Phonon drag on the conducting surface of germanium

N. V. Zavaritskiĭ and V. N. Zavaritskiĭ

S. I. Vavilov Institute of Physics Problems, Academy of Sciences of the USSR, Moscow

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The thermopower of germanium specimens with a surface of metallic conductivity has been measured between 0.4 and 20 K. The thermopower of the conducting layer, α_s, was separated out. It is anomalously large and is due to drag of the surface charges by phonons of the whole specimen. This is in accord with calculations of the interaction between phonons and the charges of a thin conducting layer (see the Appendix). The reduction in α_s on increasing the surface resistivity and also the change in the $\alpha_s(T)$ temperature dependence below 1.5 K are probably produced by a change in the nature of the phonon-electron interaction on going from $q\Lambda > 1$ to $q\Lambda < 1$ (q is the phonon wave vector and Λ the charge mean free path).

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INTRODUCTION

In the present work the thermopower, α, of a conducting layer of germanium is studied. The magnitude of α is given by the field E arising in the specimen when a temperature gradient ∇T is established in it in the absence of a current. This quantity enters into the standard transport equations, which have the following form in a crystal of cubic symmetry:

$$E=\rho j+\alpha\nabla T, \quad Q=\Pi j-\varkappa\nabla T, \quad \Pi=\alpha T. \tag{1}$$

In an experiment the voltage across the specimen is measured when a temperature difference ΔT is set up in it, so that

$$\alpha=-U_{12}/\Delta T_{12}|_{j=0}. \tag{2}$$

Unlike the resistivity, ρ, there are only a few studies of the thermopower of two-dimensional systems.[1-3] This is due both to the experimental difficulties (the necessity of measuring voltages $U\sim 10^{-8}$ to 10^{-9} V for specimen resistivities 10^5 to 10^4 Ω), and to the complexities in interpreting the results of measurements. In fact, while ρ is determined by the density of states and the scattering probability at the Fermi surface, α is determined by the energy derivatives of these quantities. Among other integral kinetic coefficients, thermopower is therefore one of the most sensitive both to the electronic structure of a metal and to the nature of the quasiparticle scattering.

The problem of accurately measuring the thermopower of metals at low temperatures has been appreciably simplified since superconducting quantum magnetometers (for example the SKIMP) have started to be used. The standard measuring arrangement[4] had to be modernized for studying a two-dimensional system. We were able to increase the current sensitivity of the apparatus by about four orders of magnitude by introducing a current transformer into the SKIMP circuit.

The objects under study were conducting surfaces of germanium. The bulk conductivity σ_v of pure germanium below 10 K falls exponentially with decreasing temperature and at 4.2 K $\sigma_v < 10^{-8}\ \Omega^{-1}\text{cm}^{-1}$. At the same time a high-conductivity surface can be produced in germanium with $\sigma_s \approx 10^{-3}$ to $10^{-4}\ \Omega^{-1}$, almost independent of temperature. We have studied two forms of such surfaces:

a) The joining plane of bicrystals with a joining angle between (100) and (100) ≈ 10 to 30°. This plane has a large density of edge dislocations (up to 6×10^6 cm^{-1}). The high conductivity arises here as a result of the capture of part ($\sim 1\%$) of the broken electron bonds from the bulk, which leads to the formation of a hole-conducting layer of thickness $\delta\sim 4\times 10^{-7}$ cm according to experiment and theoretical calculations.[5]

b) The surface obtained by cleaving a germanium single crystal at helium temperatures parallel to the easy glide plane (111). The high surface conductivity in this plane arises on subsequently annealing it to 30 to 80 K, depending on annealing conditions.[6] The formation of a conducting layer in this case seems to arise in the following way. Immediately after cleavage the broken bonds hybridize. On subsequent adsorption of gas (mainly oxygen) and heating, re-hybridization takes place and a hole-conducting layer is then formed in the same way as in the case of bicrystals.

While the high-conductivity surface of bicrystals is formed simultaneously with the specimen and exists in unchanged form for a limitless time, the conducting surface of cleaved specimens thus exists only under appropriate conditions and its conductivity can be changed in a controlled way in the course of an experiment.

As is well known,[7] thermopower can be expressed in the form

$$\alpha=\alpha_e+\alpha_{ph}, \tag{3}$$

where α_e is the diffusion part directly related to the effect of the temperature gradient on the electron system, while $\alpha_{\rm ph}$ is related to the drag on the electrons by the phonon current produced by the temperature gradient. The following relation[7] can be used to calculate the value of $\alpha_{\rm ph}$

$$\alpha_{ph}=C_L/eN_V\propto T^3. \tag{4}$$

Preliminary experiments[1] showed that phonon drag is the main process determining the thermopower of a germanium surface. However, the magnitude of $\alpha_{\rm ph}$ was more than one and a half orders of magnitude greater than the value obtained by using Eq. (4). The reason given for this[1] was that an appreciable contribution to the drag on the charges of the conducting layer comes from phonons of the whole speci-

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men. An analysis of this effect is given below with the addition of the results of a theoretical calculation carried out by Velikovich and Liberman. It was important in solving the problem to determine the phonon dynamics in the specimen and this was done from the results of measuring the thermal conductivity.

In this work several models are also discussed which can explain the departure of $\alpha_{\text{ph}}(T)$ in a number of specimens from the dependence which follows from the simplest ideas.

METHOD OF MEASUREMENT

All measurements were carried out in a vacuum jacket. The arrangement of the apparatus for studying specimens with a cleaved surface is shown in Fig. 1. Before the experiment ≈ 0.1 atm of helium remained in the jacket. Cleavage of the specimen S was carried out at 4.2 K using the quartz knives *2*. The specimen was then heated to 50 to 60 K to obtain a high conductivity in the cleavage surface. The activated charcoal *6* was then at the same temperature. The helium gas was removed from the jacket by a diffusion pump and the remains of it adsorbed by the charcoal *6* after its subsequent cooling to 4.2 K. The mean temperature of the specimen was varied by heater H_2 while the heater H_1 provided the heat flow along the specimen for measurements α and $\varkappa$. The temperature difference across the specimen was measured by thermometers T. Indium strips soldered to the specimen provided contact with the conducting layer. Leads a to the specimen for measuring the Seebeck voltage were made of superconducting wire. Heat conductivity, Seebeck voltage, and Hall effect could be measured in the apparatus over the range 1.4 to 20 K.

