Theory of magnetokinetic phenomena for small-radius scatterers in semiconductors

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The Schrödinger equation for an electron in the field of an attraction center of arbitrary depth and in a uniform magnetic field is solved under the assumption that the attraction-center effective radius a is small compared with the magnetic length $l = (c\hbar/|e|H)^{1/2}$. The spectra of the bound and quasibound states are obtained and their dependence on the depth of the potential center U is studied in detail. The width and the real part of the energy of the quasibound state in the higher Landau bands $N \neq 0$ is investigated at arbitrary values of U. A system of wave equations is constructed for the bound and quasibound states, as well as the wave functions of the continuous spectrum. On the basis of these functions it is possible to calculate any semiconductor kinetic coefficient due to interaction of the carriers with small-radius centers in a quantizing magnetic field. The transverse and longitudinal static conductivities of a carrier gas scattered by such centers in a quantizing magnetic field are calculated. Expressions for them are obtained in general form in terms of the operators for scattering by a center in an axisymmetric gauge at an arbitrary depth of the scatterer potential. Oscillations of the transverse and longitudinal conductivities with changing depth of the potential of the center are observed. It is shown that the dependences of the conductivity on the temperature and on the field are governed essentially by depth of the scatterer potential, i.e., by the type of impurity in the semiconductor.

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1. The problem of the interaction of an electron with a center having a small radius $a \ll l$ in a magnetic field was dealt with in a number of studies[1–6] (here a is the radius of the center and l is the magnetic length). This problem is of independent interest both as a study of a quantum mechanical system, such as a negative ion in a magnetic field, and from the applied viewpoint, particularly in the study of kinetic properties of semiconductors in quantizing magnetic fields. Skobov[1] solved the problem of scattering of slow particles by a short-range $a \ll l$ potential. He considered only an s wave, and the poles of his scattering amplitude gave the energy of a magnetic-field-induced bound state of an electron on a center at a zero projection $m = 0$ of the orbital momentum of the electron $L = 0$ on the direction of the magnetic field **H**. Bychkov[2] obtained the energy and wave function of a bound state of an electron with $m = 0$ and $L = 0$ by using for the center a δ-function potential and summing the ensuing geometric progression. Demkov and Drukarev[3] investigated, by the method of the zero-radius potential and using one phenomenological parameter (the amplitude of electron scattering by a center at $H = 0$ and at zero electron energy) a weakly bound state of an electron with $m = 0$ in the Landau band $N = 0$. In Ref. 3 the obtained dependence of the magnetic level on the field intensity was more accurate than in Refs. 1 and 2. Andreev[4] investigated the spectrum of bound and quasibound states of an electron in the field of a shallow center ($U \ll \hbar^2/m^*a^2$) in an arbitrary Landau band ($N \ll l^2/a^2$) for weak mixing of the Landau levels by the center. In view of this weak mixing, the problem was reduced in Ref. 4 to one-dimensional. Demkov and Drukarev[5] investigated weakly bound states of an electron with nonzero orbital momentum $L \neq 0$ in the Landau bands $N = 0$ and 1. They used in the analysis two phenomenological parameters, the electron binding energy on the center at $H = 0$ and the effective radius $r_0(H = 0)$ of the center.

Owing to the presence of a magnetic field (in particular, since there is no spherical symmetry), the problem considered calls for specifying the boundary conditions—joining the solutions of the Schrödinger equations in the region of the action of the potential of the center and outside this region. In Ref. 4, where the problem was one-dimensional, the question of the boundary conditions was resolved in elementary fashion. The method of zero-radius potential[3] for the study of the s state in a magnetic field gives very good accuracy (see below). Difficulties are encountered for states with nonzero orbital momentum at $H \neq 0$ and for a non-shallow potential ($U \gtrsim \hbar^2/m^*a^2$). The usually employed boundary condition is the matching of the logarithmic derivatives of the wave function. Near the resonance, when a level with a specified $L \neq 0$ appears in the well, such a boundary condition is very sensitive to the detuning ΔU of the well from resonance and to the value of the particle energy $F(H)$.[6] In Ref. 5 the joining of the logarithmic derivatives was replaced by joining of the wave functions. It was assumed there that the function of the "inner" region $r \lesssim a$ does not feel the magnetic field at all, and no account was taken of the change of the normalization coefficient of the wave function at $H \neq 0$. Yet the normalization coefficient is just as sensitive to ΔU and to $E(H)$ as the logarithmic derivative, and its change at $H \neq 0$ changes the joining conditions. The condition used in Ref. 5 is fully applicable when a real weakly bound state is present in the well at $H = 0$, but calls for refinement when there is no such state. In addition, the spectrum of quasibound states at arbitrary U in higher Landau bands $N \neq 0$

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was not investigated before, and the question of formulating the boundary conditions in the determination of such a spectrum remained open.

A general method of analyzing an electron spectrum in the field of a center of small radius but of arbitrary depth in a magnetic field was proposed by Andreev and Koshelkin.[7] We solve below, on the basis of this method, subject to the only assumption that $a \ll l$, the Schrödinger equation for an electron in a center of arbitrary depth and in a uniform magnetic field. The spectra of the bound and quasibound states of the electron are obtained. Their behavior was investigated in detail for the Landau bands $N = 0$ and 1 and for the angular momentum projections $m = 0$ and ± 1. The behavior of the width and energy of the quasibound states in higher Landau bands $n \neq 0$ was studied at arbitrary depths of the center, i.e., at an arbitrary relation between $\mathrm{Im}E$ and $\mathrm{Re}\,E$. A system of wave functions of these states and of the continuous-spectrum state was constructed.

The constructed system of wave functions was used as the basis for the calculation of the static transverse and longitudinal conductivities of a gas of semiconductor carriers that do not interact with one another but interact with small-radius scatterers in a quantizing magnetic field.

The conductivity for such a situation was calculated earlier in Refs. 1 and 8, but only the contribution of the s-scattering of an electron by an individual center to the conductivity was taken into account. The contributions of the higher moments with $m \neq 0$ to conductivity was taken into account in Refs. 9 and 10, but in the approximation of weak mixing of the Landau levels by an individual scatterer.

We obtain below expressions for the transverse and longitudinal conductivities in terms of the operators of electron scattering by an individual center and an axisymmetric gauge. The equations derived are valid for the potential of a center of arbitrary depth, i.e., at an arbitrary relation between T and $\hbar\omega_H$ compared with U (T is the electron temperature, $\omega_H = eH/m^*c$ is the cyclotron frequency). Account is taken of the contribution made to the conductivity by all values of the angular momentum, without using the weak-mixing approximation. Oscillations were observed in the transverse and longitudinal conductivities following changes in the depth of the potential of the scatterers. It is shown that the character of the dependence of the transverse and longitudinal conductivities on the temperature and on the field depends strongly on the depth of the scatterer potential, i.e., on the type of impurity in the semiconductor.

