Coherent bremsstrahlung of relativistic electrons in axial channeling in crystals

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The influence of the discreteness and periodicity of the potential of atomic chains on the emission spectrum of channeled electrons is investigated. The possibility of generating additional coherent radiation exceeding the bremsstrahlung background is demonstrated.

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The appearance of short-wave electromagnetic radiation from relativistic electrons and positrons channeled in a crystal, predicted in Refs. 1 and 2 and observed experimentally, particularly in Ref. 3, has been recently attracting much attention.[4,5] The theoretical models used at present to study channeling and radiation are based predominantly on the description of the motion of fast particles in the averaged potentials of atomic chains or planes.[1,5] In the present paper we consider axial channeling of relativistic electrons and show that the discreteness and periodicity of the potentials in the atomic chains can influence substantially, under certain conditions, the emission from the electrons. It is shown that additional coherent bands appear in the electromagnetic-radiation spectrum and have energies that exceed substantially the energy of the principal band of the spontaneous emission. The existence of analogous bands, produced when fast nonrelativistic electrons move through a crystal along the atomic axes, was indicated by Ter-Mikaelyan.[6] Akhiezer *et al.*[7] have considered the features of the bremsstrahlung produced when relativistic electrons are scattered by an atomic chain, but they did not discuss the specifics of the radiation from electrons moving in the channeling regime.

Let us bring to light qualitatively the causes and conditions of the appearance of the investigated coherent-bremsstrahlung bands that appear in axial channeling of electrons. We confine ourselves to electrons trapped by discrete transverse-motion levels in the channels, and assume that the crystal is thick enough for the inelastic processes to cause the states of the electrons in the different axial channels not to be coherent. Radiative spontaneous transitions of electrons from one transverse-motion level to another give rise to previously described[1,2] radiation that takes place without transfer of momentum to the crystal lattice along the channel axis. The discreteness and periodicity of the atomic-chain potential make possible also radiative transitions in which are transferred to the lattice longitudinal momenta proportional to the reciprocal-lattice vectors

$$\Delta p_{\parallel}=2\pi\hbar n/d$$

(d is the longitudinal-transformation parameter for the investigated chain, and n is an integer). If

$$\Delta p_{\parallel}\ll\hbar/\Delta \tag{1}$$

(Δ is the amplitude of the atom vibrations in the lattice), the momentum $\Delta p_{\parallel}$ is captured, with overwhelming probability, by the entire lattice as a whole. This ensures coherence of the considered additional emission. It follows from (1) that the additional emission bands can be observed only at the small values

$$n\ll d/2\pi\Delta\sim 10. \tag{2}$$

Another necessary condition for the appearance of such bands is smallness of the transverse momentum acquired by the electron as it radiates, compared with its characteristic transverse momentum in the axial channel:

$$|\varkappa|\, b\sin\theta<1 \tag{3}$$

($\varkappa$ is the wave vector of the emitted photon, b is the characteristic radius of the wave function of the transverse motion of the electron in the channel, and θ is the angle between the channel axis and the vector $\varkappa$).

It will be made clear in what follows that the appearance of the investigated additional bands is due to coherent bremsstrahlung (CBB) of the channeling electrons on the chain atoms. This bremsstrahlung is produced under usual conditions, when the radiation formation length is equal to the interatomic distance. If no account is taken of the fact that the electron is channeled along the chain, it is well known[7,8] that coherent emission is impossible in this case, because the interference terms are suppressed as a result of integration of the radiation intensity with respect to the momentum transferred to the lattice.[7] When the electron is channeled, its final state is unambiguously determined by the conservation laws, therefore the integral with respect to the momentum transfer vanishes and it becomes possible for the amplitude of the radiation to be additive in phase for all the atoms of the chain.

