Electric-field penetration into a superconductor in a Josephson microjunction

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(Submitted 2 April 1982)
Zh. Eksp. Teor. Fiz. **83**, 1851–1863 (November 1982)

The influence of penetration of a constant and alternating field into the superconducting films of the electrodes (banks) on the properties of individual Josephson junctions and of their chains is investigated theoretically and experimentally. It is found that these phenomena are of substantial significance despite the considerable current density in the film junctions. It is shown that the interaction efficiency of series-connected junctions has a nonmonotonic dependence on the distance between them, on the chain voltage, and on the temperature; this is due to the quasiwave character of the propagation of the alternating electric field in the film of the superconducting electrodes.

PACS numbers: 74.30.Gn, 85.25. + k, 74.30.Ci, 74.50. + r

1. INTRODUCTION

It is known that a longitudinal electric field can penetrate into a superconductor to a depth greatly exceeding the characteristic lengths of the superconductor, namely the coherence length $\xi(T)$ and the magnetic-field penetration depth.[1] The penetration of the electric field usually plays the most important role at temperatures near the critical T_c. This temperature region is of interest also because in a certain frequency range near T_c there exist weakly damped collective oscillations of the electric field and of the superconducting velocity.[2,3] In this frequency range the propagation of the alternating electric field in the superconductor is close to wavelike. It is shown in Ref. 4 that efffects due to the penetration of a constant and an alternating electric field in a superconductor play an important role in Josephson junctions and in their chains, in which there is no concentration of the total current (Mercereau-Notaris junctions and others).

An opinion developed that effects due to the penetration of an electric field into the superconductor can be neglected in Josephson structures with strong current density. It will be shown below, however, that in film junctions such effects can be substantial despite the appreciable current density.

We note that the penetration of a longitudinal electric field into a superconductor is accompanied by perturbation of the quasiparticle distribution. This perturbation is characterized by an asymmetric filling of electronlike and holelike branches of the excitations. Nonequilibrium phenomena of another kind were investigated in superconducting junctions with constrictions, such as stimulation of superconductivity by microwave radiation[5]; these effects are due to disturbance of the symmetric part of the quasiparticle distribution function, and are localized in the junction film itself.

The present paper is devoted to a theoretical and experimental ivestigation of phenomena due to the penetration of a constant and alternating electric field into the superconducting films of the banks of variable-thickness junctions (VTJ)[1)] and their chains. Although the results of investigations of similar effects in chains of series-connected junctions were reported in several papers,[6–8] the main laws governing the processes that take place remained unexplained.

2. THEORY

2.1. Single junction

We consider first a single junction (see Fig. 1a). We assume that the temperature T is close to the critical $T_c (T_c - T \ll T_c)$, and the voltages V are small compared with Δ (Δ is the gap in the excitation spectrum). We assume in addition the mean free path to be small compared with $\xi(0)$. We write the general expression for the current through the junction in the form

$$I = V/R_N + I_c P\{\varphi\}, \tag{1}$$

where V is the voltage on the length l of the junction, R_N is the resistance of this section, and φ is the order-parameter phase difference. We have separated in (1) the resistive component V/R_N due to the contribution made to the current by quasiparticles having energies ε of the order of T.[2)] The second term is the sum of the superconducting current and of the current of the quasiparticles with energy $\varepsilon \ll \Delta$. The actu-

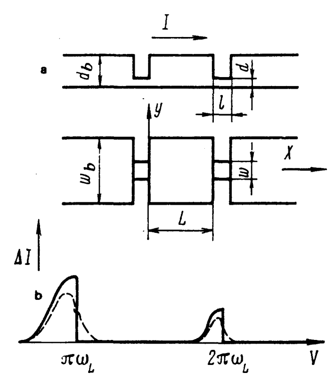

FIG. 1. (a) Chain of two variable-thickness junctions; (b) correction to the junction CVC, due to penetration of alternating electric field into the superconductor: solid line—for identical junctions, dashed—chain of junctions whose parameters differ little.

 0038-5646/82/111070-07$04.00

al form of the functional $P\{\varphi\}$, which is of no importance to us for the time being, depends on the relation between the dimension l and $\eta=(\Delta/T)^{1/2}\xi(T)$ (Refs. 9 and 10).[3)] We focus our attention on the first term, whose dependence on φ is the manifestation of the penetration of the electric field into the banks of the VTJ. This penetration produces in the banks an invariant potential $\mu=\frac{1}{2}\partial\chi/\partial t+\Phi$ (χ is the phase of the order parameter and Φ is the electric potential). The voltage differs therefore from the usual value $\varphi/2$:

$$V=\dot{\varphi}/2-[\mu], \tag{2}$$

where $[\mu]$ is the difference between the values of the invariant potentials at the ends of the constriction. To find $[\mu]$ we use the equation for the Fourier component μ_ω in the banks[1,4]

$$\nabla^2\mu_\omega-k_\omega{}^2\mu_\omega=0,$$
$$k_\omega{}^2=\frac{\pi}{4}\frac{\Delta}{T}\frac{1}{D_b\tau_\varepsilon}(1-i\omega\tau_\varepsilon)(1-i\Omega), \tag{3}$$