2. We derive an equation for the spectrum of the bound and quasibound states of an electron in the field of an attraction center of small radius $a \ll l$, but of arbitrary depth and in a uniform magnetic field $\mathbf{H} \| z$. By virtue of the axial symmetry of the problem, the projection m of the orbital angular momentum on the $\mathbf{H}$ direction is conserved. The integral equation for the wave function of an electron with a given m is

$$\psi_m(\rho, z) = -\frac{m^*}{2\pi\hbar^2}\int d^3\mathbf{r}'\,\psi_m(\rho'; z')\,U(r')\,G_m(\mathbf{r}; \mathbf{r}'; E; H). \quad (1)$$

Here E is the electron energy, G_m is the Green's function of the electron in the magnetic field. The Green's function is written in the form of a series in a product of associated Laguerre polynomials—the eigenfunctions of an electron in a uniform magnetic field in an axial gauge.[6] The series can be summed over the radial quantum number[11] at arbitrary m, after which G_m is expressed in terms of an integral in analogy with Ref. 5.

The idea of obtaining an equation for the energy spectrum of an electron consists in the following: owing to the presence of the small parameter $al^{-1} \ll 1$ it becomes possible to find the form of the wave function $\psi_m(r)$ in the effective region $r \lesssim a$ of the potential, but with an unknown energy parameter E. We substitute the obtained solution in the right and left sides of (1) and let the coordinate r tend to zero. The angles and the coordinate variables are then separated in the right-hand side of (1), and the coordinate-angle dependences of the right and left sides coincide. Canceling in both sides of (1) the common function of the coordinates and angles, we arrive at an equation for the spectrum.

For a center with a small radius $a \ll l$ in the effective region of the potential $U(r)$ we can neglect the influence of the oscillator potential of the magnetic field,

$$U_{\mathrm{osc}} = m^*\omega_H^2\rho^2/8 = {}^1/_8\hbar\omega_H(a/l)^2, \quad (2)$$

a procedure legitimate if the following inequality holds

$${}^1/_8\hbar\omega_H(a/l)^2 \ll \max\{U, E\}.$$

The dependence of the wave function on H then becomes trivial: the field enters in the Schrödinger equation of the "inner" region ($r \lesssim a$) only implicitly in the energy E (which is to be determined) and in the paramagnetic term in the combination $E + 1/2|m|\hbar\omega_H$.

The system of solutions of the Schrödinger equation of the inner region is a product of radial and spherical functions[10]:

$$\psi_{Lm}(\rho; z; E; H) = R_{Lm}(r; E; H)\,\Theta_{Lm}(\vartheta), \quad (3)$$

where the radial functions R_{Lm} must satisfy as $r \to 0$ a zero boundary condition at $L > 0$ and must be equal to a constant at $L = 0$. Since at $H \neq 0$ the angular momentum L is generally speaking not a good quantum number, the wave function of the inner region is a superposition of (3) with arbitrary coefficients $C_{L>m}$. However, by substituting in (1) the explicit expression for the Green's function it is easy to see that if there is no random degeneracy of the levels with respect to L at $H = 0$, the mixing of the states with $L \neq L'$ inside the well is small in terms of the parameter

$$\min\{a^2l^{-2};\ E_\parallel/\hbar\omega_H\}^{|L-L'|}, \quad (4)$$

where $E_\parallel$ is the energy reckoned from the bottom of the nearest Landau band.

This last circumstance has a simple physical explanation. The inequality $a \ll l$ is equivalent to the inequality $\hbar^2/m^*a^2 \gg \hbar\omega_H$, so that in the absence of random degeneracy in L a weak mixing of states with different values of the orbital momentum L takes place within the range of action of the potential. It will be shown below (see also Refs. 3 and 5) that a magnetic field alters radically the character of the energy spectrum of the electron at resonant depths of the potential, i.e., when an energy level appears in the well at $H = 0$ for

some value of L. Since by virtue of (4) states with foreign L affect this state weakly, the function of the inner region can be classified in terms of L and m.

We substitute now the function (3) in the right and left hand sides of (1) and let $r\to 0$. It becomes possible then to integrate in the right-hand side of (1) with respect to the angle variables. If the potential of the center $U(r\to 0)$ increases more slowly than r^{-2}, we have $R_{Lm}(r\to 0) = A^L r^L$. A similar coordinate dependence as $r\to 0$ is possessed also by the right-hand side of (1). The angular dependences of the right and left sides of (1) also coincide. Canceling the factor r^L in (1), we arrive at an eigenvalue equation that determines the electron spectrum:

$$A^L(m) = \lim_{r\to 0} r^{-L}\left[-\frac{m^*}{2\pi\hbar^2}\int d^3r' \sin\vartheta\, d\vartheta\, \Theta_{Lm}(\vartheta)\Theta_{Lm}(\vartheta')\right.$$
$$\left.\times U(r')\, G_m(\mathbf{r};\mathbf{r}';E;H) R_{Lm}(r';E;H)\right]. \qquad (5)$$

In view of the excessive complexity of the Green's function G_m, we shall not write out Eq. (5) in explicit form, and shall analyze it for a concrete potential and for concrete energy levels.

We point out here only two circumstances pertaining to the Green's function. First, when calculating the Green's function for energies in the upper Landau bands $N\neq 0$, all the bands break up into two groups: lower bands with energy smaller than E, and higher with energy larger than E. The lower bands are responsible for the width of the corresponding energy state, and the upper for the renormalization of the constant of the interaction of the electron with the center.[1] The lower bands, whose number is finite, remain in the form of a sum, and the higher reduce to an integral in analogy with Refs. 3 and 5.

Second, the poles of the lower bands should be circled in the case of quasibound states to yield waves that diverge from the center az $|z|\to\infty$. For bound states, however, the Green's function G_m should tend exponentially to zero as $r\to\infty$.

3. We obtain now the spectrum of the electron for the case when the potential of an individual center is a rectangular well of depth $-U$ and radius a. In this case the functions R_{Lm} and the constant A^L are given by

$$R_{Lm}(r) = (\varkappa_m r/a)^{-1/2} J_{L+1/2}(\varkappa_m r/a);$$
$$\varkappa_m^2 = 2m^* a^2\hbar^{-2}\{U+E+{}^1/_2|m|\hbar\omega_H\}, \qquad (6)$$

where $J_{L+\frac{1}{2}}$ are spherical Bessel functions and

$$A^L(m) = \varkappa_m^L/(2L+1)!! \qquad (7)$$

We shall investigate shallow state with energy $E\ll\hbar^2/m^*a^2$, since by virtue of the inequality $a\ll l$ the cyclotron energy is small, $\hbar\omega_H\ll\hbar^2/m^*a^2$, and the influence of the magnetic field on the deep states, $E\gtrsim\hbar^2/m^*a^2$, is trivial and reduces to ordinary perturbation theory.

