Electron-deformation currents in metals

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It is shown that a defect flux in a metal gives rise to an electron-deformation (ED) current. Since deformations give rise to dislocation fluxes, the metal deformations cause electron currents. The ED currents are proportional to the deformation rate $\dot{\varepsilon}$ and increase with decreasing temperature like T^{-5} in the isotropic model of the metal. Below the superconducting-transition point the ED currents are proportional to the density of the normal electrons and are equal to zero in a superconducting metal.

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If a defect flux is present in a metal, the collisions of the electrons with the mobile defects perturb the electron subsystem and, since this perturbation has a preferred direction (the direction of the defect flux J_d), one should expect the onset of anisotropy of the electron distribution function, which causes an electron current or a compensating potential difference. This was first theoretically demonstrated for point defects in Ref. 1 and for linear defects (dislocations) in Ref. 2.

Consider, for example the picture of diffusing atoms or ions. An activated ion, i.e., one executing an elementary diffusion act, as it hops over to a neighboring equilibrium position, scatters electrons that collide with it. The hop-over directions of each individual atom are random but, say owing to the ion density gradients, the electron are predominantly scattered in the direction of the ion diffusion flux. Thus, in place of external forces that perturb the electron subsystem, in this phenomenon the perturbing factor are the collisions of the defects with the electrons.

We write down the kinetic equation for this process in the form

$$(\partial f/\partial t)_{de} + (\partial f/\partial t)_{\text{coll}}^{(L)} = 0, \tag{1}$$

where the first term describes the distribution-function change due to the collisions of the mobile defects with the electrons, while the second describes the complete collision integral due to all the mechanisms of scattering and relaxation of the electrons in the metal lattice. In the relaxation-time approximation we have

$$(\partial f/\partial t)_{\text{coll}}^{(L)} = -f_1/\tau_L, \tag{2}$$

where τ_L is the electron total relaxation time due to all the scattering mechanisms. It is necessary to determine the first term of (1). We separate a group of defects having a velocity u and consider the quantity $(\partial f/\partial t)_{de}^{(\mathbf{u})}$ corresponding to this group. Let $\tilde{f}_1$ be the nonequilibrium increment to the distribution function in a coordinate system moving with the velocity $\mathbf{u}$ of the defects. Since the collisions of the electrons with the defects can be regarded as elastic, we have

$$\left(\frac{\partial f}{\partial t}\right)_{de}^{(\mathbf{u})} = -\frac{2}{(2\pi)^3}\int_{k'} [\tilde{f}_1(\mathbf{k},\mathbf{u}) - \tilde{f}_1(\mathbf{k}',\mathbf{u})]\, W_{\mathbf{k}\mathbf{k}'}\, d^3k', \tag{3}$$

where $W_{kk'}$ is the probability of the electron being scattered from a state $\mathbf{k}$ into a state $\mathbf{k}'$.

This probability is strictly speaking a function of $\mathbf{u}$, but if the ratio u/v_F (v_F is the Fermi velocity of the electron) is very small, this dependence can be neglected. The thermal velocities of the atomic defects are $u \approx 10^4$ cm/sec. Thus, $u/v_F \approx 10^{-4}$. At the same time, the motion of the defects in the periodic field of the lattice can change the scattering potential as a function of the displacement, and the probability of scattering by such a defect can differ substantially from the probability of scattering by a "normal" defect situated in one of the regular equilibrium positions, where it undergoes thermal vibrations (see scattering by an "activated" field[1,3]). To emphasize this circumstance, we shall hereafter designate the probability of scattering by a moving defect by $W^*_{kk'}$.

If the total defect density is N_i, we obtain after summing over all values of $\mathbf{u}$

$$\left(\frac{\partial \tilde{f}}{\partial t}\right)_{de} = -\sum_{\mathbf{u}} \frac{N_i(u)}{N_i}\frac{2}{(2\pi)^3} \times \int_{k'} [\tilde{f}(\mathbf{k},\mathbf{u}) - \tilde{f}(\mathbf{k}',\mathbf{u})]\, W_{\mathbf{k}\mathbf{k}'}\, d^3k', \tag{4}$$

where $N_i(\mathbf{u})$ is the density of the defects having velocities $\mathbf{u}$, and the summation extends over all possible values of the velocities $\mathbf{u}$. We determine now the function $\tilde{f}(\mathbf{k},\mathbf{u})$. We use a square-law electron-dispersion approximation, in which the entire picture is lucid. In this approximation we can consider the dependence of the distribution function of the electrons on their velocity $\mathbf{v}$. In a coordinate frame that moves together with the defect at a velocity $\mathbf{u}$, the electrons have a velocity

