Surface effects in the theory of wake potentials

Yu. V. Kononets and G. M. Filippov

Chuvash State University
(Submitted 6 November 1981; resubmitted 16 April 1982)
Zh. Eksp. Teor. Fiz. **83**, 971–987 (September 1982)

The theory of coherent states is used to solve the problem of polarization induced by a charged particle moving in a semi-infinite electron gas. The structure of the polarization fields inside and outside the medium is analyzed in the long-wave approximation by taking into account the contributions of both surface and volume single-particle and collective excitations for an arbitrary position of the particle trajectory relative to the interface. The subsurface transition regime is investigated and it is shown that the effects due to excitation and polarization of the surface plasmon system, as well as to features peculiar to the interaction between the particle and the volume modes in the presence of a metal-vacuum interface, alter significantly the properties of the wake potentials inside the metal at a distance $z \lesssim v/\omega_0$ from the surface (ω_0 is the plasma frequency of the electron gas and v is the particle velocity; it is assumed that $v_F \ll v \ll c$) compared with the situation in an infinite medium. The energy losses and their fluctuations are determined for a fast charged particle reflected from a metal sample when part of the particle trajectory passes through the interior of the sample. The plasmon part of the energy losses in the case of normal incidence and backscattering is found to undergo oscillations that depend on the scattering depth in the subsurface layer.

PACS numbers: 72.30. + q

1. INTRODUCTION

An interesting trend connected with the specific influence of the polarization of a medium on the bremsstrahlung and expansion of fast molecular ions passing through thin solid films has recently developed in the physics of interactions between charged particles and matter.[1–5] A central position in the theory of effects of this kind is occupied by the question of the polarization potential induced in the medium by one of the particles of a cluster moving in the medium and acting on the other particles of this cluster. There are also a number of other subtle phenomena[6] due to the onset of a nonequilibrium distribution of the charge in the "wake" of a charged particle moving through the medium. At present the properties of the corresponding wake potentials are actually sufficiently well understood only in one simplest case of motion of a point charge in an infinite homogeneous electron gas.[7]

Yet it is perfectly clear that the properties of the polarization potentials near the surface of a metallic sample should be substantially altered by two effects: (1) The onset of an induced surface charge that modifies substantially the field on both sides of the surface. (2) Since the fields of the elementary volume excitations do not leave the metal,[8] generation of volume modes sets in in practice only at the instant when the particle crosses the surface of the sample. This is equivalent to instantaneous turning on of the interaction with these modes and implies the existence of some nonstationary transition regime. A consistent analysis of the surface effects in the problem of the polarization of a metallic medium by a moving charged particle is in fact the subject of the present paper.

It is known that an immobile charged particle placed near a metal produces a distributed surface charge whose field outside the metal is equivalent to the field of the electrostatic image. The field inside the metal is completely screened and is equal to zero. Motion of the particle is simultaneously accompanied by motion and deformation of the electrostatic-image field. Since the eigenstates of the electromagnetic field + surface charge system are described by surface plasmons, the screening of the field of a moving particle, due to the distribution of the induced surface charge, should naturally be regarded as a problem of excitation of surface plasmons. In this case the motion of the surface charge is the response of the system of surface plasmons to an external action and manifests itself both in the polarization of the vacuum and in excitation of this system.

Suzuki *et al.*[9] have recently reported the results of an analysis of surface polarization effects accompanying the escape of a fast charged particle from a metal into vacuum. Unfortunately, the expressions for the polarization potentials, obtained in their paper by a Green's-function technique, give the correct result only for the case of escape normal to the surface. It is impossible to indicate the source of the error, since their paper does not contain the detailed calculations and cites only the final results. It appears that it is necessary also to review the results of a later paper by Ohtsuki *et al.*,[10] who used the erroneous expressions of Ref. 9 to investigate the singularities of the motion of fast charged particles traveling inside a metal at a small angle to the surface.

The method used by us to calculate the fields excited in a medium by an external particle is based on the theory of coherent states.[11] The gist of the method is to reduce the problem of finding the polarization field to the problem of the state of a system of oscillators, each of which is acted upon by a certain alternating external force. The simple form of the corresponding Hamiltonian of that interaction makes

 0038-5646/82/090547-10$04.00

it possible to write down the exact solution for each oscillator in the form of the so-called coherent state, which is then used to calculate the polarization fields produced by the external particle that executes the specified motion. This approach presupposes exact knowledge of the structure of the electromagnetic fields connected with the elementary excitations of various types.

The theory of surface excitations, based on the use of the long-wave approximation, makes it possible to find the structure of the electromagnetic field of a long-wave surface excitation and to determine the surface part of the polarization field in the most interesting region of the distances from the entrance surface, which satisfy the condition

$$|z| \gg z_c \equiv \hbar/p_F \tag{1}$$

(p_F is the Fermi momentum of the electron; the z axis is normal to the surface).

As for the volume part of the polarization potential, the structure of the electromagnetic fields of both long-wave and short-wave volume elementary excitations can be determined on the basis of the general theory of the dielectric constant of homogeneous systems. The use of the method of coherent states makes it possible then, in the case of an infinite medium, to obtain directly the known results for the wake potential, and in the presence of the boundary also to investigate the resultant transition regime.

Knowing the state of the system of elementary excitations of a metal at an arbitrary instant of time we can also calculate directly the energy losses and their fluctuations for a charged particle that moves along a given trajectory of any type. The solution of this problem is of particular interest in connection with the problem of reflection of charged particles from a metallic surface. Lucas[12] recently investigated the question of the energy lost to excitation of surface plasma oscillations when a particle moves outside a metal near its surface along a reflecting trajectory. Similar results for this case were later obtained by Núñez *et al.*[13] within the framework of the specular-reflection model. In a real situation, however, part of the reflected trajectory always passes through the volume of the sample, and the question of the energy lost to generation of surface and volume excitations under these conditions has remained open to this day.

The results reported below have a general asymptotic character, since relation (1), which defines the region of their applicability, mans actually also that they are independent of the concrete structure of the surface of a pure metallic sample.

2. STRUCTURE AND QUANTIZATION OF ELECTROMAGNETIC FIELDS OF ELEMENTARY EXCITATIONS IN A SEMI-INFINITE ELECTRON GAS

We consider in this paper nonrelativistic motion of external charged particles. This enables us to confine ourselves to a study of only longitudinal polarization fields, since effects connected with excitation of transverse electromagnetic waves, when nonrelativistic particles are stopped in homogeneous and nongyrotropic media, turn out to be smaller by approximately a factor v^2/c^2 (Ref. 14). For longitudinal fields, the vector potential can be chosen equal to zero (Coulomb gauge) and the problem reduces to a calculation of the corresponding scalar potential φ.

It is known[15] that surface excitations of different types appear in the solution of the problem of an electromagnetic field interacting with a semi-infinite medium. By virtue of the translational symmetry of the problem, the dependence of φ on the coordinates x and y and on the time t, for particular solutions, takes the form $\exp\{i(\mathbf{k}_\parallel \boldsymbol{\rho} - \omega t)\}$, where $\boldsymbol{\rho} = (x, y)$ and $\mathbf{k}_\parallel$ is the corresponding two-dimensional wave vector. At large distances from the boundary plane $z = 0$, both inside ($z > 0$) and outside ($z < 0$) of the medium, the potential $\varphi^{(s)}$ connected with the surface excitations should satisfy the usual Laplace equation, and consequently

$$\varphi^{(s)} = g \exp\{-k_\parallel |z| + i(\mathbf{k}_\parallel \boldsymbol{\rho} - \omega_s t)\}, \tag{2}$$

where the constant g, generally speaking, can have different values for positive and negative values of z. Near the boundary $z = 0$ there exists a thin near-surface layer, inside of which the dependence of $\varphi^{(s)}$ on z differs from (2) because it contains a distributed induced charge. For a semi-infinite electron gas having the usual metallic density, the dimension that characterizes the thickness of the transition layer is the quantity z_c [Eq. (1)].