We consider first the state of an electron with zero orbital momentum $L=0$ and with zero projection $m=0$ of the angular momentum for energies E below the bottom of the ground Landau band $N=0$. Substituting (6) and (7) with $L=m=0$ in (5) we obtain after straightforward but complicated transformations the equation

$$l/f_0(E) = [1+E/U\cos\varkappa_0] = 2^{1/2}\eta_0^{1/4} - \varphi_0(\eta_0). \qquad (8)$$

Here $\eta_0 = \frac{1}{2} - E/\hbar\omega_H$ and $f_0(E) = -a/((\tan\varkappa_0)/\varkappa_0 - 1)$ is the scattering length of an electron with $L=0$ by the center in the absence of a magnetic field, while the function φ_0 is defined by the integral

$$\varphi_0(\eta_0) = \sqrt{\frac{2}{\pi}}\int_0^\infty dt\left[\frac{\exp(t^2/2)}{\operatorname{sh}(t^2/2)} - \frac{1}{t^2}\right]\exp(-\eta_0 t^2). \qquad (9)$$

Equation (8) differs somewhat from that obtained by Demkov and Drukarev.[3] Thus, in the case of a well that is very close to resonance, for which $f_0(E)\to\infty$, when a bound state arises at $H=0$, the solution obtained in Ref. 3 yielded an energy $E = 0.2\hbar\omega_H$. Calculation in accord with (8) introduces a certain correction

$$E = \hbar\omega_H(0.20 + 8\cdot 10^{-3} a/l), \qquad (10)$$

and this determines the degree of accuracy of the approximation of Ref. 3.

The dependence of the energy of the state $N=m=L=0$ on the dimensionless parameter $\varkappa_0(E=0)$ is shown in Fig. 1 together with the dependence of the scattering length $f_0(E=0)$ on $\varkappa_0(E=0)$.

From Fig. 1 we can discern the following pattern of the behavior of the levels.

(1) At the values of the potential U at which the scattering length $f_0(E=0)=0$ there appear additional "magnetic" levels, which are due to the presence of the magnetic field and lie under the bottom of the Landau band $N=0$. We note that the equation of Ref. 3 that is analogous to (8) at $f_0>0$ is valid only if $f_0\gg a$, and this prevents us from investigating the pattern of the onset of the levels.

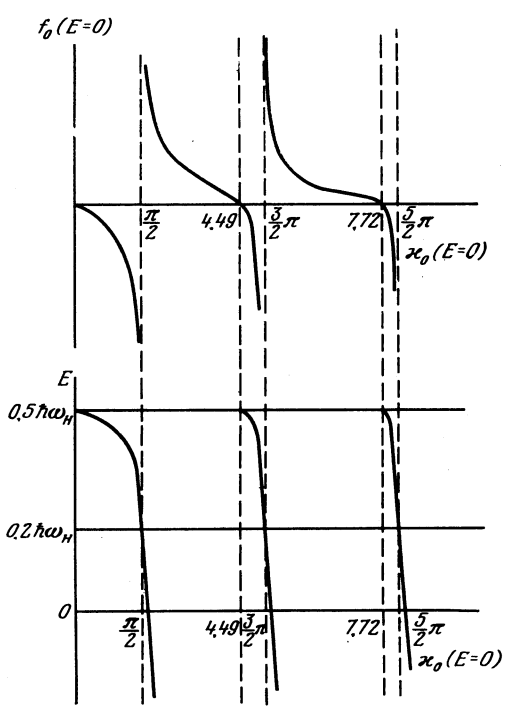

FIG. 1. Scattering length of electron from center and position of the electron energy level vs the depth of the potential of the center at $N=0$ and $L=m=0$; the parameter $\varkappa_0(E=0)$ is obtained from (6) at $m=0$ and $E=0$.

(2) When the potential lies deeper than these values (but close to them) the levels become deeper and have at exact resonance $f_0(E=0)=\infty$ one and the same value, given by Eq. (10).

(3) At values of the potential U such that $f_0(E=0)<0$, the levels are due to the presence of the magnetic field and vanish when it is turned off.

(4) Owing to the magnetic field, the levels have zero energy not at exact resonance $f_0(E=0)=\infty$ of the potential, but at larger depths of the well. It is easy to obtain from (8) the deviation of the potential from resonance, ΔU_n, at which the n-th level has zero energy:

$$\Delta U_n=\frac{\hbar^2}{m^*a^2}\frac{a}{l}\frac{(1-\varphi_0(1/2))}{2n+1}. \tag{11}$$

(5) With further increase in depth of the potential, the levels go over into the region of negative energies and coincide with the natural levels of a well with $L=0$ at $H=0$.

We note finally that all small depths of the potential, $U\ll\hbar^2/m^*a^2$, the distance of the level from the bottom of the Landau band is according to (8)

$$\Delta E=-1/2\,(f_B^2/l^2)\,\hbar\omega_H,$$

where f_B is the Born scattering amplitude of an electron with zero energy at $H=0$; this agrees with the result of Ref. 4.

Equation (5) has solutions for states with $L=m=0$ not only at energies $E\leqslant\hbar\omega_H/2$, but also at energies above the zeroth Landau band. In contrast to the ground band $N=0$, where truly stationary "magnetic" energy levels exist at $f_0(E=0)\leqslant 0$, in the higher bands $N\neq 0$ such magnetic states are quasistationary. This is due to the possibility of their relaxation to the continuous spectrum of the lower lying Landau bands.

The equation that describes the quasistationary state in the band $N=1$ for $m=L=0$ can also be obtained from (5), namely,

$$-l/f_0(E)=i[2E/\hbar\omega_H-1]^{-1/2}-2^{1/2}(3/2-E/\hbar\omega_H)^{1/2}+\varphi_0(\eta_0+1)\equiv\chi(E). \tag{12}$$

The appearance of an imaginary term in the right-hand side of (12) is due to the fact that the corresponding term in the Green's function, which pertains to the Landau band $N=0$, describes an electron that relaxes from the band $N=1$ into the continuous spectrum of the band $N=0$.