where τ_ϵ is the energy relaxation time, $\Omega=\omega/(\pi\Delta^2/2T)$, and D_b is the diffusion coefficient in the bank film and differs in general from the diffusion coefficient D in the jucntion film because of the free-path difference. When solving (3) we shall assume that the bank film thickness $d_b\ll|k_\omega|^{-1}$ (this agrees with the experimental situation). This relation means that in the principal region of variation the potential μ depends only on two coordinates that lie in the plane of the film, and can be determined by solving only the two-dimensional equation (3).[4)] The actual form of the solution depends, of course, on the relation between w_b and $|k_\omega|^{-1}$. We consider first the case of wide banks $w_b\gg l_E$ ($l_E=k^{-1}_{\omega=0}$), in which the bank can be regarded as infinite in the y direction. When solving (3) we use coordinates u and v such that

$$x={}^1/_2 w\,\mathrm{sh}\,u\sin v,\quad y={}^1/_2 w\,\mathrm{ch}\,u\cos v,\quad 0\leqslant v\leqslant\pi,\quad 0\leqslant u<\infty, \tag{4}$$

in terms of which (3) takes the form

$$\partial^2\mu_\omega/\partial u^2+\partial^2\mu_\omega/\partial v^2-(k_\omega w/2)^2\cdot(\mathrm{sh}^2u+\sin^2 v)\mu_\omega=0 \tag{5}$$

with boundary conditions $(\partial u/\partial v)_{v=0,\pi}=0$. The equation $\mathbf{E}=-\Delta\mu(1-i\Omega)^{-1}$ leads to the relation

$$V_\omega=-\frac{D_b d_b l}{D\,dw}\left(\frac{\partial\bar{\mu}}{\partial u}\right)_{u=0}\pi(1-i\Omega)^{-1},\qquad \bar{\mu}(u)=\frac{1}{\pi}\int_0^\pi\mu(u,v)\,dv. \tag{6}$$

It follows from (5) that

$$\bar{\mu}(u)=\begin{cases} C_1+C_2u, & w|k_\omega|\,\mathrm{sh}\,u\ll 1,\\ C_3K_0\left(\dfrac{k_\omega w}{4}e^u\right), & u\geqslant 3,\end{cases} \tag{7}$$

where $K_0(y)$ is a zeroth-order Bessel function of imaginary argument. Since $|k_\omega|w\ll 1$, the regions of the solutions (7) overlap. This allows us to join them and, using (6) and (2), find the connection between V and φ. Before we write down the result, we recognize that this connection can also be easily obtained in the case $w_b\ll|k_\omega|^{-1}$, when the distribution of μ is determined by the one-dimensional equation (3). Combining these two cases we obtain

$$V_\omega=\dot{\varphi}_\omega/2[1+Z(\omega)],$$

$$Z(\omega)=2(1-i\Omega)\frac{D\,dw}{D_b d_b l}\begin{cases}(k_\omega w_b)^{-1}, & |k_\omega|^{-1}\gg w_b\\ \dfrac{2}{\pi}\ln\dfrac{8}{ak_\omega w}, & w_b\gg|k_\omega|^{-1}\gg w.\end{cases} \tag{8}$$

Here $\ln a\approx 0.58$. It should be noted that when clculating $[\mu]$ in the case $w_b\gg|k_\omega|^{-1}$ we have also assumed that $d_b-d\ll w$. This restriction, however, is not very significant: It can be shown that a more general expression, valid with logarithmic accuracy at arbitrary ratio of w and d_b, will contain $\ln[1/k_\omega\max(w,d_b]$. It follows from (8) that the difference between V and $\varphi/2$ can turn out to be substantial even at an appreciable current density; this leads in particular to a deviation of the current-voltage characteristic (CVC) from the hyperbolic dependence that follows from the known resistive model.[12]

We shall not analyze the CVC in detail. We note only two essential circumstances that follow from (1) and (8) and from the rather general properties of the functional $P\{\varphi\}$. If we assume that $P\{\varphi\}$ reduces at $V>\tau^{-1}$ to a certain function of φ, of the form $\Sigma_n a_n\sin n\varphi$ (its concrete form for the case $l<\eta$ can be found in Ref. 10), we obtain from (8) that in the region of the voltages $V>\tau_\epsilon^{-1}$ and $V_c=I_cR_N$ the asympotic resistance of the junction increases and becomes equal to [5)]

$$R^*=R_N(1+Z(0)). \tag{9}$$

It follows in addition from (7) that in a VTJ with broad banks, at voltages $V_c<V\ll\Delta$, the penetration of the electric field into the bridges adds to the CVC a term $I_{\rm exc}$ independent of V, the so-called excess current. In fact, recognizing that at $V\gg V_c$ we have $\varphi=2Vt+\varphi^{(1)}$ ($\varphi^{(1)}\ll 1$), and obtaining $\varphi^{(1)}$ with the aid of (1) and (8) with allowance for the already noted properties of $P\{\varphi\}$, we obtain

$$I_{\rm exc}=c\frac{D\,dw}{D_b d_b l}I_c, \tag{10}$$

where $c\sim 1$ ($c=\frac{1}{2}$ in the case $P\{\varphi\}=\sin\varphi$). The appearance of $I_{\rm exc}$ on the CVC is due to the fact that $Z(\omega)$ contains at large ω a term proportional to $|\omega|$. In contrast to the excess-current mechanism of Ref. 10, an important factor here is the presence in the junction of Josephson oscillations with an amplitude independent of V, and the distinctive character of the alternating electric field distribution in the bank films.

