Low-temperature electrical conductivity of a quasi-one-dimensional substance $Hg_{3-\delta}AsF_6$ ("alchemist's gold")

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A derivation is given of the transport equation for the electron–phonon interaction in a quasi-one-dimensional compound $Hg_{3-\delta}AsF_6$. It is shown that this gives rise to different temperature dependences of the electrical conductivity [Eqs. (39) and (57)] in two adjoining temperature ranges below the Debye value. At the lowest temperatures the scattering by thermal phonons near singular parts of the Fermi surface is important. At higher temperatures, in the case of a sufficiently large curvature of the planes forming the Fermi surface, the scattering of electrons by short-wavelength phonons predominates. The existence of two mutually perpendicular pairs of Fermi planes in $Hg_{3-\delta}AsF_6$ determines the characteristics of the relaxation of quasimomentum in this system. The theory is in qualitative agreement with experiment.

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In a crystal of $Hg_{3-\delta}AsF_6$ a tetragonal bcc lattice composed of AsF_6^- anion complexes is interpenetrated by two families of parallel chains of the mercury atoms.[1,2] These chains are oriented along two mutually perpendicular directions parallel to the sides of the base of a unit cell of the main tetragonal lattice (Fig. 1).

The conduction process in this system is strongly anisotropic: $\sigma_\parallel \sim 10^2\sigma_\perp$ above 4° K (Ref. 3), where $\sigma_\parallel$ is the electrical conductivity in the planes of the chains and $\sigma_\perp$ is the conductivity along a perpendicular direction in which there are no chains. According to Ref. 4, we have $\sigma_\parallel(T) \propto T^{-3}$ in the range 1 °K $< T <$ 30 °K; and $\sigma_\parallel(T) \propto T^{-3/2}$ at $T >$ 30 °K. The residual resistance has not been observed right down to $T = 1.4$ °K (Ref. 3).

It is shown below that the observed $\sigma_\parallel(T)$ dependence is due to the electron–phonon interaction in the system of chains.[1)]

According to Ref. 4, the anomalously low residual resistance can be explained as follows. The main defects in the investigated samples are anion vacancies (AsF_6^-) present in an amount of 6% and distributed in the space between the mercury atom chains. On the other hand, the Ψ function of a conduction electron is localized in the vicinity of each chain and, therefore, an electron interacts only with a gently sloping tail of the vacancy field. In a one-dimensional conductor there can exist forward and backward scattering; only the latter determines the conductivity. In view of the smooth variation of the potential, this scattering should be anomalously weak, and will not be considered.

1. MODEL

The real structure of $Hg_{3-\delta}AsF_6$ is fairly complex. Below the structural transition temperature $T_c = 120$ K a long-range order is established in the conducting system of chains. The mercury atoms of each family form their own monoclinic bcc sublattices which are incommensurable with the main lattice.[1] We can show[6] that in this system the faces of the Brillouin zone and the corresponding gaps in the electron spectrum form a definite hierarchy. There are "main" faces and energy gaps of the order of the overlap energy integral (J) of states at neighboring mutually perpendicular chains (according to Ref. 4, $|J| \sim 500$ °K). In addition to them there is a "ripple" of faces and corresponding energy gaps resulting from the lattice incommensurability mentioned above.[7] The nature of their appearance allows us to assume[6] that these gaps have a much smaller energy scale ($\lesssim 1$ °K). Therefore, we shall consider a simplified (commensurable) model of two families of chains (Fig. 2), which nevertheless retains the essential features of the real structure of $Hg_{3-\delta}AsF_6$.

According to Refs. 8 and 9, the one-electron Hamiltonian H_0 of the conduction electrons in such a model system is

$$H_0 = \sum_{\mathbf{k},\alpha} \{\varepsilon_1(\mathbf{k})\, c^+_{1\alpha\mathbf{k}} c_{1\alpha\mathbf{k}} + \varepsilon_2(\mathbf{k})\, c^+_{2\alpha\mathbf{k}} c_{2\alpha\mathbf{k}} + J\cos(k_z c/2) \times (c^+_{1\alpha\mathbf{k}} c_{2\alpha\mathbf{k}} + c^+_{2\alpha\mathbf{k}} c_{1\alpha\mathbf{k}})\}, \quad (1)$$

where $c^+_{i\alpha\mathbf{k}}$ is the operator describing creation of an electron in an i-th ($i = 1, 2$) family of chains and this electron has a spin state α ("up" or "down") and a quasimomentum $\mathbf{k}$. Chains of the family 1 are parallel to the x axis whereas chains of the family 2 are parallel to the y axis (Fig. 2). More-

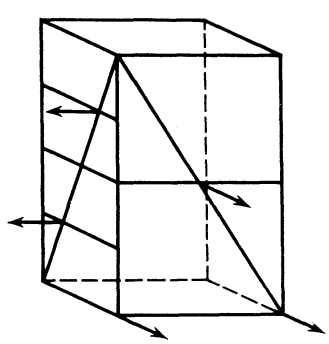

FIG. 1. Positions of chains of the Hg atoms in $Hg_{3-\delta}AsF_6$ in a unit cell of the tetragonal lattice of AsF_6^-.

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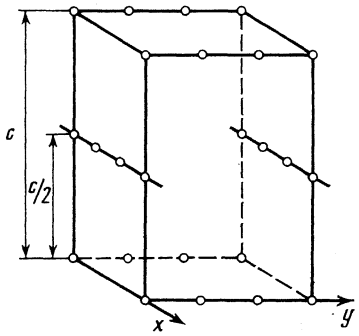

FIG. 2. Model system of chains retaining the essential features of the $Hg_{3-\delta}AsF_6$ structure.

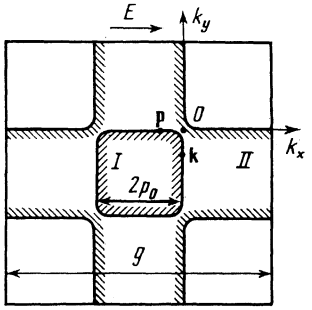

FIG. 3. Section of the Fermi surface of the model system of chains by a plane parallel to the chains. Here, I and II are the two zones.

over, $\varepsilon_i(\mathbf{k})$ can be written in the form

$$\varepsilon_1(\mathbf{k})-\mu=v_F(|k_x|-k_0)+\alpha(k_y, k_z);$$

$$\varepsilon_2(\mathbf{k})-\mu=v_F(|k_y|-k_0)+\alpha(k_x, k_z),$$

where μ is the chemical potential of the electrons; $\alpha(k_y, k_z)$ and $\alpha(k_x, k_z)$ correspond to the tunneling of an electron between chains belonging to the same family. The terms $v_F(|k_x|-k_0)$ and $v_F(|k_y|-k_0)$ correspond to one-dimensional motion along the chain. Finally, the term with J describes the tunneling between the nearest mutually perpendicular chains separated from one another by a distance $c/2$ along the z axis (Fig. 2).