However, as already noted, of greatest interest in our case is the calculation of polarization fields at sufficiently large distances $|z| \gg z_c$ from the boundary, where the principal role is played by the components of the fields with small $\mathbf{k}_\parallel$. Under these conditions, the structure of the fields in the subsurface transition region turns out to be inessential, and the problem of determining the potential $\varphi^{(s)}$ in this region can be solved by stipulating at the boundary $z = 0$ the usual continuity of the normal component of the electric-induction that corresponds to the field (2), and the continuity of the tangential component of the electric-field intensity. On the other hand, when calculating the electric induction in the case of long-wave field it is natural to assume that the properties of the medium are described everywhere, all the way to its boundary, by a dielectric function corresponding to an unbounded medium (the long-wave approximation). As a result we arrive at a dispersion equation that determines the dependence of the frequencies ω_s of the surface excitations on the twodimensional wave vector $\mathbf{k}_\parallel$:

$$1 + \frac{2k_\parallel}{L} \sum_{k_z} \frac{\varepsilon(\omega_s; \mathbf{k}_\parallel, k_z)}{k_\parallel^2 + k_z^2} = 0, \tag{3}$$

where $\varepsilon(\omega; \mathbf{k}_\parallel, k_z)$ is the longitudinal dielectric constant of the unbounded medium and corresponds to the frequency ω and to a three-dimensional wave vector $\mathbf{k} = (\mathbf{k}_\parallel, k_z)$; L is the normalization length in the z direction.

In the limit of small $\mathbf{k}_\parallel \ll p_F/\hbar$, when the spatial dispersion can be neglected and we can put $\varepsilon(\omega, \mathbf{k}) = 1 - \omega_0^2/\omega^2$, Eq. (3) leads to a well known result[16,17] that connects the frequency of the surface plasmons with the frequency ω_0 of the volume plasma oscillations:

$$\omega_s = \omega_0/\sqrt{2}. \tag{4}$$

In the general case Eq. (3) has also solutions that correspond to other surface-excitation types. Thus, in the random-phase approximation there follow from (3) a number of solutions that describe single-particle surface excitations. The dielectric constant $\varepsilon(\omega,\mathbf{k})$ should in this case have, of course, a "microscopic" form with conservation of the sum over the electronic states. We emphasize that this circumstance is of fundamental significance for the theory of surface excitations.

In fact, it is easy to verify that the use of a "macroscopic" expression for $\varepsilon(\omega,\mathbf{k})$, in which the summation over the electronic states is replaced by integration, on the one hand, leads to a complete vanishing of the branch of the single-particle surface excitations, and on the other, adds to the dispersion law of the surface plasmons a nonanalytic term, linear in $\mathbf{k}_\parallel$, with a complex coefficient. In the literature there are several attempts to understand the structure[18] and the cause[19] of the appearance of this term in $\omega_s(\mathbf{k}_\parallel)$ within the framework of the hydrodynamic model. Our analysis shows, however, that the surface plasmons arise in the theory as an auxiliary concept, convenient for the description of the system of a large number of single-particle surface excitations with respect to which the problem of the nonanalyticity of the dispersion law does not arise.

In the analysis that follows we start from exact microscopic properties of the dielectric function. At the same time, the final results for the polarization fields, as we shall see, can be expressed in terms of a "macroscopic" dielectric constant of integral type, and by the same token we can establish its important and final place in the theory. We shall also verify that at least in the case of fast external charged particles ($v \gg v_F$) the employed long-wave approximation makes it possible to describe with high accuracy the surface polarization effects of interest to us.

For what follows it is convenient to normalize the fields of the elementary excitations in such a way that the energy corresponding to double the real part of the field (2) be equal to the quantum energy $\hbar\omega_s$. Using the known expression for the energy of a longitudinal monochromatic field in a nonabsorbing medium (see, e.g., Ref. 14), we write the normalization condition in the form

$$\int \frac{d\mathbf{r}\,d\mathbf{r}'}{4\pi} \mathbf{E}^*(\mathbf{r}',t)\mathbf{E}(\mathbf{r},t)\frac{\partial}{\partial\omega}[\omega\varepsilon(\omega,\mathbf{r},\mathbf{r}')]=\hbar\omega, \tag{5}$$

where $\mathbf{E}(\mathbf{r},t)$ is the intensity of the electric field with potential (2). In the long-wave approximation it is easy to obtain from this an explicit expression for the "elementary" surface field:

$$\varphi^{(s)}_{\mathbf{k}_\parallel\alpha}=g_{\mathbf{k}_\parallel\alpha}\exp\{-k_\parallel|z|+i(\mathbf{k}_\parallel\boldsymbol{\rho}-\omega_{s\alpha}t)\},$$
$$g^2_{\mathbf{k}_\parallel\alpha}=\frac{4\pi\hbar}{k_\parallel S}\left[\frac{2k_\parallel}{L}\sum_{k_z}\frac{1}{k_\parallel^2+k_z^2}\frac{\partial\varepsilon(\omega_{s\alpha};\mathbf{k}_\parallel,k_z)}{\partial\omega_{s\alpha}}\right]^{-1}. \tag{6}$$

Here S is the normalization area of the interface, the index α numbers the branches of the surface excitations, and $\omega_{s\alpha}=\omega_{s\alpha}(\mathbf{k}_\parallel)$ are the frequencies of these excitations. In the derivation of the formula for $g^2_{\mathbf{k}_\parallel\alpha}$ in explicit form we used the condition (3). In the case of long-wave surface plasmons

$$g^2_{\mathbf{k}_\parallel}=\pi\hbar\omega_s/k_\parallel S, \tag{7}$$

where ω_s is given by (4).

In the absence of external perturbations, the arbitrary field due to the surface modes of the excitations can be written in the form of a superposition of "elementary" fields of the type (6):

$$\varphi^{(s)}(\mathbf{r},t)=\sum_{\mathbf{k}_\parallel,\alpha} g_{\mathbf{k}_\parallel\alpha}\exp(-k_\parallel|z|)[a_{\mathbf{k}_\parallel\alpha}\exp(i\mathbf{k}_\parallel\boldsymbol{\rho})+\text{c.c.}], \tag{8}$$

where the coefficients $a_{\mathbf{k}_\parallel\alpha}$ depend on the time in accordance with the equations

$$\ddot{a}_{\mathbf{k}_\parallel\alpha}+\omega_{s\alpha}^2 a_{\mathbf{k}_\parallel\alpha}=0.$$

Calculating the energy E_s of the "free" field (8) in the medium, we must obviously obtain the energy of the system of surface elementary excitations. Recognizing that the energy of the aggregate of excitations with given $\mathbf{k}_\parallel$ and α is determined by the energy of an individual monochromatic field component of the type $2\,\mathrm{Re}\{a_{\mathbf{k}_\parallel\alpha}\varphi^{(s)}_{\mathbf{k}_\parallel\alpha}(\mathbf{r},t)\}$, and taking into account the normalization condition (5) for the "elementary" field $\varphi^{(s)}_{\mathbf{k}_\parallel\alpha}(\mathbf{r},t)$, we get

$$E_s=\sum_{\mathbf{k}_\parallel,\alpha}\hbar\omega_{s\alpha}(\mathbf{k}_\parallel)\,a^*_{\mathbf{k}_\parallel\alpha}a_{\mathbf{k}_\parallel\alpha}. \tag{9}$$

Following the universally known scheme,[20] we can introduce canonically conjugate variables that are connected in standard fashion with the quantities $a_{\mathbf{k}_\parallel\alpha}$ and $a^*_{\mathbf{k}_\parallel\alpha}$, and quantize the field (8). As a result we arrive at a surface-field operator

$$\hat{\varphi}_s(\mathbf{r},t)=\sum_{\mathbf{k}_\parallel,\alpha} g_{\mathbf{k}_\parallel\alpha}\exp(-k_\parallel|z|)\times[\hat{a}_{\mathbf{k}_\parallel\alpha}\exp(i\mathbf{k}_\parallel\boldsymbol{\rho}-i\omega_{s\alpha}t)+\text{H.c.}] \tag{10}$$

where $\hat{a}^+_{\mathbf{k}_\parallel\alpha}$ and $\hat{a}_{\mathbf{k}_\parallel\alpha}$ are the usual Bose creation and annihilation operators for the surface quasiparticles. In accordance with (9), the Hamiltonian $\hat{H}^0_s$ of a system of free surface excitations is represented in standard form:

$$\hat{H}_s^0=\sum_{\mathbf{k}_\parallel,\alpha}\hbar\omega_{s\alpha}(\mathbf{k}_\parallel)\,\hat{a}^+_{\mathbf{k}_\parallel\alpha}\hat{a}_{\mathbf{k}_\parallel\alpha}+\text{const}. \tag{11}$$

In the presence of external charged particles that execute a specified motion, additional terms appear in the Hamiltonian of the system and describe the interaction of the surface excitations with the external particles,

$$\hat{H}_s^{int}=\int\tilde{\rho}(\mathbf{r},t)\hat{\varphi}_s(\mathbf{r},t)\,d\mathbf{r}. \tag{12}$$

Here $\tilde{\rho}(\mathbf{r},t)$ is the charge density of the external particles, and the operator $\hat{\varphi}_s(\mathbf{r},t)$ is defined by (10).