2.2. General relations for a chain of series-connected junctions

We proceed to analyze a chain of two series-connected junctions separated by a distance L ($L\gg w$, d_b) (Fig. 1). The current in each of the junctions is given by Eq. (1) (with separate values of R_i and I_{ci}), and V_i is connected with φ_i and $[\mu]_i$ by Eq. (2). To find the connection between $[\mu]_i$ and φ_i we must solve Eq. (3) with the corresponding boundary conditions. By virtue of the condition $d_b\ll|k_\omega|^{-1}$, just as in Sec. 2.1, it suffices to solve the two-dimensional equation (3). We consider first the case of broad banks. For the sake of brevity, we assume that $d_b \ll w$ (this restriction is of little importance for the final results). We represent the solution of (3) in the region $0 < X < L$, using the methods of the theory of potentials[13]

$$\mu(\mathbf{r}) = \int_{-w_1/2}^{w_1/2} G(\mathbf{r}, \mathbf{r}_0') \left(\frac{\partial \mu}{\partial x'}\right)_{x'=0} dy' - \int_{-w_2/2}^{w_2/2} G(\mathbf{r}, \mathbf{r}_L') \left(\frac{\partial \mu}{\partial x'}\right)_{x'=L} dy', \tag{11}$$

where $(\mathbf{r} = (x, y)$, $\mathbf{r}_0' = (0, y')$, $\mathbf{r}_L' = (L, y')$, and G is the Green's function of the two-dimensional Eq. (3) with zero values of the derivatives at the end points of the region $(x = 0, x = L)$. The mapping method yields for G the expression[13]

$$G(\mathbf{r}, \mathbf{r}') = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} [K_0(k_\omega[(x-x'-2nL)^2+(y-y')^2]^{1/2}) + K_0(k_\omega[(x+x'-2nL)^2+(y-y')^2]^{1/2})]. \tag{12}$$

Taking into account (11), (12), and the results of Sec. 2.1 we can find an expression for $[\mu]_i$ and obtain the following system of equations for the phases φ_i:

$$\begin{aligned} &\frac{1}{2}\dot{\varphi}_{1\omega} + V_c(1+a_1(\omega))P_\omega\{\varphi_1\} - V_c b_2(\omega) P_\omega\{\varphi_2\} \\ &\quad = I_\omega\left[R_1 + \frac{1+i\Omega}{1-i\Omega}(R_1 a_1(\omega) - R_2 b_2(\omega))\right], \\ &\frac{1}{2}\dot{\varphi}_{2\omega} + V_c(1+a_2(\omega))P_\omega\{\varphi_2\} - V_c b_1(\omega) P_\omega\{\varphi_1\} \\ &\quad = I_\omega\left[R_2 + \frac{1+i\Omega}{1-i\Omega}(R_2 a_2(\omega) - R_1 b_1(\omega))\right], \end{aligned} \tag{13}$$

where

$$a(\omega) = \frac{2(1-i\Omega)}{\pi} \frac{Dwd}{D_b l d_b}\left[2\ln\frac{8}{ak_\omega w} + \tilde{a}(2k_\omega L)\right],$$

$$b(\omega) = \frac{2(1-i\Omega)}{\pi} \frac{Dwd}{D_b l d_b} \tilde{b}(k_\omega L),$$

$$\tilde{a}(z) = \int_1^\infty \frac{dx}{(x^2-1)^{1/2}} \frac{e^{-zx}}{1-e^{-2zx}} = \begin{cases} \pi/2z, & |z| \ll 1 \\ (\pi/2z)^{1/2} e^{-z}, & |z| \gg 1 \end{cases},$$

$$\tilde{b}(z) = \int_1^\infty \frac{dx}{(x^2-1)^{1/2}} \frac{e^{-zx}}{1-e^{-2zx}} = \begin{cases} \pi/4z, & |z| \ll 1 \\ (\pi/2z)^{1/2} e^{-z}, & |z| \gg 1 \end{cases}.$$

We have left out of the expressions for $a(\omega)$ and $b(\omega)$ the indices of the quantities d, w, l, and d_B.