$$\tilde{\mathbf{v}} = \mathbf{v} - \mathbf{u}. \tag{5}$$

The electron distribution function in this frame is thus

$$\tilde{f}(\tilde{\mathbf{v}}) = \tilde{f}(\mathbf{v}-\mathbf{u}). \tag{6}$$

Since $u \ll v$, we have

$$\tilde{f}(\tilde{\mathbf{v}}) = \tilde{f}(\mathbf{v})\,|_{u=0} - \partial\tilde{f}/\partial\varepsilon\,|_{u=0}(\nabla_v \varepsilon \mathbf{u}), \tag{7}$$

but $\tilde{f}|_{u=0} = f_0$, i.e., $\tilde{f}|_{u=0}$ is the distribution function of the electrons in the lattice coordinate frame. In exactly the same way $\partial\tilde{f}/\partial\varepsilon|_{u=0} = \partial f_0/\partial\varepsilon$ and $\nabla_v\varepsilon = \mathbf{p}$, where $\mathbf{p}$ is the electron momentum in the lattice. Thus,

$$\tilde{f}(\tilde{v}) = f_0(v) - \frac{\partial f_0}{\partial \varepsilon}(\mathbf{pu}), \tag{8}$$

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whence

$$\tilde{f}_1 = -\frac{\partial f_0}{\partial \varepsilon}(\mathbf{pu}). \tag{9}$$

Substituting (9) in (4) we get

$$\left(\frac{\partial \tilde{f}}{\partial t}\right)_{dc}^{(u)} = -\frac{\partial f_0}{\partial \varepsilon}(\mathbf{pu})\frac{2}{(2\pi)^3}\int_{\mathbf{k}'}\left(1-\frac{k_u'}{k_u}\right)W_{\mathbf{kk}'}\,d^3k', \tag{10}$$

where k_u is the projection of the vector $\mathbf{k}$ on $\mathbf{u}$, $\mathbf{k} = \mathbf{p}/\hbar$, and

$$\frac{2}{(2\pi)^3}\int_{\mathbf{k}'}\left(1-\frac{k_u'}{k_u}\right)W_{\mathbf{kk}'}\,d^3k' = \frac{1}{\tau_{de}}, \tag{11}$$

where τ_{de} is the transport relaxation time of the electrons when scattered by a defect.

We return now to the lattice frame, replacing $\mathbf{u}$ by $-\mathbf{u}$. Using (10) and (4) and taking into account (11) as well as the fact that

$$\sum_{\mathbf{u}} N_i(\mathbf{u})\,\mathbf{u} = \mathbf{J}_d, \tag{12}$$

we obtain

$$\left(\frac{\partial f}{\partial t}\right)_{de} = \frac{\partial f_0}{\partial \varepsilon}(\mathbf{pJ}_d)\frac{1}{N_i\tau_{de}}. \tag{13}$$

Using (13) and (2), we write the kinetic equation in the form

$$\frac{\partial f_0}{\partial \varepsilon}(\mathbf{pJ}_d)\frac{1}{N_i\tau_{de}} = -\frac{f_1}{\tau_{dc}} \tag{14}$$

or

$$f_1 = -\frac{\partial f_0}{\partial \varepsilon}(\mathbf{pJ}_d)\frac{\tau_L}{N_i\tau_{de}}, \tag{15}$$

where f_1 is the total nonequilibrium increment to the distribution function in the metal. The electron current produced by the defect flux is obtained from the relation

$$\mathbf{j} = \frac{2e}{(2\pi)^3}\int \mathbf{v} f_1\,d^3k, \tag{16}$$

where f_1 is given by (15)

$$j^{(x)} = -J_d^{(s)}\frac{2e}{(2\pi)^3 N_i}\int v_x p_s\left(\frac{\tau_L}{\tau_{de}}\right)\frac{\partial f_0}{\partial \varepsilon}\,d^3k. \tag{17}$$

The coefficient of the flux $J_d^{(s)}$ is the effective charge due to the dragging of the electrons by the defects:

$$Z_{de}^{(xs)} = -\frac{2e}{(2\pi)^3 N_i}\int_{\mathbf{k}} v_x p_s\left(\frac{\tau_L}{\tau_{de}}\right)\frac{\partial f_0}{\partial \varepsilon}\,d^3k. \tag{18}$$

Thus

$$j^{(x)} = Z_{de}^{(xs)} J_d^{(s)}. \tag{19}$$

In the free-electron approximation[1,3]

$$Z_{de} = enl\sigma_{de}^*, \tag{20}$$

where σ^*_{de} is the transport cross section for electron scattering by an "activated" moving defect.