This approximation is justified by two circumstances. Firstly, we have $|J|\sim 10^{-2}\varepsilon_F\leqslant\varepsilon_F(=4\text{ eV})$ (Ref. 4), where ε_F is the Fermi energy. Secondly, measurements of the de Haas–van Alphen effect in $Hg_{3-\delta}AsF_6$ (Ref. 7) have shown that the Fermi surface does indeed consist of cylinders of different cross sections elongated along the z axis (which is normal to the plane of the chains). It is natural to diagonalize H_0 of Eq. (1) by adopting new operators c_{I}^+, c_{I} and c_{II}^+, c_{II} using the formulas

$$\begin{pmatrix} c_{\mathrm{I}} \\ c_{\mathrm{II}} \end{pmatrix}=\hat{S}^{-1}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \quad \overbrace{c_{\mathrm{I}}^+ \quad c_{\mathrm{II}}^+}=\overbrace{c_1^+ \quad c_2^+}\,\hat{S}. \tag{2}$$

The unitary matrix $\hat{S}$ diagonalizes the matrix $\hat{H}_0$ of the Hamiltonian (1):

$$\hat{s}^{-1}\hat{H}_0\hat{s}\equiv\hat{s}^{-1}\begin{pmatrix} \varepsilon_1 & J \\ J & \varepsilon_2 \end{pmatrix}\hat{s}=\begin{pmatrix} E_{\mathrm{I}} & 0 \\ 0 & E_{\mathrm{II}} \end{pmatrix}, \tag{3}$$

where $\tilde{J}=J\cos(k_zc/2)$. This matrix is easily found from the eigenvectors of the linear transformation of $\hat{H}_0$:

$$\hat{s}=\frac{1}{\sqrt{2}}\begin{pmatrix} a & b \\ c & d \end{pmatrix}, \tag{4}$$

where

$$a=\left(1+\frac{z}{(z^2+\tilde{J}^2)^{1/2}}\right)^{1/2}, \quad b=\left(1-\frac{z}{(z^2+\tilde{J}^2)^{1/2}}\right)^{1/2},$$

$$c=\frac{\tilde{J}}{|\tilde{J}|}\left(1-\frac{z}{(z^2+\tilde{J}^2)^{1/2}}\right)^{1/2}, \quad d=-\frac{\tilde{J}}{|\tilde{J}|}\left(1+\frac{z}{(z^2+\tilde{J}^2)^{1/2}}\right)^{1/2} \tag{5}$$

and $z=(\varepsilon_1-\varepsilon_2)/2$. In all the functions ε_i, z, and $\tilde{J}$ we are assuming a dependence on $\mathbf{k}$; for the sake of simplicity, the quantity J is assumed to be real.

Now, from Eqs. (1)–(5) we find H_0 expressed in terms of c_{I} and c_{II}:

$$H_0=\sum_{\mathbf{k},\alpha}\{E_{\mathrm{I}}(\mathbf{k})\,c_{\mathrm{I}\alpha\mathbf{k}}^+c_{\mathrm{I}\alpha\mathbf{k}}+E_{\mathrm{II}}(\mathbf{k})\,c_{\mathrm{II}\alpha\mathbf{k}}^+c_{\mathrm{II}\alpha\mathbf{k}}\}, \tag{6}$$

where

$$E_{\mathrm{I}}(\mathbf{k})=\frac{\varepsilon_1(\mathbf{k})+\varepsilon_2(\mathbf{k})}{2}+\left[\left(\frac{\varepsilon_1(\mathbf{k})-\varepsilon_2(\mathbf{k})}{2}\right)^2+J^2\cos^2\left(\frac{k_zc}{2}\right)\right]^{1/2}, \tag{7}$$

$$E_{\mathrm{II}}(\mathbf{k})=\frac{\varepsilon_1(\mathbf{k})+\varepsilon_2(\mathbf{k})}{2}-\left[\left(\frac{\varepsilon_1(\mathbf{k})-\varepsilon_2(\mathbf{k})}{2}\right)^2+J^2\cos^2\left(\frac{k_zc}{2}\right)\right]^{1/2}. \tag{8}$$

The functions $E_{\mathrm{I}}(\mathbf{k})$, and $E_{\mathrm{II}}(\mathbf{k})$ describe an electron spectrum in two zones: I and II. They originate from $\varepsilon_1(\mathbf{k})$ and $\varepsilon_2(\mathbf{k})$ because of the "mixing" of states of mutually perpendicular chains, represented by the term with J in Eq. (1). Therefore, c_{I} and c_{II} are electron annihilation operators for the zones I and II. Figure 3 shows sections of the Fermi surface [deduced from Eqs. (7) and (8)] by a plane normal to the z axis in the momentum space. The electron-filled states are shown shaded. In the zone I they fill a cylinder with a square base. In the zone II such a cylinder (with a base of twice the side, if we bear in mind the real occupation numbers of $Hg_{3-\delta}AsF_6$ given in Ref. 7) is occupied by "holes".[2)]

We shall write down the Hamiltonian of the interaction of the chain electrons with three-dimensional phonons of the main lattice as follows[10]

$$H_{eph}=\gamma\sum_{\mathbf{k},\mathbf{q},\alpha;j=1,2} i\left(\frac{\rho_0\hbar q}{2Vu}\right)^{1/2}$$

$$\times(c_{j\alpha\mathbf{k}}^+c_{j\alpha\mathbf{k}-\mathbf{q}}a_{\mathbf{q}}-c_{j\alpha\mathbf{k}-\mathbf{q}}^+c_{j\alpha\mathbf{k}}a_{\mathbf{q}}^+), \tag{9}$$

where $a_{\mathbf{q}}^+$ and $a_{\mathbf{q}}$ are the phonon operators; u is the velocity of sound; ρ_0 is the equilibrium density of the lattice; V is the volume of a sample; $\hbar$ is the Planck constant (we shall henceforth assume that $\hbar=1$); γ is the interaction constant. The overlap of the Ψ functions along mutually perpendicular chains is small. Therefore, Eq. (9) does not contain terms of the $c_{i\alpha\mathbf{k}}^+c_{2\alpha\mathbf{k}-\mathbf{q}}\alpha_{\mathbf{q}}$, type corresponding to electron transitions from the chain family 1 to the family 2 under the influence of phonons.

Our aim is now to express H_{eph} in terms of the operators $c_{\rm I}$ and $c_{\rm II}$ and thus find the matrix elements of the electron–phonon interaction for intraband and interband transitions. We shall do this by going over in Eq. (9) from $c_{i\alpha\mathbf{k}}$, and $c_{2\alpha\mathbf{k}-\mathbf{q}}$ to $c_{\rm I}$ and $c_{\rm II}$ taken from Eq. (2). We then obtain

$$H_{eph}=\gamma\sum_{\mathbf{kq}\alpha} i\left(\frac{\rho_0 q}{2Vu}\right)^{1/2}\{A(\mathbf{k},\mathbf{k}-\mathbf{q})\,(c^+_{\rm I\alpha k}c_{\rm I\alpha k-q}a_{\mathbf{q}}+c^+_{\rm II\alpha k}c_{\rm II\alpha k-q}a_{\mathbf{q}})$$

$$+B(\mathbf{k},\mathbf{k}-\mathbf{q})\,(c^+_{\rm I\alpha k}c_{\rm II\alpha k-q}a_{\mathbf{q}}-c^+_{\rm II\alpha k}c_{\rm I\alpha k-q}a_{\mathbf{q}})-\text{c.c.}\}. \quad (10)$$

The coefficients $A(\mathbf{k},\mathbf{k}-\mathbf{q})$ and $B(\mathbf{k},\mathbf{k}-\mathbf{q})$ are given by

$$A^2(\mathbf{k},\mathbf{p})=\frac{1}{2}\left[1+\frac{z_{\mathbf{k}}z_{\mathbf{p}}}{(z_{\mathbf{k}}^2+\mathcal{J}_{\mathbf{k}}^2)^{1/2}(z_{\mathbf{p}}^2+\mathcal{J}_{\mathbf{p}}^2)^{1/2}}+\left(\frac{\mathcal{J}_{\mathbf{k}}^2\mathcal{J}_{\mathbf{p}}^2}{(z_{\mathbf{k}}^2+\mathcal{J}_{\mathbf{k}}^2)(z_{\mathbf{p}}^2+\mathcal{J}_{\mathbf{p}}^2)}\right)^{1/2}\right], \quad (11)$$

$$B^2(\mathbf{k},\mathbf{p})=1-A^2(\mathbf{k},\mathbf{p}); \qquad z_{\mathbf{k}}=(\varepsilon_1(\mathbf{k})-\varepsilon_2(\mathbf{k}))/2. \quad (12)$$

