Activation conductivity in disordered systems with large localization length

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It is indicated that to determine the temperature dependence of the activation conductivity in films and wires it is essential to solve the electrostatic problem of the Coulomb interaction between an electron that hops over to a new localized center and the hole produced at the old center. When solving this problem, account must be taken of the differences between the polarizabilities of the film (wire) and vacuum. The dependences of the conductivity on the temperature and on the electric field are obtained for thin films, wires, and MIS structures. The dependences of the conductivity on the electric field in three-dimensional bodies near the metal-insulator transition point are also discussed.

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1. INTRODUCTION

The static conductivity σ is disordered systems with sufficiently strong disorder is zero at $T=0$. At finite temperatures, the $\sigma(T)$ dependence has an activation character and the conductivity increases exponentially with rising temperature, the argument of the exponential being proportional to a certain power of the temperature (see Refs. 1–3). Usually$^{1-3}$ one considers the case when the overlap of the wave functions corresponding to different centers is small, and the energy t_{ij} connected with the hopping from one center to another is less than the scatter of the one-center energies w_i and the Coulomb interaction $e\varphi(r_{ij})$. In this paper we consider the case when t_{ij} is of the same order or even larger than w_i. Under these conditions, at short distances or at high temperatures, the electrons behave as non-localized, and at low temperatures the states on the Fermi level have a large localization length L_c. This situation is realized in three-dimensional samples near a metal-insulator transition, and in two-dimensional and one-dimensional samples also in the case of a weak disorder.^{4,5} In one- and two-dimensional conductors the localization length L_c is large if the mean free path l or the transverse dimension d exceed the electron wavelength ($p_F l \gg$, $p_F d \gg 1$).

A large localization length means that the electrons are mobile at distances $r \lesssim L_c$. These mobile electrons screen strongly the Coulomb interaction, and therefore in the case considered the dielectric constant $\varepsilon \gg 1$. In two-dimensional and one-dimensional samples the Coulomb interaction at the largest distances is effected through vacuum and does not depend on ε, but there is an intermediate range of distances that are larger than the transverse dimension d of the plane or wire but smaller than εd ($2D$) or $\varepsilon^{1/2} d$ ($1D$), for which the electric interaction deviates greatly from Coulomb's law. As a result, at the lowest temperatures the conductivity obeys the Shklovskiĭ-Éfros law^3 $\log(T) = -(T_0/T)^{1/2}$, but in a wide temperature range there is satisfied a law close to the Arrhenius law $\log(T) = -E_a/T$. The large length of the hops leads to a strong dependence of the conductivity on the electric field $\mathscr{E}$.

We obtain in this paper the functions $\sigma(T, \mathscr{E})$ in films, MOS FET, wires, and three-dimensional conductors at a disorder close to a metal-insulator transition.

2. GENERAL RELATIONS

At absolute zero temperature, sufficiently dirty conductors cease to conduct electric current. Their conductivity σ is zero. For samples of finite size, the conduction is by tunneling and depends exponentially on the dimension, so that

$$\sigma \propto \exp\{-L/L_c\}. \tag{1}$$

The localization length L_c depends on the degree of disorder and on the dimensionality of the sample. It will be calculated below for different concrete cases. It is frequently large compared with the distance between the electrons. At distances small compared with L_c, the system behaves as a conductor, and at large distances as a dielectric. The dielectric constant of such a dielectric is large, $\varepsilon \sim L_c^2$. At low temperatures the conductivity has an activation character. An electron with an energy close to the Fermi surface hops from a region of size L_c into another region, located at a distance $x \gg L_c$ from the first. The conductivity is proportional to the probability of these hops. This probability is equal to the product of the probability of the tunneling by the activation probability

$$\mathscr{W}(x) = \exp\left\{-\frac{x}{L_c} - \frac{E_a(x)}{T}\right\}. \tag{2}$$

The activation energy consists of the energy scatter E_M (Refs. 1, 2) and the Coulomb-interaction energy $e\varphi(x)$ (Ref. 3), $E_a(x) = E_M(x) + e\varphi(x)$.