§1. CLASSICAL DESCRIPTION OF CBB IN AXIAL CHANNELING

The frequency and polarization of the considered CBB bands can be easily established from simple quasiclassical considerations. The corresponding analysis is easiest to carry out in the electron comoving reference frame. The main spontaneous emission is due to motion of the electron in this reference frame along a circle whose plane is perpendicular to the atomic chain. The electron revolution frequency ω_0', equal in the comoving frame to the frequency of the radiated electromagnetic wave, is connected with the frequency ω_0 in

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the lab by the relation $\omega_0' = \omega_0/(1-\beta_\parallel^2)^{1/2}$, where $\beta_\parallel = v_\parallel/c$ and $v_\parallel$ is the longitudinal velocity of the electron. The projections of the dipole moment **D** in the comoving system are given by

$$D_{x'}=D_0 \sin \omega_0' t', \quad D_{y'}=D_0 \cos \omega_0' t', \quad D_{z'}=0. \tag{4}$$

The discreteness of the chain potential leads to additional high-frequency radial and longitudinal oscillations of the dipole moment, $d_1(t')$ and $d_2(t')$:

$$D_{x'}=(D_0+d_1(t')) \sin \omega_0' t',$$
$$D_{y'}=(D_0+d_1(t')) \cos \omega_0' t', \quad D_{z'}=d_2(t'). \tag{5}$$

Expanding $d_1(t')$ and $d_2(t')$ in Fourier series

$$d_i=\sum_n d_i(n) \exp(in\Omega' t'), \quad i=1,2, \tag{6}$$

$$\Omega'=2\pi c/d(1-\beta_\parallel^2)^{1/2},$$

we readily see that in the electron's proper reference frame there are present, besides the fundamental radiation frequency ω_0', of additional lines with frequencies

$$\omega_n'^{(\pm)}=\Omega' n \pm \omega_0', \tag{7}$$

$$\omega_n'^{(0)}=\Omega' n. \tag{8}$$

The emission at the frequencies $\omega_n'^{(\pm)}$ has angular and polarization characteristics similar to those of the fundamental radiation. The emission at the frequency $\omega_n'^{(0)}$ is linearly polarized and none exists in the electron-motion direction. In the lab, after the appropriate frequency transformation, the radiation becomes dependent on the photon emission angle θ and equal to

$$\omega_n^{(\pm)}(\theta)=(\Omega n \pm \omega_0)/(1-\beta_\parallel \cos\theta), \tag{9}$$

$$\omega_n^{(0)}(\theta)=\Omega n/(1-\beta_\parallel \cos\theta). \tag{10}$$

It can be seen that the radiation considered is coherent bremsstrahlung that appears under the condition that the radiation formation length is equal to the interatomic distance. The appearance of the resultant resonant radiation is due in this case to the specifics of the electron motion in axial channeling and to the periodicity of the atoms [see (5)] in the chain. We emphasize once more that when (1) is satisfied the transfer of the longitudinal momentum to the lattice when the bands under consideration are emitted does not lead to loss of coherence.

Using (9) and (10) and recognizing that in the relativistic case $\Omega \gg \omega_0$ we obtain from (3) the angle regions in which CBB can be observed in the case of axial channeling of electrons:

$$\theta < \frac{d}{4\pi b}\left(\frac{m_0c^2}{E}\right)^2, \tag{11}$$

$$\theta > 2 \operatorname{arctg} \frac{2\pi b}{d}, \tag{12}$$

where E is the electron energy and m_0 is its rest mass.

§2. WAVE FUNCTION OF RELATIVISTIC ELECTRON IN AXIAL CHANNELING

Let us investigate in greater detail the wave function of a fast electron in an axial channel, with allowance for the discreteness and periodicity of the atomic-chain potential. Neglecting inelastic processes in the crystals we can assume, as shown in Ref. 9, that the electron moves in a strictly periodic lattice potential averaged over the atomic vibrations. In channeling of an electron, in contrast to that of positively charged particles, it is localized mainly in the inner regions of the atomic chains, where the longitudinal variations of the potential are appreciable, and this can lead to a noticeable deviation of the wave function of the longitudinal motion of the electron from a plane wave.

