Effects occurring near the critical points of phase transitions in a Fermi liquid as illustrated by pion condensation

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The line $n = n_c(T)$ of first-order phase transitions of neutronic matter into the inhomogeneous state with a pion condensate is determined. It is assumed that $m_\pi/p_F \ll 1$; the Thomas-Fermi approximation, which breaks down when $(n - n_c)/n_c \sim (m/p_F)^3$, is then applicable at points far from the transition line. At low T the character of the transition is determined by the quantum fluctuations. The pion-field jump is small (as small as $(m/p_F)^3$), while the entropy and compressibility jumps are large. At high T the transition is from the liquid phase of the pion condensate into the crystalline phase. In the liquid phase the characteristic pion frequency is much lower than the temperature. The pion lattice has a low melting point. Near the transition line the thermodynamic functions of the system depend on $n - n_c$ and T in a power-law manner characteristic of nearly second-order phase transitions.

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1. INTRODUCTION

1. The main theoretical ideas about pion condensation were developed by Migdal, and are expounded in Ref. 1. The physics of the phenomenon consists in the fact that in sufficiently dense nuclear matter there appear unstable spin-acoustic-type-excitation branches carrying pion quantum numbers. The instability is eliminated through a reconstruction of the medium, with the appearance of a static pionfield condensate with wave number $k \neq 0$. The nuclear spin and isospin densities after the reconstruction are periodic functions of the coordinates. The qualitative picture of the phenomenon can now be understood within the framework of the model with a $\lambda\varphi^4$-type $\pi\pi$ interaction.[1] The pion-field fluctuations that occur near the phase transition point are taken into account within the framework of this model in Ref. 2. Because of the large phase volume of the $(k \neq 0)$ fluctuations, the role of these fluctuations does not reduce only to the renormalization of the coefficients that occur in the phenomenological theory of second-order phase transitions: they change the order of the transition from second to first. This phenomenon occurs even at $T = 0$, when there are no real pion excitations in the system. In the high-T classical case, in which there are many such excitations, the role of the fluctuations has been investigated by Brazovskiĭ[3] in the analogous problem of the phase transition of a liquid crystal into the inhomogeneous state. Interest in pion condensation at finite T arose in connection with the nuclear-matter warm-up, that occurs in heavy-ion collisions.[4] Estimates of the effect of the thermal fluctuations on the phase transition have been made by Voskresenskiĭ and Mishustin.[5]

In the present paper we consider the effects occurring in the vicinity of the critical point by means of the Thomas-Fermi method.[1] We are able to determine such characteristics of the system as the entropy, the compressibility, the melting point of the lattice, and the Fermi-liquid constants. We limit ourselves to a purely neutronic medium under the assumption that m/p_F is a small parameter. The point is that for a medium with $N = Z$ it is difficult to choose the optimal isotopic composition of the condensate.[1] In a neutron medium with $m/p_F \ll 1$, the Thomas-Fermi approximation, which is a zeroth-order approximation, and against whose background the fluctuations are taken into account, is applicable at points far from the transition point. For $m/p_F \simeq 1$, we must consistently take account of the nucleon-nucleon correlations and the effect of the Δ resonance on the phase transition.[1] These effects are important in connection with the establishment of the system's instability and the determination of the critical density n_c. But the effects connected with the phase transition can be understood on the basis of a simple model in which the mean-field approximation has a broad region of applicability. They depend weakly on the local characteristics of the system, and are determined largely by the form of the universal long-range interaction that occurs in the system in the vicinity of the phase transition point. A similar model has been considered by Vaks, Larkin, and Pikin.[6]

2. The phase diagram of the system is shown in Fig. 1. For $T \ll \varepsilon_F$) the neutrons are degenerate, and there exist five characteristic (n,T) regions differing from each other in the pion states. The region 1 is a quantum liquid with a soft pion

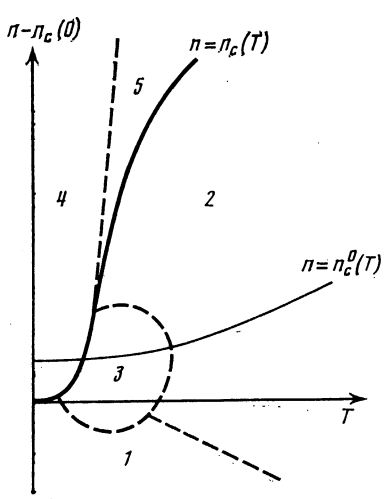

FIG. 1.

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mode and a characteristic pion frequency $\Delta \gg T$. Its properties are studied in Ref. 7, where nearly antiferromagnetic Fermi liquid is considered. The region 2 represents the classical pion gas in a background of cold neutrons; the roton gap in the pion spectrum $\Delta \ll T$. The pion excitations can be treated as static magnetic impurities. The region 3 represents a semiquantum pion gas. There occur here a cancellation of the contributions of the quantum and classical fluctuations and the appearance of a distinctive effect, a namely a proton gap $\Delta = \lambda_0 T$. The number λ_0 depends weakly on n and T. The region 4 represents a quantum pion crystal characterized by a periodic spin structure. The roton gap for the pions in the background of the condensate $\Delta \gg T$. The pion-field jump that occurs on the 1–4 transition line is small [as small as $(m/p_F)^3$], but the entropy and compressibility jumps are large. The region 5 is the region of the classical pion crystal. The periodic spin structure is superposed here with a small roton gap on a condensate background: $\Delta \ll T$. On the 2–5 transition line the jumps in all the characteristics of the system are small.

The smallness of the pion-field jump on a transition line allows us to follow how the new phase appears. Initially, the π-condensate phase is metastable and thermodynamically unstable, since its compressibility is negative. As its density increases, the sound speed goes to zero, and then the new phase becomes energetically advantageous. The melting point of the pion lattice is an order of magnitude lower than ε_F, an effect which is characteristic of the liquid-crystal transition. The thermal fluctuations of the pion field have such a strong temperature dependence that we can neglect in the entire region $T \ll \varepsilon_F$ the smearing of the momentum distribution of the neutron quasiparticles, and follow only the pion temperature. For this reason the dependence $n = n_c(T)$ has nothing in common with the dependence that corresponds to the mean-field approximation (the thin line in Fig. 1). The properties of the system do not charge on the "false" curve $n = n_c^0(T)$. Let us emphasize that the (n,T) or (V,T) plane does not contain a region of phase separation. The specific volume of one of the phases is always equal to zero. The phase transition, accompanied by pressure and chemical-potential jumps, occurs in the whole volume.

3. The main results of the paper are not connected with the specific properties of the π-condensate phase transition in neutronic matter. Only in Sec. 6 of the paper do we use the specific form of the order parameter of the pion field. In Secs. 2–5 we generalize the results obtained by the present author for liquid He^3 (Ref. 7), and develop the theory of the nearly antiferromagnetic Fermi liquid. For a neutronic medium the order parameter has the form $(\sigma\cdot\mathbf{k})\varphi_{\mathbf{k}}$, i.e., is characterized by a scalar function. For an antiferromagnetic Fermi liquid this parameter is the complex vector $\sigma\cdot\varphi_{\mathbf{k}}$; for nuclear matter it is an isotronic vector $(\sigma\cdot\mathbf{k})$ $(\tau\cdot\varphi_{\mathbf{k}})$. In the mean-field approximation this vector corresponds to the minimum of the functional:

$$F(\varphi) = \sum_k \{\xi_0^2 + \gamma^2 (k^2 - k_0^2)^2\} \varphi_{\mathbf{k}} \varphi_{-\mathbf{k}} + \Lambda_1 \left\{ \sum_k (\varphi_{\mathbf{k}} \varphi_{-\mathbf{k}}) \right\}^2 + \sum_{\mathbf{k}_i} \Lambda_2(\mathbf{k}_i) (\varphi_{\mathbf{k}_1} \varphi_{\mathbf{k}_2}) (\varphi_{\mathbf{k}_3} \varphi_{\mathbf{k}_4}). \quad (1)$$

The quantity ξ_0^2 depends on n and T, and changes sign at the transition point. The instability arises on the sphere $k = k_0$; for $k_0 \ll 2p_F$ the quantity Λ_2 depends only on the angles between the vectors $\mathbf{k}_i$. The main difficulty lies in the choice of the optimal solution. Besides the periodic solutions of the spin-density-wave type with long-range order, there exist solutions only with short-range order, i.e., of the spin-liquid or spin-glass type. An example of this solution in the coordinate representation is $\varphi(\mathbf{r}) = \Sigma_i \varphi_i(\mathbf{r} - \mathbf{r}_i)$, where $\varphi_i(\mathbf{r})$ is any solution to the equation $\Delta\varphi + k_0^2\varphi = 0$. Such a "disorder parameter" corresponds to a localized defect, with which the operation of displacement to the points $\mathbf{r}_i$ is effected. The displacement can also be accompanied by rotations of the vector φ, and we can perform both purposeful and random displacements and rotations. Furthermore, let us assume the existence of partial disorder in the background of a periodic solution. The energies of all such structures differ little from each other, and there arises the complicated problem of choosing the optimal solution with allowance for the critical fluctuations. Even a qualitative analysis of systems with a vector order parameter shows that several structural-transition-related singular lines occur in the (n,T) phase diagram. The character of the optimal solution depends strongly on the relation between k_0 and p_F. The main gain in energy for the system is connected with the softening of the spin-acoustic excitation branch, and the differences between the energies of the crystalline, liquid, and gaseous phases of the material are determined by the third term in (1), which is small, because $\Lambda_2 \ll \Lambda_1$. The computation of this small quantity takes us in a sense beyond the accuracy of the theory, since the expression (1) for the free energy approximate, and we must turn to experiment. The effect of the pion condensate on the single-nucleon absorption of pions by nuclei has been investigated by Troitskiĭ, Koldaev, and Chekunaev.[8] But such an experiment is crucial only for structures with long-range order. An isotopic-spiral type of structure[1] can be sought in experiments by the method of double charge exchange between pions. The appearance of latent itinerant antiferromagnetism in metals, as well as of a structure of the isotropic-glass type in nuclei, leads to the decrease of the mean free path of the Fermi quasiparticles. In a solid this can manifest itself as a sharp increase in the resistance at low temperatures.