Obviously, the roots of (12) are complex. The left-hand side of (12), just as Eq. (8), contains the features of the center: the scattering length $f_0(E)$ that determines the appearance and behavior of the "magnetic" levels. An analysis of (12) shows that the quasibound states appear, just as in the case of the zeroth-band levels, for potential depths at which $f_0(E=0)=0$.

Leaving out the computational details, we present the results of the analysis of (12). At small $U\ll\hbar^2/m^*a^2$ the quasibound level $N=1, L=m=0$ lies lower than the bottom of the Landau band at a distance

$$\mathrm{Re}\,\Delta E=-1/2\,(f_B^2/l^2)\,\hbar\omega_H$$

and its width is of the order of

$$\mathrm{Im}\,E\sim(f_B/l)^3\hbar\omega_H\ll\mathrm{Re}\,\Delta E,$$

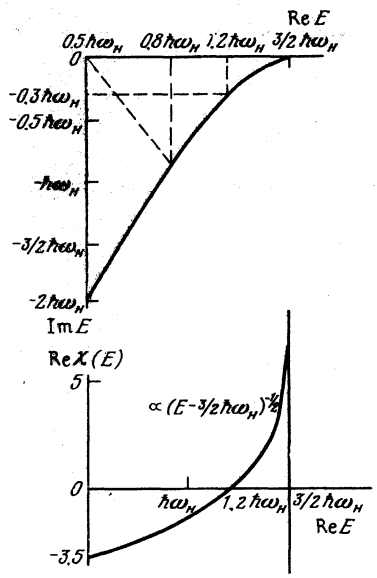

FIG. 2. Plots of the real and imaginary parts of the energy of the state of an electron quasibound to a center at $N=1, m=L=0$, and plots of $\mathrm{Re}\chi(E)$ defined in Eq. (12).

which coincides with the result of Ref. 4. With increasing U, the level goes deeper. Its depth first increases considerably more rapidly than its width. Thus, at $\mathrm{Re}\,E\approx 1.35\hbar\omega_H$ we have $\mathrm{Im}\,E/\mathrm{Re}\,E=0.09$. Next, however, starting with $\mathrm{Re}\,E\approx 0.8\hbar\omega_H$, when $\mathrm{Re}\,E$ and $\mathrm{Im}\,E$ become equal, the growth of the level width is faster than that of its depth. With further increase of the depth of the center potential, at values of U when $f_0(E=0)$, a quasistationary state with $\mathrm{Re}\,\Delta E\propto(f_0/l)^2$ and $\mathrm{Im}\,E\propto(f_0/l)^3$, again appears below the bottom of the band $N=1$, and its behavior repeats periodically the described picture when U is increased.

Plots of the real and imaginary parts of the energy of the state of an electron quasibound to a center are shown in Fig. 2 for the Landau band $N=1$ and for $L=m=0$.

We consider now a state in the lowest Landau band $N=0$, with $L=1$ and $m=-1$. We substitute in (5) the functions (3) and (6) with $L=1$ and $m=-1$ and (7). Calculating the integrals (5) we arive at an equation for the electron spectrum in the state $N=1, L=-m=1$:

$$\frac{\alpha_0^2}{2}=\frac{\alpha^2}{2}-\hbar\omega_H-a\hbar\omega_H\left(\frac{m^*\omega_H}{\hbar}\right)^{1/2}\left[\varphi_1(\eta_0)-\frac{\alpha}{(\hbar\omega_H)^{1/2}}+\frac{\alpha^3}{3}(\hbar\omega_H)^{-3/2}\right]v(\varkappa_n), \tag{13}$$

$$\varkappa_n=\left\{\frac{2U_n+3\alpha_0^2}{\hbar^2/m^*a^2}\right\}^{1/2},\quad v(\varkappa_n)=\frac{J_{3/2}(\varkappa_n)}{J_{1/2}(\varkappa_n)}, \tag{14}$$

$$\varphi_1(\eta_0)=\left(\frac{2}{\pi}\right)^{1/2}\int_0^\infty dt\{[1-\exp(-t^2)]^{-2}-t^{-2}-t^{-4}\}\exp(-\eta_0 t^2). \tag{15}$$

For convenience in the analysis of the results and for comparison with the available data we used in (13) the notation of Ref. 5: $\alpha^2/2$ is the binding energy of the electron on the center at $H\neq 0$, reckoned from the bottom of the zeroth Landau band; $\alpha_0^2/2\equiv\Delta U/3$ is the electron binding energy

on the center at $H=0$; U_n are the potential depths at which the well becomes resonant for $L=1$ ($n=0, 1, 2,\ldots$).

Although Eq. (13) is very similar in form to Eq. (3.7) of Ref. 5, it contains a substantial difference. The right-hand side of (13) contains as a factor the function $v(\varkappa_n)$ whose zeros coincide with the zeros of the scattering length of electrons with $L=1$ on the center at $H=0$. The presence of this factor determines the conditions for the onset of new "magnetic" levels in the presence of a magnetic field. In the approximations of Ref. 5 the function $v(\varkappa_n)\equiv 1$ and the question of the appearance of level was not investigated.

We examine the behavior of Eq. (13) as a function of ΔU. Let $\Delta U<0$, i.e., there is no shallow bound state with $L=1$ at $H=0$. At an arbitrary small but nonzero H, shallow ($\alpha\to 0$) magnetic levels appear under the bottom of any Landau band.[3-5] For these levels, the principal term in the square brackets of (13) and (15) is given by the function $\varphi_1(\eta_0)\sim\alpha^{-1}$. Retaining in (13) only the senior terms, we arrive at the equation

$$(a/l)\,(\hbar\omega_H)^{3/2}v(\varkappa_n)=\alpha|\alpha_0^2/2|. \qquad (16)$$

Equation (16) has solutions at $v(\varkappa_n)\geqslant 0$, when the electron scattering length $f_1(E=0)\leqslant 0$, and has no solutions at smaller $\Delta U<0$, when $f_1>0$, i.e., $v(\varkappa_n)<0$. We decrease $\Delta U<0$. At certain values $\alpha_{0k}^2<0$ the function v vanishes (the zeros of v coincides with the zeros of the Bessel function $J_{5/2}(\varkappa_n{}^k)=0$). As $\varkappa_n\to\varkappa^{k_n}$, i.e., when α_0^2 tends to α_{0k}^2, the right-hand side of (16) tends to a constant and $v(\varkappa_n)\to 0$, from which it follows that α should also vanish. In other words, magnetic levels appear at the points v_n^k. The corresponding values of $\varkappa_n^k$ and their deviations from the exact resonant values $\varkappa_n^0(H=0)$ are given in Table I (with the following notation: n is the number of the resonance in the well, $\varkappa_n^0$ are the (dimensionless) potential depths corresponding to resonance at $H=0$; $\varkappa_n^k$ are the potential depths at which "magnetic" levels appear, and $\Delta\varkappa_n$ is the deviation of the potential depth from the resonant value corresponding to the onset of the magnetic level). The behavior of the level $L=1$, $m=-1$, $N=0$ as a function of the potential. is shown in Fig. 3, which shows also the dependence of the scattering length $f_1(E=0)$ on U. We note that the behavior of the magnetic levels at higher resonances $n>0$ is the same, but differs from the behavior of the principal magnetic level with $n=0$, which exists at all values of the potential detuning $\Delta U<0$. The behavior of the level $L=1$, $m=-1$, $N=0$ near the point $\varkappa_n^k$ of its appearance is given by