The overwhelming contribution to the conductivity is made by hops over a distance x_0 determined from the condition that the exponent in Eq. (2) be a maximum

$$\sigma \sim \exp\{-x_0/L_c - E_a(x_0)/T\}, \tag{3}$$

where x_0 is determined from the equation

$$\frac{1}{L_c} + \frac{1}{T}\frac{\partial}{\partial x}E_a(x_0) = 0. \tag{4}$$

The form of the function $E_a(x)$ will be obtained below in different cases. At large distances, the principal role is played by the Coulomb interaction

$$E_a \approx e\varphi(x) = e^2/\varepsilon x. \tag{5}$$

Substituting (5) in (3) and (4) we obtain

$$x_0 = (e^2 L_c/\varepsilon T)^{1/2}, \tag{6}$$

and

$$\sigma \sim \exp\{-(T_0/T)^{1/2}\}, \quad T_0 = 4e^2/\varepsilon L_c. \tag{7}$$

This law is valid at the lowest temperatures, when x_0 defined by Eq. (6), is large and Coulomb's law (5) holds. In films and wires, at large distances, the electrons interact through vacuum, therefore the dielectric constant ε in (5) must be replaced by unity.

Owing to the large value of x_0, the conductivity depends strongly on the electric field $\mathscr{E}$. In an electric field, the activation energy decreases and in formulas (2)–(4) we must make the substitutions $\varphi \to \varphi - \mathscr{E}x$, $E_a \to E_a - e\mathscr{E}x$. In first order, x_0 does not depend on the field and the dependence of the conductivity on the field is determined by the formula

$$\sigma(\mathscr{E}) = \sigma(0)\, e^{-e\mathscr{E}x_0/T}. \tag{8}$$

In stronger fields it is necessary to take into account the field dependence, determined from (4). In this case the dependence of the conductivity on the field can be obtained by making in (3) and (4) the substitution $L_c^{-1} \to L_c^{-1} - e\mathscr{E}/T$. In particular, in the case defined by Eq. (7), we obtain

$$T_0(\mathscr{E}) = \frac{4e^2}{\varepsilon}\left[L_c^{-1} - \frac{e\mathscr{E}}{T}\right]. \tag{9}$$

These laws are valid so long as the activation energy in the field is positive for substantial hops. In a strong field this energy vanishes. With further increase of the field, the hops take place over the nearest distance, where the activation energy is zero:

$$E_a(x_{\mathscr{E}}) - e\mathscr{E}x_{\mathscr{E}} = 0. \tag{10}$$

In this case the conductivity is independent of temperature, and its dependence on the field is determined by

$$\sigma \sim \exp\left[-x_{\mathscr{E}}/L_c\right]. \tag{11}$$

At very low temperatures and relatively weak fields, we can use in (10) formula (5). As a result

$$\sigma \sim \exp\left\{-\frac{1}{L_c}\left(\frac{e}{\varepsilon\mathscr{E}}\right)^{1/2}\right\}. \tag{12}$$

In strong fields, just as at high temperatures, hops over short distances are important, where formula (5) is not valid. The temperature and field dependences are in this case different for samples with different dimensions, which we now proceed to discuss.

3. CONDUCTIVITY OF THIN FILMS

In films of normal metals of thickness d the mean free path l, at a sufficiently high temperature, the conductivity is metallic and is described by the Drude formula. Wth decreasing temperature, the conductivity decreases because of quantum effects[6,7] and is equal to

$$\sigma_\square(T) = \sigma_0 - \frac{e^2}{2\pi^2\hbar}\left[\ln\frac{L_\varphi}{l} + (1-F)\ln\frac{L_T}{l}\right], \tag{13}$$

where $\sigma_\square(T)$ is the temperature-dependent conductivity of a film of quadratic form, σ_0 is the residual conductivity without allowance for the logarithmic corrections, $L_\varphi = (D\tau_\varphi)^{1/2}$ and $L_T = (D\hbar/T)^{1/2}$ are the diffusion lengths traversed respectively during the dephasing time $\tau_\varphi \sim T^{-p}$ and $\hbar/T$. The function $F(p_F(\varkappa)$ depends on the ratio of the Fermi momentum p_F to the reciprocal Debye-screening length $\varkappa$, and determines the force of the Coulomb repulsion for electrons on the Fermi surface.