4. A characteristic of the phase transition in question is that the appearance of an order or disorder parameter is not accompanied by particle localization. The spin or isospin density nodes become localized. In this case, for underdeveloped structures, the number of nodes is much lower than the number of particles. A similar phase transitions has been considered by Kirzhnits and Nepomnyashchiĭ.[9] Another characteristic of the transition is connected with the fact that the melting curve $n = n_c(T)$ is not a line of absolute instabilty of the homogeneous phase. Therefore, the situation is more complicated than the case of ordinary second-order phase transitions, in which there is a narrow region around the transition point where the fluctuations are important, and outside which the mean-field approximation is applicable. The crystalline and liquid phases compete in a broad (n,T)

region, where one phase is stable and the other is metastable. The main difficulty is connected with the search for the transition line and the choice of the competing phases.

The properties of the homogeneous phase turn out unexpectedly to be interesting (Sec. 5). Depending on the relation between $n - n_c^0$ and T, the system turns out to be in one or another of qualitatively different states, manifesting the properties of a quantum or classical liquid. Systems with a vector order parameter will be considered in detail in subsequent publications.

2. BASIC EQUATIONS

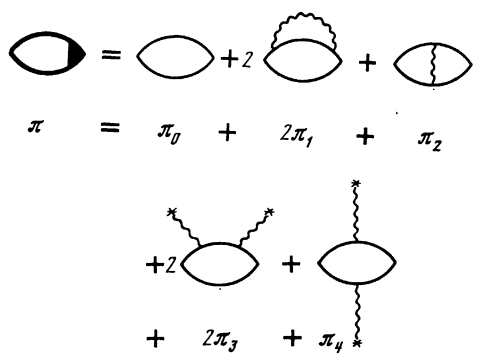
1. The long-range interaction that occurs in a neutron medium is connected with π_0-meson exchange, and is characterized by the static potential

$$V(\mathbf{k}) = -f^2(\sigma_1\mathbf{k})(\sigma_2\mathbf{k})/(k^2+m^2), \quad \hbar = c = 1.$$

The frequency dispersion of V can be neglected since the characteristic frequencies of the problem $\omega^2 \ll \varepsilon_F^2 \ll m^2$. The pion excitation spectrum in the medium coincides with the poles of the pion distribution function D:

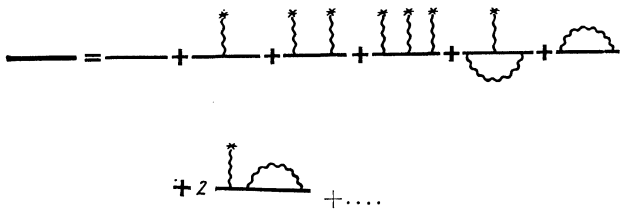
$$D(\mathbf{k}, \omega) = -f^2(\sigma_1\mathbf{k})(\sigma_2\mathbf{k})/[k^2+m^2+k^2\Pi(k, \omega)].$$

For the polarization operator $\Pi(k,\omega)$ we can use the following expansion:



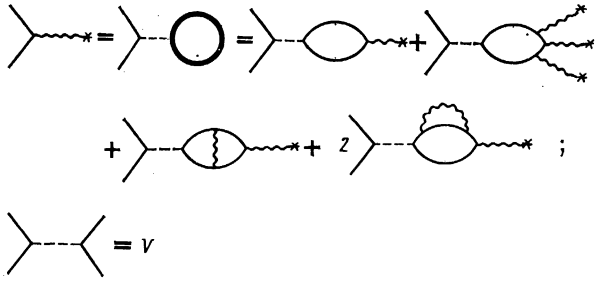
(2)

A wavy line in (2) represents the function D; a continuous line, the neutron Green function G; and a line with an asterisk, the nonequilibrium "trial" pion field $\tau_{\mathbf{k}} = \sigma \cdot \mathbf{A}_{\mathbf{k}}$. The $\mathbf{k}$ dependence of the spin vertex τ is determined by the form of D: $A_{\mathbf{k}} = \mathbf{k} \cdot \varphi_{\mathbf{k}}$. Besides the explicit dependence of Π on D and τ, there exists an implicit dependence through the chemical potential μ. The value of μ as a function of τ and D is fixed by the relation between the density and the single-particle Green function G. We can also expand G in power of G and τ:



(3)

The equilibrium value of the anomalous vertex τ is found from the consistency condition:



(4)

The heavy ring in (4) represents the neutron spin density with allowance for D and τ. In the homogeneous phase, because of the tensorial character of the one-pion exchange, $\tau = 0$. Thus, the diagrammatic expansions (2)–(4) constitute a system of nonlinear equations for the determination of Π, τ, and D. Let us emphasize that these equations are not expansions in powers of the coupling constant f^2. We shall take account of only those effects which are important for the phase transition. This means that we should retain those diagrams in (2)–(4) which are "amplified" in the vicinity of the phase transition point by the small denominator $n - n_c$, and discard the rest. In order for the model to be correct, we must "truncate" V at momenta $k \sim p_F$. If we cut off the expansions (2)–(4) at the first terms, we shall have the Thomas-Fermi approximation; the next terms are connected with the fluctuations. It is found that the phase transition occurs when the first fluctuations are switched on and the subsequent ones are still small. The expansion terms written out in (2)–(4) are sufficient for the solution of the problem. Notice that it is precisely this fact that made the exact solution of the problem in the $\lambda\varphi^4$ model possible.[2,3] An analysis of the approximations is carried out at the end of the paper.

2. The Eqs. (2)–(4) can be significantly simplified as a result of the fact that, in terms of the parameter $m/p_F \ll 1$ the pion field τ is a long-wave field, and therefore the characteristic momenta transferred via the wavy lines in (2)–(4) are small: $\omega \ll \varepsilon_F$, $k \ll 2p_F$. This makes it possible to separate the integrations over the pion and nucleon variables. We can then make the fermion loops in (2)–(4) contract to a point, and consider them to be local, their magnitude determining the $\pi\pi$ interaction via the excitation of the medium. The spatial and frequency dispersions of the pion are determined largely by the plane-polarization operator Π_0 in (2). As for the quantities Π_{1-4} in (2) they can be considered to be constants in the variables k and ω, but functions of τ, n, and T. They should be taken into account beside $(n - n_c)/n_c$, but can be neglected in comparison with unity. Taking into consideration the expression

$$\Pi_0 = -\nu\left(1 - \frac{\pi^2T^2}{12\varepsilon_F^2} - \frac{k^2}{12p_F^2} - i\frac{|\omega|}{kv}\right), \quad \nu = \frac{p_F M}{\pi^2},$$

for Π_0 for small k and ω, we parametrize the D function:

$$D = -\frac{g^2}{\nu}\frac{(\sigma_1\mathbf{n})(\sigma_2\mathbf{n})}{\xi^2+\gamma^2(n^2-1)^2-i|t|}, \quad g^2 \equiv f^2\nu. \tag{5}$$

In (5) we have introduced the variables $\mathbf{n} = \mathbf{k}/k_0$ and $t = \omega/\varepsilon_0$; $\varepsilon_0 = 2k_0v_F/\pi$. The momentum k_0 is found by minimizing the quantity $k^2[1 + f^2\Pi(k,0)]$ with respect to k^2. The quantity ξ is defined as

$$\xi^2=1+\frac{m^2}{k_0^2}+f^2\Pi(k_0,0);\qquad \xi_0^2=1+\frac{m^2}{k_0^2}+f^2\Pi_0(k_0,0).$$

It is precisely this characteristic of the system that is most sensitive to density and temperature changes. As for the quantities k_0^2 and γ^2, they depend weakly on these variables:

$$k_0^2=2\sqrt{3}p_Fm\ll p_F^2,\qquad \gamma^2=k_0^2/12p_F^2.$$

In the mean-field approximation, when $\tau=0$,

$$\xi_0^2=\frac{n_c^0(T)-n}{3n_c(0)},\qquad n_c^0(T)=n_c^0(0)\left(1+\frac{T^2\pi^2}{4\varepsilon_F^2}\right). \tag{6}$$

The function $n_c^0(T)$ determines the phase-coexistence line in the mean-field approximation: $n=n_c^0(T)$. The critical density n_c^0 (0) can be expressed in terms of the dimensionless coupling constant g^2 figuring in (5):

$$g_c^2m^2=3p_c^2(g_c^2-1)^2,\qquad g_c^2=f^2p_cM/\pi^2,\qquad n_c(0)=p_c^3/3\pi^2.$$

On account of the smallness of m^2/p_c^2, the quantity $g_c^2\sim 1$, and we can set $g^2=1$ in all the quantities that are insensitive to its precise value. Notice that the parametrization of D in the form (5) differs from the one adopted in the literature.[1] The point is that the meson system of units used in Ref. 1 is inconvenient here, since the characteristic energy scale in the vicinity of the transition point is much smaller than not only m, but also ε_F. The quantity ω_0^2 in Ref. 1 differs from ξ_0^2 in (5) by a factor $\sim p_c/m$.

