Doubly-logarithmic asymptotic of the quark scattering amplitude with nonvacuum exchange in the t-channel

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Nonlinear differential equations of the Riccati type for the t-channel partial waves $f_j(t)$ describing the scattering of quarks on the mass shell are derived by employing the dispersion relations. The derivation applies to high energies $s^{1/2}$ in the region $\alpha_s \ln^2(s/\mu^2) \sim 1$, where μ is the infrared cutoff parameter with respect to the transverse momenta of the virtual particles. For colorless channels the solutions are found in explicit form. It is shown that the singularities of partial waves with negative signature are in all cases located to the right of the singularities of partial waves with positive signature, i.e., the negative signature dominates in the asymptotic region $\alpha_s \ln^2(s/\mu^2) \gg 1$.

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1. INTRODUCTION

Knowledge of the scattering amplitudes of quarks and gluons is necessary on the parton approach for the description of inclusive hadron-hadron interactions in which the particles are produced with large transverse momenta, and for the calculation of the two-particle exclusive reactions at large momentum transfers. If the characteristic transverse momenta of the hadrons are large enough, the parton scattering amplitudes can be calculated within the framework of quantum chromodynamics by perturbation theory. Substituting the results of the calculation in the formulas of the parton model and comparing the obtained expressions with experiment, one can obtain important information on the hadron wave functions, which are determined by the interaction at large distances.

We consider in this paper the problem of calculating the scattering amplitudes of quarks and their annihilation $q\bar{q} \to Q\bar{Q}$ in the Regge region

$$s \approx -u \gg \mu^2 \gg |t|, \tag{1}$$

where, besides the ordinary invariants s, t, and u, we introduce an auxiliary parameter μ which we idenfity with the infrared cutoff parameter in the Feynman integrals with respect to the transverse components of the momenta of the virtual particles:

$$|k_\perp{}^i| > \mu. \tag{2}$$

The parameter μ is assumed to be much larger than the characteristic hadron mass scale $\Lambda \approx 100$ MeV. The effective strong-interaction coupling constant is here small:

$$\alpha_\mu = \frac{g_\mu{}^2}{4\pi} \sim 1/\ln \frac{\mu^2}{\Lambda^2} \ll 1. \tag{3}$$

There exists a region of energies $s^{1/2}$:

$$\frac{\alpha_\mu}{\pi} \ln^2 \frac{s}{\mu^2} \sim 1, \quad \frac{\alpha_\mu}{\pi} \ln \frac{s}{\mu^2} \ll 1, \tag{4}$$

in which the summation of the principal logarithmic terms $\sim \alpha_\mu [(\alpha_\mu/\pi) \times \ln^2(s/\mu^2)]^n$, which lead to the doubly logarithmic (DL) asymptotic form of the scattering amplitudes, is a valid procedure.

In our approach, the initial and final particles are assumed to lie on the mass shell ($p_1^2 = p_1'^2 = p_2^2 = p_2'^2 = 0$). At sufficiently large momentum transfers $(-t)^{1/2} \gg \Lambda$ we can assume that μ^2 is of the order of $-t$. The amplitude obtained in this manner corresponds to quark scattering accompanied by a jet of partons (quarks and gluons), and the transverse momenta of the particles in these fermion jets has an upper bound $\mu \sim (-t)^{1/2}$.

The infrared cutoff in the integrals with respect to $k_\perp$ can be effected in gauge-invariant fashion (for example, by introducing a high dimensionality of space-time, $D > 4$). The presence of gauge invariance makes our approach preferable to the usual one based on the use of the Bethe-Salpeter equations, which operate with scattering amplitudes off the mass shell. We shall show in this paper that it is possible to construct a nonlinear differential equation directly for the t-channel partial waves $f_j(t)$ on the mass shell, by tracking their variation with changing parameter μ^2 in Eq. (2).