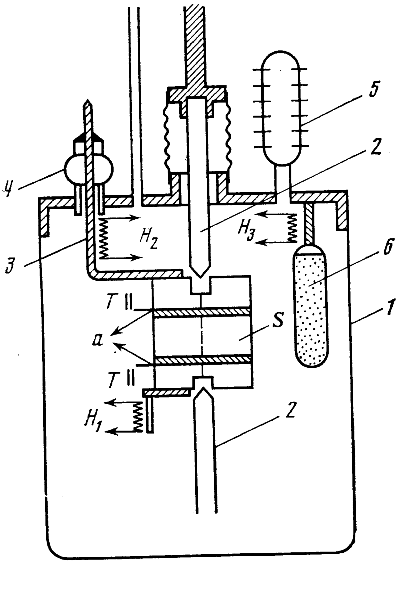

FIG. 1. Apparatus for studying germanium specimens with a conducting surface obtained by cleaving. S—specimen; the dashed line indicates the cleavage position, 1—vacuum jacket, 2—quartz knives, 3—cold finger passing through an insulator, 4,5—platinum-glass seals for introducing leads into the jacket, 6—activated charcoal, H-heaters, T—thermometers. a—potentiometer leads for measuring α.

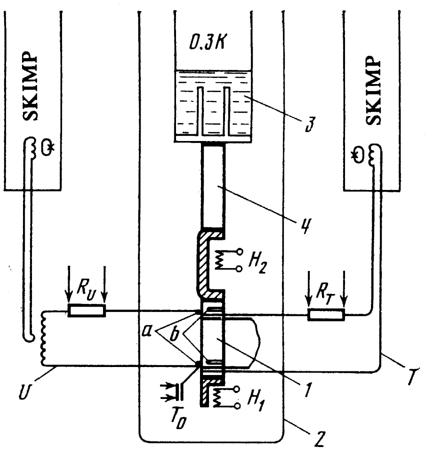

FIG. 2. Arrangement of apparatus for measuring thermopower and thermal conductivity in the range 0.4 to 3.5 K. 1—specimen, 2—vacuum jacket, 3—^{3}He chamber, 4—germanium electrical insulator and thermal resistance, H—heater, T_0— thermometer, a—indium contacts to the specimen for measuring the Seebeck voltage with circuit U, b—thermocouple junctions for measuring temperature difference across the specimen with circuit T, R_U and R_T—standard resistances.

At first, a similar apparatus was also used to study bicrystals.

The arrangement of the apparatus used in later experiments is shown in Fig. 2. A temperature $T \approx 0.3$ K was reached by pumping ^{3}He. The temperature of specimen 1 could be varied up to 3.5 K by using heater H_2 and the germanium single crystal thermal shunt.[4] Carbon thermometer T was used to measure the temperature. Heater H_1 was used to produce a heat flow along the specimen for measurements of thermal conductivity and of the Seebeck voltage in the conducting layer.

The temperature difference ΔT along the specimen was measured with a ZLZh-superconductor thermocouple using a balance method with a SKIMP device as null detector (circuit T in Fig. 2). The thermocouple was stuck to the specimen, and its junctions b were next to contacts a on the conducting layer.

Calibration of the thermocouple in subsidiary experiments showed that for $T \lesssim 1.5$ K the absolute thermopower of ZLZh $\alpha_T = 5.2 \times 10^{-6} T\,\text{V}\cdot\text{K}^{-1}$, and for higher temperatures α_T smoothly approaches a constant value $\alpha_T = 11.8 \times 10^{-6}\,\text{V}\cdot\text{K}^{-1}$ at $T \gtrsim 4.9$ K. During the experiments the temperature difference was 10^{-2} to 10^{-3} K and could easily be measured to an accuracy of $\sim 2\%$. The power $Q = \alpha_T jT$ dissipated at the thermocouple junctions due to the Peltier effect did not exceed 5×10^{-12} W and was 10^5 times smaller than the minimum power produced in the heaters. The heat conductivity of the thermometer leads between its thermal contacts with the specimen was less than 10^{-3} of the conductivity of the thinnest specimens studied.

A null method was used to measure the Seebeck voltage U (circuit U in Fig. 2). With the transformer, the sensitivity of the SKIMP device was 10^{-13} A for an input impedance of $\approx 100\ \Omega$. In preliminary experiments a galvanometer was sensitivity 10^{-10} A was used as null detector in the null circuit.

TABLE I. Characteristic of the specimens studied

Dimensions, mm $L \times S \times d$	Bicrystals 1	2	3	4	5	Cleaved $10\times1\times5$ b a
	$6\times2.4\times1.2$	$6\times0.8\times1.5$	$6\times1\times2.5$	$6\times0.2\times1.6$	$5.2\times0.9\times$ $\times1.7$	
N_s, 10^{12} cm^{-2}	7	13	9.5	8	5	6–8 *
σ_s, 10^{-4}	8	2	2.2	3		4 2
Λ, 10^{-6} cm, Eq. (11)	3	0.5	0.7	1		1
$\varkappa_0$, 10^{-2} W·cm^{-12}·K^{-4}	6.6	1.45	6.5	1.34		8
1	1.65	0.35	1.63	0.32		2
$\varkappa_0$, 10^{-2} W·cm^{-1} K^{-4} (calculated)	6.2	4.7	6.06	1.97		8.6
αT^{-3}, 10^{-5} V·K^{-4} (experimental)	8.4	4	4.1	2.5	4.2	2.5 $\gtrsim3.5$ ***
Calculated according to Eq. (A13)	1.6	0.6	1.15	1.2	2	1.9–1.4 **
Calculated according to Eq. (A13a)	1.8	0.7	1.5	2.1	3	4.7–3.5 **
$\alpha\varkappa^{-1}$, 10^{-3} V·cm·W^{-1}	1.27	2.85	0.78	1.9		0.3

*Using earlier results[6] for the carrier concentration.

**The first number corresponds to $N_s = 6\times10^{12}$, the second (a) to $N_s = 8\times10^{12}$ cm^{-2}.

***Approximate value. In the temperature range studied, $\varkappa$ had not yet reached the $\varkappa(T)\propto T^3$ dependence (Fig. 3,a).

Random errors in determining the Seebeck voltage and the thermal conductivity were not more than 5% in the main measuring range, while systematic errors could reach 30% due to the small dimensions of the specimen, which are given in Table I (L is the specimen length, S the breadth of the conducting layer and d the specimen thickness).