$$\frac{\alpha^2}{2}=\frac{\hbar\omega_H}{2}\left(\frac{\hbar\omega_H}{\alpha_0^2/2}\right)\left(\frac{a}{l}\right)^2 v'^2(\varkappa_n{}^k)\,(\varkappa_n-\varkappa_n{}^k)^2\propto H^4.$$

TABLE I. Potential depths corresponding to the onset of magnetic levels of an electron with $L=-m=1$ in the Landau band $N=0$.

n	1	2	3	4	5	6
$\varkappa_n{}^0$	3.14	6.28	9.42	12.57	15.71	18.85
$\varkappa_n{}^k$	0	5.76	9.09	12.32	15.51	18.69
$\Delta\varkappa_n$	−3.14	−0.52	−0.33	−0.25	−0.20	−0.16

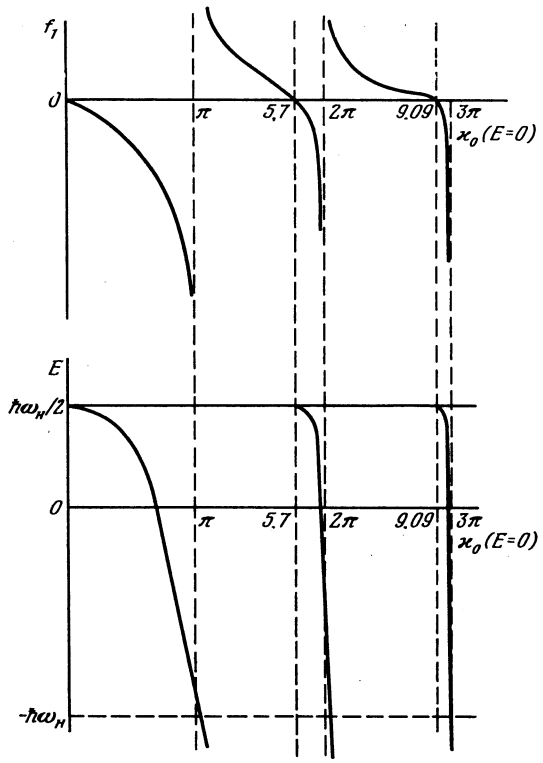

FIG. 3. Scattering length of electron on center and position of electron energy level vs the depth of the center potential at $N=0$ and $L=-m=1$.

The behavior of the level $L=m=+1$ in the band $N=1$ with change of the depth of the potential U is completely analogous to the behavior of the level $L=-m=1$ in the band $N=0$, but the energy should be reckoned from the bottom of the band $N=1$. The corresponding equation is obtained from (5) upon substitution of functions (6) and (7) with $L=1$ and $m=+1$.

To obtain the correct dependence of the energy on $\Delta U<0$ in Eq. (3.6) of Ref. 5 for the level $L=1, m=0, N=0$ it is also necessary to introduce v as a multiplier for the last two terms of the right-hand side. These levels also appear at potential depths for which $f_1(E=0)=0$.

Besides the foregoing, the left-hand side of (13) does not contain the term $-\alpha_0^3 a/3$ contained in Eq. (3.7) of Ref. 5. At $\Delta U<0$ in the region below resonance this term becomes pure maginary, leading to a small but nonzero width of the level $L=1, m=-1$ in the band $N=0$ (and $L=m=+1$ in the band $N=1$); these are certainly stationary in the single-center problem. Finally, putting $H=0$ in (13) we obtain the equation for the energy level with $L=1$ at $H=0$, accurate to terms $\propto a$.

Substituting the obtained $E(H,U)$ dependences in the functions (3) of the "inner" region, and then the functions (3) in (1), we obtain the system of wave functions of the bound and quasibound states of the electron.

The scattering problem is solved in the following manner. We substitute the function (3) of the inner region with $E_\parallel=p^2/2m^*$, multiplied by an unknown constant $B^L(m)$, in the integral equation for the inhomogeneity-containing wave function that describes the incident wave. We let $r\to 0$

and arrive at a linear equation for $B^L(m)$. The solution of this equation is equivalent to finding the amplitude for the scattering of an electron on a center in a uniform magnetic field. The poles of the obtained amplitude yield the equations of the spectrum of the bound and quasibound states considered above.

The obtained wave functions make it possible to calculate all the kinetic coefficients governed by the interaction of the carriers with small-radius centers.

4. We consider a gas of electrons interacting with spherically symmetric centers of radius a in a quantizing magnetic field in a semiconductor, at an arbitrary ratio of a and l. We express the transverse and longitudinal conductivities of such a gas in terms of the operators of electron scattering on an individual center in an axisymmetric gauge. In the calculation of the conductivity we assume the following:

(a) the carriers do not interact with one another;

(b) $\hbar\tau^{-1} \ll T$, where τ is the characteristic electron-momentum relaxation time;

(c) the scatterer density is low: $n_0 a^3 \ll 1$ and $n_0 l^2 \lambda\!\!\!^{-}_{\parallel} \ll 1$, where $\lambda\!\!\!^{-}_{\parallel} = \hbar/(m^* T)^{1/2}$.

We emphasize that the potential U of an individual center is not assumed to be small either in comparison with the characteristic energy of the longitudinal motion of the electron T, or in comparison with the characteristic energy $\hbar\omega_H$ of the transverse motion.

An expression for the transverse conductivity in a quantizing magnetic field in terms of Landau gauge functions was obtained by Magarill and Savvinykh[12]:

$$\sigma_{yy} = \frac{2\pi e^2}{\hbar} \sum_{\alpha,\alpha'} \left(-\frac{\partial f(E_\alpha)}{\partial E_\alpha}\right) \delta(E_\alpha - E_{\alpha'}) \times |\langle \varphi_\alpha^* | V | \psi_\alpha \rangle|^2 (y_\alpha - y_{\alpha'})^2. \tag{17}$$

Here $\alpha \equiv \{N, y_0, p_z\}$ is the set of Landau quantum numbers, φ_α and ψ_α are the wave functions of the free motion and of the scattering problem, respectively, taken in the Landau gauge; $V(r)$ is an arbitrary scattering potential; $f(E_\alpha)$ is the electron (assumed Boltzmann) distribution of the electrons in energy.