The equation for the wave function of a relativistic electron in a crystal, under the condition $|V/E| \ll 1$ (V is the potential of the crystal) can be easily shown in analogy with Ref. 2 to have the form of the Schrödinger equation

$$-\frac{\hbar^2}{2m}\Delta\psi + V\psi = \varepsilon\psi, \tag{13}$$

$$m=\frac{E}{c^2}, \quad \varepsilon=\frac{m^2-m_0^2}{2m}c^2, \quad V(\mathbf{r})=\sum_j V_j(\mathbf{r}-\mathbf{R}_j),$$

where $\mathbf{R}_j$ and V_j are the average coordinate and the averaged (over the atomic vibrations[9]) potential of the jth atomic center.

We seen for the channeling electron a wave function that satisfies the Bloch condition in the direction of the axis of the considered atomic chain, in the form

$$\psi_{\varepsilon,\lambda}(\boldsymbol{\rho}, z)=L^{-1/2}\exp(ik_\lambda z)\varphi_\lambda(\boldsymbol{\rho})\exp[iS(\boldsymbol{\rho}, z)], \tag{14}$$

where the z axis is directed along the chain axis, the vectors $\boldsymbol{\rho}$ are perpendicular to it, L is the length of the periodicity interval along the z axis, the phase $S(\boldsymbol{\rho}, z)$ is periodic in z with a period d:

$$S(\boldsymbol{\rho}, z+d)=S(\boldsymbol{\rho}, z), \tag{15}$$

and the function $\varphi_\lambda(\boldsymbol{\rho})$ satisfies the equation for the transverse motion

$$-\frac{\hbar^2}{2m}\Delta_\rho\varphi_\lambda+\left\{\langle V(\boldsymbol{\rho})\rangle+\frac{\hbar^2}{2m}\langle(\nabla S)^2\rangle\right\}\varphi_\lambda=\varepsilon_\lambda\varphi_\lambda,$$
$$\langle V(\boldsymbol{\rho})\rangle=\frac{1}{d}\int_0^d V(\boldsymbol{\rho}, z)\,dz, \quad \langle(\nabla S)^2\rangle=\frac{1}{d}\int_0^d(\nabla S)^2 dz. \tag{16}$$

Substituting (14) in (13) and putting

$$k_\lambda=[2m(\varepsilon-\varepsilon_\lambda)/\hbar^2]^{1/2}, \tag{17}$$

and taking (16) into account, we obtain an equation for the phase function $S(\boldsymbol{\rho}, z)$:

$$\frac{\partial S}{\partial z}=\frac{\hbar i}{mc}\frac{\nabla_\rho\varphi_\lambda}{\varphi_\lambda}\nabla_\rho S+\frac{\hbar i}{2mc}\Delta S$$
$$-\frac{\hbar^2}{2mc}[(\nabla S)^2-\langle(\nabla S)^2\rangle]-\frac{1}{\hbar c}(V-\langle V\rangle), \tag{18}$$

where account is taken of the fact that at $m \gg m_0$ we have $k_\lambda \approx mc/\hbar$, as follows from (17).

It is easily seen that Eq. (18) admits of solutions that satisfy the periodicity condition (15), thus confirming the correctness of the choice (17) for k_λ. This property of (18) is ensured by adding to the average chain potential $\langle V(\rho)\rangle$ in (16) the additional term

$$\frac{\hbar^2}{2m}\langle(\nabla S)^2\rangle.$$

We consider a simple chain with a translation parameter d, made up of atoms with atomic number Z, and show that if the condition

$$\alpha \equiv Z^* e^2/\hbar c \ll 1 \tag{19}$$

is satisfied the solution of (18) can be represented by an iteration series

$$S(\rho, z) = \sum_{n=1}^{\infty} S^{(n)}(\rho, z), \tag{20}$$

where $S^{(n)}(\rho, z) \sim \alpha^n$ and Z^* is the screened charge of the nucleus at a radius of the order of the radius of the transverse motion of the electron in the channel.