There is a close analogy between our approach to the investigation of the Regge asymptotics and the renormalization-group method[1] used in chromodynamics to calculate the amplitudes of hard processes. The main idea of our approach is to separate in the Feynman diagrams, which describe the scattering amplitude, the virtual particles with minimum values of transverse momenta $k_\perp$, followed by proving that the integration over the remaining amplitudes can be expressed again in terms of the amplitudes f_j on the mass shell, with substitution $\mu \to |k_\perp|$ in Eq. (2). A similar idea for the derivation of the renormalization-group equations from Feynman diagrams was advanced many years ago by Sudakov.[2]

The DL asymptotics of various electrodynamic processes with participation of electrons and μ mesons were obtained in Ref. 3 and are contained in a monograph on quantum electrodynamics.[4] The principal mathematical tool was in this case the construction of the Bethe-Salpeter equation for the scattering amplitudes. For the backward e^+e^- scattering process, the amplitude with negative signature was

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also calculated.[5] Summation of large singly logarithmic terms $\sim \alpha m(t-4m^2)^{-1/2}\ln s$ besides the DL terms $\sim \alpha \ln^2 s$ has made it possible to investigate the motion of the Coulomb Regge poles in the j-plane of the t-channel.[6] Algebraic equations for the partial waves $f_j(t)$ in the pseudoscalar theory were first proposed in Ref. 7. In that paper, when discussing the dependence of the amplitude on t, nonlinear differential equations of the Riccati type were formulated, but these, in contrast to our equations, contain derivatives not with respect to j but with respect to t. Equations similar to those obtained in the present article were used to calculate the asymptotic cross section for e^+e^- annihilation into an arbitrary number of $\mu^+\mu^-$ pairs and photons,[8] but the authors of that reference regarded the equations they obtained simply as a convenient technical device for the summation of the principal logarithmic terms. The same results can actually be obtained by using the Bethe-Salpeter equation technique.

The results of Ref. 3 can be easily generalized to include the non-Abelian theory[9] (this was observed independently also by Ermolaev and Lipatov). However, the approach based on the Bethe-Salpeter equation does not make it possible to obtain in this case contributions corresponding to negative signature of the partial waves in the t-channel. As will be shown below, for negative signature the singularities in the j-plane are located to the right of the corresponding singularities of the partial waves with positive signature. Therefore the amplitudes with negative signature are asymptotically more important.

2. FORMULATION OF PROBLEM

It is known[10] that the behavior of the scattering amplitudes at high energies and at fixed momentum transfers is uniquely connected with the singularities of the partial waves $f_j(t)$ in the crossing channel. Small-angle scattering in quantum chromodynamics (QCD) with exchange of a state with vacuum quantum numbers in the t-channel was investigated earlier.[11] We consider below amplitudes of scattering with exchange of nonvacuum quantum numbers, which are determined in the framework of QCD by diagrams with two fermion lines in the t channel.

A distinction must be made between two cases: diquark-state exchange with baryon number $B=2/3$, and "meson"-state exchange from a quark and an antiquark. The corresponding scattering amplitudes will be designated $D(s)$ and $M(s)$. D exchange arises in backward scattering of a quark and antiquark ($u=$ const). M exchange takes place in backward scattering of quarks of different kinds or in the $q\bar{q}\to Q\bar{Q}$ annihilation forward (the quark Q travels in the direction of the quark q). In non-Abelian gauge theory with $SU(N)$ group the wave function $q^i(p)$ of the quark is a spinor in color space ($i=1,2,\ldots,N$), whereas the antiquark is transformed like an antispinor $\bar{q}_i$.

We consider for the sake of argument the annihilation of quarks with one flavor, q, into quarks with a different flavor, Q:

$$q^{i_1}\bar{q}_{i_2}\to Q^{j_1}\bar{Q}_{j_2}. \qquad (5)$$

It is then convenient, for forward annihilation, to resolve the amplitude into two parts corresponding to exchange, in the t-channel, of the state of the colorless (M_0) quantum numbers and the state with gluon quantum numbers (M_8):

$$M_{i_1j_2}^{j_1i_2}=\frac{\gamma_\mu^\perp\gamma_\mu^\perp}{s}\left(P_{i_1j_2}^{0j_1i_2}M_0+P_{i_1j_2}^{8j_1i_2}M_8\right), \qquad (6)$$