2. TRANSPORT EQUATION

From Eqs. (10)–(12) we can obtain in the usual manner[11] the transport equation for the nonequilibrium correction $-\varphi(\mathbf{p})\partial n(\mathbf{p})/\partial\varepsilon(\mathbf{p})$ to the electron distribution function $n_{\mathbf{p}}$. We shall give separately the equations for the zones I and II:

$$-v_{\rm I}(\mathbf{k})eE\cos(\widehat{\mathbf{n}_{\rm Ik}\mathbf{x}})=\frac{1}{(2\pi)^2}\left\{\int\frac{dS_{\rm Ip}}{v_{\rm I}(\mathbf{p})}\gamma^2\rho_0A^2(\mathbf{k},\mathbf{p})\right.$$
$$\times[\varphi(\mathbf{k})-\varphi(\mathbf{p})](\mathbf{k}-\mathbf{p})^2\frac{\partial N(\mathbf{k}-\mathbf{p})}{\partial\omega}+\int\frac{dS_{\rm IIp}}{v_{\rm II}(\mathbf{p})}\gamma^2\rho_0B^2(\mathbf{k},\mathbf{p})$$
$$\left.\times[\varphi(\mathbf{k})-\varphi(\mathbf{p})](\mathbf{k}-\mathbf{p})^2\frac{\partial N(\mathbf{k}-\mathbf{p})}{\partial\omega}\right\}, \quad (13)$$

$$-v_{\rm II}(\mathbf{k})eE\cos(\widehat{\mathbf{n}_{\rm IIk}\mathbf{x}})$$
$$=\frac{1}{(2\pi)^2}\left\{\int\frac{dS_{\rm Ip}}{v_{\rm I}(\mathbf{p})}\gamma^2\rho_0B^2(\mathbf{k},\mathbf{p})[\varphi(\mathbf{k})-\varphi(\mathbf{p})]\right.$$
$$\times(\mathbf{k}-\mathbf{p})^2\frac{\partial N(\mathbf{k}-\mathbf{p})}{\partial\omega}$$
$$\left.+\int\frac{dS_{\rm IIp}}{v_{\rm II}(\mathbf{p})}\gamma^2\rho_0A^2(\mathbf{k},\mathbf{p})[\varphi(\mathbf{k})-\varphi(\mathbf{p})](\mathbf{k}-\mathbf{p})^2\frac{\partial N(\mathbf{k}-\mathbf{p})}{\partial\omega}\right\}, \quad (14)$$

where

$$v_i(\mathbf{k})=|\partial E_i(\mathbf{k})/\partial\mathbf{k}|_{E_i(\mathbf{k})=\varepsilon_F}$$

is the modulus of the vector of the velocity on the Fermi surface in the i-th zone ($i=$ I, II); $\mathbf{n}_{i\mathbf{k}}$ is the external normal to the Fermi surface at the point $\mathbf{k}$ in the i-th zone; $e<0$ is the electron charge. The index i of $dS_{i\mathbf{p}}$ means that integration is carried out over the Fermi surface in the i-th zone. Finally,

$$N(q)=\{\exp[\omega(q)/T]-1\}^{-1}$$

is the equilibrium distribution function of phonons.[3)]

The coefficients $A^2(\mathbf{k},\mathbf{p})$ and $B^2(\mathbf{k},\mathbf{p})$ in Eqs. (13) and (14) have a simple physical consequence. An electron cannot "jump" from one planar part of the Fermi surface to a perpendicular planar part bypassing a "corner" even if a phonon with the necessary quasimomentum and energy is available for this purpose. This anisotropy of the electron–phonon interaction is related directly to the weak overlap of the Ψ functions localized at different chain families[5] (see § 1).

The electrical conductivity is proportional to the time in which an electron makes a round trip on the Fermi surface.[13] We shall consider the range of low temperatures such that $T\ll\Theta_D$ ($\Theta_D=70$ K is the Debye temperature of the AsF_6^- lattice[14]). Therefore, $\sigma_\parallel$ includes contributions of two processes: the diffusion of the planar parts of the Fermi surface between the corners (Fig. 3) and the process of overcoming the corners. The former process takes the time

$$\tau_{tr}\sim\tau_{11}(p_0/q_T)^2\sim T^{-5},$$

where $\tau_{11}^{-1}\sim T^3/\Theta_D^2$ is the frequency of the electron–phonon collisions (Ref. 13); $q_T\sim T/u$ is the thermal momentum of a phonon; p_0 is the characteristic size of the large parts of the Fermi surface.

The second process is characterized by the time τ_{12} which depends in different ways on the temperature T in two limiting cases of $q_T\ll\Delta p$ and $q_T\gg\Delta p$, where Δp is the characteristic size of the corner region in the momentum space. We shall estimate it by selecting $\mathbf{k}$ in a planar part of the Fermi surface (Fig. 3). Then, in Eq. (11) we have $z_{\mathbf{k}}\gg|J|$ and

$$A^2(\mathbf{k},\mathbf{p})\approx\frac{1}{2}\left(1+\frac{z_{\mathbf{p}}}{(z_{\mathbf{p}}^2+\mathcal{J}_{\mathbf{p}}^2)^{1/2}}\right).$$

As long as the point p is still in the same planar region, we have $z_{\mathbf{p}}\gg|J|>0$ and $A^2(\mathbf{k},\mathbf{p})\approx1$. At the corner the quantity $z_{\mathbf{p}}$ changes its sign. Therefore, on a plane perpendicular to the original one we have $z_{\mathbf{p}}<0$ and $|z_{\mathbf{p}}|\gg|J|$. We thus find that

$$A^2(\mathbf{k},\mathbf{p})\sim(\mathcal{J}_{\mathbf{p}}^2/z_{\mathbf{p}}^2)\ll1.$$

However, in the corner region we have $|z_{\mathbf{p}}|\lesssim|J|$ and $A^2(\mathbf{k},\mathbf{p})\approx\frac{1}{2}$. This is a condition that determines the characteristic size of a corner region where an electron can still jump from a plane (where it can pseudointeract with a phonon):

$$\Delta p\sim(|J|/\varepsilon_F)p_0\sim10^{-2}p_0.$$

Consequently, if $q_T\ll\Delta p$, i.e., if $T\ll1$ K, an electron overcomes a corner by diffusion so that $\tau_{12}\propto T^{-5}$ and $\sigma_\parallel(T)\propto T^{-5}$.

We shall now assume that $q_T\gg\Delta p$ ($T\gg1$ °K). Then,

$$\frac{1}{\tau_{12}}\sim\frac{\Delta p}{q_T}\frac{1}{\tau_{11}}\sim\frac{T^2(\Delta p)}{up_0^2}$$

in a q_T corner region and we have $1/\tau_{12}\propto\exp(-y/q_T)$ for electrons jumping from a distance $y\gg q_T$ (in the momentum space). In this case we simply have an exponentially small number of phonons with the quasimomenta in the range $y\ll q_T$.

A corner is a strip of the Fermi surface of width $\sim\Delta p\ll q_T$. Therefore, not every phonon can facilitate the transfer of an electron to a corner from a region q_T in its vicinity. The factor $\Delta p/q_T$ in front of $1/\tau_{11}$ represents the proportion of those thermal phonons which are capable of colliding with an electron per unit time and have a suitable projection of the quasimomentum along the direction toward the corner.

It follows that the frequency of jumps from a plane to a corner averaged over the intial distance from the corner is

$$\omega_{12} \sim \frac{1}{p_0}\int_0^{p_0} \frac{dy}{\tau_{12}(y)} \sim \frac{q_T}{p_0}\frac{1}{\tau_{12}(y\simeq q_T)} \sim \frac{q_T}{p_0}\frac{\Delta p}{q_T}\frac{1}{\tau_{11}} \sim T^3.$$

It follows that

$$\sigma_{\parallel}(T) \sim \tau_{tr} + \omega_{12}^{-1} \sim (p_0/q_T)^2\tau_{11} + (q_T\tau_{11}/\Delta p)\,(p_0/q_T),$$

and hence it is clear that if $p_0/\Delta p \gg (p_0/q_T)^2$, i.e., if

$$T \gg up_0(|J|/\varepsilon_F)^{1/2} \sim 10\,\mathrm{K},$$

the conductivity obeys $\sigma_{\parallel}(T) \propto T^{-3}$ in accordance with the experimental results reported in Ref. 4. Therefore, the T^{-3} dependence is not due to the diffusion over the quasiplanar parts of the Fermi surface (see Ref. 5), but due to nondiffusion process of overcoming the corners.