We choose the origin halfway between the atoms of the chain and write out the first three terms of the iteration series (20), the principle of whose construction is clear from the following formulas:

$$S^{(0)}(\rho, z) = 0, \quad S^{(1)}(\rho, z) = -\frac{1}{\hbar c}\int_0^z [V(\rho, z) - \langle V(\rho)\rangle]\, dz + C_1(\rho), \tag{21}$$

$$S^{(2)}(\rho, z) = \left(\frac{\hbar i}{mc}\frac{\nabla_\rho \varphi_\lambda^{(0)}}{\varphi_\lambda^{(0)}}\nabla_\rho + \frac{\hbar i}{2mc}\Delta\right)\left[\int_0^z S^{(1)}(\rho, z)\, dz + C_2(\rho)\right],$$

$$S^{(3)}(\rho, z) = \frac{\hbar i}{mc}\left(\frac{\nabla_\rho \varphi_\lambda^{(0)}}{\varphi_\lambda^{(0)}}\nabla_\rho + \frac{1}{2}\Delta\right)\left[\int_0^z S^{(2)}(\rho, z)\, dz + C_3(\rho)\right]$$

$$+ \frac{\hbar i}{mc}\left(\frac{\nabla_\rho \varphi_\lambda^{(1)}}{\varphi_\lambda^{(1)}} - \frac{\Delta_\rho \varphi_\lambda^{(0)}}{\varphi_\lambda^{(0)}}\right)\nabla_\rho\left[\int_0^z S^{(1)}(\rho, z)\, dz + C_2(\rho)\right]$$

$$- \frac{\hbar}{2mc}\int_0^z \left[(\nabla S^{(1)})^2 - \langle(\nabla S^{(1)})^2\rangle\right] dz,$$

where the functions $\varphi_\lambda^{(i)}(\rho)$ satisfy the equations

$$-\frac{\hbar^2}{2m}\Delta_\rho \varphi_\lambda^{(i)} + \left[\langle V(\rho)\rangle + \frac{\hbar^2}{2m}\langle(\nabla S^{(i)})^2\rangle\right]\varphi_\lambda^{(i)} = \varepsilon_\lambda^{(i)}\varphi_\lambda^{(i)}, \tag{22}$$

and the $C_i(\rho)$ are integration constants obtained from the requirement that the phases $S^{(i)}$ be periodic in z. As follows from (15) and (21), to satisfy this condition we must have

$$\int_0^d S^{(i)}(\rho, z)\, dz = 0. \tag{23}$$

In particular, for $S^{(1)}$ the condition (23) is realized in the case of the considered chain at $C_1(\rho) = 0$.

As seen from (21), in the internal region of the chain, where the channeled electrons move for the most part, $S^{(1)}(\rho, z) \sim \alpha$. To estimate $S^{(2)}(\rho, z)$ and the succeeding terms of the iteration series we must recognize that the derivatives with respect to ρ are of the order of b_λ^{-1}, where b_λ is the characteristic scale of the spatial variation of the function $\varphi_\lambda(\rho)$. It can be concluded from (16) that $b_\lambda^2 \approx d\hbar^2/Z^* e^2 m$. It follows hence that $S^{(2)} \sim \alpha^2$, $S^{(3)} \sim \alpha^3$, etc., thus corroborating (20).

We see thus that the degree of deviation of the longitudinal-motion wave function from a plane wave is determined by the value of α and that at $\alpha \sim 1$ this deviation becomes appreciable. Understandably, in this case the expansion (20) converges poorly and we must use other methods to solve (18). Without undertaking for the time being the development of these methods, we examine the consequences of the deviation of the longitudinal-motion wave function from a plane wave for the electron radiation in axial channeling if (19) is satisfied. We confine ourselves here to the first-order approximation for the function $\psi_{\varepsilon,\lambda}(\rho, z)$:

$$\psi_{\varepsilon,\lambda}^{(1)}(\rho, z) = \frac{1}{L^{1/2}}\exp(ik_\lambda^{(0)} z)\,\varphi_\lambda^{(0)}(\rho)\exp[iS^{(1)}(\rho, z)], \tag{24}$$

where

$$k_\lambda^{(0)} = [2m(\varepsilon - \varepsilon_\lambda^{(0)})/\hbar^2]^{1/2}.$$

3. EFFECT OF DISCRETENESS OF CHAIN POTENTIAL ON THE SPONTANEOUS EMISSION OF CHANNELING ELECTRONS

We write down the matrix element of the spontaneous radiative transition, without spin flip, from a state (λ, ε) into a state (λ', ε') with emission of a photon having a momentum $\hbar\varkappa$, assuming that $\hbar c|\varkappa| \ll E$, $m_0 c^2 \ll E$:

$$\mathfrak{M}_{\lambda'\lambda} = \frac{c}{E}\int d^3 r e^{-i\varkappa r}\psi_{\varepsilon',\lambda'}^*(\rho, z)\,\hat{\mathbf{p}}\,\psi_{\varepsilon,\lambda}(\rho, z), \tag{25}$$

where $\hat{\mathbf{p}}$ is the electron-momentum operator.