where $\gamma_\mu^\perp\gamma_\mu^\perp/s$ corresponds to the spin structure of the Born term, which must be calculated in the spinor brackets, and the asymptotic form of s^0 is given only by the matrices γ_1 and γ_2 (the 3-axis is directed along the particle-collision axis). It is known[3] that the Born spin structure is preserved in calculations with DL accuracy, therefore M_0 and M_8 are scalar amplitudes. The matrices P^0 and P^8 in (6) are projectors on singlet and "octet" states:

$$P_{i_1j_2}^{0j_1i_2}=\frac{1}{N}\delta_{i_1}^{j_1}\delta_{j_2}^{i_2}, \quad P_{i_1j_2}^{8j_1i_2}=\frac{1}{2}\lambda_{i_1}^{aj_1}\lambda_{j_2}^{ai_2}. \qquad (7)$$

Here λ^a is a generalization of the Gell-Mann matrices with properties

$$\mathrm{Sp}\,\lambda^a\lambda^b=2\delta^{ab}, \quad [\lambda^a,\lambda^b]=2if^{abc}\lambda^c, \qquad (8)$$

where j^{abc} are the structure constants of the $SU(N)$ group.

For backward annihilation it is similarly convenient to resolve the amplitude into two parts that correspond to states that are symmetrical (D_6) and antisymmetrical ($D_{\bar{3}}$) in the color indices

$$D_{i_1j_2}^{j_1i_2}=\frac{\gamma_\mu^\perp\gamma_\mu^\perp}{s}\left(P_{i_1j_2}^{6j_1i_2}D_6+P_{i_1j_2}^{\bar{3}j_1i_2}D_{\bar{3}}\right), \qquad (9)$$

where

$$P_{i_1j_2}^{6j_1i_2}={}^1/_2(\delta_{i_1}^{i_2}\delta_{j_2}^{j_1}+\delta_{i_1}^{j_1}\delta_{j_2}^{i_2}), \quad P_{i_1j_2}^{\bar{3}j_1i_2}={}^1/_2(\delta_{i_1}^{i_2}\delta_{j_2}^{j_1}-\delta_{i_1}^{j_1}\delta_{j_2}^{i_2}) \qquad (10)$$

are the projectors on the corresponding color states.

In the Born approximation, the color structure is $1/4\lambda_{i_1}^{ai_1}\lambda_{j_2}^{aj_2}$, and we therefore have for small g^2

$$M_0=\frac{N^2-1}{2N}g^2, \quad M_8=-\frac{1}{2N}g^2,$$
$$D_{\bar{3}}=-\frac{N+1}{2N}g^2, \quad D_6=\frac{N-1}{2N}g^2. \qquad (11)$$

We divide the amplitudes $M_{0,8}$ and $D_{\bar{3},6}$ into parts that are symmetrical and antisymmetrical with respect to the permutation $s\rightleftarrows u$

$$M_{0,8}=M_{0,8}^{+}+M_{0,8}^{-}, \quad M_{0,8}^{\pm}(s)=\pm M_{0,8}^{\pm}(-s),$$
$$D_{\bar{3},6}=D_{\bar{3},6}^{+}+D_{\bar{3},6}^{-}, \quad D_{\bar{3},6}^{\pm}(s)=\pm D_{\bar{3},6}^{\pm}(-s). \qquad (12)$$