As for the volume excitations in a semi-infinite medium, in the long-wave approximation the dependence of the frequencies $\omega_\beta(\mathbf{q})$ on the total vector $\mathbf{q}$ is determined by the zeros of the volume dielectric function $\varepsilon(\omega,\mathbf{q})$. It is easily seen then that the electric induction for the volume modes in the medium must be equal to zero, and consequently the fields of the volume excitations cannot leave the medium (the refinements contained in Ref. 8 are of no importance for the region of large distances from the boundary surface, which is of interest to us) and are represented by expressions of the type

$$\varphi_{q\beta}^{(V)}(\mathbf{r},t)=A_{q\beta}\,\theta(z)\sin(q_z z)\exp\{i[\mathbf{q}_{\parallel}\rho-\omega_\beta(\mathbf{q})t]\}. \quad (13)$$

Here $\theta(z)$ is the unit step function

$$\theta(z)=\begin{cases}1, & z\geqslant 0\\ 0, & z<0\end{cases}.$$

The normalization coefficients $A_{q\beta}$ are determined by the condition (5) with an intensity $\mathbf{E}(\mathbf{r},t)$ corresponding to the field (13), and are equal to

$$A_{q\beta}=\left[8\pi\hbar/\Omega q^2\frac{\partial\varepsilon(\omega,\mathbf{q})}{\partial\omega}\Big|_{\omega=\omega_\beta(\mathbf{q})}\right]^{1/2}, \quad (14)$$

where Ω is the normalization volume.

The quantization of the field due to the volume excitations leads to the "volume" field operator

$$\hat{\varphi}_V(\mathbf{r},t)=\sum_{\beta,\mathbf{q}_{\parallel},q_z>0}A_{q\beta}\theta(z)\sin(q_z z)$$
$$\times[\hat{b}_{q\beta}\exp(i\mathbf{q}_{\parallel}\rho-i\omega_\beta(\mathbf{q})t)+\text{H.c.}], \quad (15)$$

where $\hat{b}^+_{q\beta}$ and $\hat{b}_{q\beta}$ are the operators of the creation and annihilation of volume elementary excitations of type β that obey the Bose commutation relations. The Hamiltonian of the system of volume excitations, which takes into account their interaction with external particles, is formally of the same form as in the case of a system of surface excitations.

When describing systems of surface and volume elementary excitations within the framework of the long-wave approximation, a number of far reaching beautiful analogies can be easily discerned. In particular, if we introduce with the aid of the relation

$$\varepsilon_s(\omega,\mathbf{k}_{\parallel})=1+\frac{2k_{\parallel}}{L}\sum_{k_z}\frac{\varepsilon(\omega;\mathbf{k}_{\parallel},k_z)}{k_{\parallel}^2+k_z^2} \quad (16)$$

a "surface" dielectric constant $\varepsilon_s(\omega,\mathbf{k}_{\parallel})$ that depends on the frequency ω and on the two-dimensional wave vector $\mathbf{k}_{\parallel}$, then, on the one hand, the frequencies of the surface elementary excitations are determined by the zeros of the function $\varepsilon_s(\omega,\mathbf{k}_{\parallel})$ [see Eq. (3)], and on the other, the factors $g_{\mathbf{k}_{\parallel}\alpha}$ [Eq. (6)], which determine the structure of the surface field (8) and play the most important role in the theory, will be expressed in terms of frequency derivatives of the "surface" dielectric constant, in full accord with the expressions for the structure factors $A_{q\beta}$ (14) of the volume excitations.

3. SURFACE PART OF POLARIZATION POTENTIAL FOR A POINT CHARGE ENTERING THE METAL ALONG A NORMAL TO THE SURFACE

We assume that an external point particle with charge Ze moves along the z axis with velocity $\mathbf{v}$ and crosses the vacuum-metal interface at the instant of time $t=0$. For the Hamiltonian of the interaction of the system of surface excitations with such a particle we have from (12) (we use the interaction representation throughout)

$$\hat{H}_s^{int}=Ze\sum_{\mathbf{k}_{\parallel},\alpha}g_{\mathbf{k}_{\parallel}\alpha}e^{-k_{\parallel}v|t|}[\hat{a}_{\mathbf{k}_{\parallel}\alpha}(t)+\hat{a}^+_{\mathbf{k}_{\parallel}\alpha}(t)]. \quad (17)$$

To find the state of the system at an arbitrary instant of time, we use the solution of the problem of the motion of a quantum oscillator under the influence of an alternating external field. We denote by the symbol $|\text{vac}\rangle$ the ground state of the system of surface excitations at $t=-\infty$. Then, at the instant of time t, the state of the system will be described by the direct product of the so called coherent states (see, e.g., Ref. 11) of the type

$$|\Psi(t)\rangle=\prod_{\mathbf{k}_{\parallel},\alpha}\exp\Big\{-i\Phi_{\mathbf{k}_{\parallel}\alpha}(t)-\frac{1}{2}|Q_{\mathbf{k}_{\parallel}\alpha}(t)|^2$$
$$+Q_{\mathbf{k}_{\parallel}\alpha}(t)\hat{a}^+_{\mathbf{k}_{\parallel}\alpha}(t)\Big\}|vac\rangle, \quad (18)$$

where the complex functions $Q_{\mathbf{k}_{\parallel}\alpha}(t)$ and $\Phi_{\mathbf{k}_{\parallel}\alpha}(t)$ are defined by the expressions

$$Q_{\mathbf{k}_{\parallel}\alpha}(t)=-i\frac{Ze}{\hbar}g_{\mathbf{k}_{\parallel}\alpha}\exp[-i\omega_{s\alpha}(\mathbf{k}_{\parallel})t]$$
$$\times\int_{-\infty}^{t}\exp\{i\omega_{s\alpha}(\mathbf{k}_{\parallel})t'-k_{\parallel}v|t'|\}dt',$$
$$\Phi_{\mathbf{k}_{\parallel}\alpha}(t)=\int_{-\infty}^{t}\text{Im}[\dot{Q}_{\mathbf{k}_{\parallel}\alpha}(t')Q_{\mathbf{k}_{\parallel}\alpha}(t')]dt'+\text{const}. \quad (19)$$

We emphasize that $|\Psi(t)\rangle$ (18) is the exact dolution of the corresponding time-dependent Schrödinger equation, which satisfies the condition $|\Psi(-\infty)\rangle=|\text{vac}\rangle$.

The surface polarization field $\varphi_s(\mathbf{r},t)$ should now be defined as the mean value of the operator (10) averaged over the obtained coherent state of the system of surface excitations. Since the mean values of the operators $\hat{a}_{\mathbf{k}_{\parallel}\alpha}(t)$ and $\hat{a}^+_{\mathbf{k}_{\parallel}\alpha}(t)$ in this state differ from zero and are equal respectively to $Q_{\mathbf{k}_{\parallel}\alpha}(t)$ and $Q^*_{\mathbf{k}_{\parallel}\alpha}(t)$, we have for $\varphi_s(\mathbf{r},t)$ simply

$$\varphi_s(\mathbf{r},t)=\sum_{\mathbf{k}_{\parallel},\alpha}g_{\mathbf{k}_{\parallel}\alpha}\exp(-k_{\parallel}|z|)\{\exp(i\mathbf{k}_{\parallel}\rho)Q_{\mathbf{k}_{\parallel}\alpha}(t)+\text{c.c.}\}. \quad (20)$$

When analyzing the obtained expression, we change from summation over α at a fixed $\mathbf{k}_{\parallel}$ to integration over the frequencies, in acordance with the formulas

$$\sum_\alpha f(\omega_{s\alpha})\left[\frac{\partial\varepsilon_s(\omega_{s\alpha},\mathbf{k}_{\parallel})}{\partial\omega_{s\alpha}}\right]^{-1}=-\frac{1}{\pi}\int_0^\infty d\omega f(\omega)\,\text{Im}\frac{1}{\bar{\varepsilon}_s(\omega,\mathbf{k}_{\parallel})}, \quad (21)$$

where $\bar{\varepsilon}_s(\omega,\mathbf{k}_{\parallel})$ is the "integral" dielectric function obtained from the microscopic expression by integrating over the electronic states. Relation (21) is proved in the same manner as used in the theory of oscillations of an impurity atom to prove the formula for the transition from summation over the shifted frequencies to integration (see, e.g., Ref. 21).