We can determine how the anisotropy of the electron–phonon interaction affects the electron distribution function and then present the above qualitative conclusions in terms of $\varphi(\mathbf{p})$, by considering Eqs. (13) and (14).

3. INVESTIGATION OF THE TRANSPORT EQUATION

We shall consider low temperatures when $\Delta p \ll q_T \ll p_0$, i.e.,

$$(|J|/\varepsilon_F)\Theta_D \ll T \ll \Theta_D.$$

We shall postulate that $\mathbf{k}$ lies on a planar part of the Fermi surface far from corners. Then, integration with respect to $\mathbf{p}$ in Eqs. (13) and (14) is limited to the same planar region because $\mathbf{p} = \mathbf{k} - \mathbf{q}$ and $|q| \lesssim q_T \ll p_0$, where $\mathbf{q}$ is the phonon quasimomentum. However, for such pairs of values of $\mathbf{k}$ and $\mathbf{p}$ the relevant factors are $A^2(\mathbf{k}, \mathbf{p}) \approx 1$ and $B^2(\mathbf{k}, \mathbf{p}) \approx 0$ [see Eqs. (11) and (12)]. Therefore, we can use the diffusion approximation[13] to solve Eqs. (13) and (14) in this range of values of k.

We shall allow for the fact that $\varphi(\mathbf{k}) = \varphi(k_y)$ or $\varphi(\mathbf{k}) = \varphi(k_x)$ for the regions parallel to the k_y or k_x axis (see Fig. 3).[4] Considering Eqs. (13) and (14) and expanding the difference $\varphi(\mathbf{k}) - \varphi(\mathbf{p})$ in the integrand with respect to q_T/p_0, we obtain

$$-eE\cos(\widehat{\mathbf{n}_{\mathbf{k}}\mathbf{x}}) = \pi D\frac{d^2\varphi(k_\alpha)}{dk_\alpha^2}, \qquad (15)$$

where

$$D = \frac{30T^5}{\pi^2 v_k^2 u^5}\xi(5)\left(\frac{\gamma^2\rho_0}{2u}\right) \qquad (16)$$

[$\xi(5) = 1.037$], $i = \mathrm{I}, \mathrm{II}$; $\alpha = x$ or y, depending on the orientation of a planar region.[5]

For example, if the point $\mathbf{k}$ lies in the zone I in a region normal to $\mathbf{E}$ (Fig. 3), Eq. (15) gives

$$\varphi(k_y) = -\frac{eE}{\pi D}k_y(k_y + 2p_0) + C_{\mathrm{I}}, \qquad (17)$$

where C_{I} is a constant defined below. This form satisfies the self-evident requirement that the solution be symmetric about a line parallel to $\mathbf{E}$ and dividing the square in Fig. 3 into two halves (this is the symmetry axis of the square).

If the point $\mathbf{k}$ lies in the zone I in a region parallel to $\mathbf{E}$, then (if we ignore quantities of the order of $\sim v_F(|J|/\varepsilon_F)$ compared with v_F) we find from Eq. (15) that

$$\varphi(k_x) = G_{\mathrm{I}}(k_x + p_0). \qquad (18)$$

Here, G_{I} is another constant also defined below.

The function $\varphi(k_x)$ in Eq. (18) is odd in $k_x + p_0$. This property is a consequence of two factors. First, we have $\varphi(-\mathbf{k}) = -\varphi(\mathbf{k})$, if the origin of the coordinate system lies at the center of inversion of the square. This is associated with the odd (with respect to k) nature of the left-hand side of Eq. (15), where $\mathbf{n}_{\mathbf{k}}$ is an external normal to the Fermi surface. Second, $\varphi(-k_y) = \varphi(k_y)$ because of the symmetry of the Fermi surface relative to the axis of the square parallel to $\mathbf{E}$. Consequently,

$$\varphi(-k_x, -k_y) = \varphi(-k_x, k_y) = -\varphi(k_x, k_y).$$

Since we have selected the origin of the coordinate system outside the center of the square in Fig. 3, we obtain Eq. (18).

The linearity of Eq. (18) in k_x is a simple consequence of the conservation of the electron flux ($\sim -\nabla\varphi$) in the course of diffusion over the planar parts of the Fermi surface parallel to $\mathbf{E}$.

A calculation which allows for the zone II contributes nothing basically new to the relaxation mechanism[6] or to the nature of the dependence of $\sigma_{\parallel}$ on T. Therefore, we shall ignore the presence of the second zone and assume that there is only zone I. Naturally, all that we have said about $A^2(\mathbf{k}, \mathbf{p})$ still applies. Then, the whole electron flux from a planar part of the Fermi surface normal to $\mathbf{E}$ is directed through the corners to regions parallel to $\mathbf{E}$ in the same zone. The constant G_{I} is found from the condition of conservation of this flux. The constant C_{I} is also found from the same condition, but applied together with the definition of the behavior of $\varphi(\mathbf{k})$ in the vicinity of the corners due to the "transmission" of the latter.

We shall begin by finding G_{I}. We shall cut a cylindrical surface into two halves by a plane parallel to z (this is the axis normal to the plane in Fig. 3) and parallel to the field $\mathbf{E}$. We shall denote the right-hand half by S^+ and the left-hand one by S^-. Obviously, $S = S^+ \cup S^-$ represents the whole of the Fermi surface. We shall rewrite Eq. (13) in the equivalent form dropping the term with $\int dS_{\mathrm{II}_{\mathbf{p}}}$ in accordance with the adopted simplification:

$$-eE\cos(\widehat{\mathbf{n}_{\mathbf{k}}\mathbf{x}}) = \frac{1}{(2\pi)^2}\int_S \frac{dS_{\mathbf{p}}}{v(\mathbf{k})\,v(\mathbf{p})}\gamma^2\rho_0 A^2(\mathbf{k},\mathbf{p})\,[\varphi(\mathbf{k}) - \varphi(\mathbf{p})] \times (\mathbf{k} - \mathbf{p})^2\frac{\partial N(\mathbf{k} - \mathbf{p})}{\partial\omega}. \qquad (19)$$

Integrating both sides of Eq. (19) over the surface S^+, we obtain

$$-\int_{S_+} eE\cos(\widehat{\mathbf{n}_{\mathbf{k}}\mathbf{x}})\,dS_{\mathbf{k}} = \frac{1}{(2\pi)^2}\gamma^2\rho_0\int_{S_+} dS_{\mathbf{k}}\int_S dS_{\mathbf{p}} \times \frac{A^2(\mathbf{k},\mathbf{p})\,[\varphi(\mathbf{k}) - \varphi(\mathbf{p})]}{v(\mathbf{k})\,v(\mathbf{p})}(\mathbf{k} - \mathbf{p})^2\frac{\partial N(\mathbf{k} - \mathbf{p})}{\partial\omega}. \qquad (20)$$

We can easily see that the integrand on the right is antisymmetric under the transposition $k \leftrightarrow p$ (this is a common property and it is not due to the fact that we have ignored the zone II Ref. 13). Therefore, if the contribution to the left-hand side of Eq. (20) made by regions parallel to E is negligible, we find from Eq. (20) that

$$-2eEp_0l_z=\frac{1}{(2\pi)^2}\gamma^2\rho_0\int_{S^+}dS_{\mathbf{k}}\int_{S^-}dS_{\mathbf{p}}\frac{A^2(\mathbf{k},\mathbf{p})}{v(\mathbf{k})v(\mathbf{p})}$$

$$\times[\varphi(\mathbf{k})-\varphi(\mathbf{p})](\mathbf{k}-\mathbf{p})^2\frac{\partial N(\mathbf{k}-\mathbf{p})}{\partial\omega}, \tag{21}$$

where $l_z=2\pi/c$ is the length of the Brillouin zone along the z axis in the quasimomentum space. Since the line separating S^+ from S^- lies far from the corners ($p_0\gg q_T$), we can substitute on the right-hand side of Eq. (21) the function $\varphi(\mathbf{k})$ from Eq. (18). We then obtain

$$eEp_0l_z=l_z\pi DG_1, \tag{22}$$

where we have ignored the corrugations of the surface along z. Hence, we find that

$$G_1=eEp_0/\pi D, \tag{23}$$

where D is defined by Eq. (16).