We note that the effective charge Z_{de} of the electrons dragged by the defects is equal to the effective charge Z_{ed} produced by dragging of the defects by electrons[1]:

$$Z_{de} = Z_{ed}. \tag{21}$$

This leads to an important and quite general consequence: any defect is acted upon in a current-carrying metal a force that it would have were its effective charge Z_{ed}. Conversely, any defect moving in a metal produces a current $j = Z_{de} V_d$.

Equation (21) can be obtained from the principle of the symmetry of the kinetic coefficients, and is consequently independent of the electron dispersion law $\varepsilon(\mathbf{k})$. Therefore, using the results of electric transport theory, we write for the dragging of "holes" by defects

$$Z_{de}^{(h)} = -en_h l_h \sigma_{hd}^*. \tag{22}$$

If the flux $\mathbf{J}_d$ is produced by the density gradient of point defects, we have

$$j^{(x)} = -Z_{de} D\,\partial N_i/\partial x. \tag{23}$$

In liquid metals the gradient of the impurity atoms will produce a current or its compensating potential difference; this is the so-called electrodiffusion effect. The experiment agrees with the theory, i.e., with Eq. (23). In liquid metals, where the diffusion coefficient $D \sim 10^{-5}$–10^{-6} cm^2/sec the electrodiffusion potential is of the orde of microvolts and is perfectly observable.[4–6]

In solid metals at low temperatures the diffusion coefficient is exceedingly small ($D < 10^{-12}$ cm^2/sec), so that observations of the diffusion potentials are either highly complicated or practically impossible. Particular interest attaches in solid metals, especially at low temperatures, to electron-deformation currents produced by dislocation motion due to deformation of the metal.

The theory developed above applies also to dislocation fluxes. The dislocation flux density J_{ik}^{dis} is given by (see Ref. 7)

$$J_{ik}^{\text{dis}} = e_{ilm}\sum_s \rho_{lk}^{(s)} v_m^{(s)}, \tag{24}$$

where e_{ilm} is a unit antisymmetric tensor of third rank, $\rho^{(s)}$ is the flux density of the dislocations of type s, and $V_m^{(s)}$ is the velocity of the dislocations of type s. If we neglect the anisotropy of electron scattering by dislocations, the effective charge of electron dragging by a dislocation per unit dislocation length is

$$Z_{de} = enl\sigma_{\text{dis}}^*, \tag{25}$$

where σ^*_{dis} is the transport cross section for electron scattering by a moving dislocation. The electron current produced by the dislocation flux is

$$\mathbf{j} = Z_{de}\mathbf{J}^{\text{dis}}. \tag{26}$$

Associated with the dislocation flux, however, is a metal deformation rate

$$\dot{\varepsilon} = \mathbf{bJ}^{\text{dis}}, \tag{27}$$

where $\mathbf{b}$ is the Burgers vector. In the simplest case of dislocations of one type moving in one direction with equal velocities, we have

$$\dot{\varepsilon} = \mathbf{b}\rho_{\text{dis}}\mathbf{v}_{\text{dis}} = \mathbf{bJ}^{\text{dis}}, \tag{28}$$

ρ_{dis} is the dislocation density and $\mathbf{v}_{\text{dis}}$ is the dislocation velocity. Using (27), we express the dislocation flux in terms of the sample deformation rate

$$J^{\text{dis}} \approx \dot{\varepsilon}/b. \tag{29}$$

Using (26) and (29) we obtain a simple connection between the electron-deformation current and the deformation rate

$$j = \frac{Z_{de}}{b}\dot{\varepsilon} = \frac{enl\sigma^{*}_{\text{dis}}}{b}\dot{\varepsilon}. \tag{30}$$

The quantity σ^{*}_{dis} cannot as yet be readily calculated theoretically. It is usually assumed that $\sigma^{*}_{\text{dis}} \sim b$ but direct measurements frequently yield considerably larger values of σ^{*}_{dis}. Thus, for edge dislocations in Zn we have $\sigma^{*}_{\text{dis}}/b \approx 28$ (Ref. 8). We shall put hereafter $\sigma^{*}_{\text{dis}}/b = \varkappa$ With decreasing temperature the mean free path of the electrons increases, therefore the effective charge increases, since $Z_{de} \propto l$. Taking into account the known temperature dependences of l, we can expect the following temperature dependence[9]

$$\frac{j(T)}{j(\Theta)} \approx \begin{cases} \Theta/T, & T \gg \Theta \\ (\Theta/T)^{5}, & T \ll \Theta, \end{cases} \tag{31}$$