By substituting in (20) the values of $Q_{\mathbf{k}_{\parallel}\alpha}(t)$ calculated in accordance with (19) we arrive, after integrating over the directions of the vector $\mathbf{k}_{\parallel}$, at the expression

$$\varphi_s(\mathbf{r},t)=\frac{4Ze}{\pi}\int_0^\infty d\omega\int_0^\infty dk_{\parallel}\frac{\omega J_0(k_{\parallel}\rho)}{\omega^2+k_{\parallel}^2v^2}\exp(-k_{\parallel}|z|)\Big\{\exp(-k_{\parallel}v|t|)$$
$$+\theta(t)\frac{2k_{\parallel}v}{\omega}\sin\omega t\Big\}\text{Im}\frac{1}{\bar{\varepsilon}_s(\omega,k_{\parallel})}. \quad (22)$$

The integral of the first term in the curly brackets with respect to the frequencies can be calculated in the explicit for and yields

$$\int_0^\infty \frac{d\omega\omega}{\omega^2+k_\parallel^2v^2}\,\mathrm{Im}\frac{1}{\varepsilon_s(\omega,\mathbf{k}_\parallel)}=-\frac{\pi}{2}\left[\frac{1}{2}-\frac{1}{\varepsilon_s(ik_\parallel v,\mathbf{k}_\parallel)}\right]. \quad (23)$$

At small $\mathbf{k}_\parallel$ using the Lindhard expression for $\bar{\varepsilon}(\omega,\mathbf{k})$ (Ref. 22), we get

$$\varepsilon_s(ik_\parallel v,\mathbf{k}_\parallel)\approx 2 + \frac{k_{TF}^2}{\pi k_\parallel^2}\int_{-\infty}^{\infty}\frac{dx}{(1+x^2)^2}\left[1-\frac{v}{2v_F}\frac{1}{(1+x^2)^{1/2}}\arcsin\frac{2vv_F(1+x^2)^{1/2}}{v^2+v_F^2(1+x^2)}\right], \quad (24)$$

where $k_{TF}=\sqrt{3}\omega_0/v_F$ is the Thomas-Fermi wave number. If the velocity is high, $v\gg v_F$, then

$$\varepsilon_s(ik_\parallel v,\mathbf{k}_\parallel)\approx 2+\omega_0^2/(k_\parallel v)^2, \quad (25)$$

and in the case of low velocities, $v\to 0$,

$$\varepsilon_s(ik_\parallel v,\mathbf{k}_\parallel)\approx 2+k_{TF}^2/2k_\parallel^2. \quad (26)$$

The integral, with respect to the frequencies, of the second oscillating term in the right-hand side of (22) can be calculated with sufficient accuracy at small $\mathbf{k}_\parallel$ with the aid of the following considerations. We note that the function $\mathrm{Im}\,1/\bar{\varepsilon}_s(\omega,\mathbf{k}_\parallel)$ has at small $\mathbf{k}_\parallel$ a sharp peak in the vicinity of the point $\omega=\omega_s$, with a width Γ that depends linearly on $\mathbf{k}_\parallel$. This becomes obvious after representing the quantity $\bar{\varepsilon}_s(\omega,\mathbf{k}_\parallel)$ at $\mathbf{k}_\parallel\to 0$, $\omega\approx\omega$, accurate to terms of first order in $\mathbf{k}_\parallel$, in the form

$$\varepsilon_s(\omega,\mathbf{k}_\parallel)\approx 2-\omega_0^2/\omega^2+l(\omega)k_\parallel, \quad (27)$$

where

$$l(\omega)=l_1(\omega)+il_2(\omega)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{dk_z}{k_z^2}[\varepsilon(\omega;0,k_z)-\varepsilon(\omega;0,0)]. \quad (28)$$

It follows from (27) that in the vicinity of $\omega=\omega_s$

$$\mathrm{Im}\frac{1}{\varepsilon_s(\omega,\mathbf{k}_\parallel)}\approx-\frac{\omega_s}{4}\frac{\Gamma(k_\parallel)/2}{\{\omega-\omega_s[1-{}^1/_4l_1(\omega_s)k_\parallel]\}^2+\Gamma^2(k_\parallel)/4}, \quad (29)$$

where

$$\Gamma(k_\parallel)={}^1/_2\omega_sl_2(\omega_s)k_\parallel. \quad (30)$$

The expression in the right-hand side of (29) can be regarded as the imaginary part of the reciprocal dielectric function $\bar{\varepsilon}_s^{-1}(\omega,\mathbf{k}_\parallel)$, corresponding to the presence in the system of only one type of surface excitation—surface plasmons having at small $\mathbf{k}_\parallel$ energies $\hbar\omega_s[1-\frac{1}{4}l_1(\omega_s)\mathbf{k}_\parallel]$ and finite lifetimes $\tau=\hbar/\Gamma(\mathbf{k}_\parallel)$. The damping of these excitations is not due to their decay into some other excitations, and has an entirely different physical meaning. Indeed, the surface plasmons appear as a concept that describes effectively the result of generation of a large number of undamped surface excitations. In the course of time a packet made up of these excitations and equivalent to one surface plasmon spreads out because of the difference between their energies and momenta. These processes of spreading of the packet is perceived by the observer as the damping of the surface plasmon. In the Lindhard approximation we easily obtain from (28) and (30)

$$\Gamma(k_\parallel)=\frac{\sqrt{6}}{32\chi}\left[1+\left(\frac{8}{3}\right)^{1/2}\chi\right]^{3/2}v_Fk_\parallel,\quad \chi^2=\frac{e^2}{\pi\hbar v_F}. \quad (31)$$

Using (23), (25), and (29) and retaining in the second oscillating term of (22) terms up to first order in $\mathbf{k}_\parallel$ in the pre-exponential factors, we arrive at the result

$$\varphi_s(\mathbf{r},t)=-Ze\left\{[\rho^2+(|z|+v|t|)^2]^{-1/2}-\int_0^\infty\frac{dk_\parallel k_\parallel^2J_0(k_\parallel\rho)}{k_\parallel^2+\omega_s^2/v^2}\times\exp[-k_\parallel(|z|+v|t|)]+2\theta(t)\frac{v}{\omega_s}\int_0^\infty dk_\parallel k_\parallel J_0(k_\parallel\rho)\times\exp[-k_\parallel|z|-\Gamma(k_\parallel)t/2]\sin\left[\omega_st\left(1-\frac{1}{4}l_1k_\parallel\right)\right]\right\}. \quad (32)$$

In (32) we singled out explicitly the potential corresponding to the electrostatic-image field of the charge. The second integral term is a correction and decreases at large distances like $v^2\omega_s^{-2}(|z|+v|t|)^{-3}$. It vanishes as $v\to 0$, but becomes substantial at high velocities $v\gg v_F$ and cancels out the electrostatic-image field at distances from the surface which satisfy the condition $|z|+v|t|\lesssim v/\omega_s$. The physical meaning of this cancellation is that the electrons in the subsurface region, located at short distances (comparable with v/ω_s) from the external particle, do not have time to be displaced enough to produce around themselves in vacuum a polarization field equivalent to the electrostatic-image field. At the same time, the electrons located farther, which acquire only small displacements, turn out to be perfectly capable of "keeping up" with the rapidly moving particle.