3. The quantity ξ_0^2 has the meaning of a dimensionless bare roton gap for the diffusion spectrum of pions. This follows from (5); D has, when analytically continued from the right semiaxis of ω, a pole at

$$\omega=-i\frac{2kv_F}{\pi}\left[\xi_0^2+\gamma^2\left(\frac{k^2}{k_0^2}-1\right)^2\right].$$

If $\xi_0^2>0$, i.e., if $n<n_c^0$, then this pole is located in the lower half-plane under the branch cut in the unphysical sheet of the ω plane. But if $\xi_0^2<0$, the pole lies on the physical sheet in the upper ω half-plane, thus indicating that the system is unstable when $n>n_c^0$. To determine the dependence of the true roton gap ξ^2 on n and T, let us first determine the quantities Π in (2). These quantities can be factorized with respect to the parameter $k_0^2/4p_F^2\ll 1$:

$$\Pi_1=\Lambda_1B,\quad \Pi_2=-\frac{1}{3}\Pi_1,$$
$$B=\frac{T}{\nu}\frac{k_0^3}{(2\pi)^3}\int\sum_{\omega_n}\frac{d\mathbf{n}}{\xi^2+\gamma^2(n^2-1)^2+|\omega_n|/\varepsilon_0},\quad \omega_n=2n\pi T. \tag{7}$$

In (7) Λ_4 is the fermion ring with four external pion lines, and is a slowly varying function of the pion momenta:

$$\Lambda_4=-2i\int G^4(p)\frac{d^4p}{(2\pi)^4}=\frac{\nu}{24\varepsilon_F^2}.$$

The quantities Π_3 and Π_4 can be computed in the same way:

$$\Pi_3=\Lambda_4\sum_{\mathbf{k}_i}(\mathbf{A}_{\mathbf{k}_i}\mathbf{A}_{-\mathbf{k}_i}),$$
$$\Pi_4=-\frac{\Pi_3}{3}+\frac{2\Lambda_4}{k^2}\sum_i\left\{(\mathbf{kA}_{\mathbf{k}_i})(\mathbf{kA}_{-\mathbf{k}_i})-\frac{k^2}{3}\right\}.$$

The dependence of Π_4 on the angle between $\mathbf{k}$ and the reciprocal-lattice vectors $\mathbf{k}_i$ is due to the tensor character of the one-pion exchange and the noncommutability of the spin operators. For this reason, $\Pi_1\neq\Pi_2$, $\Pi_3\neq\Pi_4$, and the roton gap ξ^2 depends on the angle between $\mathbf{k}$ and $\mathbf{k}_i$. But for the three-dimensional lattice, which, in the mean-field approximation, corresponds to the minimum energy,[1] this dependence disappears, and $\Pi_4=-\Pi_3/3$. The relation connecting ξ^2 with $\mathbf{A}$ and D follows from the expressions obtained:

$$\xi^2=\xi_0^2+\Lambda\left[\sum_i(\mathbf{A}_{\mathbf{k}_i}\mathbf{A}_{-\mathbf{k}_i})+B\right]. \tag{8}$$

We have, in deriving (8), also taken into account the dependence of the chemical potential on D, τ, and n:

$$n=(2M\mu)^{3/2}/3\pi^2+\Lambda_3\left[\sum_i(\mathbf{A}_{\mathbf{k}_i}\mathbf{A}_{-\mathbf{k}_i})+B\right]. \tag{9}$$

The relation (9) follows from the relation between μ and G: the d^4p integral of the diagonal—in the spin and the momentum—G is equal to the mean density $n_{\mathbf{k}=0}$. It is natural that the scalar quantity μ depends on the square of the pion field $\mathbf{A}$. The quantity Λ_3 in (9) is the fermion ring with three external pion lines, and Λ in (8) can be expressed in terms of Λ_3 and Λ_4:

$$\Lambda_3=-2i\int G^3(p)\frac{d^4p}{(2\pi)^4}=\frac{3n}{8\varepsilon_F^2},$$
$$\Lambda=\frac{5}{3}\Lambda_4\frac{1}{\nu}+\frac{1}{3}\frac{\Lambda_3}{n}=\frac{7}{36\varepsilon_F^2}. \tag{10}$$

In the $\lambda\varphi^4$ model the dependence of μ on the pion field is neglected, and the effect of the noncommutability of the spin operators also gets lost; therefore, in this model $\Lambda=3\Lambda_4/\nu$.

4. Let us introduce the following convenient—for what follows—notation:

$$\Lambda B=\Phi,\quad \Lambda\sum_i(\mathbf{A}_{\mathbf{k}_i}\mathbf{A}_{-\mathbf{k}_i})=a^2.$$

Then (8) is an equation for the determination of ξ^2:

$$\xi^2=\xi_0^2+a^2+\Phi(\xi^2). \tag{11}$$

The quantity a^2 has the meaning of a dimensionless pion-field amplitude and the quantity Φ is connected with the fluctuations. To determine it, we need only compute the integral in (7). The sum over the frquencies ω_n in (7) is computed, using the standard procedure: by going over to integration over ω. Let us, omitting the computations, give the result:

$$\Phi(\xi^2)=2\beta\xi[J(\xi^2)-1],\quad J(\xi^2)=\int_0^\infty\frac{\operatorname{sh}(x/2)\,dx}{\exp(T^{-1}\xi^2\varepsilon_0\operatorname{sh}x)-1}. \tag{12}$$

Since the integral in (7) formally diverges, it is necessary to carry out a substraction procedure, which only renormalizes the quantity n_c^0 (0), i.e., shifts the curve $n_c^0(T)$ as a whole. The quantity Φ is normalized by the following condition: at $T=0$, $\Phi\sim\xi$; the parameter β in (12) is small [as small as $(m/p_F)^{3/2}$: $\beta=7\varepsilon_0m/6\varepsilon_Fp_F$. At low T the value of Φ is determined by the quantum fluctuations, since the quantity $J(\xi^2)$ in (12) is small for $T\to 0$: $J=\pi^2T^2/12\varepsilon_0^2\xi^4$. At high T the

dominant contribution to Φ is made by the classical fluctuations:

$$\Phi=\frac{2\alpha}{\xi}, \quad \alpha=\beta\frac{\pi}{2}\frac{T}{\varepsilon_0}\sim\frac{T}{\varepsilon_F}\frac{m}{p_F}\ll 1. \tag{13}$$

In the classical limit, (11) coincides, when $a^2=0$, with the analogous equation obtained by Brazovskiĭ in Ref. 3. The expression (13) can also be obtained by retaining in the sum over the ω_n in (7) only the leading term with $\omega_n=0$. Notice that we can choose the scale of the quantities ξ^2, ξ_0^2, and T such that the parameters β and ε_0 will not enter into the equation for ξ^2.

3. THE FREE ENERGY

1. Let us now proceed to determine the equilibrium value of τ from (4). The computational procedure is the same as for the derivation of the equation for ξ^2. The optimal three-dimensional solution has in the coordinate representation the form

$$\mathbf{A}(\mathbf{r})=2\left(\frac{6}{7}\right)^{1/2}\frac{a\varepsilon_F}{k_0}\{\mathbf{k}_1\sin\mathbf{k}_1\mathbf{r}+\mathbf{k}_2\sin\mathbf{k}_2\mathbf{r}+\mathbf{k}_3\sin\mathbf{k}_3\mathbf{r}\},$$

where the $\mathbf{k}_i$ are three orthogonal vectors: $|\mathbf{k}_i|=k_0$. Using the values obtained above for Π_{1-4}, we find from (4) the consistency condition for the field amplitude a^2:

$$\xi_0^2+\Phi(\xi^2)+(1-\delta^2)a^2=0 \quad \text{or} \quad \xi^2=\delta^2a^2. \tag{14}$$

The parameter δ^2 depends on the type of lattice: for the three-dimensional lattice $\delta^2=3/28$; for the one-dimensional, $\delta_1^2=1/28$; and for the two-dimensional, $\delta_2^2=5/56$. For $a^2\ll 1$, when the pion field is weak and the next harmonics in the expansion of $\mathbf{A}(\mathbf{r})$ in terms of the reciprocal-lattice vectors need not be taken into account, the equation for a^2 is closed on the sphere $|\mathbf{k}|=k_0$. The dependence on the type of lattice has a purely combinatorial origin, and affects only the quantity δ^2. The quantity δ^2 is small, and the consistency condition for a^2 coincides with the equation for ξ^2 when $\delta^2=0$, and, as can be seen from (14), the simultaneous solution of (11) and (14) should yield the value $\xi^2=0$. In order to understand the cause of the smallness of δ^2, let us determine the dependence of δ^2 on the parameters of the system. For a three-dimensional lattice

$$\delta^2=3\Lambda_4/2(5\Lambda_4+\Lambda_3\nu/n)=\Lambda_4/(2\nu\Lambda). \tag{15}$$

Thus, the smallness of δ^2 is due to the weak nonlinearity, which arises as a result of the interaction of the pions via the excitation of the medium. When $\Lambda_4=0$, we have a theory that is linear in the pion variables, and the instability is eliminated by the redistribution of the nucleons in the condensate fields, a redistribution which leads to the decrease of the chemical potential and the Fermi momentum and stabilizes the system. As can be seen from the expression (10) for the effective value of Λ, the quantity $\Lambda\neq 0$ when $\Lambda_4=0$. This fact, which is unexpected for the phenomenological approach, leads to a number of interesting consequences. The actual small parameter of the theory is not $\beta\sim(m/p_F)^{3/2}$, but $\delta^2\beta^2$; this parameter is small even when $m\sim p_F$. As can be seen from (14), the value of the roton gap ξ^2 is small even at points far from the supercritical region with a highly developed condensate. As will be shown below, the smallness of δ^2 leads to a very low pion-lattice melting point. It is precisely this parameter which makes the phase transition a nearly second-order transition: even when $m\sim p_F$ we have $a^2\ll 1$ on the phase-coexistence line. On the face of it, it appears that a consistent allowance for the interaction of the nucleons can greatly affect the magnitude of δ^2 and change its sign. But perturbation-theory estimates show that this is not case, and the sign of δ^2 is surprisingly stable. In particular, the magnitude of δ^2 is not affected when allowance is made for the weakening of the pion-nucleon vertex by a factor $(1+q^{-1})^{-1}$ (Ref. 1). Let us emphasize that the smallness of δ^2 is not a consequence of the allowance for the fluctuations: this fact is noticeable even in the mean-field approximation.[10] Below we shall consider δ^2 to be a free parameter.

