obtained in the cited references 7 and 8 in a basis of atomic states that are unperturbed by an external field.

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