After the particle crosses the interface ($t>0$), free oscillations of the surface charge set in and are responsible for the appearance of the time-oscillating terms in the polarization field (32). The additional oscillating force, due to these terms, during the initial period of time $0<t<\pi/\omega$ after the entry, exerts a retarding action on the external particle.

In the case of small v, Eq. (22), with allowance for (23) and (26), determines the field of the slowly moving polarization charges that "follow" the displacement of the external charge. In particular, for a charge that is at rest at a distance $|z_0|$ from the surface, the polarization field (22) takes the form

$$\varphi_s(\mathbf{r})\approx-Ze\left\{\frac{1}{[\rho^2+(|z|+|z_0|)^2]^{1/2}}-\int_0^\infty\frac{dk_\parallel k_\parallel^2}{k_\parallel^2+k_{TF}^2/4}J_0(k_\parallel\rho)\exp[-k_\parallel(|z|+|z_0|)]\right\}. \quad (33)$$

For the case of a charge at rest outside the metal, it follows from (33) that the polarization field in vacuum is determined, as it should, by the field of the electrostatic image, and inside the metal it is equal to the self-field of the charge taken with a minus sign. Thus, the total field inside the metal is zero in this case. This statement does not pertain to the narrow subsurface layer, where the total field decreases like $(|z|+|z_0|)^{-3}$. The correction to the field of the electrostatic image in vacuum behaves in similar fashion, in full accord with the results of the analysis[23] of the situation with a charge at rest outside the metal.

We note that the total energy of a charge lying on the surface, $z=0$, is estimated according to (33) at $-\pi Z^2e^2k_{TF}/4$.

4. VOLUME FIELD IN THE PRESENCE OF AN INTERFACE AND THE TOTAL POLARIZATION POTENTIAL

In analogy with the situation with surface excitations, the evolution of the wave function of a system of volume excitations will be described by the direct product of the corresponding coherent states. Repeating the arguments of Sec. 3, we easily obtain with the aid of (15) the volume part of the polarization potential

$$\varphi_V(\mathbf{r},t)=\theta(t)\theta(z)\frac{Ze}{2\hbar}\sum_{\beta,\mathbf{q}_\parallel,q_z>0}A_{\mathbf{q}\beta}{}^2\sin(q_zz)\times\left\{ie^{i\mathbf{q}_\parallel\rho}\left[\frac{\exp(-i\mathbf{q}\mathbf{v}t)-\exp(-i\omega_\beta(\mathbf{q})t)}{\mathbf{q}\mathbf{v}-\omega_\beta(\mathbf{q})}-\frac{\exp(i\mathbf{q}\mathbf{v}t)-\exp(-i\omega_\beta(\mathbf{q})t)}{\mathbf{q}\mathbf{v}+\omega_\beta(\mathbf{q})}\right]+\text{c.c.}\right\}. \quad (34)$$

Replacing the summation over β by integration with respect to ω in accordance with the volume analog of (21), we can write (34) in the form

$$\varphi_V(\mathbf{r},t)=Ze\theta(z)\left[V(x,y,z;t)-V(x,y,-z;t)\right], \quad (35)$$

where

$$V(\mathbf{r},t)=-\theta(t)\frac{1}{2\pi^3}\int\frac{d^3\mathbf{q}}{q^2}\times\int_0^\infty d\omega\,\mathrm{Im}\frac{1}{\varepsilon(\omega,\mathbf{q})}\left\{\frac{e^{-i\mathbf{q}\mathbf{v}t}-e^{-i\omega t}}{\mathbf{q}\mathbf{v}-\omega}e^{i\mathbf{q}\mathbf{r}}+\text{c.c.}\right\} \quad (36)$$

is the field of a unit point charge moving with velocity v along the z axis in an infinite homogeneous medium, if the interaction of this charge with the particles of the medium was turned on instantaneously at the instant of time $t=0$. In other words, the volume part of the polarization potential inside the metal, in the presence of a plane boundary with a vacuum, is determined by a superposition of two fields produced in the infinite metal by the point charge itself and by its mirror image, just as if they were produced at the instant $t=0$ at the point $\mathbf{r}=0$ and then moved apart from this point.

Using the Kramers-Kronig relations, we can reduce the field (36) to the form

$$V(\mathbf{r},t)=\int\frac{d^3\mathbf{q}\,d\omega}{(2\pi)^4}\frac{4\pi}{q^2}\left(\frac{1}{\varepsilon(\omega,\mathbf{q})}-1\right)\rho_{\mathbf{q},\omega}e^{i\mathbf{q}\mathbf{r}-i\omega t} \quad (37)$$

which demonstrates the possibility of calculating $\varphi_V(\mathbf{r},t)$ with the aid of the usual "macroscopic" considerations. Here $\rho_{\mathbf{q},\omega}$ is the Fourier transform of the charge density $\theta(t)\delta(\mathbf{r}-\mathbf{v}t)$.

Regarding the field (36) at large distances from both the charge itself and from the point where the interaction is turned on, we estimate first the contribution of the single-particle excitations with small q. Under the condition $r\gg v_Ft$, the t-dependent exponentials in (36) can be expanded in a series and the first three terms retained, and the corresponding contribution can be written in the form

$$V_{ind}(\mathbf{r},t)\approx-\theta(t)\frac{t^2}{2\hbar}\sum_{\mathbf{q},p}A_{\mathbf{q}p}{}^2\omega_p(\mathbf{q})e^{i\mathbf{q}\mathbf{r}}, \quad (38)$$

where the index p numbers the single-particle excitations. The summation over p can be carried out with the aid of the rule

$$-\frac{1}{\pi}\int_{-\infty}^{\infty}d\omega\omega\,\mathrm{Im}\frac{1}{\varepsilon(\omega,\mathbf{q})}=\frac{i}{\pi}\int_{-\infty}^{\infty}d\omega\omega\left(\frac{1}{\varepsilon(\omega,\mathbf{q})}-1\right)=\omega_0{}^2 \quad (39)$$

and leads to the result

$$\frac{1}{2\hbar}\sum_p A_{\mathbf{q}p}{}^2\omega_p(\mathbf{q})=\frac{4\pi}{q^2\Omega}\left\{\frac{\omega_0{}^2}{2}-\omega\Big/\frac{\partial\varepsilon(\omega,\mathbf{q})}{\partial\omega}\bigg|_{\omega=\omega_{pl}(\mathbf{q})}\right\}. \quad (40)$$

Here $\hbar\omega_{\text{pl}}(\mathbf{q})$ is the energy of a plasmon with momentum $\mathbf{q}$.

In the random-phase approximation[22] it is easy to find that the first term of the expansion of the right-hand side of (40) in powers of q is the term of order q^2, and (38) takes the form

$$V_{ind}(\mathbf{r},t)\approx-\theta(t)\frac{12\omega_0{}^2t^2}{175\pi r}\frac{v_F{}^4}{\omega_0{}^4}\int_0^\infty dqq^3f(q)\sin qr, \quad (41)$$

where $f(q)$ is the certain continuous function that ensures convergence of the integral with respect to q and has the property $f(0)=1$. It follows from (41) that the contribution of the single-particle excitations with small q to the potential (36) at large $r\gg v_F\,t$ falls off like r^{-6}

$$V_{ind}(\mathbf{r},t)\approx-\theta(t)\frac{288}{175\pi}\frac{v_F{}^4t^2}{\omega_0{}^2}\frac{f'(0)}{r^6}. \quad (42)$$

The contribution of the single-particle excitations with large $q\sim p_F$ can turn out to be much more substantial; this contribution is dictated by the corresponding singularities of the dielectric constant. If we start from (41), this contribution oscillates in space with an amplitude that decreases, as in the case of the known Friedel oscillations, like

$$z_c{}^2/r^3. \quad (43)$$

Actually, however, the approximation leading to (41) is incorrect for single-particle excitations with large q, and integration over the frequencies and over the directions of the vector $\mathbf{q}$ in (36) leads to an effective weakening of the singularities of $\varepsilon(\omega,\mathbf{q})$, and consequently to a faster decrease of the contribution of such excitations compared with (43).