If (19) is satisfied, we confine ourselves to the first-order approximation for the function $\psi_{\varepsilon,\lambda}$ (24). Noting that $S^{(1)}(\rho, z)$ is practically independent of λ and ε in the relativistic region, we can readily show that

$$\mathfrak{M}_{\lambda'\lambda} = \mathbf{M}_{\lambda'\lambda} + \mathbf{m}_{\lambda'\lambda}, \tag{26}$$

$$\mathbf{M}_{\lambda'\lambda} = \frac{c}{EL}\int d^3 r e^{-i\varkappa r}\varphi_{\lambda'}^*(\rho)\exp(-ik_{\lambda'} z)\,\hat{\mathbf{p}}\,\varphi_\lambda(\rho)\exp(ik_\lambda z), \tag{27}$$

$$\mathbf{m}_{\lambda'\lambda} = \frac{ic}{EL}\int d^3 r \varphi_{\lambda'}^*(\rho)\,\varphi_\lambda(\rho)\exp[i(k_\lambda - k_{\lambda'})z - i\varkappa\mathbf{r}]\,\hat{\mathbf{p}}S(\rho, z), \tag{28}$$

where $k_\lambda = k - \varepsilon_\lambda/\hbar c$, $k = mc/\hbar$, and we omit here and elsewhere the superscripts of the functions $\varphi^{(0)}$ and $S^{(1)}$, which indicate the order of the approximation. The matrix element $\mathbf{M}_{\lambda'\lambda}$ describes the usual spontaneous emission in channeling,[2] and $\mathbf{m}_{\lambda'\lambda}$ are the additional CBB bands.

We expand $S(\rho, z)$ in a Fourier series:

$$S(\rho, z) = \sum_g S(\rho, g)\, e^{-igz}, \tag{29}$$

where

$g=2\pi n/d,$

$$S(\rho,g)=\frac{1}{d}\int_0^d S(\rho,z)e^{igz}dz$$

$$=\frac{1}{d}\int_0^d\left[-\frac{1}{\hbar c}\int_0^z (V(\rho,z')-\langle V(\rho)\rangle)dz'\right]e^{igz}dz. \quad (30)$$

Integrating (30) by parts, we get

$$S(\rho,g)=\begin{cases}-(i\hbar\Omega_g)^{-1}V_g(\rho), & g\neq 0\\ 0, & g=0\end{cases}, \quad (31)$$

where

$$\Omega_g=gc,\quad V_g(\rho)=\frac{1}{d}\int_0^d V(\rho,z)e^{igz}dz.$$

Substituting (29) and (31) in (28) and integrating with respect to z we obtain

$$\mathbf{m}_{\lambda'\lambda}^{\perp}=-\frac{c}{E}\sum_{g\neq 0}\frac{1}{\hbar\Omega_g}\int d^2\rho\exp(-i\varkappa_\perp\rho)\varphi_{\lambda'}^*(\rho)\varphi_\lambda(\rho)\mathbf{p}_\perp V_g(\rho)$$
$$\times\delta(k_\lambda-k_{\lambda'}-\varkappa_\parallel-g), \quad (32)$$

$$m_{\lambda'\lambda}^{\parallel}=-\frac{1}{E}\sum_{g\neq 0}\int d^2\rho\exp(-i\varkappa_\perp\rho)\varphi_{\lambda'}^*(\rho)\varphi_\lambda(\rho)V_g(\rho)$$
$$\times\delta(k_\lambda-k_{\lambda'}-\varkappa_\parallel-g), \quad (33)$$

where δ is the Kronecker delta. It is important that the expansions in terms of g in (32) and (33) do not contain zeroth harmonics. There is therfore no interference between the additional and fundamental radiation.