The surface conductivity σ_s and the surface charge density N_s for the specimens were determined from measurement of the Hall voltage, carried out in a magnetic field up to 20 kOe directed perpendicular to the high-conductivity surface. The specimen characteristics obtained are shown in Table I.

The results of measuring the thermal conductivity $\varkappa$ and thermopower α of specimens with a high-conductivity surface obtained by cleavage are shown in Fig. 3. The results of measuring the same specimens with surface conductivity reduced by additional heating are indicated by the dashed lines. Specimen 1 was heated at 160 K for 10 h to remove completely the surface conductivity.

The results of measuring the thermal conductivity $\varkappa$ and thermopower α of some bicrystals are shown in Figs. 4 and 5. The filled circles indicate results of measurements in the apparatus with ^{3}He, the open circles show results obtained during preliminary measurements. The measurements made with different apparatus are seen to agree.

DISCUSSION OF THE RESULTS

1. An unusual form of temperature dependence of thermopower is observed in a number of n-type germanium

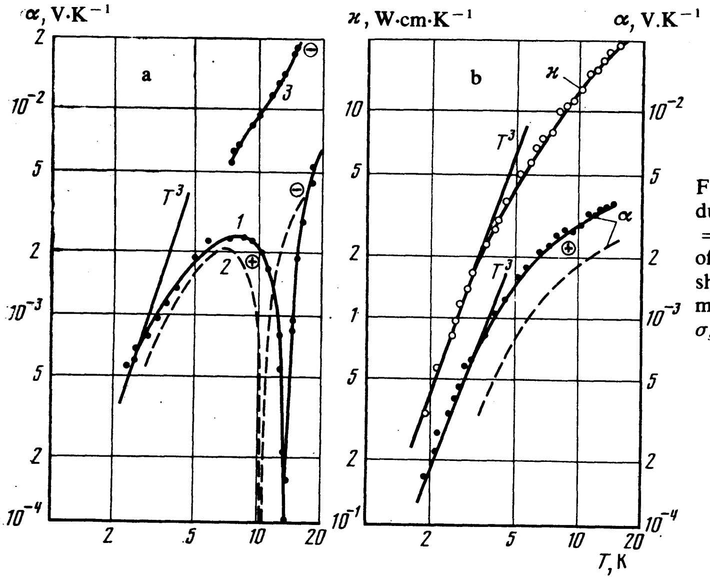

FIG. 3. Results of studying specimens with a conducting surface produced by cleavage. a—specimen 1: 1—$\sigma_s = 2\times10^{-4}\,\Omega^{-1}$, 2—$\sigma_s = 7\times10^{-5}\Omega^{-1}$, 3—the same specimen after destroying the conductivity of the surface, $\sigma_{4.2\mathrm{K}} < 10^{-7}\Omega^{-1}$cm^{-1}, the sign of the thermopower is shown in the circles. b—specimen 2: $\varkappa$—thermal conductivity, α—thermopower, the continuous line is for $\sigma_s = 4\times10^{-4}\Omega^{-1}$, the dashed line $\sigma_s = 6\times10^{-5}\Omega^{-1}$.

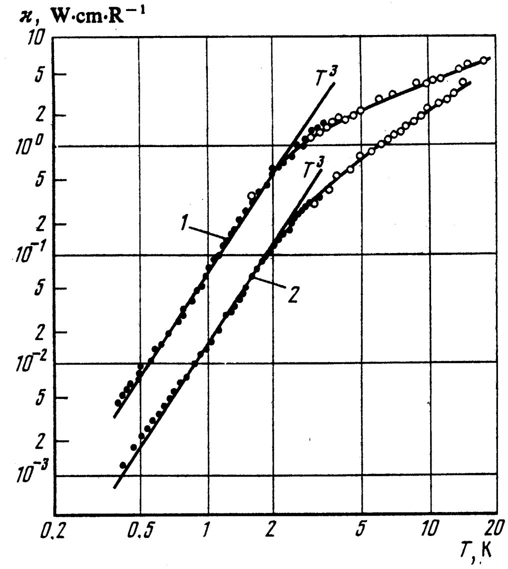

FIG. 4. Thermal conductivity of bicrystal specimens 1 and 2. Measurements in apparatus of the type of Fig. 1, open circles; type of Fig. 2, full circles.

specimens with a conducting layer (Figs. 3 and 5). Above 10 K the thermopower changed sharply in magnitude and the sign changed. The following explanation can be given for this phenomenon. The thermopower measured in an experiment is naturally determined by the charge carriers not only in the conducting layer but also in the bulk of the specimen. It is easy to show that for non-rectifying contacts

$$\alpha=\frac{\alpha_s R_V+\alpha_V R_s}{R_V+R_s},\tag{5}$$

where R_v and R_s, α_v and α_s are the resistivity and thermopower of the bulk and of the conducting layer. At $T\lesssim 6$ K we have $R_v \gg R_s$ and the quantity measured in experiment $\alpha\equiv\alpha_s$. However, R_v decreases by 3 to 4 orders of magnitude between 4.5 and 10 K and at $T>6$ the contribution of the bulk of the specimen to α has to be taken into account. At sufficiently high temperature the sign of α is determined by the sign of α_v i.e. by the carriers in the bulk of the specimen. Correspondingly the thermopower of an n-type specimen is negative and of a p-type specimen positive.

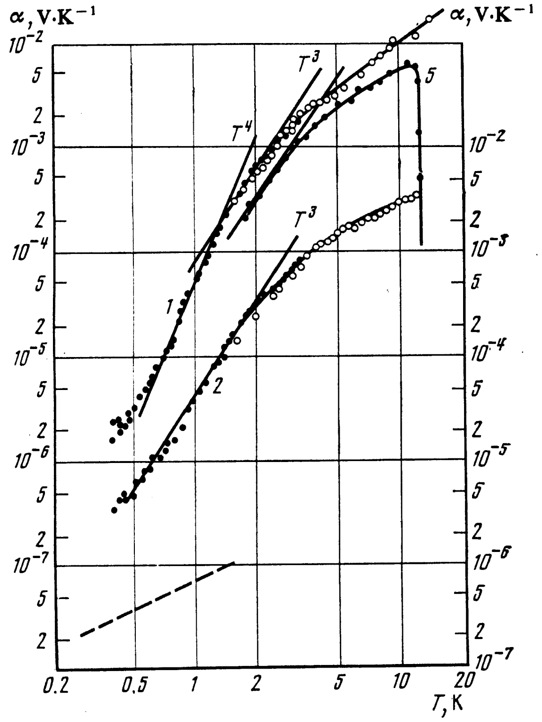

FIG. 5. Thermopower of bicrystals; specimens 1, 2 and 5 (5 is an n-type specimen). The symbols are the same as in Fig. 1. Dashed line—the diffusion part of the thermopower for specimen 2 (right hand scale).