In the vicinity of a corner we have

$$\Delta p\sim p_0(|J|/\varepsilon_F),$$

where $\varphi(\mathbf{k})$ varies rapidly (this is demonstrated below), but we have $\Delta\mathbf{p}\ll q_T$ and the diffusion approximation can no longer be used. However, the same circumstance ($\Delta p\ll q_T$) allows us to find $\varphi(\mathbf{k})$ in a region within a distance of Δp from a corner and this can be done using directly the integral equation (19) with the results subject to an error $\Delta p/q_T\ll 1$.

It is therefore convenient to use the fact that $\varphi(\mathbf{k})$ depends in reality only on the distance from a corner. Therefore, we shall introduce a coordinate y on the Fermi surface: the absolute value of this coordinate is equal to the distance from a corner [here, $k_x(y)$, $k_y(y)$, and $k_z(y)$ are the components of the radius vector $\mathbf{k}$ of a point located at a distance $|y|$ from a corner]. The right-hand vertical part of the Fermi surface corresponds to $y<0$, the right-hand one to the horizontal region $y>0$, and in the vicinity of a corner we have $-p_c<y<p_c$ (Fig. 4). We shall now ignore the nonlinearity of the mapping which transfers a small (compared with p_0) region of size Δp to a segment $[-p_c,p_c]$. We can now rewrite Eq. (19) in the form

$$-eE\cos(\widehat{\mathbf{n}_y\mathbf{x}})$$

$$=\int_{-\infty}^{+\infty}\frac{dy'}{v(y)v(y')}A^2(y,y')F(y,y')\cdot[\varphi(y)-\varphi(y')], \tag{24}$$

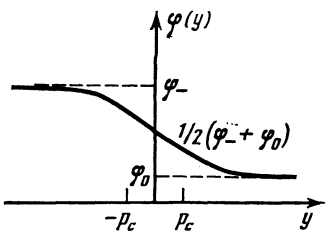

FIG. 4. Behavior of $\varphi(y)$ in the vicinity of a corner of the Fermi surface. The $\mathbf{k}$ faces in Fig. 3 correspond to $y<-p_c$, and the $\mathbf{p}$ faces to $y>p_c$, whereas the corner occurs at $-p_c<y<p_c$.

where

$$F(y,y')=\frac{\gamma^2\rho_0}{(2\pi)^2}$$

$$\times\int_{-\infty}^{+\infty}[(k_x(y)-p_x(y'))^2+(k_y(y)-p_y(y'))^2+(k_z-p_z)^2]\frac{\partial N}{\partial\omega}dp_z, \tag{25}$$

$$A^2(y,y')=\frac{1}{2}\left[1+\frac{yy'}{(y^2+p_c^2)^{1/2}((y')^2+p_c^2)^{1/2}}+\frac{p_c^2}{(y^2+p_c^2)^{1/2}((y')^2+p_c^2)^{1/2}}\right]; \tag{26}$$

$$v(y)=\frac{v_F}{\sqrt{2}}\left(1+\frac{y^2}{y^2+p_c^2}\right)^{1/2} \tag{27}$$

and $p_c=2|J|/v_F$. We shall now explain the meaning of Eqs. (24)–(27). Equation (24) is obtained from Eq. (19) by going over from dS_p to dp_xdp_z and dp_ydp_z on the relevant parts of the Fermi surface, subject to the notation change

$$dp_y\to dy'\,(y'<0);\quad dp_x\to dy'\,(y'>0)$$

in accordance with Fig. 4. This procedure gives also Eq. (26) instead of Eq. (11) if we note that

$$z_{\mathbf{k}}=(\varepsilon_1(\mathbf{k})-\varepsilon_2(\mathbf{k}))/2$$

and $z_{\mathbf{p}}$ in Eq. (11) changes to $-v_Fy/2$ and $-v_Fy'/2$ in the vicinity of a corner ($y=0$) if $|y|,|y'|\ll p_0$. Finally, using $p_c=2|J|/v_F$ instead of the exact expression

$$p_c(k_z)=2|J\cos(k_zc/2)|/v_F,$$

we ignore the unimportant corrugation of the Fermi surface along the z axis, since the corrugation period is $\sim p_0\gg q_T$ and we have $|k_z-p_z|\lesssim q_T$. The expression (27) for $v(y)$ is obtained from the definition

$$v(\mathbf{k})=|\partial E(\mathbf{k})/\partial\mathbf{k}|$$

by the same method.

We shall rewrite Eq. (24) in the form

$$\frac{\varphi(y)}{v(y)}\int_{-\infty}^{+\infty}\frac{dy'}{v(y')}A^2(y,y')F(y,y')=\int_{-\infty}^{+\infty}\frac{dy'}{v(y)v(y')}A^2(y,y')\varphi(y')F(y,y')-eE\cos(\widehat{\mathbf{n}_y\mathbf{x}}). \tag{28}$$

We are interested in the behavior of $\varphi(y)$ in the vicinity of a corner, i.e., when $|y|\lesssim p_c$. It follows from Eq. (25) that the characteristic scale of a change in $F(y,y')$ along y and y' is of the order of $q_T\gg p_c$. Completing the integration with respect to y' on the left-hand side of Eq. (28), we obtain ($|y|\ll q_T$)

$$-\frac{\varphi(y)}{2\tau_{11}}=-v(y)eE\cos\theta_y+\int_{-\infty}^{+\infty}\frac{dy'}{v(y')}A^2(y,y')\varphi(y')F(y,y'), \tag{29}$$

where

$$\frac{1}{\tau_{11}}=\frac{3\gamma^2\rho_0}{2\pi v_Fu}\left(\frac{T}{u}\right)^3\int_0^\infty\frac{x^2dx}{e^x-1} \tag{30}$$

and $\cos\theta_y=\cos(\widehat{\mathbf{n}_y\mathbf{x}})$. In view of the presence of the factor $F(y,y')$, the integration with respect to y' in Eq. (29) is over a region $|y'|\sim q_T$. For such values of y' we find that $A^2(y,y')$ of

Eq. (26), regarded as a function of y, changes greatly in an interval $\sim p_c \ll q_T$. The same property is exhibited by $v(y)$ of Eq. (27). However, it follows then from Eq. (29) that the scale of change of $\varphi(y)$ is also $\sim p_c$. Therefore, substituting in Eq. (29) the function $\varphi(y')$ in the form

$$\varphi(y')=\begin{cases}\varphi_-, & y'<0\\ \varphi_0, & y'\geqslant 0\end{cases}, \tag{31}$$

where φ_- and φ_0 have to be found and the inequality $|y'| \lesssim q_T$, is obeyed, we readily obtain from Eq. (29)

$$\varphi(y)=2\tau_{11}v(y)eE\cos\theta_y+\frac{1}{2}\left\{\varphi_-\left[1-\frac{y}{(y^2+p_c^2)^{1/2}}+O\left(\frac{p_c^2}{q_T(y^2+p_c^2)^{1/2}}\right)\right]+\varphi_0\left[1+\frac{y}{(y^2+p_c^2)^{1/2}}+O\left(\frac{p_c^2}{q_T(y^2+p_c^2)^{1/2}}\right)\right]\right\}. \tag{32}$$

The roughness of the "unrenormalized" function (31) in the region $|y| \lesssim p_c$ does not affect the result (32) because the integration in Eq. (29) is over a large part of the region $|y'| \sim q_T$. The error introduced in Eq. (32) by the replacement of $\varphi(y')$ with constants in an interval $|y'| \lesssim q_T$, is also small in terms of the parameter p_c/q_T, as shown below.