In the other limiting case of narrow banks ($w_b \ll |k_\omega|^{-1}$) the distribution of μ in the banks is obtained from the one-dimensional equation (3), so that the calculations are similar in many respects to those of Ref. 4. As a result we arrive again at a system in the form (13), but with different coefficients:

$$a(\omega) = \frac{2(1-i\Omega)}{k_\omega l}(\mathrm{cth}\, k_\omega L + 1)\frac{Dwd}{D_b w_b d_b} = (\mathrm{ch}\, k_\omega L + \mathrm{sh}\, k_\omega L)\, b(\omega),$$

$$|k_\omega| w_b \ll 1, \quad L \gg w_b - w, \quad d_b - d. \tag{13'}$$

It was assumed in (13) that, as is usually the case in experiment, the characteristic voltages V_c of both junctions are equal, and the reason why the parameters are not identical is that the normal resistances, and hence the critical currents, are different.

2.3. Interaction of identical junctions

We consider now some consequences of the obtained system, for the simplest case of identical junctions. In the voltage region $V \gg V_c$ (V is the total voltage on the chain) the system (13) can be solved for any value of $a(\omega)$ and $b(\omega)$. We note that hereafter, unless otherwise stipulated, we assume $P\{\varphi\} = \sin\varphi$. As a result we obtain for the CVC at $V = V_c$ the expression

$$I = \frac{V}{R^*} + \frac{V_c^2}{VR_N}[1 + \mathrm{Re}(a(V) - \cos\psi_0 b(V))],$$
$$R^* = 2R_N(1 + a(0) - b(0)), \tag{14}$$

where ψ_0 is the phase shift of the Josephson oscillations in the junctions; it can be shown to be given by

$$|\cos\psi_0| = 1, \quad \cos\psi_0\, \mathrm{Im}\, b(V) > 0. \tag{15}$$

We note that at $V_c < V \ll \Delta$ the frequency of the Josephson oscillations lies in the region in which collective oscillations exist in the superconductor: At these frequencies the alternating magnetic field propagates in the superconductor in the form of a weakly damped wave. As already noted,[4] in the considered voltage range, singularities should appear on the CVC of chains of Josephson junctions. We shall examine these singularities, using relations (14) and (15). It can be seen from (13) and (13') that the function $b(\omega)$ has a nonmonotonic dependence on ω (as well as on the temperature and on the distance between the junctions), owing to the wave character of the propagation of the alternating field in the banks. In particular, at $L \gg \delta$ we have

$$b(\omega) = [\cos(2\pi L/\lambda(\omega) + \vartheta(\omega)) + i\sin(2\pi L/\lambda(\omega) + \vartheta(\omega))]\,|b(\omega)|, \tag{16}$$

where

$$k_\omega = \delta^{-1} - i2\pi/\lambda(\omega),$$

$$e^{i\vartheta(\omega)} = \frac{1-i\Omega}{(1+\Omega^2)^{1/2}}(|k_\omega|/k_\omega)^s, \quad s = \begin{cases} 1/2, & |k_\omega| w_b \gg 1 \\ 1, & |k_\omega| w_b \ll 1 \end{cases}.$$

Here $\lambda(\omega)$ and δ are respectively the wavelength and the damping length of the collective oscillations at the frequency ω. The alternation of the signs of $\mathrm{Im}\, b(V)$, as seen from (15), causes the junctions (because of the interaction between them) to be either in phase ($\psi_0 = 0$) or in antiphase ($\psi_0 = \pi$). At voltages V_m such that $\mathrm{Im}\, b(V_m) = 0$ the sign of $\cos\psi_0$ is reversed, and this leads in accordance with (15) to the appearance of a step on the $I(V)$ curve. In real experiments the current is usually specified rather than the voltage. Voltage jumps should then be observed at the currents $I_m^{(+)}$ (for forward passage through the section near V_m) and $I_m^{(-)}$ (for backward passage), defined by the relation

$$I_m^{(\pm)}=\frac{V_m}{R^*}+\frac{V_c^2}{V_m R_N}[1+\mathrm{Re}\,a(V_m)\pm|\mathrm{Re}\,b(V_m)|]\qquad (I_m^{(+)}>I_m^{(-)}).$$

Thus, in the regime with given current, the CVC should exhibit hysteresis. The CVC singularities due to excitation of collective oscillations in the banks will be most strongly manifest in junctions with narrow banks ($|k_V|^{-1}\gg w_b$). Indeed, taking (13') into account we obtain (we write down only that part of the current ΔI which contains the oscillating components)

$$\Delta I(V)\sim\frac{Dwd}{D_b w_b d_b}\frac{\eta}{l}\frac{\Delta}{V}I_c\frac{\mathrm{sh}(2L/\delta)}{\sin^2(V/\omega_L)+\mathrm{sh}^2(L/\delta)}\times\left[1-\frac{\cos(V/\omega_L)}{\mathrm{ch}(L/\delta)}\,\mathrm{sign}\,\mathrm{Im}\,b(V)\right],\qquad (17)$$

where $\omega_L=(2D\Delta)^{1/2}/L$, $V/\omega_L=2\pi L/\lambda(V)$. It can be seen from (17) that at $L\ll\delta$ the CVC have resonant singularities at voltages $V_m=m\omega_L(m=1,2,...)$ at which L spans an integer number of half-waves of the collective oscillations. In addition, it turns out that $\mathrm{Im}\,b(V_m)$ reverses sign at $V=V_m$, therefore $\Delta I(V)$ takes the form of a step near V_m (see Fig. 1). In the regime with given current, as already noted, one observes not steps but jumps of the voltage and hysteresis.