For all specimens the thermopower of the charge carriers of the conducting surface is positive below 6 K, which provides additional confirmation of the fact that the charge carriers in the conducting layer of germanium are holes.[5] On raising the temperature of n-type specimens the transition from α_s to α_v should be accompanied by a change in the sign of the thermopower, which is observed for a number of specimens in the temperature range studied, see Fig. 3 and Fig. 5 curve 5 (there is an error in the sign of α in the figures of the earlier publication[1]).

The explanation proposed for the change in sign of α was confirmed by direct measurement of the bulk thermopower of one of the n-type specimens with a cleaved surface which had the conductivity completely removed by heating. For this specimen (Fig. 3) α_v was indeed negative and different in sign from α_s. The $\alpha_v(T)$ dependence and the magnitude of α_v were not really different from the results of earlier measurements of α made on pure germanium.[8] At the same time, experiments showed that $R_s\approx R_v$ at 16 K.

2. As we have already mentioned, the thermopower of the carriers is made up of the diffusion part and of phonon drag. The magnitude of the diffusion part, α_e, can be estimated from the well-known relation

$$\alpha\approx\pi^2\frac{k^2}{e\varepsilon_F}T\ .\tag{6}$$

We shall use the results of measuring the surface density N_s and data on the thickness, δ, of the conducting layer in order to calculate α_e.

We will assume that the properties of the conducting layer with $N_s=8\times10^{12}$ cm^{-2} are the same as of doped germanium with the same volume concentration of carriers

$$N_V=N_S\delta^{-1}=2\cdot10^{19}\ \mathrm{cm}^{-3}.\tag{7}$$

At such concentrations the holes form a degenerate Fermi system with $\varepsilon_F=40$ mV. Substituting this value into Eq. (6), we obtain

$$\alpha_e\approx10^{-6}\ T\ \mathrm{V\cdot K^{-1}}.\tag{8}$$

The values of α_e calculated for one of the specimens is shown in Fig. 5 by the dashed line. It can be seen that the experimental values of α_s exceed α_e appreciably and have a different temperature dependence. It is possible that the contribution from α_e only appears in the $\alpha_s(T)$ dependence at $T\approx0.4$ to 0.5 K, where it amounts to ~10 to 20% of the measured value.

Over most of the range of the measurements, the thermopower of the conducting surface of germanium is, thus, produced by phonon drag.

3. We can use Eq. (4) to estimate the magnitude of the thermopower associated with phonon drag. The carriers of the conducting layer, in fact, form a degenerate system with Fermi momentum $\mathbf{p}_F = \hbar \mathbf{k}_F = 6\times 10^6 \hbar$, which is greater than the momentum of a thermal phonon $\mathbf{p} = \hbar q = 3.5\times 10^5 T\hbar$ over the whole range of measurements. The nature of the interaction between charges and phonons is thus rather metallic, where $\mathbf{p}_F \gg \mathbf{p}$, than semiconducting, where $\mathbf{p}_F \ll \mathbf{p}$.

Substituting the specific heat capacity of germanium, $C_L = 3\times 10^{-6} T^3 \mathrm{J\cdot cm^{-3}\cdot K^{-1}}$, and the mean velocity of sound $u = 4\times 10^5\ \mathrm{cm\cdot sec^{-1}}$ into Eq. (4), we obtain

$$\alpha \sim 10^{-6} T^3\ \mathrm{V\cdot K^{-1}}, \tag{9}$$

which is 20 to 80 times less than the experimental value. Consequently, if the charges occupy a narrow layer in the bulk or on the surface of the specimen, then they obtain an additional impulse from the out-of-equilibrium phonons of the whole specimen volume. (We remember that phonon drag in fact consists of the transfer of momentum from out-of-equilibrium phonons to the electrons.)

Before going on to a more detailed discussion of this problem, we shall elucidate how the existence of a conducting layer influences the kinetic characteristics of the phonons.

4. We shall obtain information on the phonon kinetics from the results of measuring the thermal conductivity (Figs. 3 and 4). The thermal conductivity of germanium specimens with a high conductivity surface agrees with results on the conductivity of pure specimens inthe literature.[9] In the range 0.4 to 2 K the thermal conductivity $\varkappa$ follows the relation

$$\varkappa = {}^1/_3 C_L u l \sim \varkappa_0 T^3, \tag{10}$$

where $u \sim 4\times 10^5\ \mathrm{cm\cdot sec^{-1}}$ is the mean phonon velocity $(3/u = 2/u_t + 1/u_l)$, l is the phonon mean free path which is of the order of the minimum transverse dimension of the specimen, as can be seen from Table I.

The conductivity of specimen No. 2 is an example of a departure from the general rule; l for it is appreciably less than the minimum dimension. This is probably evidence of macroscopic defects in the specimen (cracks). For a number of specimens some increase in conductivity compared with relation (10) is observed in the region of the lowest temperatures covered by the measurements. This deviation is specially clearly seen from Fig. 6 where the magnitude of $\varkappa T^{-3}$ is shown. The values of $\varkappa T^{-3}$ at 1 and 2 K are shown in Table I. Such departures are often met with in dielectric crystals and are usually connected with specular reflection of phonons from the specimen boundary.[10]

At $T > 2$ K appreciable departures of $\varkappa(T)$ from what would follow from Eq. (10) are observed in all specimens studied, similar to those found in earlier studies of the thermal conductivity of germanium. This is apparently related to a reduction in the phonon mean free path in the bulk of the specimen.

Both the absolute value of the thermal conductivity and its temperature dependence can thus be completely explained within the framework of existing views on phonon kinetics in an ideal crystal. Consequently phonon scattering by electrons of the conducting layer is small. This conclusion also follows from comparing the phonon mean free path for scattering by charges, $l_e \sim 10^{-3}\ T^{-1}$, with the thickness of the conducting layer δ.