The foregoing considerations allow us to state that outside the subsurface layer of thickness z_c, and at large distances from the particle compared with z_c, the volume part of the polarization potential is determined only by the plasmon contribution, which can be calculated with the aid of the relation

$$\left(\mathrm{Im}\frac{1}{\varepsilon(\omega,\mathbf{q})}\right)_{pl}=-\pi F(q)\delta(\omega-\omega_{pl}(q))\theta(p_c-q), \quad (44)$$

where p_c is the maximum plasmon momentum, and

$$F(q)=\left[\frac{\partial\varepsilon(\omega,\mathbf{q})}{\partial\omega}\bigg|_{\omega=\omega_{pl}(q)}\right]^{-1}. \quad (45)$$

With the random-phase approximation as the example, when

$$F(q)=\frac{\omega_0}{2}\left(1-\frac{3}{10}\frac{v_F^2}{\omega_0^2}q^2+\ldots\right),$$
$$\omega_{pl}(q)=\omega_0\left(1+\frac{3}{10}\frac{v_F^2}{\omega_0^2}q^2+\ldots\right), \quad (46)$$

we can see that allowance for the dependence of the function F and of the plasmon frequency $\omega_{\rm pl}$ on the momentum $\mathbf{q}$ is of no importance for the calculation of the potential (36); in particular, on the symmetry axis z we can easily obtain under the formulated conditions

$$V(z,t)\approx\frac{\omega_0}{v}\left\{2\theta(z)\theta(z')\ln\frac{p_cv}{\omega_0}\sin\frac{\omega_0z'}{v}\right.$$
$$-\frac{\omega_0}{v}\int_0^\infty d\xi\frac{e^{-\xi|z'|}}{\xi^2+\omega_0^2/v^2}+\operatorname{sign}z\operatorname{ci}\frac{\omega_0z}{v}\sin\frac{\omega_0z'}{v}$$
$$\left.-\operatorname{si}\frac{\omega_0z}{v}\cos\frac{\omega_0z'}{v}\right\}, \quad (47)$$

where $z'=z-vt$.

The integral in (47) is a function that changes monotonically from its maximum value $\pi\omega_0/2v$ at $z'=0$ to zero, and its asymptotic value at $|z'|\gg v/\omega_0$ is $1/|z'|$. This part of the polarization potential ensures screening of the Coulomb field at large distances from the moving particle. The last two terms in (47) constitute the contribution due to the instantaneous turning on of the interaction with the medium.

Using (47), we obtain for the volume potential (35) on the z axis

$$\varphi_V(z,t)\approx Ze\theta(z)\left\{2\frac{\omega_0}{v}\theta(z')\ln\frac{p_cv}{\omega_0}\sin\frac{\omega_0z'}{v}+\frac{1}{|z+vt|}-\frac{1}{|z'|}\right.$$
$$\left.+2\frac{v}{\omega_0z^2}\sin\omega_0t\right\}\quad\text{at}\quad z,|z'|\gg\frac{v}{\omega_0}. \quad (48)$$

On the other hand, the total polarization potential in the medium, which takes into account the contribution of both the surface and the volume excitations, is determined at large distances from the surface and from the particle on the z axis by the expression [see (32)]

$$\varphi(z,t)=\varphi_V(z,t)+\varphi_s(z,t)$$
$$\approx Ze\left\{2\frac{\omega_0}{v}\Theta(z')\ln\frac{p_cv}{\omega_0}\sin\frac{\omega_0z'}{v}-\frac{1}{|z'|}\right.$$
$$\left.+\frac{2v}{\omega_0z^2}\left(\sin\omega_0t-\sqrt{2}\sin\frac{\omega_0t}{\sqrt{2}}\right)\right\}. \quad (49)$$

The fact that the potentials of the "volume" and "surface" electrostatic images of the charge cancel each other in the obtained expression confirms the important role of the surface polarization in our problem.

The foregoing analysis shows that a stationary regime with a polarization potential typical of an infinite medium,

$$\bar{\varphi}(\mathbf{r},t)=\frac{1}{2\pi^2}\int\frac{d^3\mathbf{q}}{q^2}\left(\frac{1}{\varepsilon(\mathbf{qv},\mathbf{q})}-1\right)e^{i\mathbf{q}(\mathbf{r}-\mathbf{v}t)}, \quad (50)$$

is reached within a time $t>\omega_0^{-1}$. The difference between $\varphi(\mathbf{r},t)$ and the "stationary" value (50) is due to excitation of surface and volume modes of oscillations at the instant when the particle crosses the vacuum–metal interface. The influence of the additional terms in the vicinity of the particles becomes weaker like z^{-2} with increasing distance from the particle to the boundary. At $t<\omega_0^{-1}$ the surface effects play an important role and alter substantially the polarization field near the particle compared with the situation in an infinite medium.

We note that establishment of a stationary regime takes place independently of the presence of damping of the quasiparticles. The time $t\gg\omega_0^{-1}$ needed to make the influence of the boundary conditions on the field small near the particle, can still be much shorter than the lifetime of the quasiparticle.

Of importance in our problem is the establishment of a stationary value, corresponding to the potential (50), of the stopping force that is exerted on the particle by the medium. It is easily understood that the corresponding establishment time is estimated at ω_0^{-1}. However, part of this force is due to the so called "close" collisions that set in even earlier—within times of the order of $\hbar/m_ev^2$, where m_e is the electron mass.

5. POLARIZATION FIELD IN OBLIQUE INCIDENCE OF THE PARTICLE ON THE METAL SURFACE

Let a charged particle move with high velocity $v\gg v_F$ at an angle ϑ to the plane surface of a metal, crossing the surface at the instant $t=0$ at the point $\mathbf{r}=0$. Calculations similar to those given above lead to the following compact expression for the potential due to the polarization and to the excitation of the system of surface plasmons, whose dispersion and damping will be neglected in this section:

$$\varphi_s(\mathbf{r},t)=-Ze\omega_s\int_{-\infty}^{t}d\tau\frac{\sin\omega_s(t-\tau)}{[(|z|+|v_\perp\tau|)^2+(\boldsymbol{\rho}-\mathbf{v}_\parallel\tau)^2]^{1/2}}, \quad (51)$$

where $\mathbf{v}_\perp$ and $\mathbf{v}_\parallel$ are respectively the normal and tangential components of the velocity $\mathbf{v}$ of the particle relative to the vacuum–metal interface, and ω_s is the frequency of the surface plasmons (4).

The volume field, in analogy with the results of the preceding section, is given by the formula

$$\varphi_V(\mathbf{r},t)=Ze\theta(z)\,[V(x,y,z;t)-V(x,y,-z;t)]. \quad (52)$$

Here $V(\mathbf{r},t)$ is the polarization potential corresponding to the instantaneous turning on of the interaction with the medium at $t=0$ for a unit point charge moving in an infinite metal with constant velocity $\mathbf{v}$ at an angle ϑ to the plane $z=0$.

In particular, when moving in a vacuum parallel to the surface at a distance z_0 from the latter, the total polarization field

$$\varphi^{(\parallel)}(\mathbf{r},t)=\varphi_s^{(\parallel)}(\mathbf{r},t)=-\frac{Ze\omega_s}{v}$$
$$\times\int_0^\infty d\zeta\frac{\sin(\omega_s\zeta/v)}{[(|z|+|z_0|)^2+(\zeta\mathbf{v}/v+\boldsymbol{\rho}-\mathbf{v}t)^2]^{1/2}} \quad (53)$$

takes the form of the Vager—Gemmell potential, proposed in Ref. 2 for the description of the polarization produced by a point charge in an infinite homogeneous medium of metallic type. As shown in Ref. 2, the field (53) is characterized by intense oscillations in the wake of the moving particle. When using the numerical results cited in Ref. 2, it is necessary, however, to bear in mind that the unit of measurement of the potential indicated there is approximately three times larger than the true value.

It follows from (53) that the energy losses of the particle in the case considered are determined by the relation

$$-\frac{dE}{dt}=\frac{(Ze\omega_s)^2}{v}K_0\left(2|z_0|\frac{\omega_s}{v}\right), \quad (54)$$

where K_0 is a cylindrical function of imaginary argument. Formula (54) is valid at $|z_0|>z_c$. With decreasing $|z_0|$ in the region $z_c<|z_0|<v/\omega_s$, the energy losses increase logarithmically, and when z_0 increases in the region $|z_0|>v/\omega_s$, they decrease exponentially.