where Θ is the Debye temperature, $j(\Theta)$ is the electron–deformation current at $T = \Theta$, and no account is taken of the relatively weak dependence of $l(T)$ on the electron dispersion law and of the temperature dependence of the electron scattering by a dislocation. Let us estimate the electron-deformation currents in zinc at low temperature. If $l = 10^{-2}$ cm at $T = 10$ K, $n = 3\times 10^{22}$ cm^{-3}, $\varkappa = 30$, and $b = 3\times 10^{-8}$ cm, we have $Z_{de} \approx 3\times 10^{14} e$ per unit dislocation length, i.e., approximately $10^{6} e$ per atom in the dislocation. We note that in the purest tungsten $l \approx 10^{-1}$ cm (see Ref. 8), i.e., $Z_{de} \sim 10^{7}$ e per atom in the dislocation. Substituting the values of Z_{de} and b for zinc in (30) we obtain

$$j = 10^{3}\dot{\varepsilon}\ \text{A/cm}^{2}$$

Using (31), we can express the field intensity E as

$$E = \frac{j}{\sigma} = \frac{J_d}{e} m v_F \sigma_{\text{dis}}^{*} = \frac{m v_F}{e}\sigma_{\text{dis}}^{*}\frac{\varepsilon}{b}.$$

For zinc we have under the same conditions $E \approx 10^{-8}$ V/cm.

We now obtain for j an estimate suitable at not too low temperatures, for an ideal defect-free crystal. We consider the expression for $j(\Theta)$ at the Debyre temperature Θ:

$$j(\Theta) \approx enl(\Theta)\varkappa\dot{\varepsilon}, \tag{32}$$

where $l(\Theta)$ is of the form (see, e.g., Ref. 8):

$$l(\Theta) \approx \hbar v_F/k\Theta. \tag{33}$$

Thus,

$$j(\Theta) \approx \frac{en\hbar v_F \varkappa\dot{\varepsilon}}{k\Theta} \approx 10^{-1}\frac{\varkappa\dot{\varepsilon}}{\Theta}. \tag{34}$$

Then

$$j(T) \approx j(\Theta)\left(\frac{\Theta}{T}\right)^{5} \approx 0.1\frac{\varkappa\dot{\varepsilon}}{\Theta}\left(\frac{\Theta}{T}\right)^{5}. \tag{35}$$

This estimate is meaningful so long as $l(T) < l_L^{(s)}$, where $l_L^{(s)}$ are the mean free paths determined by electron scattering by lattice defects or by lattice boundaries. It overvalues j if $l(T) > l_L^{(s)}$. For example for zinc ($\Theta = 305$ K) at $T = 10$ K we obtain from (35) $j \approx 10^{5}$ A/cm^{2}; this means that $l(T) \gg l_L^{(s)}$. If the additional resistivity ρ^{*}_{dis} introduced in the metal by the dislocations is known, we can estimate j from an equation that follows from (30) if the anisotropy of the scattering of the electrons by the dislocations is neglected:

$$j \approx \frac{e}{c_{\text{dis}}\rho_0}\frac{\rho_{\text{dis}}}{b}\dot{\varepsilon}. \tag{36}$$

where $c_{\text{dis}} = N_{\text{dis}}/N_i$ is the relative density of the dislocations in the crystal and ρ_0 is the resistivity of the metal at the deformation temperature.

Directed dislocation fluxes can be produced by various methods, say by nonuniform loading of the sample. One must not think, however, that the phenomena considered will not be observable if the average dislocation flux $\bar{J}^{\text{dis}} = 0$. Deformation by constant loading, say in tension, frequently develops jumpwise; the microdeformation jumps correspond to dislocation microfluxes of the same sign, i.e., a jumplike $\dot{\varepsilon}_\mu$ curve should correspond to a "jumping" and fluctuating current

$$j_\mu \propto \dot{\varepsilon}_\mu; \tag{37}$$

where $\dot{\varepsilon}_\mu$ is the microdeformation rate.

As already noted, j increases like T^{-5} with decreasing temperature, but below the superconducting-transition temperature j begins to decrease in proportion to the density n_n of the "normal" electrons not bound into Cooper pairs:

$$j \propto n_n = \frac{2n}{\exp(\Delta/kT)+1}, \tag{38}$$

where Δ is the width of the superconducting gap and $j|_{T=0} = 0$. Although the observation of ED currents is not a simple experimental problem, one can hope that a study of electron-deformation phenomena will contribute to a better understanding of the deformation mechanism. Dislocation motion is accompanied by generation and motion of point defects (interstitial atoms, vacancies, and others). According to the general conclusion of the theory, electron currents correspond also to a flux of point defects as well as any other defects. An analysis of currents due to generation of point defects and accompanying the deformation is beyond the scope of the present article.

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