The following qualitative explanation of the increase in phonon drag on charges in the conducting layer can be proposed on the basis of the results obtained. The directed motion of the phonons arising from the presence of ∇T is limited by their diffuse scattering, mainly at the specimen boundaries. In the course of their motion phonons cross the conducting layer; they then give up some of their momentum, $\sim \delta / l_e \sim 10^{-4}$, to the charges. However, the number of phonons which can exchange momenta with the carriers contained in a narrow layer is increased by d/δ. As a result of

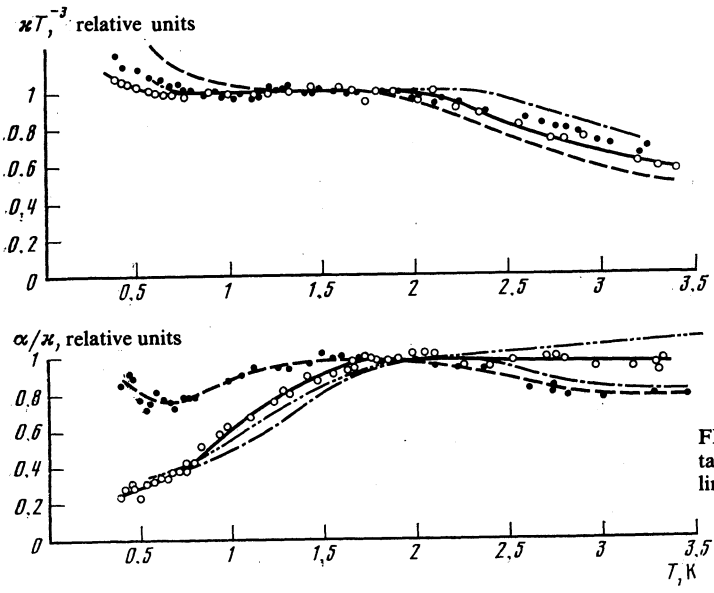

FIG. 6. Temperature dependence of $\varkappa T^{-3}$ and $\alpha/\varkappa$ for different bicrystals. Specimen 1—open circles and full line, 2—full circles and dashed line, 3—double dot-dash line, 4—dot-dashed line.

the process considered, an increase in phonon drag by $d/l_e \sim 10^2$ can thus be expected.

5. In order to understand this result quantitatively, starting from first principles, Velikovich and Liberman calculated the drag on charges in a thin conducting layer δ inside or on the surface of a germanium specimen of finite dimensions (see Appendix). It is assumed that $q\Lambda > 1$ as is usually done in the theory of metals. As has been indicated above, the phonon mean free path for scattering by charges is much greater than the thickness of the conducting layer, and it could be assumed that the existence of the charges does not change substantially the non-equilibrium phonon distribution. It is assumed that phonons are scattered by the specimen boundaries, so that the geometrical dimensions of the specimen can be introduced into the calculations. The charge (hole)-phonon interaction is, as usual, determined by the germanium deformation potential. It is clear that the thermal conductivity of the specimen can also be calculated by the same means as this is determined by the geometrical dimensions and shape of the specimens. It can be seen from Table I that the thermal conductivity calculated from Eq. (A4) agrees well with the experimental results; this shows that the geometrical factor in the problem has been correctly taken into account in the calculations.

An additional arbitrariness arises in calculating the phonon drag due to the dependence of the results calculated on the deformation potential ε_d. According to present ideas, $\varepsilon_d \approx 10$ to 20 eV in germanium. Values of the thermopower, or more exactly the coefficient of T^3, are given in Table I for $\varepsilon_d = 15$ eV, calculated on the assumption of two- and three-dimensional charge distributions in the conducting layer [Eqs. (A13a) and (A13)]. The calculated results are closer to the experimental results for a two-dimensional charge distribution, although there is a large divergence between calculation and experiment in the case of bicrystals. The existence of this divergence seems quite natural. The deformation potential for pure germanium was used in the calculations while it is well known that the boundary of a bicrystal is surrounded by a strongly deformed layer. In this layer the value of ε_d will be noticeably different (most likely in the direction of higher ε_d) from the value typical for undeformed material. It would therefore be more correct to compare the results of the calculation with experimental results on cleaved specimens in which deformation of the crystal near the conducting layer is not so great. As can be seen from Table I, the best agreement between calculation and experiment is indeed observed in this case.

We should remember that the system studied experimentally differs rather considerably from the theoretical model considered. The conducting layer is probably not smooth; islands can appear on it during the growth of the bicrystal or on cleaving the specimen.[1)] The results of measuring the electrical conductivity of the layer which may vary by up to a factor of three are clear evidence of the presence of non-uniformities. There is a similar large scatter also observed in the magnitude of the mean free path, calculated on the assumption of a two-dimensional charge distribution[2)] from the relation (see Table I)

$$\Lambda \approx (2\pi)^{1/2} \hbar \sigma_s / N_s^{1/2} e^2. \tag{11}$$

The value of Λ obtained in this way evidently indicates a lower limit to Λ.

Taking all this into account, we can consider the results of calculating the absolute magnitude of the phonon drag on charges in the conducting layer completely satisfactory, all the more so because there are no adjustable constants in the calculations.[3)]

6. However, some experimental facts cannot be explained by these calculations, namely:

1) For $T < 1.5$ K, α_{ph} for a number of specimens starts to vary according to a steeper relation than follows from Eq. (4) and is nearer to a T^4 proportionality (see Figs. 5 and 6);

2) The thermopower of specimens with conducting surfaces formed by cleaving decreases after their conductivity has been removed by additional heating (see Fig. 3) while the reduction in concentrations N_s which occurs on heating[6] should be accompanied by an increase in thermopower [see Eqs. (4) and (6)].

(a) It is difficult to explain these phenomena starting only from the properties of the phonon and electron systems. The phonon wavelength at these temperatures is actually 2×10^{-5} cm, i.e., much larger than the main geometrical dimensions for the problem, which is the thickness of the conducting layer, $\delta \sim 4 \times 10^{-7}$ cm. One could think up a model in which the change in $\alpha_{\text{ph}}(T)$ in bicrystals is explained. For example, if the vibration spectrum in the layer near the joining plane is so distorted that it approximates to that typical of a glass, then at sufficiently low temperatures one could really expect a decrease in the transfer of phonon momentum to the electrons. However, the characteristics of specimens with a cleaved conducting surface cannot be explained by this model, since the reduction in the drag in this case is clearly related to a change in the hole system of the conducting layer.

(b) In considering the hole system, one must first of all take into account that localization effects occur in it.[6] They show up, for example, in the logarithmic or activated increase in resistivity on decreasing the temperature.