Matching $\varphi(k_x)$ to $\varphi(y)$ from Eq. (18) in the range $y \gg p_c$, we obtain [using Eq. (23)]

$$\varphi_0=G_1p_0=eEp_0^2/\pi D. \tag{33}$$

We shall find φ_- integrating both sides of Eq. (19) over the surface S_1 of the right-hand planar part of the Fermi surface all the way to the corners. We then obtain, by analogy with Eqs. (20) and (21),

$$-2eEp_0l_z=\frac{1}{(2\pi)^2}\gamma^2\rho_0\int_{S_1}dS_{\mathbf{k}}\int_{S-S_1}dS_{\mathbf{p}}\frac{A^2(\mathbf{k},\mathbf{p})}{v(\mathbf{k})v(\mathbf{p})}\times[\varphi(\mathbf{k})-\varphi(\mathbf{p})](\mathbf{k}-\mathbf{p})^2\frac{\partial N(\mathbf{k}-\mathbf{p})}{\partial\omega}, \tag{34}$$

where $S-S_1$ is the whole Fermi surface with the exception of S_1. In view of the presence of the factor $\partial N(\mathbf{k}-\mathbf{p})/\partial\omega$, the integration on the right-hand side of Eq. (34) is carried out in the vicinity of the lines on the Fermi surface separating S_1 from the corners. In view of the symmetry of the Fermi surface half the flux $-2eEp_0l_z$ passes through each of the two lines. Therefore, we shall consider only the upper right-hand corner in Fig. 3. As in the case of going over from (19) to (24), we shall represent Eq. (34) in the equivalent form for the selected corner:

$$-eEp_0l_z=l_z\int_{-\infty}^{y_0}\frac{dy}{v(y)}\int_{y_0}^{\infty}\frac{dy'}{v(y')}A^2(y,y')F(y,y')[\varphi(y)-\varphi(y')], \tag{35}$$

where $y_0(<0)$ is the coordinate of the line representing the boundary (in the right-hand upper part) of the surface S_1; the limits $\pm\infty$ in the integral (35) are inserted for convenience because $q_T \ll_0$. Selecting $p_c \ll |y_0| \ll q_T$ and substituting $\varphi(y)$ from Eq. (32) and $\varphi(y)$, and also taking $A^2(y,y')$ from Eq. (26), we obtain from Eq. (35)

$$-2eEp_0=-\left(\frac{\varphi_--\varphi_0}{2}\right)\int_{-\infty}^{y_0}\frac{dy}{v(y)}\int_{y_0}^{\infty}\frac{dy'}{v(y')}\frac{p_c^2}{[(y')^2+p_c^2]}F(y,y'). \tag{36}$$

The factor $p_c^2/[p_c^2+(y')^2]$ in the integral (36) justifies the use of Eq. (32) as $\varphi(y')$. After integration in Eq. (36), we obtain

$$\varphi_-=\frac{2eEp_0\tau_{11}v_F}{\nu p_c}+\varphi_0=\frac{2eEp_0\tau_{11}v_F}{\nu p_c}+\frac{eEp_0^2}{\pi D}, \tag{37}$$

where

$$\nu=\frac{1}{4}\int_0^{\infty}\frac{dx}{[(x^2+1)(x^2+1/2)]^{1/2}}\approx 1, \tag{38}$$

and $1/\tau_{11}$ is defined in Eq. (30). The matching of $\varphi(y)=\varphi_-$ for $y \lesssim -q_T$ with Eq. (17) gives $\varphi_- \approx C_1$.

Knowing now $\varphi(k)$ for all parts of the Fermi surface and using the expression for the current density[13]

$$\mathbf{j}=\frac{2e}{(2\pi)^3}\int_{S_F}\varphi(\mathbf{k})\mathbf{n}(\mathbf{k})\,dS_{\mathbf{k}},$$

we find the conductivity

$$\sigma_{\parallel}(T)=\frac{e^2S_F}{(2\pi)^3}\left[\frac{2p_0\tau_{11}v_F}{\nu p_c}+\frac{5}{3}\frac{p_0^2}{\pi D}\right]\equiv\frac{e^2S_F}{(2\pi)^2}\frac{v_F^2u^4p_0}{\gamma^2\rho_0}T^{-3}\times\left[\frac{v_F}{6\nu\xi(3)|J|}+\frac{p_0}{18\xi(5)}\left(\frac{u}{T}\right)^2\right], \tag{39}$$

where S_F is the Fermi surface and $\xi(3)=1.202$. It follows from Eq. (39) that $\sigma_{\parallel} \propto T^{-3}$ at temperatures

$$T \gg \frac{up_0}{\sqrt{10}}\left(\frac{|J|}{p_0v_F}\right)^{1/2}\approx\frac{up_0}{30}\sim 1\ ^{\circ}\mathrm{K}.$$

Allowance for both zones adds factors of the order of unity to the terms in Eq. (39) and S_F then denotes the total Fermi surface area in the two zones.

Finally, we note that, in accordance with Eq. (17), the increment $\Delta\varphi$ in an interval $\sim q_T$ is

$$\Delta\varphi\sim\frac{eE}{D}p_0q_T.$$

The change in the function $\varphi(y)$ in the interval $|y|<p_c$ obtained from Eq. (32) is

$$\Delta\varphi(p_c)\sim\varphi_-\sim eEp_0\tau_{11}v_F/p_c$$

if $\varphi_- \gg \varphi_0$. Therefore, the relative error in the determination of $\varphi(y)$ from Eq. (32) introduced by the replacement of $\varphi(y)$ with φ_- is of the order of

$$\Delta\varphi/\Delta\varphi(p_c)\sim p_c/q_T \ll 1.$$

The same error results from the replacement of $\varphi(y)$ with φ_0 in the case when $q_T \gg y \gg p_c$, and this justifies the above simplifications.

4. INFLUENCE OF SHORT-WAVELENGTH PHONONS

As temperature increases, the backscattering of electrons by short-wavelength phonons with $q=2p_0$ increases in frequency. Since in the case of $Hg_{3-\delta}AsF_6$ we have $p_0 \approx 0.3g$, where g is the reciprocal vector of the AsF_6^- lattice,[7] it follows that such phonons have energies $\Omega \sim \Theta_D \approx 70$ K (Ref. 5).

If $T \ll \Theta_D$, we obtain the following estimates of the integrals of the phonon–phonon collisions I_{pp} involving phonons with a momentum $2p_0$ (Ref. 11):

$$I_{pp}^{(1)}(2p_0) \sim \frac{\partial N_0}{\partial \Omega} \chi \frac{\Theta_D^3}{\varepsilon_F g^3} \times \int \delta[\Omega(\mathbf{k}_1)+\Omega(\mathbf{k}_2)-\Omega(\mathbf{k})]\, d^3\mathbf{k}_1 \sim \frac{\partial N_0}{\partial \Omega} \chi \frac{\Theta_D^3}{\varepsilon_F g^3} \left(\frac{g^3}{\Theta_D}\right) = \frac{\partial N_0}{\partial \Omega} \chi \frac{\Theta_D^2}{\varepsilon_F} \tag{40}$$

for the decay of a phonon with a momentum $2p_0$ into two short-wavelength phonons and

$$I_{pp}^{(2)}(2p_0) \sim \frac{\partial N_0}{\partial \Omega} \frac{T^3}{\varepsilon_F \Theta_D} \chi \tag{41}$$

in the case of the scattering by a thermal phonon $q \sim T/u$; here, χ is defined as

$$\delta N = -\chi \partial N_0 / \partial \Omega$$

representing a nonequilibrium correction to the equilibrium phonon distribution function N_0. Naturally, in Eqs. (40) and (41) the symbol χ simply represents the order of the corresponding quantity.