The Landau gauge is usually employed to obtain the general equations of conductivity,[13] since the Schrödinger equation for an electron in crossed field can be solved in it exactly. We change over in (17) to functions of the axisymmetric gauge, in which we know how to solve the scattering problem.

The coefficients of the transition from the wave functions $\varphi N_m(\rho)$ of free motion in an axisymmetric gauge to the functions $\psi_{Ny_0}(\rho)$ in the Landau gauge, i.e., the expansion coefficients:

$$\varphi_{Ny_0}(\rho) = \exp(ixy/2l^2) \sum_{m=-\infty}^{N} C_m^N(y_0) \varphi_{Nm}(\rho), \tag{18}$$

were calculated by Pogosyan and Ter-Antonyan[14] and are of the form

$$C_m^N(y_0) = (-1)^N i^{|m|} (2\pi)^{1/2} \varphi_{N-m}(y_0); \tag{19}$$

where φ_k are the oscillator wave functions.[15]

Knowing the coefficients C_m^N(19), we obtain the connection between the functions of the scattering problems in the two gauges:

$$\psi_{Ny_0 p}(r) = \exp(ixy/2l^2) \sum_{m=-\infty}^{N} (-1)^N i^{|m|} (2\pi)^{1/2} \varphi_{N-m}(y_0) \psi_{Nmp}(\mathbf{r}). \tag{20}$$

Substituting (20), (19), and (18) in (17), retaining in the sums over α and α' only the terms with $N = N' = 0$ (quantizing magnetic field), we have an equation for the transverse conductivity in terms of the operators $T^m_{kk'}$ for electron scattering by an individual center in the axisymmetric gauge:

$$\sigma_{yy} = 2\pi e^2 n_0 \int_{-\infty}^{+\infty} dk_z\, dk_z'\, \delta(E_\parallel - E_\parallel') \left(-\frac{\partial f(E_\parallel)}{\partial E_\parallel}\right) \times \sum_{m=0}^{\infty} |T^{-m}_{k_z k_z'} - T^{-m-1}_{k_z k_z'}|^2, \tag{21}$$

with

$$T^m_{k_z k_z'} = \int d^3 r \psi_{0mk_z}(\mathbf{r})\, U(r)\, \varphi^*_{0mk_z'}(\mathbf{r});$$

here $E_\parallel$ is the longitudinal energy of the electron and n_0 is the density of the scatterers.

In the approximation of weak mixing of the Landau level by an individual scatterer, $U/\hbar\omega_H \ll 1$, Eq. (21) coincides with the result of Ref. 9.

The longitudinal conductivity is calculated in analogy with Ref. 16 and is expressed in terms of the T-operator in the form

$$\sigma_{zz} = \frac{e^2\hbar^2 l^2}{\pi m^* n_0} \int_0^\infty dE_\parallel E_\parallel \left(-\frac{\partial f(E_\parallel)}{\partial E_\parallel}\right) \left\{\sum_{m=0}^{\infty} |T^{-m}_{k_z - k_z'}|^2\right\}^{-1}. \tag{22}$$

In the approximation lowest in $U/\hbar\omega_H \ll 1$, Eq. (22) coincides with the result of Ref. 16.

Equations (21) and (22) are valid for a spherically symmetric potential of an individual center of arbitrary depth U and radius a. If the solution of the problem of electron scattering by such a center in a magnetic field is known, it yields the temperature-field dependences of the transverse and longitudinal conductivities $\sigma_{yy}(U,H,T)$ and $\sigma_{zz}(U,H,T)$, respectively.

5. We investigate now the behavior of the conductivity as a function of the depth of an individual scatterer in the case of centers of small radius $a \ll l$. Hereafter, generally speaking, U is everywhere much larger than the electron temperature. We shall show that the conductivity has an oscillatory dependence on the well depths. With rising temperature, the oscillations become stronger—their amplitude increases and their width decreases.

Solution of the scattering problem leads in the case of rectangular well to the following expression for the scattering operator ($m = L$):

$$T^m_{k_z k_z'} = \frac{\hbar^2 a (2\pi)^{1/2}}{m^* \varkappa_m} \frac{U}{\hbar^2/m^* a^2} \left(\frac{a}{l}\right)^{|m|} J_{|m|+1/2}(\varkappa_m) \times \left[F\left(\frac{a}{l}, \varkappa_m, \eta_0\right)\right]^{-1}, \tag{23}$$

where

$$F\left(\frac{a}{l}, \varkappa_m, \eta_0\right) = \varkappa_m^{|m|+1/2} - (2\pi)^{1/2}\frac{U}{\hbar^2/m^* a^2}\left(\frac{a}{l}\right)^{2|m|+1} \times \int_0^1 dx x^{|m|+3/2} J_{|m|+1/2}(\varkappa_m x) \times\left\{\frac{i}{(-2\eta_0)^{1/2}} + \sqrt{\frac{2}{\pi}}\int_0^\infty dt \exp\left(-\eta_0 t^2 - \frac{a^2}{l^2}\frac{x^2}{2t^2}\right) \times [[1-\exp(-t^2)]^{-m-1} - 1]\right\}. \quad (24)$$

In the approximation linear in $U/\hbar\omega_H \ll 1$ Eq. (23) goes over into the corresponding expression of Ref. 16. The poles of (23) yield the equations considered above for the bound states. Equation (23) does not depend on the relative directions of the momenta p_z and p'_z, i.e., the scattering along the z axis, is, just as in the case of a one-dimensional δ potential, isotropic.

For the lowest angular momentum projection $m = 0$ ($L - 0$) it is easy to obtain from (23) and (24)

$$T^0_{k_z k_z'} = -\frac{\hbar^2}{m^*} f_0(E)\left[1 + \frac{f_0(E)}{l}\left(\frac{i}{(-2\eta_0)^{1/2}} + \varphi_0^1(\eta_1)\right)\right]^{-1}, \quad (25)$$

where

$$\varphi_0^1(\eta_1) = -(2\eta_1)^{1/2} + \frac{2^{1/2}}{\pi^{1/2}}\int_0^\infty dt \exp[-\eta_1 t^2] \times [[1-\exp(-t^2)]^{-1} - t^{-2}],$$

$$\eta_1 = {}^3/_2 - E/\hbar\omega_H. \quad (26)$$

Equation (25) for the scattering operator $T^0_{k_z k'_z}$ differs from Skobov's results,[1] first in the presence of a second term in the square bracket of the denominator. This term gives for the bound states a somewhat different dependence on the magnetic field intensity[3] than the poles of the amplitude of Ref. 1. Second, Eq. (25) contains the amplitude for scattering of the electron at $H = 0$ not as a function of a zero electron energy, but of the true one. At $a \ll l$, except for very narrow regions of resonances of the potential U, the behavior of the conductivity is determined by the states $m = 0$ and $m = -1 (L = 0$ and $L = 1)$.