It is well known that all these effects are increased on lowering the temperature or reducing the specimen conductivity, so that it is very tempting to ascribe the change in phonon drag to a manifestation of localization effects. However, in order to describe quantitatively the effects discussed, it must be assumed that localization effects are manifest in a more violent form in thermopower than, for example, in the specimen resistivity. A logarithmic fall in conductivity was observed in the specimens studied (Fig. 7) which can be described satisfactorily by the relation

$$\Delta\sigma_s [\Omega^{-1}] = -2.5 \cdot 10^{-5} p \ln T. \tag{12}$$

According to localization theory,[11] $\Delta\sigma_s$ should really vary according to Eq. (12) with $p \approx 1$ in the two-dimensional case, in a theory which takes account of the different nature of the charge interaction,[12] $p < 1$ and depends on the nature of the interaction. Experiment showed that in a hole-type conducting layer of a germanium bicrystal $p \approx 0.3$. It can

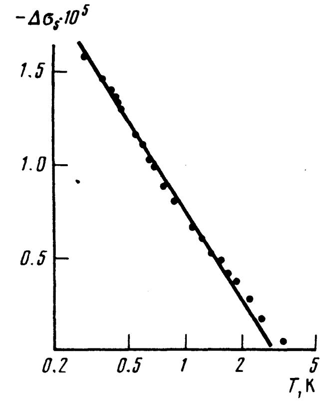

FIG. 7. Typical temperature variation of the conductivity of bicrystals (curve for specimen No. 3).

easily be seen that the relative change in $\alpha_{\rm ph}$ is an order of magnitude greater than the change in σ_s.

The only qualitative difference between experiments to measure resistivity and phonon drag lies in the fact that the additional energy of a charge differs by 10^4 to 10^5 in them (in measuring the conductivity the voltage drop along the specimen in 10 mV, while it is $\sim 10^{-7}$ V when measuring α). An explanation of a change in $\alpha_{\rm ph}(T)$ through localization effects can, consequently, have real foundations only if it is shown that the magnitude of these effects can change appreciably with energy. So far such an effect is only observed in the region of the activated dependence of resistivity on temperature.

(c) We shall now turn to a discussion of the phonon-electron interaction. It seems that a detailed consideration of just this problem will enable the problem posed at the start of this section to be answered.[4)] As is well known, the attenuation, Γ, of a sound wave depends on the relation between its wave vector q and the charge mean free path, Λ, according to:[13]

$$\begin{aligned} &\Gamma_1 \sim 0{,}5 p_F \omega && \text{where } q\Lambda > 1, \\ &\Gamma_2 \sim 0{,}2 p_F \Lambda \omega^2 / v && \text{where } q\Lambda < 1, \end{aligned} \tag{13}$$

and it easy to see that $\Gamma_1 > \Gamma_2$. For other conditions being equal it appears that the momentum transferred by a phonon to the charges of the conducting layer is proportional to the magnitude of Γ. In view of Eq. (13), phonon drag must, consequently, depend on the value of $q\Lambda$ for the phonons which are most important in this process. We may note that comparison of the magnitude of Λ (see Table I) and the phonon wave vector $q = 3.5 \times 10^5\ T \mathrm{cm}^{-1}$ shows that $q\Lambda = 1$ for phonons with energy ~ 1 K. This is one of the most important properties of the system we studied. In metals Λ is usually so large at low temperatures that the condition $q\Lambda > 1$ is always satisfied with plenty to spare. In our specimens the holes are mainly scattered by charged centers with $\Lambda \approx 10^{-6}$ cm and on decreasing the temperature, i.e. on reducing q, the transition to the region where $q\Lambda < 1$ can easily be achieved and this is accompanied (since $\Gamma_2 < \Gamma_1$) by a reduction in $\alpha_{\rm ph}$. In addition, since Γ_1 and Γ_2 have different ω dependences it should be expected that $\alpha_{\rm ph}$ will decrease more like T^4 in the region $q\Lambda < 1$.

The magnitude of $q\Lambda$ can also be reduced by changing the resistivity of specimens obtained by cleaving. The magnitude of $\alpha_{\rm ph}$ should then decrease, as is observed experimentally (Fig. 3). We could not follow the change in the $\alpha_{\rm ph}(T)$ dependence for this case because of insufficient sensitivity of the system used when studying cleaved specimens and the insufficiently wide temperature range of the measurements.

By taking account of the dependence of the phonon-charge interaction on $q\Lambda$, it is thus possible, at least qualitatively, to understand the whole aggregate of phenomena which are inexplicable within the framework of the simplest model discussed at first.

7. As can be seen from Fig. 5, some change in the $\alpha\varkappa^{-1}(T)$ dependence is observed for all specimens in the region of 0.4 to 0.5 K. This change can be explained quantitatively by the contribution of the diffusive part of the thermopower α_e of the charge carriers to the effect. It should be remembered however that at these temperatures the accuracy of measurement is not great.

CONCLUSIONS

As a result of the study carried out it has thus been established that phonons of the whole specimen are responsible for the phonon drag, and not only phonons in the conducting layer. This leads to an appreciable increase in the effect, up to a hundred-fold. The magnitude of the effect depends strongly on $q\Lambda$, where q is the phonon wave vector and Λ is the electron mean free path. This leads to an unusual temperature dependence, $\alpha_{\rm ph} \propto T^4$ of the thermopower associated with phonon drag. An unresolved possible influence of localization effects in the thermopower of a two-dimensional system remains in the end.

Although the work was devoted to a study of the role of phonon drag in the thermopower of the conducting layer, its results are of considerably wider interest. This is because if scattering of electrons occurs in the specimen studied mainly at defects in the system, charged centers, etc., as generally occurs in two-dimensional conducting systems, reliable information on the interaction between the charges and the phonon system can be obtained just from results of measuring $\alpha_{\rm ph}$, since this is only determined by the direct action of phonons on the electrons. It can therefore be expected that the results obtained will allow one to understand how the properties of two-dimensional systems which are connected with electron-phonon interaction, and not previously understood, can be explained.

We shall consider this using the example of a metal oxide-semiconductor structure (MOS). As is known,[14] the resistivity of a MOS structure is due to the scattering of charges by phonons, but for unknown reasons is almost an order of magnitude greater than all theoretical estimates. In our view it is natural to connect this result, just as the increase in $\alpha_{\rm ph}$, with the appearance in the resistivity of a contribution from phonons of the bulk of the specimen. In these structures an appreciable heating of the electrons in the channel is observed at low temperatures.[15] The hypothesis of the glassy spectrum of the material adjacent to the channel is attractive for explaining this phenomenon, in a way similar to our use of it in considering the results in section 6a. The assumption of the $q\Lambda$ dependence of the energy exchange with phonons provides a simpler explanation of this phenomenon. Other examples could also be produced which indicate that a clear answer about the interaction between charges and phonons is essential in considering problems of the dynamics of a two-dimensional system of charges.