The dispersion relations in the s plane are equivalent at high energies $s^{1/2}$ to the following Mellin represetation for the amplitudes $M^\pm$ and $D^\pm$ with definite signature $p=\pm 1$:

$$M_{0,8}^{p}=\int_{a-i\infty}^{a+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{\mu^2}\right)^{\omega}\xi_\omega{}^{p}f_{0,8}^{p}(\omega),$$
$$D_{\bar{3},6}^{p}=\int_{a-i\infty}^{a+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{\mu^2}\right)^{\omega}\xi_\omega{}^{p}f_{\bar{3},6}^{p}(\omega), \qquad (13)$$

where ξ_ω^p is a signature factor, conveniently chosen in the form

$$\xi_{\omega}{}^{p}=(e^{-i\pi\omega}+p)/2. \tag{14}$$

In the DL approximation (4), small values of ω are significant:

$$\omega\sim\sqrt{g^2}, \tag{15}$$

in which case expressions (13) coincide with the Watson-Sommerfeld representation. The variable ω has the meaning of the complex angular momentum j in the t channel, and the functions

$$\varphi_i{}^{p}(\omega)=\sin\pi\omega\ f_i{}^{p}(\omega) \tag{16}$$

constitute the t-channel partial waves. We note that the contribution of the negative signature $p=-1$ to the scattering amplitude in the energy region (4) is suppressed (one of the $\ln s$ is replaced by $i\pi$). This is due to the smallness of the corresponding signature factor (14) in the region (15).

For the imaginary part of the amplitudes M and D in the s and u channels we have the following representation:

$$\mathrm{Im}_s M_{0,8}^{p}=p\ \mathrm{Im}_u M_{0,8}^{p}=-\frac{1}{2}\int\frac{d\omega}{2\pi i}\left(\frac{s}{\mu^2}\right)^{\omega}\varphi_{0,8}^{p}(\omega),$$
$$\mathrm{Im}_s D_{\bar{3},6}^{p}=p\ \mathrm{Im}_u D_{\bar{3},6}^{p}=-\frac{1}{2}\int\frac{d\omega}{2\pi i}\left(\frac{s}{\mu^2}\right)^{\omega}\varphi_{\bar{3},6}^{p}(\omega). \tag{17}$$

The inverse Mellin transformations make it possible to obtain $\varphi_i^{\pm}$ in terms of the imaginary parts of the corresponding amplitudes in the s and u channels:

$$\varphi_{0,8}^{p}(\omega)=-2\int\limits_{\mu^2}^{\infty}\frac{ds}{s}\left(\frac{s}{\mu^2}\right)^{-\omega}\frac{\mathrm{Im}_s M_{0,8}+p\ \mathrm{Im}_u M_{0,8}}{2},$$
$$\varphi_{\bar{3},6}^{p}(\omega)=-2\int\limits_{\mu^2}^{\infty}\frac{ds}{s}\left(\frac{s}{\mu^2}\right)^{-\omega}\frac{\mathrm{Im}_s D_{\bar{3},6}+p\ \mathrm{Im}_u D_{\bar{3},6}}{2}. \tag{18}$$

In the Born approximation (11), the t-channel partial waves $\varphi^{+}(\omega)$ are not analytic in ω (they contain $\delta_{\omega 0}$ singularities). Therefore equations (13) should have been written with nonintegral terms $\sim g^2s^0$. It is known,[3] however, that the higher radiative corrections $\sim g^2(g^2/\omega^2)^n$ give for the partial waves analytic expressions that coincide as $\omega\to 0$ with their values at the physical point $\omega=0$. This means that if the nonintegral terms are included in the integral, as is done in (13), the suitably redefined functions $f_i^{+}(\omega)$ (which we shall hereafter call for brevity partial waves) should tend to a constant as $\omega\to 0$:

$$f_i{}^{+}(\omega)\,|_{\omega\to 0}\to\text{const}. \tag{19}$$

As will be shown in the following sections, the property (19) does indeed hold.

The functions $f_i^{p}(\omega)$ take in the Born approximation the form (see (11))

$$f_0^{\pm(1)}(\omega)=\frac{N^2-1}{2N}\frac{g^2}{\omega},\quad f_8^{\pm(1)}(\omega)=-\frac{1}{2N}\frac{g^2}{\omega},$$
$$f_{\bar{3}}^{\pm(1)}(\omega)=-\frac{N+1}{2N}\frac{g^2}{\omega},\quad f_6^{\pm(1)}(\omega)=\frac{N-1}{2N}\frac{g^2}{\omega}. \tag{20}$$