In the case of the integral of the phonon–electron collisions $I_{pe}(2p_0)$ associated with electron transitions between the opposite vertical "planes" of the Fermi surface (Fig. 3), we obtain the estimate

$$I_{pe}(2p_0) = \frac{\partial N_0}{\partial \Omega} \int \omega (n_0' - n_0) \times [\varphi(\mathbf{p}') - \varphi(\mathbf{p}) + \chi_{2p_0}] \delta[\varepsilon(\mathbf{p}) - \varepsilon(\mathbf{p}') - \Omega_{2p_0}] \frac{2d^3\mathbf{p}}{(2\pi)^3} \approx \frac{\partial N_0}{\partial \Omega} \int \omega (n_0' - n_0) \times (\varphi^- - \varphi^+ + \chi_{2p_0}) \delta(2v_F p_x - v_F k_x + \alpha(p_y) - \alpha(p_y - k_y) - \Omega_{2p_0}) \frac{2d^3\mathbf{p}}{(2\pi)^3} \sim \frac{\partial N_0}{\partial \Omega} \times (-2\varphi^+ + \chi_{2p_0}) \begin{cases} \Theta_D; & J' \ll \Theta_D \\ \Theta_D^2/J; & J' \gg \Theta_D \end{cases} \tag{42}$$

where the argument of the δ function is obtained allowing for

$$\varepsilon(\mathbf{p}) - \mu = v_F(|p_x| - p_0) + \alpha(p_y, p_z)$$

describing the electron energy near the Fermi surface; $J' \sim |\alpha|$ represents the curvature of the Fermi surface "planes"; the unimportant dependence of α and p_z is dropped from Eq. (42). Finally, $\varphi(\mathbf{p}) \sim \varphi^+$ on the right Fermi surface plane and $\varphi(\mathbf{p}') \sim \varphi^- \approx -\varphi^+$ on the left plane.

The occurrence of two horizontal Fermi surface "planes" (Fig. 3) has the effect that in addition to $I_{pe}(2p_0)$ of Eq. (42) there is also $I_{pe}'(2p_0)$ with electron transitions along the horizontal planes of the Fermi surface:

$$I_{pe}'(2p_0) = \frac{\partial N}{\partial \Omega} \int \omega (n_0' - n_0) \times [\varphi(\mathbf{p}') - \varphi(\mathbf{p}) + \chi'_{2p_0}] \delta(\varepsilon(\mathbf{p}) - \varepsilon(\mathbf{p}') - \Omega_{2p_0}) \times \frac{2d^3\mathbf{p}}{(2\pi)^3} \approx \frac{\partial N_0}{\partial \Omega} \times \int \omega (n_0' - n_0) \chi'_{2p_0} \delta(v_F k_y + \alpha(p_x) - \alpha(p_x - k_x) - \Omega_{2p_0}) \frac{2d^3\mathbf{p}}{(2\pi)^3} \sim \frac{\partial N_0}{\partial \Omega} \frac{\Theta_D^2}{J'} \chi'_{2p_0}, \tag{43}$$

where in writing down the argument of the delta function we have used the equality

$$\varepsilon(\mathbf{p}) - \mu = v_F(|p_y| - p_0) + \alpha(p_x, p_z)$$

for the electron energy near the horizontal Fermi surface "planes." The prime of χ'_{2p_0} denotes the important circumstance that only the phonons with the projection

$$k_y \sim \Omega_{2p_0}/v_F \sim p_0\Theta_D/\varepsilon_F, \quad \Theta_D \gg J', \quad |k_y| \leqslant J'/v_F \sim p_0 J'/\varepsilon_F, \quad \Theta_D \ll J' \tag{44}$$

may be absorbed by electrons on the horizontal Fermi surface "planes" [in the case of other phonons the argument of the δ function in Eq. (43) cannot vanish]. The scatter of the permissible values of k_y is governed by the curvature J' and is equal to[7]

$$\delta k_y \sim (J'/\varepsilon_F) p_0. \tag{45}$$

In the case of these phonons the steady-state condition for the distribution function

$$I_{pp}^{(1)} + I_{pp}^{(2)} + I_{pe} + I_{pe}' = 0 \tag{46}$$

gives [when Eqs. (42) and (43) are used and $J' \geqslant \Theta_D$]

$$(-2\varphi^+ + \chi'_{2p_0})\Theta_D^2/J' \sim -(\Theta_D^2/J')\chi'_{2p_0}, \tag{47}$$

i.e.,

$$\chi'_{2p_0} \sim \varphi^+, \quad -2\varphi^+ + \chi'_{2p_0} \sim -\varphi^+. \tag{48}$$

It should be noted that in obtaining the estimate (47) from Eq. (46) we have dropped the terms $I_{pp}^{(1)}$ and $I_{pp}^{(2)}$. The point is this: a comparison of Eq. (43) with Eqs. (40) and (41) shows that the phonon–phonon collisions with participation of a "resonance" phonon [Eqs. (44) and (45)] cannot convert it into a "nonresonance" phonon in the time between the emission of such a phonon in a system of vertical "planes" and its absorption in a system of horizontal "planes" of the Fermi surface. It follows that the phonon–phonon collisions represent only a small perturbation.

In the case of short-wavelength phonons that to not satisfy the conditions (44) and (45) we have $I_{pe}' \equiv 0$, and only the first three terms remain in Eq. (46), so that we can no longer ignore the integrals I_{pp} and $I_{pp}^{(2)}$ as in the estimates given by Eqs. (47) and (48). We now find from Eqs. (46) and (40)–(42)

$$\frac{\partial N_0}{\partial \Omega}(-2\varphi^+ + \chi_{2p_0}) \frac{\Theta_D^2}{J'} \sim \frac{\partial N_0}{\partial \Omega} \frac{\Theta_D^2}{\varepsilon_F} \chi_{2p_0}, \tag{49}$$

i.e.,[8]

$$-2\varphi^+ + \chi_{2p_0} \ll \chi_{2p_0} \approx 2\varphi^+. \tag{50}$$

The relationship (48) has an important physical conse-

quence. Approximately half the short-wavelength phonons emitted in electron transitions between the vertical Fermi surface "planes" and satisfying the conditions (44) and (45) are absorbed in the horizontal "planes" of the Fermi surface, giving rise to electron transitions along the open directions, including those accomplished by umklapp processes. Therefore, the quasimomentum acquired from the field by electrons on the vertical "planes" of the Fermi surface is transferred by these phonons to the zone with the horizontal "planes" and is lost in electron–phonon processes accompanied by umklapp transitions [the field-induced electron transitions on the "horizontal" planes can be ignored completely because of the smallness of $J'/\varepsilon_F \ll 1$, and this has been done in obtaining the estimate represented by Eq. (43), where the terms with $\varphi(\mathbf{p})$ and $\varphi(\mathbf{p}')$ are omitted and only the term with χ_{2p} is retained].[9]

An estimate of the integral of the electron–phonon collisions for electrons on vertical "planes" gives[11]

$$I_{ep}=-\int\frac{\partial N_0}{\partial\Omega}\omega(n_0'-n_0)$$
$$\times\{[\varphi(\mathbf{p}')-\varphi(\mathbf{p})+\chi_{\mathbf{k}}]\delta[\varepsilon(\mathbf{p})-\varepsilon(\mathbf{p}')-\Omega_{\mathbf{k}}]$$
$$-[\varphi(\mathbf{p}')-\varphi(\mathbf{p})-\chi_{-\mathbf{k}}]\delta[\varepsilon(\mathbf{p})-\varepsilon(\mathbf{p}')+\Omega_{\mathbf{k}}]\}\frac{d^3\mathbf{k}}{(2\pi)^3}$$
$$\sim-\int\frac{\partial N_0}{\partial\Omega}$$
$$\times\omega(\varphi^--\varphi^++\chi'_{2p_0})\delta(2v_Fp_x-v_Fk_x+\alpha(p_y)-\alpha(p_y-k_y)-\Omega_{2p_0})$$
$$\times\frac{d^3k}{(2\pi)^3}\sim\left(\frac{\Theta_D}{T}\right)^{1/2}\frac{J'}{\varepsilon_F}\exp\left(\frac{-\Omega_{2p_0}}{T}\right)(-2\varphi^++\chi'_{2p_0}). \quad (51)$$