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APPENDIX

A. Thermal conductivity and thermopower for Ge at low temperatures

We shall calculate the thermopower arising in a conducting layer on the surface (cleaved or in the bulk (bicrystal) and the thermal conductivity of a germanium specimen of finite dimensions. It is assumed that charge carriers with mass $m^* = 0.4m_e$ are distributed in a conducting layer of thickness $\delta \simeq 4\times 10^{-7}$cm with a surface density N_s. The phonon wavelength in germanium for $\bar{u} = 4\times 10^5$cm·sec^{-1} is $\lambda = \hbar\bar{u}/kT = 1.8\times 10^{-5}T^{-1}$, so that for not too low a temperature ($T \gtrsim 1$ to 2 K) for $\Lambda q > 1$ the problem of phonon drag comes down to a quantum problem of the scattering of a phonon gas by carriers in the conducting layer.

It can easily be seen that at low temperatures ($T \lesssim 20$ K) the phonon mean free paths in germanium for scattering by defects, impurities and due to anharmonicity of the lattice[16] are large compared with the specimen dimensions (~ 1 cm). The phonon mean free path for scattering by carriers is[17]

$$l_{ph,e}=\frac{\pi^3\hbar^4\bar{u}^2\rho}{6\zeta(3)\varepsilon_d^2 m^{*2}T}. \tag{A1}$$

In the temperature region which interests us, taking the deformation potential in germanium as $\varepsilon_d =$ 10 to 20 eV, we find $l_{\rm ph,e} \approx 8\times 10^{-4}/T \gg \delta$, so that the scattering by carriers in the conducting layer does not affect the phonon distribution function. Consequently, the distribution function is determined by phonon scattering at the specimen boundaries, and the flow of the phonon gas is collisionless Knudsen flow. For diffuse scattering at the boundaries the non-equilibrium part of the phonon distribution function is of the form[18]

$$\delta n_q=-\frac{\partial n_q}{\partial T}\, l(\mathbf{q}/q)\frac{\mathbf{q}}{q}\nabla T, \tag{A2}$$

where n_q is the equilibrium distribution function and $l(\mathbf{q}/q)$ is the distance from the given point to the scattering point on the surface in the $\mathbf{q}$ direction.

We have for the heat flow connected with acoustic phonons of one kind

$$\mathrm{Q}=\int \hbar\omega_q C_q\left(\frac{\mathbf{q}}{q}\right)\delta n_q\, d^3q/(2\pi)^3. \tag{A3}$$

Transforming this integral and summing over the types of acoustic phonons, we obtain the following expression for the thermal conductivity:

$$\varkappa=\frac{\pi^2k^4}{30\hbar^3\bar{u}^2}\lambda_1 d\left(\frac{1+p}{1-p}\right)\left[\left(\frac{\bar{u}}{c_l}\right)^2+2\left(\frac{\bar{u}}{c_t}\right)^2\right]T^3. \tag{A4}$$

Here p is the specular coefficient; comparison with the measured values of $\varkappa$ show that the scattering is almost completely diffuse, so that in what follows we shall take $p = 0$. The factor λ_1 in Eq. (A4) is determined by the geometrical dimensions of the specimen:

$$\lambda_1=\frac{4}{\pi L_\parallel L_\perp}\int_0^{L_\parallel}dx\int_0^{L_\perp}dy\,\frac{x^2}{(x^2+y^2)}\left\{\frac{(L_\parallel-x)(L_\perp-y)}{1+x^2+y^2}\right.$$
$$\left.+\frac{(L_\parallel L_\perp-xy)}{(x^2+y^2)^{1/2}}\operatorname{arctg}\left(1/(x^2+y^2)^{1/2}\right)\right\}; \tag{A5}$$
$$L_\parallel=L/d,\quad L_\perp=S/d.$$

The expression (A5) for λ_1 is averaged over the specimen volume corresponding to the assumption of the one-dimensionality of the problem.

In calculating the thermopower we will also take account of the contribution to the phonon drag of surface phonons (Rayleigh waves). The mean free path of surface phonons is

$$l_{ph,e}^{(2)}=\frac{5gc_t^2c_R\hbar^5\rho}{\pi\varepsilon_d'^2m^{*2}\delta T^2}\simeq 5\cdot 10^{-3}/T, \tag{A6}$$

where c_R is the velocity of Rayleigh waves and $g = 0.79$, $\varepsilon_d' \approx 5$ eV for germanium.

The kinetic equation for the carriers is

$$-e\mathbf{E}\frac{\partial N_0}{\partial \mathbf{p}}-\frac{\varepsilon-\mu}{T}\frac{\partial N_0}{\partial\varepsilon}(\mathbf{v}\nabla T)=\mathrm{St}_{e,ph}^{(2)}N+\mathrm{St}_{e,ph}^{(3)}N. \tag{A7}$$

Correspondingly we have for the surface phonons

$$-\frac{\hbar\omega}{T}\frac{\partial N_0'^{(2)}}{\partial\mathbf{q}}\nabla T=\mathrm{St}_{ph,e}^{(2)}N_0{}^2, \tag{A8}$$

where N_0, N, $n_0{}^{(2)}$ and $n'^{(2)}$ are, respectively, the equilibrium and non-equilibrium distribution functions of the carriers and surface phonons, and $\mathrm{St}^{(2)}$ and $\mathrm{St}^{(3)}$ are the corresponding collision integrals (see Gurevich[17]).