Equation (13) is valid so long as the logarithmic corrections term is smaller than the Drude term. When the temperature is lowered, this relation is violated and the film goes over into the localization regime. The localization length can then be estimated at

$$L_c \sim l \exp\left\{\frac{\sigma_0}{\sigma_T}\frac{1}{2-F}\right\}, \tag{14}$$

where $\sigma_T = e^2/2\pi^2\hbar \approx 0.01\ (\mathrm{k}\Omega)^{-1}$. The temperature

$$T_* = DL_c^{-2} \sim \frac{1}{\tau}\exp\left\{-\frac{\sigma_0}{\sigma_T}\frac{2}{2-F}\right\} \tag{15}$$

is the temperature scale such that at $T > T_*$ the conductivity depends logarithmically on the temperature, and at $T < T_*$ it is determined by activation processes.

To determine the temperature dependence of the conductivity in the hopping-conduction region, it is necessary, as shown in Sec. 2, to solve the problem of the electrostatic potential $\varphi(r)$ of a charge placed in such a film. It must be understood here that over dimensions $r \ll L_c$ the field in the film is screened, with a screening length $\varkappa^{-1}(\varkappa^2 = 4\pi e^2\nu$, ν is the state density), while at larger distances $r \gg L_c$ there is no screening:

$$\varepsilon(q) = \begin{cases} 1 + \varkappa^2/q^2, & qL_c \gg 1 \\ \varepsilon, & qL_c \ll 1 \end{cases} \tag{16}$$

$$\varepsilon = \alpha\varkappa^2 L_c^2, \quad \alpha \sim 1. \tag{16'}$$

The potential of an electric charge e placed in a film of thickness d having dielectric constant ε, at a distance $\rho \gg d$ inside the film, is equal to[8]

$$\begin{aligned} \varphi(\rho) &= \frac{2e}{\varepsilon d}\int_0^\infty \frac{J_0(t)\,dt}{t + (\varepsilon_1+\varepsilon_2)\rho/\varepsilon d} \\ &= \frac{\pi e}{\varepsilon d}\left[\mathscr{H}_0\left(\frac{\varepsilon_1+\varepsilon_2}{\varepsilon}\frac{\rho}{d}\right) - N_0\left(\frac{\varepsilon_1+\varepsilon_2}{\varepsilon}\frac{\rho}{d}\right)\right] \\ &= \begin{cases} \dfrac{2e}{\varepsilon d}\left[\ln\left(\dfrac{2\varepsilon}{\varepsilon_1+\varepsilon_2}\dfrac{d}{\rho}\right) - C\right]; & d \ll \rho \ll \dfrac{\varepsilon d}{\varepsilon_1+\varepsilon_2}, \\ \dfrac{2e}{(\varepsilon_1+\varepsilon_2)\rho}; & \rho \gg \dfrac{\varepsilon d}{\varepsilon_1+\varepsilon_2}. \end{cases} \end{aligned} \tag{17}$$

Here $J_0(x)$, $\mathscr{H}_0(x)$ and $N_0(x)$ are the Bessel, Struve, and Neumann functions, $C = 0.55...$ is the Euler constant, ε_1 and

and ε_2 are the dielectric constants of the media on the two sides of the investigated film ("substrate" and "vacuum").

In the case when the film thickness d is of the dorder of one monatomic layer, Eq. (17) can still be used if εd is replaced by $\varkappa_2 L_c^2$, where $\varkappa_2 = 4\pi e^2 \nu_2$, and ν_2 is the state density per unit area.

Using (17), (3), and (4) we obtain the temperature dependence of the film conductivity

$$\sigma(T) \sim \exp(-T_0/T), \tag{18}$$

where T_0 depends logarithmically on the temperature

$$T_0 = \frac{2e^2}{\varepsilon d}\left(\ln\frac{\varepsilon^2 d^2 T}{(\varepsilon_1+\varepsilon_2) e^2 L_c} - C + 1\right). \tag{19}$$

Equations (18) and (19) are valid if the length x_0 of the hop satisfies the conditions $L_c \gg x_0 \gg \varepsilon d/(\varepsilon_1 + \varepsilon_2)$, which are satisfied at temperatures.

$$\frac{e^2}{\varepsilon d} = T_1 \gg T \gg T_2 = \frac{e^2 L_c(\varepsilon_1+\varepsilon_2)}{\varepsilon^2 d^2}.$$

At a temperature $T = T_1$ the length x_0 of the hop becomes comparable with the localization length L_c. The activation energy is then $T_0(T_1) \sim T_* \gg T_1$. In the temperature interval $T_* > T > T_1$, hops are produced over a length L_c, and the activation energy is equal to T_0 (T_1) and does not depend on the temperature. Alternately, when using (16′) and (14), this interval can be restricted by the inequality