2. For the analysis of the phase transition, the equations for ξ^2 and a^2 are not sufficient: we must also derive an expression for the free energy F. The quantity F can be found from (14), since it is known from general principles that this equation determines the extremum of F. Therefore, we find from (14) with allowance for the relation

$$\frac{\partial\xi^2}{\partial a^2}\left(1-\frac{\partial\Phi}{\partial\xi^2}\right)=1,$$

that

$$F-F_0=\delta F$$

$$=E_0\left\{\xi_0^2a^2+\frac{(1-\delta^2)a^4}{2}-\frac{\Phi^2}{2}-\frac{4}{3}\beta\xi^2-2\beta\int_{\xi^2}^{\infty}J(\xi^2)\xi\,d\xi^2\right\}. \tag{16}$$

It can be verified that the variation of F with respect to a^2 leads to the condition (14). The constant E_0 in (16) is determined from the requirement that the density derivative of F at a fixed value of T give the correct value of the chemical potential μ, (9): $E_0=27n_c\varepsilon_F/7$. The quantity F_0 in (16) is the free energy of the ideal Fermi gas and the value of $J(\xi^2)$ is given by formula (12). The direct calculation of F through the summation of the ring diagrams is complicated. This is done for the homogeneous ($a^2=0$) phase in the linear ($\Lambda_4=0$) theory in Ref. 7, where the analogous problem for He^3 is considered. It can be seen from (16) that, when the field a^2 appears, the functional dependence of F on the roton gap does not change, but ξ^2 becomes a function of a^2 in accordance with (11).

4. EXACT RELATIONS

1. Let us show that the system is stable against small variations in the field a^2, and that the pion field can start out only from a finite value, discontinuously. To do this, let us expand F in powers of a^2:

$$F=F(0)+F'(0)a^2+\frac{a^4}{2}F''(0)+\frac{a^6}{6}F'''(0). \tag{17}$$

The derivatives of F with respect to a^2 can be found from (16) if the relation (11) between ξ^2 and Φ is taken into account:

$$\frac{F'(0)}{E_0}=\xi^2,\quad \frac{F''(0)}{E_0}=\frac{1}{1-\Phi_{\xi^2}'}-\delta^2,\quad \frac{F'''(0)}{E_0}=\frac{\Phi_{\xi^2}''}{(1-\Phi_{\xi^2}')^3}.$$

The quantities ξ^2 and Φ have been referred to the homogeneous phase, i.e., the roton gap is the solution to the equation: $\xi^2=\xi_0^2+\Phi$. As ξ^2 decreases, the coefficient attached to a^4 changes sign before ξ^2 vanishes. This can be seen from the expression (12) for Φ. At low T, $\Phi\sim\xi$, and Φ' diverges like $1/\xi$; the divergence is even stronger at high T: $\Phi'\sim 1/\xi^3$. Notice that Φ is a monotonic function of ξ^2: $\Phi'<0$ and $\Phi''>0$; therefore, the sign in front of a^6 in (17) is always positive. At low T, F''' has a finite limit at $\xi\to 0$; at high T, $F'''\sim\xi^4$. Therefore, for $T\to 0$ the terms written out in (17) are sufficient for the determination of the optimal value of a^2, but at high T such an expansion is sufficient, and the exact expression (16) for F should be minimized. The physical cause of the change of sign of F'' is the same as in the phenomenological theory. The second variational derivative with respect to a^2 of F is proportional to the effective-interaction amplitude $\Gamma_{\pi\pi}$, which changes sign as $\xi\to 0$. For the one-dimensional lattice in the scalar $\lambda\varphi^4$ model, $\delta^2=1/2$ and

$$F''/E_0=(1+\Phi')/2(1-\Phi').$$

At $T=0$

$$F''/E_0=(\xi-\beta)/2(\xi+\beta);$$

at high T (Refs. 2 and 3)

$$F''/E_0=(\xi^3-\alpha)/2(\xi^3+\alpha).$$

2. For the determination of the entropy and the compressibility of the system, it is convenient to eliminate Φ from (16), using Eq. (11). For the homogeneous phase F_1 has the form

$$\frac{F_1-F_0}{E_0}=\xi_1^2\xi_0^2-\frac{1}{2}\xi_1^4-\frac{1}{2}\xi_0^4-\frac{4}{3}\beta\xi_1^3-2\beta\int_{\xi_1^2}^{\infty}J(\xi^2)\xi\,d\xi^2, \tag{18}$$

$$\xi_1^2=\xi_0^2+\Phi(\xi_1^2). \tag{18'}$$

For the phase with a pion condensate

$$\frac{F_2-F_0}{E_0}=\xi_2^2\xi_0+\frac{1}{2}\xi_2^4\frac{(1-\delta^2)}{\delta^2}-\frac{1}{2}\xi_0^4-\frac{4}{3}\beta\xi_2^3-2\beta\int_{\xi_2^2}^{\infty}J(\xi^2)\xi\,d\xi^2, \tag{19}$$

$$\xi_0^2+\xi_2^2\frac{(1-\delta^2)}{\delta^2}+\Phi(\xi_2^2)=0. \tag{19'}$$

In computing the T derivatives of $F_{1,2}$ we need to take into account only the explicit dependence of these quantities on T, since the variations of $F_{1,2}$ with respect to $\xi_{1,2}^2$ are equal to zero on account of Eqs. (19) and (19'). We have

$$S_{1,2}=S_0+2\beta E_0\int_{\xi_{1,2}^2}^{\infty}\frac{\partial J(\xi^2)}{\partial T}\xi\,d\xi^2,\quad S_0=-\frac{\partial F_0}{\partial T}; \tag{20}$$

F depends on T also through the quantity ξ_0^2, (6), but this dependence is significantly weaker than (20), and we shall not take it into consideration. The entropy of the homogeneous phase is greater than that of the π-condensate phase at all n and T, since the phase transition leads to an increase in the roton gap, while S, as can be seen from (20), decreases with increasing ξ^2. Thus, the entropy of the system is greatest on the transition line, and its decrease occurs discontinuously with heat release. At low T the entropy $\sim T$. This can be seen after substituting the low-temperature expansion for J into (20):

$$S_{1,2}=\frac{1}{3}p_F\dot{M}_{1,2}T,\quad \frac{\dot{M}_{1,2}}{M}=1+\frac{36}{7}\frac{\beta\varepsilon_F^2}{\xi_{1,2}\varepsilon_0^2}.$$

Near the transition line the quantities $M_{1,2}^*$ are large (as large as $1/\xi$). This result is obtained in Ref. 2 by a direct computation of the contribution of the pion to the self-energy part of the nucleon. At high T the entropy S has the form

$$S=\frac{1}{3}p_FMT+2\beta E_0\int_0^{\infty}\frac{\partial J}{\partial T}\xi\,d\xi^2-2\beta E_0\int_0^{\xi^2}\frac{\partial J}{\partial T}\xi\,d\xi^2. \tag{21}$$

The first integral in (21) does not depend on ξ^2; into the second we can substitute the high-temperature expansion of J:

$$S_{1,2}=\frac{1}{3}p_FMT+\beta E_0\frac{T^{1/2}}{\omega_1^{1/2}}-2\beta E_0\frac{\pi\xi_{1,2}}{\varepsilon_0},\quad \omega_1\sim\varepsilon_0. \tag{22}$$

In the region $T<(m/p_F)^{3/2}\varepsilon_F$ the dominant term in the expression for the entropy is $\sim T^{1/2}$; at $T>(m/p_F)^{3/2}\varepsilon_F$ the fluctuations are unimportant, and $S\sim S_0$. Thus, in both phases the entropy and the specific heat are linear functions of the temperature T in both the low and high ($T>\beta\varepsilon_F$) temperature regions. As shown in Ref. 7, this is due to the existence of well defined quasiparticles near and far from the Fermi surface. In the medium-T region the thermal fluctuations destroy the quasiparticles, and $S\sim T^{1/2}$. In the classical region the difference between the entropies of the phases at small ξ^2 stems from the small correction $\sim\xi$ in (22). This means that the entropy jump is small at high T: $\Delta S\ll S$.