6. ENERGY LOSSES OF FAST PARTICLES REFLECTED BY A SEMI-INFINITE METAL

We analyze now the energy losses and their fluctuations when a fast charged particle is reflected from a metallic sample. We assume that at $t=\pm\infty$ the particle is infinitely far from the semi-infinite metal, and that at finite t its trajectory passes near the surface or through the volume of the sample.

From equations such as (18) follows directly the Poisson law for the excitation probability of n surface quasiparticles, each of which has a momentum $\hbar\mathbf{k}_\parallel$ and an energy $\hbar\omega_{s\alpha}(\mathbf{k}_\parallel)$:

$$W_n(\mathbf{k}_\parallel,\alpha)=\frac{[\bar{n}(\mathbf{k}_\parallel,\alpha)]^n}{n!}\exp\{-\bar{n}(\mathbf{k}_\parallel,\alpha)\}. \quad (55)$$

For the average number of excited quasiparticles of this type we have

$$\bar{n}(\mathbf{k}_\parallel,\alpha)=\left|\frac{Ze}{\hbar}g_{\mathbf{k}_\parallel\alpha}\int_{-\infty}^{\infty}\exp\{-k_\parallel|z_0(t)| -i\mathbf{k}_\parallel\boldsymbol{\rho}_0(t)+i\omega_{s\alpha}(\mathbf{k}_\parallel)t\}dt\right|^2. \quad (56)$$

Here $\mathbf{r}_0(t)\{\boldsymbol{\rho}_0(t),z_0(t)\}$ is the radius vector of the external particle as a function of the time t, while $g_{\mathbf{k}_\parallel\alpha}$ is given by Eq. (6).

The probabililty of exciting n volume quasiparticles characterized by a wave vector $\mathbf{q}$ and energy $\hbar\omega_\beta(\mathbf{q})$ has the same form (55), with $\bar{n}(\mathbf{k}_\parallel,\alpha)$ replaced by $\bar{n}(\mathbf{q},\beta)$:

$$\bar{n}(\mathbf{q},\beta)=\left|\frac{Ze}{\hbar}A_{\mathbf{q}\beta}\int_{-\infty}^{\infty}\theta[z_0(t)]\sin[q_zz_0(t)]\times\exp\{-i\mathbf{q}_\parallel\boldsymbol{\rho}_0(t)+i\omega_\beta(\mathbf{q})t\}\,dt\right|^2, \quad (57)$$

where $A_{\mathbf{q}\beta}$ is defined in (14). For the energy losses of the external particle and for the square of their fluctuations we easily obtain then

$$\delta^\lambda E=\delta^\lambda E_s+\delta^\lambda E_V=\sum_{\mathbf{k}_\parallel,\alpha}[\hbar\omega_{s\alpha}(\mathbf{k}_\parallel)]^\lambda\bar{n}(\mathbf{k}_\parallel,\alpha)+\sum_{\beta,\mathbf{q}_\parallel,q_z>0}[\hbar\omega_\beta(\mathbf{q})]^\lambda\bar{n}(\mathbf{q},\beta). \quad (58)$$

Here

$$\delta^\lambda E=\begin{cases}\overline{\Delta E} & \text{at } \lambda=1\\ \overline{(\Delta E)^2}-(\overline{\Delta E})^2 & \text{at } \lambda=2\end{cases}.$$

Equations (56)–(58) solve in general form the problem of the energy lost by a fast particle moving along an arbitrary "reflecting" trajectory, including the situation when part of the trajectory of the particle passes through a volume occupied by a semi-infinite electron gas.

To reveal the main features of the energy losses in scattering in a thin subsurface layer, we carry out concrete calculations for the case of motion along a symmetrical trajectory of the "corner" type, made up of two rays and corresponding to entry of the particle from the vacuum into the metal and to scattering at a depth z_0, followed by escape to the vacuum. The particle velocities $\mathbf{v}=(\mathbf{v}_\parallel,v_\perp)$ and $\mathbf{v}'=(\mathbf{v}_\parallel,-v_\perp)$ respectively at the entrance and exit sections of the trajectory will be assumed constant, with $v_\perp>0$. Neglecting the dispersion and damping of the surface plasmons, we obtain for the "surface" part of the energy losses

$$\delta^\lambda E_s=\frac{1}{\pi}\left(\frac{Zev_\perp}{\hbar}\right)^2(\hbar\omega_s)^{\lambda+1}\int_0^{2\pi}d\varphi\times\int_0^\infty dk_\parallel k_\parallel^2\left\{\frac{2\cos[(\omega_s-k_\parallel v_\parallel\cos\varphi)z_0/v_\perp]-e^{k_\parallel z_0}}{k_\parallel^2v_\perp^2+(\omega_s-k_\parallel v_\parallel\cos\varphi)^2}\right\}^2. \quad (59)$$

The main contribution to the integral with respect to $k_\parallel$ in the right-hand side of (59) is made by the region of small $k_\parallel\lesssim\omega_s/v$, so that the long-wave approximation used to derive (59) is adequate in the case of fast particles ($v\gg v_F$).

If $z_0\gg v/\omega_s$, we get from (59) a simple result for the case of particle reflection from the surface of the metal:

$$\delta^\lambda E_s|_{z_0=0}=\frac{\pi}{2}\frac{Z^2e^2}{\hbar v_\perp}(\hbar\omega_s)^\lambda, \quad (60)$$

i.e., in accord with Ref. 12, the surface energy losses and the square of their fluctuations increase like $1/v_\perp$ with decreasing transverse velocity of the particle.

In the analysis of the inverse limiting case $z_0\gg v/\omega_s$, the trajectories of the considered type can be broken up into two groups.

1) Trajectories with $v_\perp\lesssim v_\parallel$. For these trajectories Eq. (59) with $z_0\gg v/\omega_s$ gives a physically natural result, exactly double the value (60) for the case of reflection from a surface:

$$\lim_{z_0\to\infty}\delta^\lambda E_s=2\delta^\lambda E_s|_{z_0=0}=\pi\frac{Z^2e^2}{\hbar v_\perp}(\hbar\omega_s)^\lambda. \quad (61)$$

2) Of particular interest are trajectories that are almost normal to the surface of the metal and are characterized by the relation $v_\parallel<v$. It is clear from (59) that the trajectories of this group execute in the region $z_0\ll v^2/v_\parallel\omega_s$ oscillations with amplitudes $\delta^\lambda E_s$ that are functions of the depth of scattering z_0:

$$\delta^\lambda E_s\approx2\pi\frac{Z^2e^2}{\hbar v}(\hbar\omega_s)^\lambda\cos^2\frac{z_0\omega_s}{v}. \quad (62)$$

Thus, in the case when the depth z_0 spans an odd number of "quarter-wave" segments $\pi v/2\omega_s$, the surface energy losses and their fluctuations are close to zero. This phenomenon is due to the "resonant" energy transfer from the excited surface plasmons to the particle emitted from the metal.

At very small values of $v_{\parallel}$ the dispersion and damping of the surface plasmons play an essential role, restricting from above the region of values of z_0 where the oscillations $\delta^{\lambda}E_s$ can be observed. The corresponding condition takes the form

$$1 \ll z_0\omega_s/v \ll v/(v_{\parallel}+v_F). \tag{63}$$

An increase of z_0 outside the region (63) leads to the limit (61).

If the scattering takes place at a point with coordinate $z_0 < 0$, i.e., when the particle has still not reached the surface of the metal, it is necessary to use in place of (59) the relation

$$\delta^{\lambda}E=\delta^{\lambda}E_s=\frac{1}{\pi}\left(\frac{Zev_{\perp}}{\hbar}\right)^2(\hbar\omega_s)^{\lambda+1} \times\int_0^{2\pi}d\varphi\int_0^{\infty}dk_{\parallel}\frac{k_{\parallel}^2e^{-2k_{\parallel}|z_0|}}{[k_{\parallel}^2v_{\perp}^2+(\omega_s-k_{\parallel}v_{\parallel}\cos\varphi)^2]^2} \tag{64}$$

which shows that in the region of negative z_0 at $|z| \gg v/\omega_s$ the total energy losses of the particle and the square of their fluctuations decrease like $|z_0|^{-3}$.