2.4. Nonidentical junctions

Under real conditions the junctions have unequal resistances. We consider the case when the difference in the parameters is not large, $|R_1-R_2|\ll R_{1,2}\approx R$, and the interaction between the junctions is weak, $b_i\approx b\ll 1$, $a_i\approx a\ll 1$. Equation (13) reduces then to the form

$$\begin{aligned}&{}^1/_2\dot\varphi_{1,2}+V_cP\{\varphi_{1,2}\}=IR_{1,2}+(\Delta iV)_{1,2},\\&(\Delta iR)_{1,2}(\omega)={}^1/_2[(\dot\varphi_{1,2})_\omega a(\omega)-(\dot\varphi_{2,1})_\omega b(\omega)].\end{aligned}\qquad (18)$$

Since $\Delta i\ll I_c$, Eqs. (18) can be solved by perturbation theory. In the case $P\{\varphi\}=\sin\varphi$ such a system of equations was analyzed in Ref. 14, where the method of slowly changing amplitudes was used for the solution. In this method, the initial system is reduced to a system of equations for the phase θ_i, which are obtained from φ_i after averaging over frequencies of the order of the Josephson-oscillation frequency. Proceeding as in Ref. 14, we easily obtain

$$\frac{1}{2}\dot\theta_{1,2}=V_{1,2}-\frac{2V_c^2}{2IR+V}\{\mathrm{Re}[a(V)-\cos\psi b(V)]\pm\sin\psi\,\mathrm{Im}\,b(V)\},\qquad (19)$$

where

$$\psi=\theta_1-\theta_2,\quad V_{1,2}=[(R_{1,2}I)^2-V_c^2]^{1/2}+2IR(a(0)-b(0)).$$

From (19) follows an equation for the phase difference ψ:

$$\begin{aligned}&{}^1/_2\dot\psi+B\sin\psi=\delta V,\qquad \delta V=V_1-V_2,\\&B=\frac{4V^2}{2IR+V}\,\mathrm{Im}\,b(V).\end{aligned}\qquad (20)$$

The junctions are then in synchronism ($\bar{\dot\theta}_1=\bar{\dot\theta}_2$) at $|\delta V|\leqslant|B|$ and out of synchronism at $|\delta V|>|B|$. Using (19) and (20) we obtain an expression for the CVC:

$$V=V_1+V_2-\frac{4V_c^2}{2IR+V}\mathrm{Re}[a(V)-\overline{\cos\psi}\,b(V)],\qquad (21)$$

where

$$\overline{\cos\psi}=\begin{cases}\cos\psi_0=[1-(\delta V/B)^2]^{1/2}\,\mathrm{sign}\,B, & |\delta V|\leqslant|B|\\ 0, & |\delta V|>|B|\end{cases}$$

In the case of identical junctions Eq. (21) leads again to the conclusion that the CVC are suject to hysteresis and to voltage jumps, even when the difference between the parameters is relatively small: $|\delta V|\ll\max B$. At $\delta V\sim\max B$ there are no jumps or hysteresis, but even in this case the voltage region in which the junctions are synchronized can nevertheless still be quite large. Examination of the mutual synchronization of the junctions is therefore the simplest method of experimentally investigating the interaction between them in the case when this interaction is weak.

Our theoretical analysis has therefore shown that the increase of the junction resistance, due to penetration of the electric field into the banks, can be quite appreciable also at a high current density. The situation is similar also in chains of junctions. Then, however, more interesting singularities appear and are due to the interaction that is produced between the junctions as a result of penetration of the alternating electric field into the banks. These singularities include voltage jumps and hysteresis of the CVC (in the given-current regime) in chains of junctions with close values of the parameters, and also repeated alternation of the states (synchronous–synchronous, phases–antiphased) when the voltage (or temperature) is varied.

3. EXPERIMENT

3.1. Experimental technique

We used in the experiment single VTJ made of tin and chains of two series-connected VTJ. The single VTJ and their chains were produced by scribing tin films.[5] The parameters of the single junctions were: length $l=0.5\ \mu$m, width $w=(1–3)\ \mu$m, junction film thickness $d=(300–1000)$ Å, bank-film thickness $d_b=(2-5)\ 10^3$ Å, and bank film width $w_b\approx 500\ \mu$m. The distance between the junctions in the chains ranged from 3 to 30 μm. The geometric dimensions of the junctions in the chains were estimated not to differ usually by more than 20%. We measured the autonomous CVC of the individual VTJ and their chains, as well as the CVC when acted upon by mocrowave radiation of frequencies $f_e=10$ and 35 GHz. All the measurements were made in the temperature interval 3–4 K.

3.2. Single VTJ

We consider those singularities of the CVC of a single VTJ, which follow from the theoretical relations (8) and (9).