3. Let us now ascertain how the compressibility of the system depends on n and T. From (18) and (19) we can determine the chemical potentials μ_1 and μ_2 of the phases:

$$\mu_{1,2}=\varepsilon_F\{1-{}^9/_7(\xi_{1,2}^2-\xi_0^2)\}. \tag{23}$$

It can be seen from (23) that the μ jump is negative, i.e., that $\mu_2<\mu_1$, since $\xi_2^2>\xi_1^2$. For the velocity of sound we have from (23) the expressions

$$u_1^2=u_0^2\left(1+\frac{9}{14}\frac{\Phi_{\xi^2}'}{1-\Phi_{\xi^2}'}\right),\quad u_0^2=\frac{n_c}{M}\frac{\partial\mu_0}{\partial n}=\frac{p_c^2}{3M^2}$$
$$u_2^2=\frac{5}{14}u_0^2\left(1-\frac{9}{5}\frac{1}{(1-\delta^2)/\delta^2+\Phi_{\xi^2}'}\right). \tag{24}$$

The phase transition leads to a decrease in the velocity of sound:

$$u_2^2<u_1^2.$$

5. THE PROPERTIES OF THE HOMOGENEOUS PHASE

The properties of the homogeneous phase are determined by the relation between the roton gap and the tem-

perature. Let us analyze Eq. (11) for ξ^2 in the case when $a^2 = 0$. It is convenient to do this is the variables.

$$\Delta = \frac{\xi^2}{\beta^2}; \quad \Delta_0 = \frac{\xi_0^2}{\beta^2}; \quad \theta = \frac{T}{\varepsilon_0 \beta^2};$$

$$\beta^2 \approx 10 \left(\frac{m}{p_F}\right)^3; \quad \varepsilon_0 \beta^2 \approx 20 \left(\frac{m}{p_F}\right)^{1/3} \varepsilon_F.$$

After such a choice of scale for the quantities ξ^2, ξ_0^2, and T, the equation for the gap Δ does not contain dimensional quantities:

$$\Delta = \Delta_0 + 2\Delta^{1/2}[J(\lambda) - 1]; \quad J(\lambda) = \int_0^\infty \frac{\operatorname{sh}(x/2)\, dx}{\exp(\lambda \operatorname{sh} x) - 1},$$

$$\lambda = \frac{\Delta}{\theta}. \tag{25}$$

1. In the quantum case, i.e., for $\Delta \gg \theta$, we can treat the quantity J in (25) as a small correction, and solve Eq. (25):

$$\Delta^{1/2} = -1 + (1+\Delta_0)^{1/2} + \frac{\pi^2 \theta^2}{12(1+\Delta_0)^{1/2}[(1+\Delta_0)^{1/2} - 1]^3}. \tag{26}$$

The region $\Delta_0 \gg 1$ corresponds to the mean-field approximation, i.e., for

$$\frac{n_c^0 - n}{n_c^0} \gg \left(\frac{m}{p_F}\right)^3, \quad \Delta = \Delta_0 + \frac{\pi^2 \theta^2}{6\Delta_0^{1/2}}, \quad \Delta_0 > 1, \quad \theta < \Delta_0.$$

In the region $\Delta_0 \ll 0$, i.e., for $(n_c^0 - n)/n_c^0 \ll (m/p_F)^3$

$$\Delta = \frac{\Delta_0^2}{4} + \frac{2}{3}\frac{\pi^2 \theta^2}{\Delta_0^2}, \quad 0 < \Delta_0 \ll 1, \quad \Delta_0 > \sqrt{\theta}. \tag{27}$$

The fluctuations significantly alter the bare dependence $\Delta_0 \sim (n_c^0 - n)/n_c^0$: the true roton gap depends quadratically on $n_c^0 - n$. The mean-field approximation has no region of applicability when $\beta \sim 1$. According to (27), the region $\Delta_0 > \theta^{1/2}$ corresponds to the quantum case when $\Delta_0 \to 0$ and $\theta \to 0$. This is due to the fact that the limit for $\Delta \to 0$, $\theta \to 0$ of the quantity J in (25) depends on the relationship between Δ and θ.

2. In the classical case the gap is smaller than T and Δ is the solution to th^ equation

$$\Delta = \Delta_0 + \pi\theta, \qquad ^\gg \Delta.$$

There are two characteristic regions; in the first

$$\Delta = (\pi\theta)^{2/3}, \quad \theta \gg 1, \quad |\Delta_0| \ll \theta^{2/3}. \tag{28}$$

In this region the roton gap does not depend on the density. The condition $\Delta/\theta \ll 1$ is fulfilled better at high θ values: $\Delta/\theta \sim \theta^{-1/3}$. For $m \sim p_F$ the region $\theta \gg 1$ corresponds to the temperature region $T \gg \varepsilon_F$, where the neutrons can no longer be considered to be degenerate. In the second region

$$\Delta = \pi^2\theta^2/\Delta_0^2, \quad \Delta_0 < 0, \quad |\Delta_0| > \theta^{1/2}, \quad |\Delta_0| > \theta^{2/3}. \tag{29}$$

This region for the mean-field approximation is supercritical: $\Delta_0 < 0$ when $n > n_c^0$. The limitations $|\Delta_0| > \theta^{1/2}, \theta^{2/3}$ can be combined with the requirement that $\theta < 1$; therefore, (29) is valid in the relativistic case $m \sim p_F$, $T \ll \varepsilon_F$.

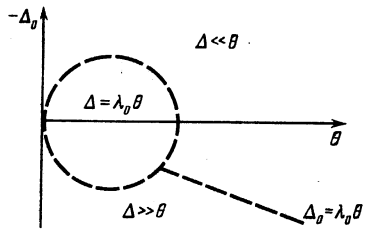


FIG. 2.

3. There exists a third interesting region where $\theta \ll 1$ and $|\Delta_0| \ll \theta^{1/2}$:

$$\Delta = \theta\lambda_0 + c_0\theta^{1/2}(\Delta_0 - \theta\lambda_0). \tag{30}$$

Here λ_0 is a root of the equation $J(\lambda) = 1$; $\lambda_0 \sim 1$ and the number $c_0 \sim 1$. In this region $\Delta = \theta\lambda_0$ for $\theta \to 0$, $\Delta_0 \to 0$. Thus, in the semiquantum case the gap depends weakly on the density, and is proportional to the temperature. The relation (30) is valid near the line, i.e., the boundary separating the regions of applicability of the quantum and classical statistics for the pions. This line is characterized by the cancellation of the contributions of the quantum and classical fluctuations. For the real case $m \sim p_F$ the semiquantum region corresponds to $T \ll \varepsilon_F$, i.e., it is most interesting region. The strong singularity of Eq. (25) at $\Delta \to 0$, $\theta \to 0$ is, of course, fictitious. As will be shown below, all this region belongs to the crystalline phase of the material, for which all the quantities are single-valued, and do not depend on the relationship between Δ and θ for $\Delta \to 0$ (see Fig. 2).

4. At $T \gg \Delta$ the system contains real pion excitations. Their distribution function $n(\mathbf{k})$ is characterized by a delta-function peak on the sphere $k = k_0$:

$$n(\mathbf{k}) \approx \frac{T}{\omega(k)}, \qquad \omega(k) = \varepsilon_0 \left\{ \xi^2 + \gamma^2 \left(\frac{k^2}{k_0^2} - 1 \right)^2 \right\}.$$

Let us determine the total pion density:

$$n_\pi = \frac{T}{\varepsilon_0} \frac{k_0^3}{(2\pi)^3} \int \frac{d\mathbf{n}}{\xi^2 + \gamma^2 (n^2 - 1)^2} = n_N \frac{T}{\varepsilon_0} \frac{m}{p_F} \frac{9\pi}{\xi}, \quad n_N = \frac{p_F^3}{3\pi^2}. \tag{31}$$

The pion density n_π increases with decreasing ξ. The quantity $n_\pi/n_N = c_\pi$ has the meaning of a pion concentration (the number of pions per neutron). Neutrons are scattered by classical pions in the same way as they are scattered by static magnetic impurities. In order to show this, let us determine the decrement of the neutron-like quasi-particles:

$$G(p) = (\varepsilon - E_p + i\varepsilon_1 \operatorname{sign}(\varepsilon))^{-1}.$$

The quantity ε_1 can be expressed in terms of the imaginary part of the mass operator Σ:

$$\Sigma = -T \int \sum_{\omega_n} \frac{d^3k}{(2\pi)^3} D(k, i\omega_n) G(\mathbf{p} - \mathbf{k}, \varepsilon - \omega_n).$$

For $\varepsilon > \Delta$, we can retain in the sum over the frequencies the dominant $\omega_n = 0$ classical term, and take out the function G from under the integral sign at the point $k = k_0$:

$$\varepsilon_1 = n_\pi / \nu = {}^2/_3 c_\pi \varepsilon_F.$$

We have obtained a natural result: the quantity $1/\varepsilon_1$, which

has the meaning of the time between collisions with the "pion impurities," is proportional to the density ν of states of the Fermi surface, and inversely proportional to the pion density n_π. The next orders of perturbation theory in terms of the pion concentration c_π can be found by means of the usual cross technique.[11] Let us emphasize that the analogy with impurities is almost total. In the homogeneous phase the sphere $k = k_0$ is uniformly populated by pions, and the scattering on them occurs in random fashion. In the inhomogeneous phase the pions separate out and form a condensate at six (for a three-dimensional lattice) points on the sphere, and the scattering on them occurs coherently. As will be shown below, not all the pions precipitate out into the condensate: there remains a classical pion gas in its background. The density of this gas decreases with decreasing T, and vanishes at $T = 0$. The pion-number conservation law is approximately observed in the process: the number of condensate pions + the number of pions in the gas is a function of only $n - n_c$, and does not depend on T. This law can be illustrated by writing down the diagrammatic expression for ξ^2:

The first diagram represents the density of the condensate particles; the second, that of the epicondensate particles. Since the pions are soft at all n and T in the crystalline phase, and $\xi^2 \ll \xi_0^2$, the total pion density depends only on ξ_0^2, i.e., on $n - n_c$. In the quantum region $\xi_0^2 > 0$, and the number of epicondensate pions is negative. This is not surprising, since pions are produced virtually at $T = 0$. In the semiquantum region the total density of the real and virtual pions is equal to zero.