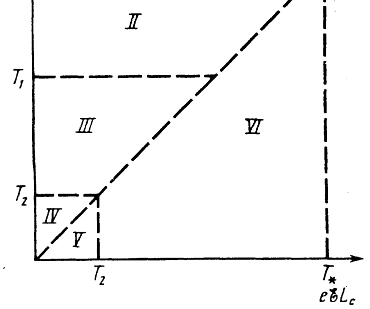
$$(e^2/\varkappa^2 L_c^2 d)\ln(\varkappa^2 L_c d) \sim T_* \gg T \gg T_1 = e^2(\varepsilon_1+\varepsilon_2)/\varkappa^4 L_c^4 d^2. \tag{20}$$

At $T \ll T_2$ the following relation holds

$$\ln\tau = -2\left[\frac{2e^2}{(\varepsilon_1+\varepsilon_2) L_c T}\right]^{1/2}, \tag{21}$$

and coincides with (7) at $\varepsilon_1 = \varepsilon_2 = \varepsilon$. Thus, in films at high temperatures $T > T_*$ we have metallic conductivity with logarithmic corrections; at $T_1 < T < T_*$ there is produced a region in which the conductivity is described by the Arrhenius law (18) with an activation energy of the order of T_*; at $T_1 > T > T_2$ the activation energy decreases logarithmically with decreasing temperature. Finally, at $T < T_2$ the Shklovskiĭ-Éfros law (21) is satisfied. In a strong electric field, the conductivity is determined by the general formulas (8)–(11), with allowance for the law governing the decrease of the electric potential (17). Diffferent regions of fields and temperatures are shown in the figure. In the metallic region (I) the nonlinearity is connected only with the heating of the electron gas.[6] In region II, the conductivity is

$$\sigma \sim \exp\left\{-\frac{T_* - e\mathscr{E} L_c}{T}\right\}. \tag{22}$$



To find the field dependence $\sigma(\mathscr{E})$ in the region III, it is necessary to replace $1/L_c$ in (18) and (19) by $1/L_c - e\mathscr{E}/T$. As a result we obtain

$$\sigma \sim \exp\left\{-\frac{T_1}{T}\left(\ln\left[\frac{\alpha\varkappa^2 L_c d^2 T}{e^2}\left(1 - \frac{e\mathscr{E} L_c}{T}\right)\right] - C + 1\right)\right\}. \tag{23}$$

The conductivity is described by Eqs. (7) and (9) in region IV and by Eq. (12) in region V. In region IV, just as in region V, the conductivity is determined by the activationless hops in the electric field, but the length of the hop in this case corresponds to a logarithmic decrease of $\varphi(\rho)$ (17). As a result we have in region IV

$$\sigma(\mathscr{E}) \sim \exp\{-T_1/e\mathscr{E} L_c[\ln(\varepsilon^2 e\mathscr{E} d^2) - C + 1]\}. \tag{24}$$

4. CONDUCTIVITY OF MIS STRUCTURES

The metal-dielectric junction was investigated in detail in layered metal-insulator-semiconductor (MIS) structures (see the review by Atkins[9]). When the general equations of Sec. 2 are applied to these structures, it must be remembered that the electrostatic interaction of two charges in a surface layer in the semiconductor is screened by the metal, and at large distances this is a dipole-dipole interaction

$$e\varphi = \frac{e^2(2d)^2}{\varepsilon_s r^2}, \tag{25}$$

where d is the thickness of the dielectric layer and ε_s is the static dielectric constant of the smeiconductor without allowance for the screening influence of the electrons. The interaction decreases more rapidly than the distance to the first excited level, and

$$E_a \approx E_M = 1/\nu_2 r^2,$$

where ν_2 is the two-dimensional state density, therefore at the lowest temperatures the Coulomb interaction is negligible and Mott's law should hold. It follows from (3) and (4) that

$$\sigma \sim \exp\{-(\theta/T)^{1/3}\}, \quad \theta = 1/\nu_2 L_c^2. \tag{26}$$

At short distances the screening of the Coulomb interaction is less significant so that in intermediate temperatures, just as in ordinary films, a law similar to the Arrhenius law may hold. To derive this law we obtain the potential produced by the charge at arbitrary distances. This potential satisfies the equation

$$e\Delta\varphi = -\frac{4\pi e^2}{\varepsilon_s}\delta(\mathbf{r}) - \frac{4\pi e^3}{\varepsilon_s}\alpha\nu_2 L_c^2 \Delta\varphi\,\delta(z) \tag{27}$$

and the boundary condition $\varphi(z = -d) = 0$. The three-dimensional coordinate is here $\mathbf{r} = (z, \boldsymbol{\rho})$. Equation (27) expresses the fact that the electrons in the semiconductor screen the Coulomb interaction, but at large distances this screening leads to a redefinition of the dielectric constant [see (16)]. The boundary conditions take into account the influence of the metallic substrate.