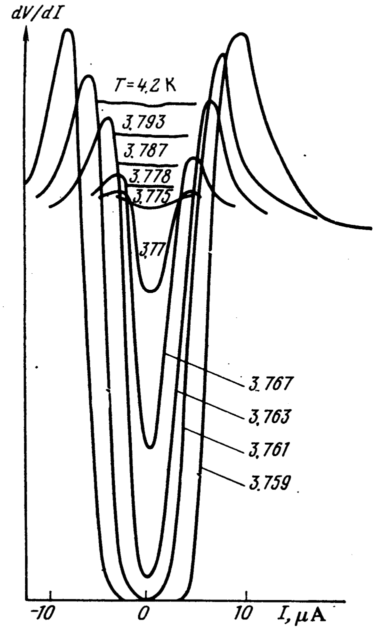

FIG. 2. Experimental dependences of the differential resistance of VTJ CS-75 on the current at various temperatures near the critical temperature.

The penetration of a constant electric field causes the asymptotic junction resistance R^* to differ from R_N. Let us estimate $\Delta R = R^* - R_N$ for the VTJ CS-75: $d = 600$ Å, $d_b = 5000$ Å, $w = 0.5\,\mu$m, $l = 0.5\,\mu$m, and $D/D_b = 0.5$. At $T_c - T = 0.02$ K and $\tau_\epsilon = 2\times 10^{-10}$ sec we obtain $l_E \approx 10\ \mu$m. In our experiments we dealt with junctions having broad banks, for which the condition $l_E \ll w_b$ was satisfied in the investigated temperature range. Taking this into account and using the corresponding part of Eq. (7), we obtain $\Delta R/R_N \approx 0.3$. Such an increase of the asymptotic resistance is experimentally observable.

Figure 2 shows the experimental dependences of the differential resistnce $R_d(I) = dV/dI$ of the VTJ CS-75 on the current I at different temperatures. The upper horizontal line $R_d =$ const corresponds to the resistance of the entire sample in the normal state at $T > 3.8$ K. It can be seen that as the temperature is lowered the sample resistance is first decreased as a result of the transition of the bank film into the superconducting state,[5] and at $T \approx 3.775$ K the $R_d(I)$ dependence becomes nonlinear. The absence of a nonlinear section on the CVC (for low currents) at temperatures close to T_c, and in particular at $T = T_1 = 3.778$ K, is explained by the strong influence of the fluctuations. In this case $R_d(0)$ practically coincides with the asymptotic value $R_{as} = R^*$. It follows from (8), R^* depends on T, albeit weakly [inasmuch as the argument of the logarithm in (8) contains the temperature-dependent quantity $l_E(T)$], and decreases with decreasing temperature. This agrees with experiment; it can be seen from Fig. 2, R_{as} at $T_2 = 3.758$ K is smaller than R_{as} at $T = T_1$. We now estimate $R^* - R_N$ from the experimental data. The value of R_N can be obtained by measuring the temperature dependence of the critical current, which is known to be determined for VTJ near T_c by the expression

$$I_c = \pi\Delta^2/4TR_N \sim T_c - T \equiv \Delta T \qquad (22)$$

in a wide range of $l/\xi(0)$.[15] Indeed, the experimental $I_c(T)$ dependence varies linearly with ΔT.[5] The value of R_N determined with the aid of (22) from the experimental $\Delta I_c/\Delta T$ turns out to be less than the asymptotic R_{as} obtained from experiment, as it should in accord with Eq. (8). In particular, the VTJ CS-75 has $\Delta R/R_N \approx 0.4$ at $T = 3.758$ K, corresponding to our estimates from (7) and (8) when account is taken of the errors in the parameters of the VTJ.

As for the excess current, it should be noted that in our experiment an excess current is indeed observed with a temperature dependence similar to (9) (Ref. 16). The measured I_{exc}, however, exceeds as a rule the theoretical I_{exc} (9). This is apparently due to the influence of other mechanisms on the value of I_{exc}.

3.3. CVC of chains of junctions

Figure 3 shows the family of CVC of the chain D3 at various temperatures. It can be seen that the observed CVC of the chain are on the whole similar to the CVC of a single VTJ (see Fig. 5 of Ref. 5). With increasing voltage, a number of singularities appear on the chain CVC, in the form of abrupt increases of the differential resistance R_d. The voltages at which the singularities appear, however, do not lie in the voltage regions corresponding to singularities due to penetration of an alternating electric field into the superconductor. Figure 3 shows at $T = 3.52$ K also the influence of microwave signal of frequency $f_e = 10$ GHz. It can be seen that when the microwave signal is applied the CVC acquires Josephson current steps at voltages:

$$V_n = \tfrac{1}{2} n f_e \quad (n = 1, 2, 3\ldots), \qquad (23)$$

with the steps horizontal at small n and inclined at $n \gtrsim 8$. The

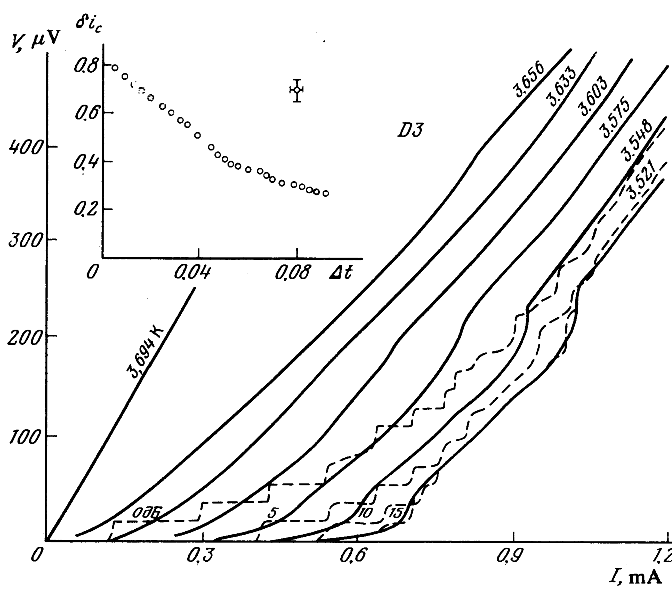

FIG. 3. Family of CVC of a junction chain at various temperatures. CVC for $T = 3.521$ K at different levels of microwave power of frequency $f_e \approx 10$ GHz. The inset shows the dependence of the experimentally observed difference δi_c between the critical currents on $\Delta t = (T_c - T)/T_c$.

appearance of a step with $n = 1$ at low microwave power is evidence that only one VTJ of the chain has gone into the resistive state, at least at the currents corresponding to the voltage V_1 [Eq. (23)] at our value of the frequency f_e. Indeed, under the action of a weak microwave signal the CVC of the chain, after several steps, resumes the autonomous form. At currents $I > 900\,\mu$A the CVC undergoes a new change coorresponding to formation of an inclined symmetric step. This form of the CVC of a chain acted upon by a weak mocrowave signal is evidence that when the current is increased above the critical value only one VTJ (which we number 1), with the smaller critical current, goes over into the resistive state, and the second junction is still superconducting. Further increase of the critical current ($I > I^*_{c2} = 900\,\mu$A at $T = 3.521$ K) causes the second junction to go over into the resistive state. The inset of Fig. 3 shows the temperature dependence of the quantity $\delta i_c(T) = 2(I^*_{c2} - I_{c1})/(I^*_{c2} + I_{c1})$. It can be seen that as T is lowered δi_c decreases and approaches at low temperature the value 0.2 that follows from our estimate of the difference between the parameters. Similar results were obained in experiments on chains of indium junctions in Refs. 6 and 7.

Let us analyze the considered case ($I_{c1} < I < I^*_{c2}$) with the aid of the system (13). Then $\varphi_2 = \varphi_0 + \delta\varphi_2$, $\delta\varphi_2 \ll 1$, $\overline{\delta\varphi_2} = 0$. The connection of φ_0 with the current is determined by the relation

$$I_{c2}P(\varphi_0)+2b^*V(I)/R_2=I,$$

$$P(\varphi_0)=\overline{P\{\varphi_0+\delta\varphi_2\}},\quad b^*=b(0)/[(1+a(0))^2-b^2(0)], \tag{24}$$

which is obtained after time-averaging the second equation of (13). For $P\{\varphi\} = \sin\varphi$ it follows from (24) that the second junction becomes resistive at a current

$$I_{c2}^*=I_{c2}(1+2b^*V(I_{c2}^*)/V_c). \tag{25}$$

The physical reason is that the penetration of the constant electric field of the first junction into the banks produces in the second a normal excitation current I_N even at $\dot{\varphi}_2 = 0$. As a result the superconducting current flowing through the second junction is only a fraction of the total current $(I - I_N)$. The second junction therefore becomes resistive at a current $I^*_{c2} = I_{c2} + I_N > I_{c2}$. When the polarity of the bias voltage in one of the junctions is reversed [$V \to -V$ in (23)] a decrease of I^*_{c2} compared with I_{c2} will be observed. This was indeed observed in experiment.[6,7] As seen from (23), the excess of I^*_{c2} over I_{c2} turns out to be proportional to b^*. In our experiment it was found, however, that δi_c near T_c exceeds the estimate (23). This may be due to the following circumstance. As seen from the system (13), the presence of Josephson oscillations in the first junction leads to oscillations of the phase in the second. It is known that in junctions of length $l > \eta$ even small oscillations of this kind increase the critical current of the second junction at temperatures close to T_c (Ref. 17) [max $P(\varphi_0) > 1$]. This stimulation of the second critical current of the second junction by the alternating electric field of the first can also delay the transition of the second junction into the resistive state. We emphasize

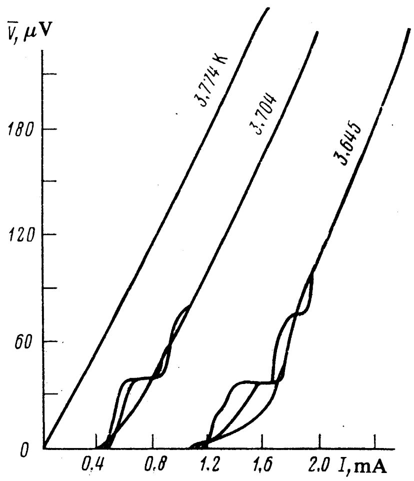

FIG. 4. CVC of a chain in which mutual synchronization of the Josephson oscillations is observed, at two values of the temperature.

that the stimulation can increase I_{c2} substantially even if the interaction between the junctions is weak.