In accord with (57) and (58), the contribution of the volume excitations to the energy loss of a particle moving along a symmetrical "corner" trajectory with $z_0 > 0$ is described by the expression

$$\delta^{\lambda}E_V=-\frac{16}{\pi^3}\frac{(Zev_{\perp})^2}{\hbar}\int d^3\mathbf{q}\int_0^{\infty}d\omega\frac{q_z^2(\hbar\omega)^{\lambda}}{q^2[(\omega-q_{\parallel}v_{\parallel})^2-q_z^2v_{\perp}^2]^2} \times\sin^2\left(\frac{\omega-q_{\parallel}v_{\parallel}-q_zv_{\perp}}{2v_{\perp}}z_0\right)\sin^2\left(\frac{\omega-q_{\parallel}v_{\parallel}+q_zv_{\perp}}{2v_{\perp}}z_0\right)\mathrm{Im}\frac{1}{\varepsilon(\omega,\mathbf{q})}. \tag{65}$$

In the analysis of (65), particular interest attaches to the plasmon part of the energy losses, separation of which from (65) with the aid of (44)–(46) and allowance for the weak dependence of the frequency of the volume plasmons on the wave vector enables us to write

$$\overline{\Delta E}_{V,pl}=2\left(\frac{Ze\omega_0v_{\perp}}{\pi}\right)^2 \times\int_{q<p_c}d^3\mathbf{q}\frac{q_z^2\{\cos[(\omega_0-q_{\parallel}v_{\parallel})z_0/v_{\perp}]-\cos(q_zz_0)\}^2}{q^2[(\omega_0-q_{\parallel}v_{\parallel})^2-q_z^2v_{\perp}^2]^2}. \tag{66}$$

Simple calculations in the case of normal incidence ($\mathbf{v}_{\parallel}=0$) under the conditions $z_0 \gg v/\omega_0$ and $v \gg v_F$ lead to the result

$$\overline{\Delta E}_{V,pl}=\frac{Z^2e^2\omega_0}{v}\left\{2\frac{\omega_0z_0}{v}\ln\frac{p_cv}{\omega_0}-\pi+\left[\left(\ln\frac{p_cv}{\omega_0}-1\right)^2+\frac{\pi^2}{4}\right]^{1/2} \times\sin\left(2\frac{\omega_0z_0}{v}-\gamma\right)\right\}, \tag{67}$$

where

$$\gamma=\mathrm{arctg}\frac{\pi}{2[\ln(p_cv/\omega_0)-1]}.$$

For the plasmon part of the effective deceleration force we have then

$$F_{V,pl}=\frac{1}{2}\frac{\partial\overline{\Delta E}_{V,pl}}{\partial z_0}=\left(\frac{Ze\omega_0}{v}\right)^2\left\{\ln\frac{p_cv}{\omega_0} +\left[\left(\ln\frac{p_cv}{\omega_0}-1\right)^2+\frac{\pi^2}{4}\right]^{1/2}\cos\left(2\frac{\omega_0z_0}{v}-\gamma\right)\right\}. \tag{68}$$

Thus, in the case of backscattering noticeable oscillations of the plasmon part of the volume energy losses, as functions of the scattering depth z_0, are produced in a thin subsurface layer of the metal; these oscillations manifest themselves particularly strongly in the stopping force (68). Simple estimates with allowance for the dispersion of the plasmons yield the interval of incidence angles $\vartheta \approx v_{\parallel}/v$ and depth z_0 where these oscillations take place:

$$1 \ll z_0\omega_0/v \ll (\vartheta+v_F^2/v^2)^{-1}. \tag{69}$$

At values $z_0\omega_0/v$ larger than the upper limit in (69), we can put in the entire reigon of ω and $\mathbf{q}$

$$\left\{q_zv_{\perp}\sin\left(\frac{\omega-q_{\parallel}v_{\parallel}-q_zv_{\perp}}{2v_{\perp}}z_0\right)\sin\left(\frac{\omega-q_{\parallel}v_{\parallel}+q_zv_{\perp}}{2v_{\perp}}z_0\right) \times[(\omega-q_{\parallel}v_{\parallel})^2-q_z^2v_{\perp}^2]^{-1}\right\}^2 =\frac{\pi}{16}\frac{z_0}{v_{\perp}}[\delta(\omega-q_{\parallel}v_{\parallel}-q_zv_{\perp})+\delta(\omega-q_{\parallel}v_{\parallel}+q_zv_{\perp})], \tag{70}$$

and substitution of (70) in (65) leads to total energy losses and a stopping force that correspond to motion in an unlimited medium.

As for the energy losses due to single-particle volume excitations and for fluctuations of the total energy loss, the transition to the corresponding results that are typical of the situation in an unbounded medium is realized already at negligible depths $z_0 \sim z_c v_F/v$.

[1]W. Brandt, A. Ratkowski, and R. H. Ritchie, Phys. Rev. Lett. **33**, 1325 (1974); **35**, 130(E) (1975).
[2]Z. Vager and D. S. Gemmell, *ibid.* **37**, 1352 (1976).
[3]Yu. Kagan, Yu. V. Kononets, and N. K. Dzhamankyzov, Zh. Eksp. Teor. Fiz. **74**, 288 (1978) [Sov. Phys. JETP **47**, 148 (1978)].
[4]N. R. Arista, Phys. Rev. **B18**, 1 (1978).
[5]P. J. Cooney, D. S. Gemmell, W. J. Pietsch, A. J. Ratkowski, Z. Vager, and B. J. Zabransky, Phys. Rev. **A24**, 746 (1981).
[6]D. B. Rogozkin and M. I. Ryazanov, Zh. Eksp. Teor. Fiz. **80**, 2250 (1981) [Sov. Phys. JETP **53**, 1175 (1981)].
[7]P. M. Echenique, R. H. Ritchie, and W. Brandt, Phys. Rev. **B20**, 2567 (1979).
[8]A. G. Eguiluz, Phys. Rev. **B23**, 1542 (1981).
[9]K. Suzuki, M. Kitagawa, and Y. H. Ohtsuki, Phys. Stat. Sol. (b) **82**, 643 (1977).
[10]Y. H. Ohtsuki, K. Koyama, and Y. Yamamura, Phys. Rev. **B20**, 5044 (1979).
[11]A. M. Perelomov, Usp. Fiz. Nauk **123**, 23 (1977) [Sov. Phys. Usp. **20**, 703 (1977)].
[12]A. A. Lucas, Phys. Rev. **B20**, 4990 (1979).
[13]R. Núñez, P. ;M. Echenique, and R. H. Ritchie, J. Phys. **C13**, 4229 (1980).
[14]V. P. Silin and A. A. Rukhadze, Élektromagnitnye svoĭstva plazmy i plazmopodobnykh sred (Electromagnetic Properties of Plasma and of Plasmalike Media), Gosatomizdat, 1961, §4.
[15]V. M. Agranovich and V. L. Ginzburg, Spatial Dispersion in Crystal Optics and the Theory of Excitons, Wiley, 1967 (Second Russ. Ed., Nauka, 1979, §11).
[16]R. H. Ritchie, Phys. Rev. **106**, 874 (1957).
[17]E. A. Stern and R. A. Ferrell, Phys. Rev. **120**, 130 (1960).
[18]A. Eguiluz, S. C. Ying, and J. J. Quinn, Phys. Rev. **B11**, 2118 (1975).

[19]A. Ya. Blank and V. L. Berezinskiĭ, Zh. Eksp. Teor. Fiz. **75**, 2317 (1978) [Sov. Phys. JETP **48**, 1170 (1978)].
[20]V. B. Berestetskiĭ, E. M. Lifshitz, and L. P. Piatevskii, Kvantovaya élektrodinamika (Quantum Electrodynamics), Nauka, 1980, Chap. 1.
[21]Yu. Kagan and Ya. Iosilevskiĭ, Zh. Eksp. Teor. Fiz. **44**, 1375 (1963) [Sov. Phys. JETP **17**, 926 (1963)].
[22]J. Lindhard, Danske Vid. Slsk. Mat.-Fyz. Medd. **28**, No. 8, (1954).
[23]N. D. Lang and W. Kohn, Phys. Rev. **B7**, 3541 (1973).

Translated by J. G. Adashko