5. Thus, the allowance for the fluctuations has led to the appearance of a qualitatively new phenomenon: the homogeneous phase is stable at all n and T, since the equation for the roton gap $\Delta > 0$ possesses solutions. In the far supercritical region, at low temperatures, this phase is metastable, and is a supercooled liquid. For the classical case, this result was first obtained by Brazovskiĭ,[3] and has been reobtained by Bunatyan and Mishustin.[12]

6. THE PHASE TRANSITION

To determine the phase-coexistence line $n = n_c(T)$ and the melting points $T = T_c(n)$ of the lattice, we must compute the roton gap $\xi_{1,2}^2$ for each point of the (n,T) plane, and determine the $F_{1,2}$ from (18) and (19). By comparing F_1 and F_2, we can determine the transition line: $F_2 < F_1$. Such an approach turns out to be too complicated, and we shall limit ourselves to the consideration of the limiting cases, using an expression for F in the form (16), which includes the explicit dependence of F on the field amplitude. A significant simiplification arises because the curve $n = n_1(T)$, which determines the density n_1 at which the metastable inhomogeneous phase first appears, is close to the curve $n = n_c(T)$. As for the curve $n = n_1(T)$, we can determine it from the consistency condition (14) without computing the free energy. Thus, we can give a reliable estimate for n_c: $n_c(T) > n_1(T)$.

The low-temperature expansion

1. The expression for F gets simplified at $T = 0$:

$$\frac{F-F_0}{E_0} = \xi_0^2 a^2 + \frac{(1-\delta^2)a^4}{2} - 2\beta^2\xi^2 - \frac{4}{3}\beta\xi^3. \tag{32}$$

The equation for the roton gap can also be solved:

$$\xi = -\beta + (\beta^2 + \xi_0^2 + a^2)^{1/2}.$$

Let us again use the fact that we are free to choose the scale, and perform the analysis in the variables Δ, Δ_0, and φ^2:

$$\Delta = \xi^2/\beta^2, \quad \Delta_0 = \xi_0^2/\beta^2, \quad \varphi^2 = a^2/\beta^2.$$

In those variables we obtain from (32) the expression

$$\frac{F-F_0}{F_0\beta^4} = (2+\Delta_0)\varphi^2 + \frac{(1-\delta^2)\varphi^4}{2} - \frac{4}{3}(1+\Delta_0+\varphi^2)^{3/2} + 2\Delta_0 + \frac{4}{3}. \tag{33}$$

Thus, in the variables φ and Δ_0 the expression for F is universal; the dependence on β^2 enters only as constraints, $\varphi^2 \ll 1/\beta^2$ and $\Delta_0 \ll 1/\beta^2$, which are connected with the inapplicability of (33) at high pion field values (i.e., for $a^2 \sim 1$) and at points far from the transition point (i.e., at points where $\varphi_0^2 \sim 1$). The region $1 \ll \Delta_0 \ll 1/\beta^2$ corresponds to the mean-field approximation; it is essential that the fluctuations be taken into account in the region $\Delta_0 \ll 1$. Let us use the fact that δ^2 is small, and expand δF for $\Delta_0 \ll 1$ and $\varphi^2 \ll 1$:

$$\delta F/E_0\beta^4 = {}^1/_{12}(\Delta_0+\varphi^2)^3 - {}^1/_2\delta^2\varphi^4 - {}^1/_2\Delta_0^2. \tag{34}$$

The optimal φ_0 is found from the condition $\partial F/\partial\varphi^2 = 0$:

$$\varphi_0 = \delta \pm (\delta^2 - \Delta_0)^{1/2}.$$

Let us also give the expression for $\tilde{\varphi}_0$, that minimizes the exact expression (33) for F:

$$\tilde{\varphi}_0(1-\delta^2) = \delta \pm [\delta^2 - \Delta_0(1-\delta^2)]^{1/2}.$$

A comparison of φ_0 and $\tilde{\varphi}_0$ shows that indeed the smallness of δ^2 causes them to be close in value. Below we limit ourselves to the analysis of the aproximate expansion (34). For $\Delta_0 < \delta^2$, F possesses two extremal points, one of which corresponds to the minimum, and the other to the maximum, of F. For $\Delta_c < \Delta_0 < \delta^2$, where $\Delta_c = 2\sqrt{3}\delta^2/(2+\sqrt{3})$, the crystalline phase is metastable, and at $\Delta_0 = \Delta_c$ there arises discontinuously a pion field of strength $a_c^2 = \beta^2\varphi_c^2$, where $\varphi_c^2 = 6\delta^2/(2+\sqrt{3})$.

Thus, the quantum fluctuations facilitate the phase transition, which occurs at $n_c < n_c^0$: $(n_c^0 - n_c)/3n_c^0 = \Delta_c\beta^2$. The density at which the new phase first appears is close to the equilibrium density $\delta^2 - \Delta_c = \delta^2/(2+\sqrt{3})^2 \ll \delta^2$. Qualitatively, this is also the case when the thermal effects are taken into account; therefore, the function $n_c(T)$ for low T can be approximately determined from (14):

$$\Delta_0 + \varphi_0^2 - 2\varphi_0\delta + \frac{T^2}{T_0^2\varphi_0^3\delta^3} = 0, \quad T_0^2 = \frac{6\varepsilon_0^2\beta^4}{\pi^2}.$$

Let us, considering the term $\sim T^2$ to be a small correction, find φ_0:

$$\varphi_0 = \delta\left(1 + \frac{3T^2}{2T_0^2\delta^8}\right) + \left(\delta^2 - \frac{T^2}{T_0^2\delta^6} - \Delta_0\right)^{1/2}.$$

The metastable state appears at $\Delta_c(T) = \delta^2 - T^2/T_0^2\delta^6$. Now we can estimate $n_c(T)$:

$$n_c(T)\approx n_c(0)\,(1+T^2/T_1^2),\quad T_1=T_0\delta^3/\beta\sqrt{3}\approx 0{,}1\varepsilon_F m^2/p_F^2.$$

The fluctuations significantly modify the bare temperature dependence $n = n_c^0(T)$, since $T_1 \ll \varepsilon_F$. Even when $m \sim p_F$, the temperature $T_1 \sim 0.1\ \varepsilon_F$. Let us also give the density dependence of the melting point:

$$T_c(n)=T_1[(n-n_c(0))/n_c(0)]^{1/2},\quad n>n_c(0).$$

2. To ascertain how the thermodynamic quantities vary on the melting curve, we need only find the roton gap as a function of n and T, and substitute its value into the formulas (20), (23), and (24). Let us limit ourselves to the consideration of the $T\to 0$ limit. For the homogeneous phase

$$\frac{\xi_1}{\beta}=\Delta_1^{1/2}=(1+\Delta_0)^{1/2}-1,\quad \Delta_1^{1/2}(n_c)=\delta^2\frac{\sqrt{3}}{2+\sqrt{3}},\quad n<n_c.$$

For the condensate phase:

$$\frac{\xi_2}{\beta}=\Delta_2^{1/2}=\delta^2+\delta(\delta^2-\Delta_0)^{1/2},\quad \Delta_2^{1/2}(n_c)=\delta^2\frac{3+\sqrt{3}}{2+\sqrt{3}},\quad n>n_c.$$

The roton gap changes sharply at the transition point: $\Delta_2/\Delta_1 = (1+\sqrt{3})^2$. For the μ jump we obtain from (23) the expression

$$\frac{\mu_1(n_c)-\mu_2(n_c)}{\varepsilon_F}=\frac{27}{7}\beta^2\delta^4\frac{\sqrt{3}}{2+\sqrt{3}}.$$

Let us given the density dependence of the velocity of sound:

$$\begin{aligned}
u_1^2&=u_0^2\left[1-\frac{9}{14(1+\Delta_0)^{1/2}}\right],\\
u_1^2(n_c)&=\frac{5}{14}u_0^2\left(1+\frac{9}{5}\frac{\sqrt{3}}{2+\sqrt{3}}\delta^2\right),\\
u_2^2&=\frac{5}{14}u_0^2\left(1-\frac{9}{5}\delta^2\frac{\delta+(\delta^2-\Delta_0)^{1/2}}{(\delta^2-\Delta_0)^{1/2}}\right),\\
u_2^2(n_c)&=\frac{5}{14}u_0^2\left[1-\frac{9}{5}(3+\sqrt{3})\delta^2\right.
\end{aligned}\qquad (35)$$

At the transition point u_2^2 is small: $u_2^2(n_c)\approx 0.03u_0^2$; let us also give the value of u_2^2 at the "old" critical point n_c^0 and in the Thomas-Fermi approximation:

$$u_2^2(n_c^0)=0{,}07u_0^2,\quad u_{2TF}^2=\frac{5}{14}u_0^2\left(1-\frac{9}{5}\frac{\delta^2}{1-\delta^2}\right)=0.28u_0^2.$$

At the phase temperature point, the quantity u_1^2 is close to its minimum value $u_1^2 = (5/14)u_0^2$. In the condensate phase, this value is, up to the term $\sim\delta^2$, the maximum value, and is attained at points far from the transition point. The sound velocity jump is high: $\Delta u^2 \sim u_1^2$. Let us note that μ jump is small, i.e., as small as δ^2, and that the u^2 jump does not contain this parameter at all, being dependent only on the type of lattice through δ^2. In the mean-field approximation (i.e., for $\beta = 0$), the jump $\Delta\mu = 0$, and u^2 no longer depends on the density. Since the condensate phase, at the moment of its appearance, has $u_2^2 = -\infty$, (35), it is thermodynamically unstable, i.e., it collapses.