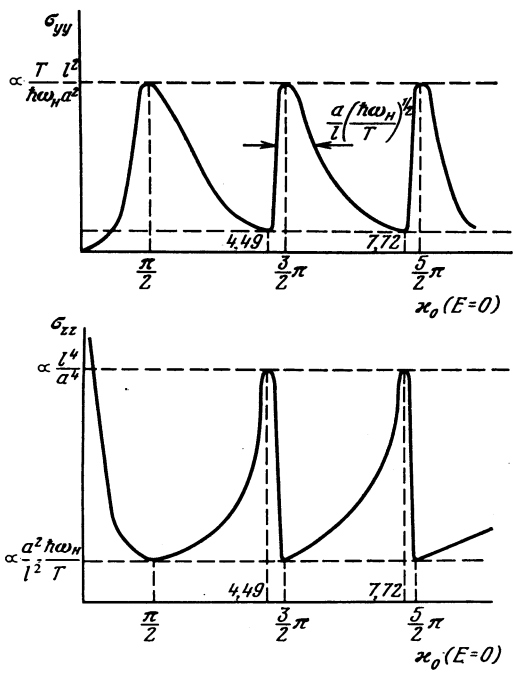

FIG. 4. Plots of the transverse and longitudinal conductivities vs the depth of the scatterer potential at high temperatures: $(a/l)[\hbar\omega_H/T]^{1/2} \ll 1$.

Without presenting the general cumbersome equations, we shall investigate the asymptotic behavior of the conductivity. Greatest interest attaches to two limiting cases:

(a) case of very low temperatures:

$$(a^2/l^2)\hbar\omega_H \gg T; \quad (27)$$

(b) case of high temperatures:

$$\hbar\omega_H \gg T \gg (a^2/l^2)\hbar\omega_H. \quad (28)$$

In the first limiting case the conductivity depends little on the scatterer potential:

$$\sigma_{zz} = 2m^{*2}\omega_H n_e T^{-1/2}/(2\pi m^*)^{3/2} n_0 \hbar, \quad (29)$$

$$\sigma_{yy} = \frac{4e^2 n_e n_0 (2\pi)^{1/2}\hbar^3}{T^{3/2} m^{*5/2}\omega_H}\int_0^\infty \frac{d\varepsilon}{\varepsilon}\exp\left[-\varepsilon\frac{\hbar\omega_H}{T}\right] \times\left[\frac{\varepsilon^{-1} + 2(\varphi_1^1(1) - 2\varphi_0^1(1))^2}{\varepsilon^{-1} + 2(\varphi_0^1(1))^2(\varepsilon^{-1} + 2(\varphi_1^1(1))^2)} + \frac{8}{\varepsilon^{-1} + (\varphi_1^1(1))^2}\right]; \quad (30)$$

here n_e is the electron density.

We consider now the inverse limiting case (28) of high temperatures. We investigate first the behavior of the transverse conductivity. Except for the situation when $f_0(E = 0) \to 0$, the transverse conductivity is determined at $al^{-1} \ll 1$ by the scattering operator $T^0_{k_z k'_z}$. If $f_0(E = 0) \neq \infty$, the transverse conductivity depends weakly on the magnetic field and is given by

$$\sigma_{yy} = \frac{2^{5/2}\pi^{1/2} e^2 n_0 n_e \hbar^2}{(m^* T)^{3/2}} f_0^2(E=0)\ln\frac{l}{a}. \quad (31)$$

If, however, the scatterer potential has a depth close to resonance, i.e.,

$$f_0(E=0) \gtrsim l(T/\hbar\omega_H)^{1/2} \gg a,$$

the transverse conductivity increases abruptly by $(l^2/a^2)(T/\omega_H) \gg 1$ times (see Fig. 4). In this case the conductivity equals

$$\sigma_{yy} = \frac{2^{5/2} e^3 n_e n_0 \pi^{1/2} a^2 \hbar^2}{T^{3/2} m^{*5/2}}\int_0^\infty d\varepsilon \exp\left[-\varepsilon\frac{\hbar\omega_H}{T}\right] \times\left\{\left[\frac{\Delta U}{\hbar^2/m^* a^2} + \frac{a}{l}\varphi_0^1(1)\right]^2 2\varepsilon + a^2/l^2\right\}^{-1}. \quad (32)$$

As $f_0(E = 0) \to 0$ the conductivity σ_{yy} is determined by the scattering of carriers with $m = -1$ by the center and is smaller by a factor $(a/l)^4$ than in (31).

We consider now the behavior of the longitudinal conductivity in the limiting case of high temperatures (28). In the case of the nonresonant situation, $f_0(E = 0) \neq \infty$, the conductivity is determined by the scattering operator $T^0_{k_z k'_z}$:

$$\sigma_{zz} = \frac{e^2\hbar m^*\omega_H}{2(2\pi m^* T)^{3/2}}\frac{n_e}{n_0} f_0^{-2}(E=0)\left(\frac{2T}{\hbar\omega_H}\right)^2. \quad (33)$$

If, however, $f_0(E=0)\to\infty$, the longitudinal conductivity is determined by the scattering of carriers with $m=-1$ and increases sharply compared with (33) (see Fig. 4):

$$\sigma_{zz}=\frac{e^2\hbar m^*\omega_H}{2a^2(2\pi m^*T)^{1/2}}\frac{n_e}{n_0}\left(\frac{2T}{\hbar\omega_H}\right)^2[B_k^2(\varkappa_{-1}-B_k)^2+(a/l)^4]^{-1}, \tag{34}$$

where B_k is a root of the equation $(\tan B)/B-1=0$.