The solution of Eq. (27) with the boundary condition is of the form

$$e\varphi_q=\frac{2\pi e^2}{\varepsilon_s}\left\{\frac{q}{1-e^{-2qd}}+2\pi\alpha e^2\nu_2L_c{}^2q^2\right\}^{-1}$$
$$\approx\frac{2\pi e^2}{\varepsilon_s}\left[\frac{1}{2d}+\varkappa_2L_c{}^2q^2\right]^{-1},\qquad qd\ll1, \tag{28}$$

$$e\varphi(\rho)=\int e\varphi_qe^{iq\rho}\frac{d^2q}{(2\pi)^2}$$
$$=\frac{e^2}{\varepsilon_s\varkappa_2L_c{}^2}K_0\left(\frac{\rho}{L_c(2d\varkappa_2)^{1/2}}\right),\qquad \varkappa_2=2\pi\alpha e^2\nu_2. \tag{29}$$

Here $K_0(z)$ is a Macdonald function

$$K_0(z)=\begin{cases}-\ln z+C; & z\ll1\\ e^{-z}; & z\gg1\end{cases}.$$

The dipole-dipole character of the decrease of $e\varphi(\rho)$ at the largest distances follows from (28) and (29) if $\exp(-2qd)$ is expanded in powers of $qd\ll1$ up to terms of order $(qd)^2$. If the oxide thickness d is small enough, so that $\varkappa_2d\ll1$, the Coulomb interaction is weak at all distances, and Mott's law should hold at all temperatures at which the activation regime is valid. Even at high temperatures in the metallic phase the Coulomb interaction is weak in this case and can be disregarded.

In the other limiting case $\varkappa_2d\gg1$, Mott's law is valid only at the lowest temperatures $T<T_1$, while at $T>T_1$ the activation conductivity is determined by the Coulomb interaction. If $d\ll(L_c^2/\varkappa_2)^{1/3}$, then we can use for the electrostatic potential Eq. (29), and disregard the dipole-dipole "tail" at the largest distances. Under these conditions $T_1=T_0(\varkappa_2d)^{-3/2}\ln^{-3}\varkappa_2d$. In the temperature region $T_0(\varkappa_2d)^{-1/2}=T_2>T>T_1$ it is necessary to use the asymptotic form of the function $K_0(z)=e^{-z}$. The conductivity in this region is

$$\sigma\propto\exp\left\{-(\varkappa_2d)^{1/2}\ln\left(\frac{T}{T(\varkappa_2d)^{1/2}}\right)\right\}. \tag{30}$$

In this region the conductivity has a power-law dependence on the temperature.

At higher temperatures $T>T_2$, smaller distances, where $K_0(z)\approx-\ln z+C$, become important. In this region

$$\sigma\propto\exp\left\{-\frac{T_0}{T}\ln\frac{T(\varkappa_2d)^{1/2}}{T_0}\right\}. \tag{31}$$

The dependence on the electric field is obtained in accordance with the general scheme, just as in the case of ordinary films. For example, in (31) it is necessary to add under the sign of the natural logarithm the factor $(1-e\mathscr{E}L_c/T)$.

5. CONDUCTIVITY OF THIN FILMS

In metallic films, the localization length corresponds to a wire resistance of the order of $\pi^2\hbar/e^2\sim50\ \mathrm{k}\Omega$. Hence

$$L_c\sim l(p_F{}^2S), \tag{32}$$

where S is the cross-section area.