3. Let us now determine the dependence of $M_{1,2}^*$ on n:

$$\frac{M_1^*(n)}{M}=1+\frac{b}{(1+\Delta_0)^{1/2}-1},\quad b=\frac{36}{7}\frac{\varepsilon_F^2}{\varepsilon_0^2}\gg 1,\quad n<n_c,$$

$$\frac{M_2^*(n)}{M}=1+\frac{b}{\delta^2+\delta(\delta^2-\Delta_0)^{1/2}},\quad n>n_c.$$

Both M_1^* and M_2^* are maximal at the transition point:

$$\frac{M_1^*(n_c)}{M}\approx b\frac{2+\sqrt{3}}{\sqrt{3}\,\delta^2},\quad \frac{M_2^*(n_c)}{M}\approx b\frac{2+\sqrt{3}}{\delta^2(3+\sqrt{3})}.$$

The entropy jump can be found in terms of these values:

$$(S_1-S_2)/S_1=1-M_2^*(n_c)/M_1^*(n_c)=\sqrt{3}/(1+\sqrt{3}).$$

For the physically interesting $m \sim p_F$ case, quantity M^*/M attains a value of 10^2 at the transition point. From the expressions for u^2 and M^* we can, using the formulas of the theory of the Fermi liquid, determine two harmonics Φ_0 and Φ_1 of the neutron-like quasiparticle scattering amplitude. In the vicinity of the transition line $\Phi_0 \gg 1, \Phi_1 \gg 1$, with $\Phi_0 = \Phi_1/3$ in the region of applicability of the mean-field approximation. This result is obtained in Ref. 7 through a direct computation of Φ. Let us emphasize that the Fermi surface does not disappear at the transition point, and the Landau theory of the Fermi liquid remains valid there, but it has a narrow region of applicability: $T \ll \varepsilon_F\beta^2\delta^4$; for $m \sim p_F$, $T \ll 10^{-2}\varepsilon_F$.

The high-temperature expansion

1. The contribution of the thermal fluctuations begins to predominate at $T > \varepsilon_F(m/p_F)^{7/2}\delta^4$. The fluctuations melt the lattice, and the phase transition occurs at a point far from the "old" supercritical region $n(T) > n_c^0(T)$. The mean-field approximation does not possess a region of applicability right up to $T \sim \varepsilon_F$. Neglecting the quantum fluctuations, we find from (16) that

$$\frac{\delta F}{E_0}=\xi_0^2a^2+\frac{(1-\delta^2)a^4}{2}-\frac{2\alpha^2}{\xi^2}+4\alpha\xi.\qquad (36)$$

The roton gap is a solution of the cubic equation

$$\xi^2=\xi_0^2+a^2+2\alpha/\xi,\quad \alpha=7\pi Tm/12\varepsilon_F p_F.$$

In (36) we have retained only those terms which depend on the field amplitude a^2. It is now convenient to choose the active variables in the form

$$\frac{a^2}{\alpha^{2/3}}=\eta^2,\quad \frac{\xi^2}{\alpha^{2/3}}=\chi^2,\quad Z=-\frac{\xi_0^2}{\alpha^{2/3}}=\frac{n-n_c^0(T)}{3n_c^0(0)\alpha^{2/3}}.$$

The minus sign in the definition of Z is connected with the fact that the transition occurs at $n > n_c^0$. From (36) we find in these variables that

$$\frac{\delta F}{E_0\alpha^{4/3}}=-Z\eta^2+\frac{(1-\delta^2)\eta^4}{2}+4\chi-\frac{2}{\chi^2},\quad \chi^2+Z=\eta^2+\frac{2}{\chi}.\qquad (37)$$

Let us, to begin with, consider the supercritical region, where the mean-field approximation is applicable to the condensate phase. We obtain

$$\frac{\delta F_2}{E_0\alpha^{4/3}}=-Z^2\frac{1}{2(1-\delta^2)},\quad \eta_0^2=\frac{Z}{1-\delta^2}. \tag{38}$$

Let us compare this value of δF_2 with δF_1 for the homogeneous phase, which is metastable at points deep in the supercritical region. For the homogeneous phase $\eta=0, \chi\ll 1$, and we can approximately solve the cubic equation (37):

$$\chi=\frac{2}{Z},\quad \frac{\delta F_2}{E_0\alpha^{4/3}}=-\frac{Z^2}{2},\quad Z\gg 1.$$

On account of the smallness of δ^2, the values of δF_2 and δF_1 are close; therefore, it is natural to expect that the transition will occur at $Z_c\gg 1$, when the qunatities δF_1 and δF_2 are equal. In order to give an estimate for Z_c, let us first of all find the value of Z_1 at which the new phase first appears. In the variables η and Z, the equation for the equation for the optimal η_0 has the form

$$Z=(1-\delta^2)\eta_0^2+\frac{2}{\eta_0\delta}.$$

Taking the smallness of δ^2 into account, we obtain the solution to this equation in the vicinity of the threshold for appearance of the new phase:

$$\eta_0^2=\frac{1}{\delta^{2/3}}\pm\frac{2}{\sqrt{3}\,\delta^{1/3}}(Z-Z_1)^{1/2},\quad Z_1=\frac{3}{\delta^{2/3}}. \tag{39}$$

The estimate for Z_c has the form $Z_c>Z_1$; in fact, Z_1 turns out ot be large: $Z_1\sim 7$. This enables us to approximately solve the equation (37) for χ, and obtain for F a simplified expression that is applicable in the vicinity of the transition line:

$$\chi\approx\frac{2}{Z-\eta^2},\quad \frac{\delta F}{4E_0\alpha^{4/3}}=\frac{\eta^2}{Z(Z-\eta^2)}-\frac{\delta^2\eta^4}{8}. \tag{40}$$

The expansion (40) for small η^2 coincides with the general expression (17):

$$\frac{\delta F}{4E_0\alpha^{4/3}}=\frac{\eta^2}{Z^2}+\eta^4\left(\frac{1}{Z^3}-\frac{\delta^2}{8}\right)+\frac{\eta^6}{Z^4}.$$

By rewriting (40) in the form

$$\frac{\delta F}{E_0\alpha^{4/3}}=\frac{\eta^2\delta^2}{2(Z-\eta^2)}(\eta^2-\eta_1^2)(\eta^2-\eta_2^2),$$

$$\eta_{1,2}^2=\frac{Z}{2}\left\{1\pm\left(1-\frac{Z_c^3}{Z^3}\right)^{1/2}\right\},$$

we can give a more accurate estimate: $Z_c^3=32/\delta^2$, and Z_1 and Z_c are close. Now from the relation $Z_c=-\xi_0^2/\alpha^{2/3}$ we can find the transition line:

$$n_c(T)=n_c(0)\left(1+\frac{T^2}{T_2^2}+\frac{T^{2/3}}{T_3^{2/3}}\right),$$
$$T_2=\frac{2\varepsilon_F}{\pi},\quad T_3=\frac{2\varepsilon_0}{\pi\beta(3Z_c)^{2/3}}\approx 0.01\frac{p_F}{m}\varepsilon_F. \tag{41}$$

Since $Z_c\gg 1$, T_3 is low; T_3 is equal to ε_F only in the uninteresting region $m/p_F<10^{-2}$, where the largeness of Z_c is canceled out by the smallness of m/p_F. Thus, the T^2 term in (41) can be neglected when $T\ll\varepsilon_F$. The melting point of the lattice is low, i.e., as low as T_3:

$$T_c(n)=T_3\left(\frac{n-n_c(0)}{n_0(0)}\right)^{2/3},\quad n>n_c(0); \tag{42}$$

for $m\sim p_F$, $T_3\sim\varepsilon_F(10^{-2}$–$10^{-1})$.

2. Let us estimate the jumps that occur in the principal characteristics of the system on the melting curve. The pion-field jump $a_c^2=\alpha^{2/3}Z_c/2$ increases like $T^{2/3}$. For comparison, let us give the value of a^2 on the melting curve in the Thomas-Fermi approximation: $a^2(n=n_c)=\alpha^{2/3}Z_c/(1-\delta^2)$,. The factor-of-two discrepancy between these two values is due to the fact that, on the transition line, only one half of the pions drop out into the condensate; the other half is in the gaseous phase, and has a roton gap $\ll T$. Although the thermal fluctuations shift the transition line a great distnace away from n_c^0, the quantity a^2 increases with increasing T. The jump in the roton gap is large on the transition line:

$$\xi_1^2(n_c)=\frac{4}{Z^2}\alpha^{2/3}\sim T^{2/3},\quad \xi_2^2(n_c)=\frac{16}{Z_c^2}\alpha^{2/3}\sim T^{2/3},$$

$$\frac{\xi_2^2(n_c)}{\xi_1^2(n_c)}=4,\quad \xi_{2TF}^2=\frac{32}{Z_c^2}\alpha^{2/3}. \tag{43}$$

For comparison, we give in (43) the value of ξ_2^2 in the mean-field approximation. The density of the classical pion gas in the background of the condensate can be found from the formula (31). Since, according to (43), the roton gap increases discontinuously by a factor of four, the density of the epicondensate pions decreases by a factor of two on the transition line. This is a manifestation of the pion-number conservation law discussed above.