The investigated conductivity oscillations (see Fig. 4) have a simple physical explanation. We shall discuss the oscillations of the transverse conductivity. As seen from Fig. 4, the transverse conductivity increases abruptly for well depths U that are at resonance at $L=0$. The relative oscillation amplitude increases with rising temperature:

$$\propto(T/\hbar\omega_H)\,(l/a)^2\gg 1, \tag{35}$$

while the width of the peak decreases

$$\propto(a/l)\,(\hbar\omega_H/T)^{1/2}\ll 1. \tag{36}$$

As follows from (21) and (25), the transverse conductivity is proportional to the product

$$\sigma_{yy}\propto|f_0(E)|^2\left[1+\left|\frac{f_0(E)}{l}\right|^2\frac{\hbar\omega_H}{E_\parallel}\right]^{-1}$$

$$\propto|f_0|^2\left[1+\left|\frac{f_0}{l}\right|^2\frac{\hbar\omega_H}{T}\right]^{-1}. \tag{37}$$

The factor, which increase linearly $\propto|f_0|^2$, is the square of the electron-center interaction constant, the second term that decreases with increasing $|f_0|^2$ is the coefficient of the passage of an electron having a characteristic kinetic energy $\propto T$ through the effective one-dimensional δ potential due to the joint action of the center and of the magnetic field.[10]

With increasing $|f_0|^2$ the right-hand side of (37) increases and tends at $f_0\to\pm\infty$ to a constant

$$\propto(T/\hbar\omega_H)\,l^2\propto(T/\hbar\omega_H)\,(l/a)^2,$$

which corresponds to (35) and to the relative amplitude of the transverse-conductivity oscillation (see Fig. 4). The width (36) of the oscillation peak can be easily obtained from (37). Indeed, near resonance we have $f_0\propto a/\Delta U$ [see Eqs. (8) and (9)]; hence, estimating the effective vlaue of ΔU from the condition that the first and second terms in the denominator of (37) be equal, we obtain the estimate (36).

The longitudinal conductivity has a maximum at the points U were $f_0(E=0)=0$. In this case the electrons with $m=0$ do not "feel' the center ($T^0=0$) and the contribution to the conductivity is made by the state with $m=-1$. The operator T^{-1} is in this case of the order of a^2/l^2, making the relative amplitude of the oscillation $\sim|T^{-1}|^2\propto l^4/a^4$.

6. The problem considered by us, of the energy spectrum of an electron in the field of a short-range attracting potential, was recently investigated experimentally by Taniguchi and Narita.[17] They investigated isolated D^- states and D^- complexes in Ge doped with Sb or As, in a uniform magnetic field of intensity in the interval 0–25 kG. The samples were taken at a temperature 0.35 K and at impurity densities 5×10^{13}, 9.9×10^{14}, and 5×10^{15} cm^{-3} for Sb and 6×10^{13} and 6×10^{14} for As impurities. At impurity densities 5×10^{13} cm^{-3} the D^- states were well isolated.[18,19] The photoconductivity was measured for a transition of an additional electron bound to the impurity from the ground state to the N-th Landau band, and the maximum number of distinctly observable peaks was four. The ground-state energy of the additional electron was measured in Ref. 18 and was equal to 1.15 and 1.33 meV for Sb and As, respectively. The absorption peaks observed in Ref. 17 were spaced $\hbar\omega_H$ apart with an effective mass $m^*=0.082$ m in single-valley Ge obtained for samples under pressure, and $m^*=0.135$m in multivalley Ge. The parameter al^{-1} at $H=5$ kG is of the order of $0.14\ll1$ for the indicated impurities. Absorption peaks can appear only when the final state of the electron $N\neq0$, $m=\pm1$ is of magnetic character, i.e., when in the absence of a magnetic field there are no bound states of the additional electron on a neutral atom of the impurity with $L=1$ and $m=\pm1$. In the opposite case, as shown by simple calculation with the aid of the obtained functions (1), the Landau levels are strongly mixed by the center and the peaked absorption picture vanishes. Estimates of the bound-state energies of the additional electron, obtained by us from a comparison of the energy difference between the principal absorption peak and the satellite peaks[17] with the corresponding formulas calculated from the wave functions (1) and the functions of the scattering problem, yield 0.83 meV for Sb impurities and 0.94 meV for As impurities.

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[1]V. G. Skobov, Zh. Eksp. Teor. Fiz. **37**, 1467 (1959) [Sov. Phys. JETP **10**, 1039 (1960)].
[2]Yu. A. Bychkov, Zh. Eksp. Teor. Fiz. **39**, 689 (1960) [Sov. Phys. JETP **12**, 483 (1961)].
[3]Yu. N. Demkov and G. F. Drukarev, Zh. Eksp. Teor. Fiz. **49**, 257 (1965) [Sov. Phys. JETP **22**, 182 (1966)].
[4]S. P. Andreev, Zh. Eksp. Teor. Fiz. **75**, 1056 (1978) [Sov. Phys. JETP **48**, 532 (1978)].
[5]Yu. N. Demkov and G. F. Drukarev, Zh. Eksp. Teor. Fiz. **81**, 1218 (1981) [Sov. Phys. JETP **54**, 650 (1981)].
[6]L. D. Landau and E. M. Lifshitz, Quantum Mechanics, Nonrelativistic Theory, Pergamon, 1978 [Russ. orig. p. 525].
[7]S. P. Andreev and A. V. Koshelkin, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 187 (1982) [JETP Lett. **35**, 229 (1982)].
[8]V. G. Skobov, Zh. Eksp. Teor. Fiz. **38**, 1304 (1960) [Sov. Phys. JETP **11**, 941 (1960)].
[9]S. P. Andreev, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 665 (1979) [JETP Lett. **30**, 630 (1979)].
[10]S. P. Andreev and S. V. Tkachenko, Zh. Eksp. Teor. Fiz. **82**, 915 (1982) [Sov. Phys. JETP **55**, 537 (1982)].
[11]A. Erdelyi, ed., Higher Transcendental Functions, McGraw, 1953 [Russ. transl., Nauka, 1965, Vol. 1, p. 54].
[12]L. I. Magarill and S. K. Savvinykh, Zh. Eksp. Teor. Fiz. **60**, 175 (1971)

[Sov. Phys. JETP **33**, 97 (1971)].
[13]E. Adams and T. Holstein, Problems in Quantum Theory of Irreversible Processes [Russ. transl.], IIL, 1961.
[14]G. S. Pogosyan and V. M. Ter-Antonyan, Teor. Mat. Fiz. **40**, 140 (1979).
[15]I. S. Gradshtein and I. M. Ryzhik, Tables of Integrals, Sums, Series, and Products, Academic, 1965.
[16]S. P. Andreev, Zh. Eksp. Teor. Fiz. **77**, 1046 (1979) [Sov. Phys. JETP **50**, 526 (1979)].
[17]M. Taniguchi and S. Narita, J. Phys. Soc. Japan **47**, 1503 (1979).
[18]M. Taniguchi and S. Narita, J. Phys. Soc. Japan **43**, 1262 (1977).
[19]M. Taniguchi, S. Narita, N. Hasegawa, and M. Kobayashi, J. Phys. Soc. Japan **45**, 545 (1978).

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