3.4. Mutual stimulation of the junctions

At currents $I > I^*_{c2}$ both junctions are in the resistive state. Penetration of the alternating electric field into the superconductor leads to an interaction of the junctions, which causes under certain conditions mutual synchronization of the Josephson oscillations in the chain. Figure 4 shows for two values of the temperature the CVC of the chain D_2, where such interactions are most clearly observed. It can be seen that a weak microwave signal produces on the CVC of this chain steps at $V = V_{2n}$, corresponding to double the frequency in (23). Since the step observed at low microwave powers is symmetric relative to the autonomous CVC, it can be concluded that the processes in the chains are synchronous,[18] i.e., that mutual synchronization takes place.

Experimental data on chains in which the junctions of close parameter values are shown appear in Fig. 5(a). Here △ marks the distances between the junctions in chains in which steps at $V = V_{2n}$ are produced by weak microwave power, and O marks distances at which there are no steps. Figure 5a shows also the results of an investigation of the mutual syn-

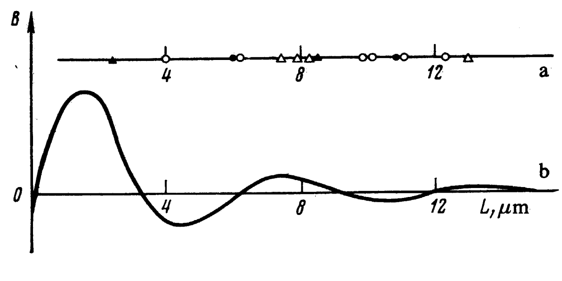

FIG. 5. Results of an investigation of the effectiveness of the interaction between junctions with slightly different parameters, at a fixed temperature $\Delta t = 0.025$ and at a voltage $V = f_e/2$ ($f_e \approx 10$ GHz). ▲—chains in which inphase synchronization is observed, O—antiphase synchronization or none, △, O—the corresponding data taken from Ref. 8.

chronization of tin junctions, obtained by others at the same temperatures and voltages in the course of registration of Josephson radiation.[8] It can be seen that the interactions of the junctions in the chain have a nonmonotinic dependence on the distance between them. This is in good agreement with our theoretical results. The dependence of the junction-interaction effectiveness on the distance between them, obtained in experiment at $T_c - T \approx 0.1$ K and $f_e \approx 10$ GHz, is shown in Fig. 5b. Since the condition $\Delta \ll T$ used in the theory is not satisfied at these temperatures, the experimental wavelength is smaller than the theoretical. It can be seen from Fig. 5 that the appearance of steps at $V = V_{2n}$ and the in-phase increase of the Josephson radiation power[8] are observed in those ranges of L in which $B > 0$. The mutual synchronization is here in phase ($\psi_0 \approx 0$) and, as shown in Ref. 19, a weak microwave signal produces steps at $V = V_{2n}$. In the case of antiphase synchronization ($B < 0$, $\psi = \pi$) this does not take place. $|B|$ decreases with increasing L, the difference between the junction parameters exceeds max $|B|$, and there is no mutual synchronization at $L \gg \delta$. This, too, takes place in our experiment.

Finally, a few remarks concerning the chain-CVC singularities due to the interaction between the junctions. In our experiment we did not observe the chain-CVC steplike singularities described by Eqs. (14) and (16) even for junctions with close parameter values and operating in the synchronous regime. The point is that to observe such singularities it is necessary to satisfy the very strong inequality $|\delta V| \ll |B|$. This condition was not satisfied in our experiment. At the same time, to observe mutual synchronization of the junctions it suffices to have $|\delta V| \sim \max |B|$. The mutual synchronization makes it therefore possible to assess the interaction of the junctions even if it is weak (B is small). We note that for better observation of the singularities (14) on the CVC in the case of weak interaction it is preferable to use an additional mutual-synchronization mechanism, say via electrodynamic coupling of the junctions.[14]

The authors thank V. P. Koshelets for help with the experiment and for helpful discussions.

[1)] Variable-thickness junctions consist of a narrow film (constriction) connected to thicker and broader films called banks.

[2)] We use $e = \hbar = k_B = 1$ (k_B is the Boltzmann constant).

[3)] The expression for the current depends, generally speaking, on the junction width w (Ref. 11). We shall assume w to be small enough to be able to neglect the dependences of all the constriction parameters on the coordinate perpendicular to the current direction.

[4)] We disregard the voltage drop ΔV in the bank region in which the current flow becomes two-dimensional. Allowance for ΔV leads to a small (independent of T and ω) change in the resistance.

[5)] Expression (9) determines also (at $l \gg w$) the asymptotic resistance of Dayem junctions, for which it is necessary to put $d_b = d$ in (8).

[6)] By "resistive" we mean here the state with $\bar{\dot{\varphi}} \neq 0$.

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Translated by J. G. Adashko