We shall now write down the transport equation for electrons:

$$I_{ep}=eEv\partial n_0/\partial\varepsilon, \quad e<0, \quad (52)$$

and integrate both its sides with respect to $d^3\mathbf{p}$, for example, in the vicinity of the right vertical "plane" of the Fermi surface, which gives [subject to Eq. (51)]

$$-eEg^2=\int I_{ep}d^3\mathbf{p}\sim g^2\left(g\frac{\Theta_D}{\varepsilon_F}\right)\frac{J'}{\varepsilon_F}\left(\frac{\Theta_D}{T}\right)^{1/2}$$
$$\times\exp\left(\frac{-\Omega_{2p_0}}{T}\right)(-2\varphi^++\chi'_{2p_0}), \quad (53)$$

and hence it follows from Eq. (48) that

$$\varphi^+\sim eEv_F\tau_{+-}, \quad (54)$$

where

$$\tau_{+-}^{-1}=\frac{J'\Theta_D}{\varepsilon_F}\left(\frac{\Theta_D}{T}\right)^{1/2}\exp\left(-\frac{\Omega_{2p_0}}{T}\right). \quad (55)$$

On the other hand, the mechanism of relaxation via the corners gives, in accordance with Eq. (37)

$$\varphi^+\sim eE\tau_{11}v_Fg/p_c,$$

where

$$\frac{1}{\tau_{11}}\sim\frac{T^3}{\Theta_D^2}, \quad p_c\sim\frac{|J|}{\varepsilon_F}g.$$

Since, subject to the condition

$$\frac{|J|}{\varepsilon_F}\frac{T^3}{\Theta_D^2}\ll\frac{J'}{\varepsilon_F}\Theta_D\left(\frac{\Theta_D}{T}\right)^{1/2}\exp\left(\frac{-\Omega_{2p_0}}{T}\right)$$

or in the equivalent form (it is assumed that $J'\lesssim|J|$)

$$x^{7/2}\exp(-\varepsilon x)\gg1, \quad (56)$$

where

$$x=\Theta_D/T, \quad \varepsilon=\Omega_{2p_0}/\Theta_D\lesssim1,$$

the mechanism of relaxation accompanied by backscattering predominates over the corner mechanism and this results in a change from the dependence $\sigma_\parallel^{-1}\propto T^3$ [Eq. (39)] to the dependence [see (55)]:

$$\sigma_\parallel^{-1}\sim1/\tau_{+-}\sim T^{-1/2}\exp(-\Omega_{2p_0}/T) \quad (57)$$

subject to the condition

$$\Theta_D2\varepsilon/7<T\ll\Theta_D \quad (58)$$

[the condition (58) is obtained from the inequality (56)].

In comparing the $\sigma_\parallel(T)$ given by Eqs. (39) and (57) with the experimental results, we find that—according to Refs. 3 and 4—the $\sigma_\parallel^{-1}(T)$ can be approximated by a T^n law with the exponent n found by plotting $(\sigma_\parallel^{-1})$ as a function of $\ln T$. Such a procedure shows that in the range $T\gtrsim30$ °K there is a change from $n=3$ to $n=3/2$. We can easily see that in our case the change from Eq. (39) to Eq. (57) corresponds to a change in n from $n=3$ to

$$n=-1/2+\Omega_{2p_0}/T<3$$

when

$$T>2\Omega_{2p_0}/7\sim10\text{ °K}.$$

We can thus see that n also decreases. To determine whether (57) agrees with the experimental results in the interval defined by Eq. (58), we have to plot along the abscissa not $\ln T$ but T.

5. CONCLUSION

We shall conclude be noting that the scattering by short-wavelength phonons [Eq. (50)] not satisfying Eqs. (44) and (45) may give rise to a finite resistance only if we allow for the phonon–phonon collisions (including those of the umklapp type). Estimates using Eqs. (40) and (50) show that this mechanism may give rise to a value of φ^+ of the same order as that given by Eq. (54). A more accurate estimate requires the knowledge of the function $\Omega(\mathbf{k})$ corresponding to $k\sim2p_0$, which then makes it possible to integrate correctly in the presence of the δ function:

$$\delta[\Omega(\mathbf{k}_1)+\Omega(\mathbf{k}_2)-\Omega(\mathbf{k})]d^3\mathbf{k}_1,$$

contained in $I_{pp}^{(1,2)}$ described by Eqs. (40) and (41).

A structural transition takes place at $T>T_c=120$ K in $Hg_{3-\delta}AsF_6$. (Ref. 1): the correlation between the positions of the mercury atoms in various chains disappears. Therefore, at temperatures $T>T_c$ our model with a three-dimensional lattice of the mercury atoms is no longer suitable and we have to solve the problem of conduction of a quasi-one-dimensional Fermi liquid.

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[1)] This idea was put forward in Ref. 5, where the authors began from an intuitive concept of "one-dimensional" diffusion of electrons over the planar parts of the Fermi surface in toward the boundaries of these parts. The estimate $\rho(T) \equiv \sigma^{-1}(T) \propto T^3$ was obtained in Ref. 5 by making the transition $\int dq_x dq_y dq_z \to \int dq_x$ in the phonon momentum integral, but this is not well justified. Here, and in § 2, we shall show that it is precisely the nondiffuse motion of electrons in the region of contact between planar parts of the Fermi surface which gives rise to the dependence $\rho(T) \propto T^3$.

[2)] We note *in passim* that measurements of the residual resistance of $Hg_{3-\delta}AsF_6$ indicate[4] that the localization length of an electron state in a chain is $\lambda > 10^{-4}$ cm. Therefore, the backscattering frequency is $1/\tau_i \sim \varepsilon_F/\lambda p_F \sim 1\,°\text{K} \ll 1/\tau \sim |J| \sim 500\,\text{K}$, where $1/\tau$ is the frequency of electron transitions from chain to chain. Therefore, a system of chains is quasi-one-dimensional only in the case of a small curvature of the parts of the Fermi surface ($|J|/\epsilon_F \sim 10^{-2}$); see Fig. 3. Therefore, we shall ignore the localization of the electron states.

[3)] In Eqs. (13) and (14) it is assumed that phonons are in equilibrium. However, it is well known that their drag in quasi-one-dimensional conductors should be allowed for.[12] There is no contradiction here. The situation is due to the special nature of the Fermi surface of $Hg_{3-\delta}AsF_6$ due to the presence of two families of mutually perpendicular chains. We shall return to this question in § 4.

[4)] We are ignoring here the unimportant corrugations of the Fermi surface of amplitude $\sim p_0(|J|/\varepsilon_F)$ and with a characteristic scale of variation $\sim p_0$.

[5)] Equations (15) and (16) are identical with the results of Ref. 13 in the absence of phonon drag.

[6)] This is not true in the case of the phonon drag (see § 4).

[7)] We shall consider only the case $J' \gtrsim \Theta_D$. If $J' \lesssim \Theta_D$, then the "unrenormalized" vertex of the electron–phonon interaction cannot be used in the case of phonons with quasimomenta satisfying the conditions (44) and (45) and the situation requires a separate study (see Ref. 15). Otherwise, it would have followed from Eq. (43) that $1/\tau_{ph,e} \sim \Theta_D^2/J' \gtrsim \Theta_D$, which makes it meaning less to consider such excitations. The author is grateful to I. B. Levinson for pointing this out.

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