If we confine ourselves to consideration of the states $\varphi_\lambda(\rho)$ localized on an isolated atomic chain whose averaged potential has axial symmetry $V_g(\rho)=V_g(\rho)$, the index λ includes the projection of the orbital angular momentum l of the channeling electron on the z axis [$\lambda\equiv(n,l)$, where n is the principal quantum number]. The amplitudes $\mathbf{m}_{\lambda'\lambda}^{\perp}$ and $m_{\lambda'\lambda}^{\parallel}$ satisfy in this case different selection rules and in the dipole approximation (we have $\varkappa_\perp b_\lambda\ll 1$) $\mathbf{m}_{\lambda'\lambda}^{\perp}\neq 0$ at $l'=l\pm 1$ and $m_{\lambda'\lambda}^{\parallel}\neq 0$ at $l'=l$ (we note that in the latter case a photon can be emitted without a change of the transverse-motion state, $\lambda'=\lambda$).

It can be seen from (32) and (33) that the emission act is accompanied by a transfer of a longitudinal momentum $\hbar g$ to the lattice. If (1) is satisfied this momentum is transferred to the lattice as a whole, so that the coherence of the radiation is not violated.

Of course, besides the longitudinal-momentum conservation law the expression for the radiation intensity contains the energy conservation law

$$E'+\hbar c|\varkappa|-E=0. \quad (34)$$

From (32)–(34) we easily obtain in analogy with Ref. 2 for the additional-band frequencies expressions that coincide with those obtained in §1:

$$\omega_g^{(\pm)}=[\Omega_g\pm\omega_0(\lambda',\lambda)]/(1-\beta_\parallel\cos\theta), \quad (35)$$

$$\omega_g^{(0)}=\Omega_g/(1-\beta_\parallel\cos\theta), \quad (36)$$

where

$$\omega_0(\lambda',\lambda)=\hbar^{-1}|\varepsilon_{\lambda'}-\varepsilon_\lambda|.$$

The conservation laws require likewise, as seen from (35) and (36), that g be positive. The subscripts and superscripts in (35) correspond to $\varepsilon_{\lambda'}-\varepsilon_\lambda>0$ and $\varepsilon_{\lambda'}-\varepsilon_\lambda<0$.

In the dipole approximation, the probability of photon emission into a solid angle do per unit time at the frequencies (35) and (36) and at the fundamental frequency is equal to

$$\frac{dP_0}{do\,dt}=\frac{e^2}{2\pi\hbar c}\frac{\omega_0(\lambda',\lambda)}{1-\beta_\parallel\cos\theta}\sum_i\left|\frac{c}{E}\int d^2\rho\varphi_{\lambda'}^*\hat{\mathbf{p}}_\perp\varphi_\lambda\beta_i^\perp\right|^2, \quad (37)$$

$$\frac{dP_g^{(\pm)}}{do\,dt}=\frac{e^2}{2\pi\hbar c}\frac{\Omega_g\pm\omega_0(\lambda',\lambda)}{1-\beta_\parallel\cos\theta}$$
$$\times\sum_i\left|\frac{c}{E}\frac{1}{\hbar\Omega_g}\int d^2\rho\varphi_{\lambda'}^*\varphi_\lambda\hat{\mathbf{p}}_\perp V_g(\rho)\beta_i^\perp\right|^2, \quad (38)$$

$$\frac{dP_g^{(0)}}{do\,dt}=\frac{e^2}{2\pi\hbar c}\frac{\Omega_g}{1-\beta_\parallel\cos\theta}\sum_i\left|\frac{1}{E}\int d^2\rho\,\varphi_{\lambda'}^*V_g(\rho)\varphi_\lambda\beta_i^\parallel\right|^2, \quad (39)$$

where β_i is the polarization vector of the emitted photon, and $\beta_i^\perp$ and $\beta_i^\parallel$ are the projections of β_i on the plane perpendicular to the channel axis and on the z axis.