3. The μ and S jumps are small on the transition line:

$$\Delta S=\frac{18\pi}{Z_c}\frac{m}{p_F}n_c\alpha^{1/3}\sim T^{1/3},\quad \frac{\Delta\mu}{\varepsilon_F}=\frac{3}{7}\left(\frac{6}{Z_c}\right)^2\alpha^{2/3}\sim T^{2/3}.$$

Let us recall that the dominant term in the expression (22) for the entropy is $\sim T^{1/2}$ or T; therefore, $\Delta S\ll S$. The sound velocity u^2 depends only on the parameter Z, and not on n and T separately:

$$u_1^2=u_0^2\left(1-\frac{9}{14}\frac{1}{1+\chi_1^3}\right),\quad \chi_1^2+Z=\frac{2}{\chi_1};$$
$$u_2^2=\frac{5}{14}u_0^2\left(1-\frac{9}{5}\frac{\chi_2^3\delta^2}{(1-\delta^2)\chi_2^3-1}\right),\quad Z=\left(\frac{1}{\delta^2}-1\right)\chi_2^2+\frac{2}{\chi_2}. \tag{44}$$

The expression (44) gives the implicit dependence of u^2 on the density and the temperature. It follows from (44) that the curve $Z=$ const are constant-compressibility lines, since χ_1 and χ_2 depend only on Z. The melting curve $Z=Z_c$ is such a line; therefore, the compressibility is constant on it. Since the quantities χ_1 and χ_2 are small on this curve, the sound velocity jump is also small:

$$\chi_1^2(n=n_c)=\frac{\delta^2}{4}\ll 1,\quad \chi_2^2(n=n_c)=2\delta^2\ll 1,$$
$$u_1^2(Z_c)\approx u_2^2(Z_c)\approx{}^5/_{14}u_0^2;\quad \Delta u^2\approx 0.15\,u_0^2.$$

Let us give the function u_2^2 in the vicinity of the threshold $Z\approx Z_1$:

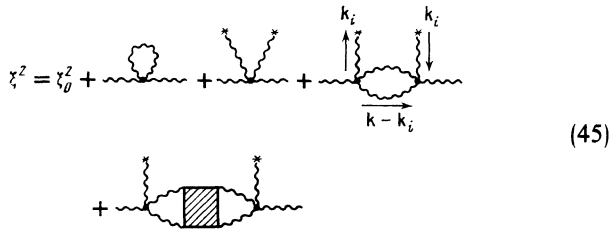
$$u_2^2(Z)=\frac{5}{14}u_0^2\left[1-\frac{3\sqrt{3}\delta^{5/3}}{5(Z-Z_1)^{1/2}}\right].$$

4. It follows from the formulas (41) that the critical density is already two times higher than $n_c(0)$ at a temperature $T\sim 0.1\varepsilon_F$. Since the analysis is based on the assumption that $(n-n_c)/n_c^0<1$, we cannot consider the high-temperature region $T>0.1\varepsilon_F$. The energies of the crystalline and homogeneous phases are close; therefore, the lattice melts at $T\ll\varepsilon_F$ in the region where the neutrons are still degenerate. The system does not undergo any drastic changes on the melting curve: the jumps in all the thermodynamic characteristics are small.

We were not able to obtain the analytic solution in the region of medium T. The melting curve for these T in Fig. 1 was obtained through extrapolation from the low- and high-temperature regions. Since cancellation of the contributions of the classical and quantum fluctuation to the free energy occurs here, allowance for even the weak effects that are lost in the Thomas-Fermi approximation can greatly affect the character of the transition.

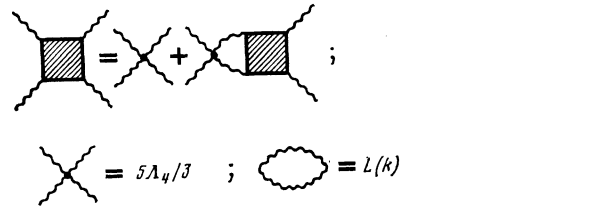
7. ANALYSIS OF THE APPROXIMATIONS MADE

To determine the region of applicability of the theory, let us write down the diagrammatic equation for the roton gap:



(45)

We took only the first two diagrams into account. The next terms are important in the vicinity of the singular points on the sphere $|\mathbf{k}|=k_0$ when $\mathbf{k}=\mathbf{k}_i$. Here the singularities of the two D functions are closer to each other, and these diagrams are large. Let us determine the $\pi\pi$ scattering amplitude $\Gamma_{\pi\pi}$ for the case in which in one channel the momentum transfer $k=0$ and in the two others $k\sim k_0$:



$$\Gamma_{\pi\pi}=\frac{5\Lambda_4/3}{1-(5\Lambda_4/3)L(0)},\qquad L(0)=-T\sum_{\omega_n}D^2(\omega_n,k)\frac{d^3k}{(2\pi)^3}.$$

(46)

There is no need to compute the quantity $L(0)$ in (46): it is connected by an identity with the function Φ:

$$L(0)=(\Lambda\nu)^{-1}\partial\Phi/\partial\xi^2.$$

The equation for the gap $\xi^2(\mathbf{k}_i)\equiv\xi_i^2$ now has the form

$$\xi_i^2=\xi_0^2+\Phi(\xi_i^2)+a^2\left(1+\frac{5}{42}\frac{5\Phi_{\xi^2}'/14}{1-5\Phi_{\xi^2}'/14}\right).$$

The correction is a maximum, and is equal to 5/42, when $\Phi'=-\infty$. This number determines the scale of the anisotropy of the roton gap ξ^2; the cause of its smallness is the same as for the quantity, δ^2: the first two diagrams in (45) are proportional to the total Λ, which is large; the corrections contain the small quantity Λ_4. Let us now give a symbolic estimate, unrelated to this smallness. Since the region $\mathbf{k}\sim\mathbf{k}_i$ is integrally small, and ξ^2, averaged over the broad range of angles of the vector $\mathbf{k}$, enters into all the equations, we should compare the third diagram in (45) with the second for $\mathbf{k}\neq\mathbf{k}_i$. For $T=0$, when

$$1\gg\frac{m}{p_F}\ln\frac{1}{\xi^2},$$

the third diagram is small. Then $\Lambda_4 L(k\sim k_0)\ll 1$, and the ladder approximation can be used to compute $\Gamma_{\pi\pi}$. The quantity ξ^2 has its minimum value at the transition point, where $\xi^2\sim(m/p_F)^3$. Therefore, for the theory to be applicable at $T=0$, it is sufficient that the weak inequality $m/p_F<1$ be satisfied. In the classical case, the corrections are small if $\xi^2\gg(T/\varepsilon_F)(m/p_F)^{1/2}$. Substituting the minimum value of ξ^2 on the transition line into this inequality, we arrive at the criterion $T\ll\varepsilon_F(m/p_F)^{1/2}$. Then, as can be seen from (21), the pion concentration c_π is again small.

8. CONCLUSION

1. In the paper we have considered the effects connected with the appearance in a neutronic medium of soft pion excitations near the phase transition point. The material then differs greatly from the normal Fermi liquid. The main results of the paper are not related to the smallness of the adiabaticity parameter m/p_F. This parameter is "expelled" from the theory by the transformation of the n and T scales. At $T\ll\varepsilon_F$ the neutrons constitute a cold active medium. Their activity is manifested in strong striction effect. Allowance for the nucleon-nucleon repulsion at small distances will enhance this effect even more. In a dense neutron gas the inhomogeneous spin structure cannot be strongly pronounced, since even in second-order perturbation theory in terms of the condensate field a total-density modulation is imposed on the system, and this results in an energetically highly disadvantageous situation. The difference between the homogeneous and crystalline phases is purely superficial: their energies are close, which results in a small melting point for the lattice. Even at points deep in the supercritical region a temperature of several MeV is enough to destroy the lattice. A classical analysis is then suitable for the pion gas. The state of the pion-excitation gas changes abruptly on the melting curve. In the homogeneous phase, the system is in a precondensate state, a fact which manifests itself in the high occupation numbers $n_{\mathbf{k}}$ of pions with $k\sim k_0$. This essentially means that there is short-range order in the homogeneous phase. In its properties, the homogeneous phase resembles a

dirty metal with magnetic impurities. The mean-field approximation cannot be used to describe its properties, and perturbation theory cannot reveal the difference between a dense gas and a liquid in which there is short-range order.

2. Allowance for the finiteness of the system may lead to the appearance of qualitatively new effects. In a finite system the pion condensate is not a classical, but a quantum, object.[1] The pion-field fluctuations does not now have such a strong influence on the phase transition, since the long-range interaction due to the soft-pion exchange is inevitably cut off at distances of the order of the nuclear dimension. Furthermore, since the energies of all the phases of the material are close, even a small surface energy can lead to the lifting of this degeneracy. The surface effect is not a weak one: the correction $\sim A^{-1/3}$ should be compared not with unity, but with the parameter δ^2. There exist in an isotropically symmetric medium (i.e., in a medium in which $N = Z$) π-condensate phases in which the spin and isospin densities are modulated, but the total density is a constant[1]: spiral pion-field structures. The striction effect is less important for these structures, and they can be highly developed. An effect similar to the above-considered thermal effect can manifest itself in a finite system. The equation for the roton gap can have solutions for which $\Delta \sim 1/R^2$, where R is the nuclear radius. The result then depends on the relationship between $(n - n_c)/n_c$ and $1/R^2$. Such a solution corresponds to a coherence state of the entire system, and is a low-lying collective level with pion quantum numbers.

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