To determine the electrostatic potential it is necessary to solve the equation

$$\Delta\varphi=-4\pi e\delta(\mathbf{r})-4\pi e^2\alpha\nu SL_c{}^2\frac{\partial^2\varphi}{\partial z^2}\delta(\rho). \tag{33}$$

Here $\mathbf{r}$ is the dimensionless coordinate, and z is the coordinate in the direction of the wire axis. The solution (33) can be represented at $\rho=0$ in the form

$$e\varphi=2e^2\left[\frac{1}{\ln(qd)^{-1}}+\frac{\varepsilon Sq^2}{2\pi}\right]^{-1}, \tag{34}$$

where ε is given by (16′), $d=(4S/\pi)^{1/2}$ is the diameter of the wire,

$$e\varphi(z)=\int_{-\infty}^{+\infty}\frac{dz}{2\pi}e\varphi_qe^{iqz}$$
$$=e^2\begin{cases}2\pi\left(\pi S\varepsilon\ln\left(\dfrac{S\varepsilon}{\pi d^2}\right)\right)^{-1/2}\exp\left\{-|z|\left(\dfrac{\pi}{S\varepsilon}\ln\dfrac{S\varepsilon}{\pi d^2}\right)^{1/2}\right\}\quad\text{at}\\ |z|\ll\left(\dfrac{S\varepsilon}{\pi\ln(S\varepsilon/\pi d^2)}\right)^{1/2};\\ \dfrac{1}{|z|}\quad\text{at}\quad|z|\gg\left(\dfrac{S\varepsilon}{\pi\ln(S\varepsilon/\pi d^2)}\right)^{1/2}.\end{cases} \tag{35}$$

The general scheme described in Sec. 2 for the calculation of the average conductivity cannot be applied to the one-dimensional case. In this case it is necessary to calculate not the average conductivity but the average resistance. It is determined by the places with very large resistance, where there is no energy level lower than E over a large length x. The probability of existence of such a section is

$$W(E,\ x)\sim e^{-\nu_1Ex}, \tag{36}$$

where ν_1 is a one-dimensional state density, $\varphi)\nu_1=\nu S$, S is the wire cross-section area. Without allowance for the Coulomb interaction, the average resistance was obtained by Kurkijarvi.[10] Let us determine the change introduced in his result by the Coulomb interaction.

The resistance of a segment of length x is the equivalent of two parallel resistances: the tunnel resistance $\sim\exp(x/L_c)$ and the activation resistance $\exp[(E+e\varphi-e\mathscr{E}x)/T]$, where φ is the Coulomb potential and $\mathscr{E}$ is the external electric field. The average resistance is

$$R=\int\frac{W(x,E)\,dE\,dx}{\exp(-x/L_c)+\exp\{-(E+e\varphi-e\mathscr{E}x)/T\}}. \tag{37}$$

The main contribution to the integral with respect to x is made by those energies at which both terms in the denominator are of the same order, therefore

$$R\sim\int\exp\left\{\frac{x}{L_c}-\nu_1x\left(T\frac{x}{L_c}-e\varphi+e\mathscr{E}x\right)\right\}dx. \tag{38}$$

The integration region with respect to x is bounded in this case by the condition

$$E=Tx/L_c-e\varphi+e\mathscr{E}x>0. \tag{39}$$

At the lowest temperature, hops over large distances are of importance, where the Coulomb potential is equal to $\varphi=e/x$.

Calculating the integral by the saddle-point method, we obtain

$$R\sim\exp\left(\frac{T_0}{T+e\mathscr{E}L_c}+2\nu_1e^2\right),\quad T_0\sim\frac{1}{\nu_1L_c}. \tag{40}$$

Thus, at low temperatures, the Arrhenius law should be satisfied. The Coulomb interaction leads to the appearance in the argument of the exponential of an additional temperature-independent large term $2e^2\nu_1\sim\varkappa^2S$. Formula (40) is valid at $T<T_0/\varkappa^2S$. At higher temperatures, the integral (38) is determined not by the saddle point but by the edge of the integration region, which can be obtained from the condition (39). At $T_0/\ln(S\varepsilon/\pi d^2)>T>T_0/\varkappa^2S$ we have

$$R\sim\exp[(e^2L_cT)^{1/2}]. \tag{41}$$

Equation (41) coincides with expression (7). At $T>T_0/\ln^2(S\varepsilon/\pi d^2)$ account must be taken of the exponential character of the decrease of the electrostatic potential (35), while at $T_0>T>T_0/\ln^2(S\varepsilon/\pi d^2)$ the resistance increases with increasing temperature in power-law fashion