Comparison of (38) and (39) is possible on the basis of the estimate

$$\left|\int d^2\rho\,\varphi_{\lambda'}^*\{\hat{\mathbf{p}}_\perp V_g(\rho)\}\varphi_\lambda\right|\sim\frac{\hbar}{b_\lambda}\left|\int d^2\rho\,\varphi_{\lambda'}^*V_g(\rho)\varphi_\lambda\right|, \quad (40)$$

where b_λ is defined in §2. It follows from (40) that

$$\frac{dP_g^{(\pm)}}{do\,dt}\sim\left(\frac{d}{2\pi nb_\lambda}\right)^2\frac{dP_g^{(0)}}{do\,dt}\gg\frac{dP_g^{(0)}}{do\,dt}. \quad (41)$$

To compare (37) and (38) we transform in (37) the radiative-transition matrix element

$$\left|\int d^2\rho\,\varphi_{\lambda'}^*\hat{\mathbf{p}}_\perp\varphi_\lambda\right|=\frac{1}{\hbar\omega_0(\lambda',\lambda)}\left|\int d^2\rho\,\varphi_{\lambda'}^*\varphi_\lambda\hat{\mathbf{p}}_\perp\langle V(\rho)\rangle\right|. \quad (42)$$

At the same time, since we are interested as a rule in small n, for which $gR\ll 1$ (R is the radius of the potential), we can, as is clear from the definition of $V_g(\rho)$ in (31), replace with good accuracy $V_g(\rho)$ in (38) by $\langle V(\rho)\rangle$. Recognizing that $\Omega_g\gg\Omega_0$, we easily obtain then a sufficiently accurate relation between the probabilities dP_0 and $dP_g^{(\pm)}$:

$$\frac{dP_0}{do\,dt}=\frac{\Omega_g}{\omega_0}\frac{dP_g^{(\pm)}}{do\,dt}. \quad (43)$$

By collimating in the measurements the flux of the emitted photons, we can make the spectrum of the considered CBB monochromatic (in view of the firm connection between ω and θ). The bremsstrahlung background has then a continuous spectrum, so that with a detector having a suffi-

ciently high resolution we can always register the bands of the considered CBB. Since usually the detector resolution $\sim\omega^{1/2}$, the conditions for observation of the CBB will be $(\Omega_g/\Omega_0)^{1/2}$ times worse than those for spontaneous emission.

Even without allowance for collimation, however, an estimate shows that the radiation considered exceeds the bremsstrahlung background. Indeed, the spectral energy density of the bremsstrahlung from one particle moving in an amorphous medium is equal to (see, e.g., Ref. 10)

$$\frac{dE}{d\omega\, dt} \approx \hbar\omega N c \frac{d\sigma}{d\omega}, \quad \frac{d\sigma}{d\omega} \approx \frac{M}{L\omega}, \tag{44}$$

where N is the crystal density (in cm^{-3}), M is the mass of the nucleus, and L is the radiation length (in g/cm^2). At the same time the spectral energy density of the additional CBB bands, as can be easily seen from the formulas above, is equal to

$$\frac{dE_g^{(\pm)}}{d\omega\, dt}\Big|_{\omega=\omega_{\max}} \approx \frac{2e^2}{E^2}\frac{c}{\Omega_g}|(V_g')_{\lambda',\lambda}|^2, \quad \omega_{\max}=2\Omega_g\gamma^2. \tag{45}$$

The matrix element $(V'_g)_{\lambda',\lambda}$ can be estimated with the aid of the relation

$$(V_g')_{\lambda',\lambda} \approx V_0/b_\lambda,$$

where V_0 is the depth of the chain potential, and b_λ is defined in §2. Substituting next in (44) and (45) the values $E = 20$ MeV, $V_0 = 100$ eV, $L = 30$ g/cm^2, and recognizing that $b_\lambda < R$, we obtain

$$\frac{dE_g^{(\pm)}/d\omega\, dt}{dE/d\omega\, dt}\Big|_{\omega=\omega_{\max}} \approx 5,$$

i.e., the additional CBB bands exceed the bremsstrahlung background.

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