We choose the direction of ∇T in the x axis along the conducting surface, multiply Eq. (A7) by p_x and Eq. (A8) by q_x and integrate them respectively over $2d^3p/(2\pi\hbar)^3$ and over $d^3q/(2\pi\hbar)^3$, after which we add the results obtained termwise. As a result of a transformation, similar to that carried out in Gurevich,[17]§82, we obtain

$$-eE_xN_V-\frac{m^*p_x}{3\hbar}T\frac{dT}{dx}-\frac{3\zeta(3)}{2\pi\hbar^2c_R{}^2}T^2\frac{dT}{dx}=\int p_x\,\mathrm{St}_{e,ph}^{(3)}N\frac{d^3p}{(2\pi\hbar)^3}, \tag{A9}$$

where $N_v = N_s/\delta$ is the volume density of carriers. In obtaining Eq. (A9) it was taken into account that umklapp processes can be neglected at low temperatures, and because of the conservation of quasimomentum in collisions of the carriers with surface phonons, the total contribution of the first term on the right-hand side of Eq. (A7) and the right-hand side of Eq. (A8) are zero, while the bulk phonons transfer a quasimomentum to the carriers per unit time of magnitude which is determined by the right hand side of Eq. (A9).

When evaluating the integral in Eq. (A9) only the nonequilibrium part of the phonon distribution function has to be retained in the collision integral, since in the absence of a current the contribution from the change in the carrier distribution function is small. As a result, we obtain for the thermopower $\alpha = \nabla U / \nabla T$

$$\alpha = \alpha_D + \alpha_{ph}^{(2)} + \alpha_{ph}^{(3)}. \tag{A10}$$

The first, diffusive term, in Eq. (A10) is

$$\alpha_D = \frac{k^2 m^*}{e^2 \hbar^2} \left(\frac{\pi}{3N_V} \right)^{2/3} T. \tag{A11}$$

The second term in Eq. (A10) is related to the surface phonon drag

$$\alpha_{ph}^{(2)} = \frac{k}{e} \frac{c_S}{2N_S k} = \frac{3\zeta(3) k^3}{2\pi \hbar^2 c_R{}^2 e N_S} T^2. \tag{A12}$$

The last term in Eq. (A10) is the contribution to the drag from bulk phonons:

$$\alpha_{ph}^{(3)} = \frac{k}{e} \frac{\pi \gamma^2 m^{*3}}{30 N_V \hbar^7 \rho \bar{u}^3} \lambda_2 d (kT)^3, \tag{A13}$$

where

$$\gamma^2 = \sum_{\substack{\text{over the} \\ \text{phonon types}}} \varepsilon_d{}^2 \left(\frac{\bar{u}}{c} \right)^3.$$

The parameter λ_2 depends on the shape of the specimen, and using the same symbols as in Eq. (A5), has for a rectangular parallelepiped the form

$$\lambda_2 = \frac{2}{\pi L_\parallel L_\perp} \int_0^{L_\parallel} dx \int_0^{L_\perp} dy x^2 \left\{ \frac{2(L_\parallel - x)(L_\perp - y)}{(1+x^2+y^2)^2} + \frac{(yL_\parallel + xL_\perp - 2xy)}{(x^2+y^2)} \times \left[\frac{1}{1+x^2+y^2} + \frac{1}{(x^2+y^2)^{1/2}} \operatorname{arctg}(1/(x^2+y^2)^{1/2}) \right] \right\}. \tag{A14}$$

In the case of a bicrystal, as distinct from a conducting cleaved surface, phonon reflexion from both sides of the conducting layer has to be taken into account, and

$$L_\parallel = 2L/d, \quad L_\perp = 2S/d,$$

have to be substituted into Eq. (A14).

For the specimens considered here, the main contribution to phonon drag in the conducting layer for $T \gtrsim 0.2$ K comes from bulk phonons, so that in the present case $\alpha \simeq \alpha_{\rm ph}$.3)

It was assumed in deriving Eq. (A10) that the carriers in the conducting layer have a three-dimensional Fermi distribution function. If the conditions

$$k_F \delta \ll 1, \quad k_F = (2\pi N_S)^{1/2}, \tag{A15}$$

are fulfilled, a two-dimensional Fermi distribution function has to be chosen for the carriers. Calculations similar to those given above then give

$$\alpha_2 = \alpha_{D2} + \alpha_{ph}^{(2)} + \alpha_{ph2}^{(3)}, \tag{A10a}$$

$$\alpha_{D2} = \frac{k}{e} \frac{\pi^2 kT}{3\varepsilon_F}, \tag{A11a}$$

$$\alpha_{ph2}^{(3)} = \frac{k}{e} \frac{\pi \gamma^2 (m^*)^3}{15 N_S \hbar^7 \rho \bar{u}^3 k_F} \lambda_2' d (kT)^3. \tag{A13a}$$

The expression for λ_2' is similar to Eq. (A14) and takes the form

$$\lambda_2' = \frac{4}{\pi L_\parallel L_\perp} \int_0^{L_\parallel} dx \int_0^{L_\perp} dy \frac{x^2}{(x^2+y^2)^{3/2}} \left\{ \frac{(L_\parallel - x)(L_\perp - y)}{(1+x^2+y^2)^{1/2}} + (xL_\perp + yL_\parallel - 2xy) \ln \frac{1+(1+x^2+y^2)^{1/2}}{(x^2+y^2)^{1/2}} \right\}.$$

We find for $\delta = 4 \times 10^{-7}$ cm and $N_s = (4 \text{ to } 12) \times 10^{12}\ \text{cm}^{-2}$ that $k_F \delta = 2$ to 3, so that strictly speaking one cannot make an unambiguous choice between a two-dimensional or three-dimensional Fermi distribution for the carriers. Correspondingly, the numerical values of α obtained from Eqs. (A13) and (A13a) did not differ much in the present case. A difference between a two-dimensional and a three-dimensional distribution for the carriers could be seen from a different dependence of α on N_s. According to Eq. (A13) we have $\alpha \propto 1/N_s$, while for a two-dimensional carrier distribution it follows from Eq. (A13a) that $\alpha \propto 1/N_s{}^{3/2}$.

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1)Electron microscope investigation showed that the cleavage surface consisted of ~1 μm plane sections separated by sharp steps.

2)The conducting surfaces of germanium can be considered as three- (3*D*) or two-dimensional (2*D*) conductors. Calculations of all the characteristics according to both models give similar values. However, the results of comparing $\alpha_{\rm ph}$ with the calculations and also the logarithmic form of the change in resistivity [Eq. (12)] indicate that the conducting layer should rather be regarded as an example of a 2*D* hole conductor.

3)It is clear that the calculations carried out (see Appendix) are only applicable for the temperature region where phonon scattering in the bulk of the specimen is negligibly small, i.e., $\varkappa \propto T^3$. We note that on raising the temperature above 2 K both $\alpha(T)$ and $\varkappa(T)$ depart from a cubic dependence.

4)The possibility of explaining the experimental results in this way was pointed out to the authors by I. M. Suslov.

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