$$R\sim T^{(\varkappa^2S)}. \tag{42}$$

At $T_0(\varkappa^2S)^{1/2}>T>T_0$ we have

$$R\sim\exp\left[\frac{T_0}{T}(\varkappa_2S)^{1/2}\right]. \tag{43}$$

6. THREE-DIMENSIONAL CONDUCTORS

A large localization radius is realized in a three-dimensional conductor only when the impurity density n is close to the critical density $n_c(|(n-n_c)/n_c|\ll1)$ at which the metal-insulator transition takes place. In this region, the similarity laws are satisfied,[11] and the localization region L_c has a power-law dependence on $n-n_c$

$$L_c\sim\left|\frac{n-n_c}{n_c}\right|^{-\nu}. \tag{44}$$

The electrostatic potential is in this case

$$e\varphi(r)\propto\begin{cases} r^{-\eta}, & r\ll L_c,\\ \dfrac{1}{rL_c^{\eta-1}}, & r\gg L_c\quad(\text{insulator}),\\ \dfrac{1}{rL_c^{\eta-1}}e^{-r/L_c}, & r\gg L_c\quad(\text{metal}),\\ 1<\eta<3, \end{cases} \tag{45}$$

Equations (45) show that near the transition $\varepsilon\sim L_c\eta^{-1}\gg1$, and $r_D\sim L_c$.

At the lowest temperatures, using (5)–(7) and (45), we obtain

$$\sigma\propto\exp\left\{-\left(\frac{E_0}{T}\right)^{1/2}\right\},\quad E_0\sim L_c^{-\eta}\sim\left|\frac{n-n_c}{n_c}\right|^{\nu\eta}. \tag{46}$$

Formula (46) is valid at $T<E_0$. At $T>E_0$ the length of the hop is $x_0\sim L_c$, therefore at $T>E_0$ the conductivity does not have an activation character, but is determined by scattering and by Coulomb interaction in the critical region, and is independent of $|n-n_c|$:

$$\sigma(T)\sim\frac{e^2}{\hbar L(T)}\sim\frac{e^2p_F}{\hbar^2}\left(\frac{T}{E_F}\right)^{1/\eta}. \tag{47}$$

Thus, in three-dimensional samples near a metal-insulator junction there are no grounds for expecting the Arrhenius law for the conductivity.

The dependence of the conductivity on the electric field $\mathscr{E}$ is determined by Eqs. (9) and (12), in which the dielectric constant ε must be substituted. The exponential dependence on the field is valid at

$$e\mathscr{E}L_c<E_0;\quad \mathscr{E}<\mathscr{E}_0\sim L_c^{-(\eta+1)}. \tag{48}$$

At $\mathscr{E}>\mathscr{E}_0$ the conductivity is independent of $|n-n_c|$ and is equal to

$$\sigma(\mathscr{E})\sim\frac{e^2}{\hbar L(\mathscr{E})}\sim\mathscr{E}^{1/(\eta+1)}. \tag{49}$$

Here $L(\mathscr{E})$ is determined from the relation $e\mathscr{E}L(\mathscr{E})\sim L^{-\eta}(\mathscr{E})$.

7. CONCLUSION

In the extensive experimental material on the activation conductivity of disordered conductors, the case of large localization rates is particularly clearly realized in experiments on films and on MIS structures (see also Ref. 12). In all such cases there is a temperature interval in which the Arrhenius law is satisfied. The explanation of this law[2] was usually formulated as follows: there is a mobility threshold E_g at which $E_F<E_g$ but $E_g-E_F\ll E_g$, and the conductivity is effected by activation at the mobility threshold. Serious objections to this explanation have been raised recently. First, weighty arguments have been presented[4] in favor of the assumption that in the two-dimensional case there is no mobility threshold and all the states are localized. Second, the very concept of mobility threshold for localized electrons was borrowed from the problem of noninteracting electrons in a random potential. Recent studies by Al'tshuler and Aronov[7,13] and by McMillan[11] indicate that the electron-electron interaction alters very strongly the problem, and the language of single-particle levels ceases to be correct.

In the present paper a law close to the Arrhenius law was obtained without resorting to the concept of mobility threshold. The nonlinearity of the current-voltage characteristics is observed practically in all the experiments on activation conductivity. A study of the field and temperature dependences of the conductivity $\sigma(T,\mathscr{E})$ can provide a check on the proposed theory, as well as offer an experimental determination of such a fundamental quantity as the localization length L_c.

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