The degeneracy in signature is connected with the absence in this approximation of singularities of the amplitude in the u channel. In the general DL case, contributions to the Feynman gauge of the gluon Green's function

$$D_{\mu\nu}(k)\sim\delta_{\mu\nu}/k^2$$

are made by diagrams of the ladder type in the t-channel with arbitrary gluon inserts[3,9] (see Fig. 1). The gluons in the ladder have polarizations perpendicular to the (p_1,p_2) plane of the initial particles:

$$D_{\mu\nu}\sim\delta_{\mu\nu}{}^{\perp}/k^2,$$

whereas the remaining gluons are polarized in the (p_1,p_2) plane:

$$D_{\mu\nu}(k)\sim(p_{1\mu}p_{2\nu}+p_{1\nu}p_{2\mu})/(p_1p_2)k^2. \tag{21}$$

Following the tradition,[3] we shall call these ladder and bremmstrahlung gluons, respectively. In the single-loop approximation the DL contribution comes from the five diagrams of Fig. 2 in the case of exchange of a state of the meson type (M) in the t-channel, and from analogous diagrams with coinciding directions of the arrows on the fermion lines in the diquark case (D). Using the fact that the matrices P in (6) and (9) are projectors on states with definite color in the t-channel, we can express the contribuions to $M_{0,8}$ and $D_{\bar{3},6}$ from the diagram of Fig. 2a in terms of the corresponding electrodynamic expressions (the spin-structure simplification is effected similarly[3]):

$$M_{0,8}^{a}=(M_{0,8}^{(1)})^2J^a(s),\quad D_{\bar{3},6}^{a}=-(D_{\bar{3},6}^{(1)})^2J^a(s), \tag{22}$$

$$J^a(s)=s\frac{(-i)}{(2\pi)^4}\times\int\frac{|s|\,d\alpha\,d\beta\,d^2k_\perp\cdot 2k_\perp{}^2}{2(-s\alpha+k_\perp{}^2+i\varepsilon)(s\beta+k_\perp{}^2+i\varepsilon)(s\alpha\beta+k_\perp{}^2+i\varepsilon)}=\frac{1}{16\pi^2}\ln^2\left(-\frac{s+i\varepsilon}{\mu^2}\right), \tag{23}$$

where we have introduced the Sudakov parameters

$$k=\alpha p_2+\beta p_1+k_\perp,\quad |\alpha|\ll 1,\quad |\beta|\ll 1,\quad \mu^2\ll -k_\perp{}^2\ll|s|. \tag{24}$$

The plot of Fig. 2a has singularities only in the s channel. Therefore its contribution to the s-channel imaginary

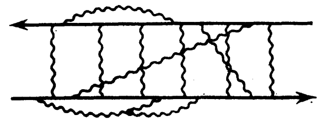

FIG. 1.

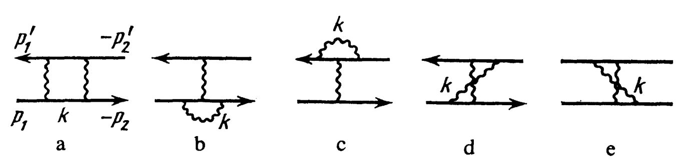

FIG. 2.

part M and D can be easily obtained from Eqs. (22) and (23) by the substitution

$$\ln\left(-\frac{s+i\varepsilon}{\mu^2}\right)\to\ln\frac{|s|}{\mu^2}-i\pi.$$

From the representation (13) we obtain partial waves $f_i^{\pm a}(\omega)$ that are degenerate in signature (see (20)):

$$f_{0,8}^{\pm a}(\omega)=\frac{1}{8\pi^2\omega}\left(f_{0,8}^{\pm(1)}(\omega)\right)^2,\quad f_{\bar{3},6}^{\pm a}(\omega)=-\frac{1}{8\pi^2\omega}\left(f_{\bar{3},6}^{\pm(1)}(\omega)\right)^2. \tag{25}$$

Since the gluon carries color degrees of freedom, the amplitudes with different color spin in the t-channel are intermixed in the diagrams of Figs. 2b–2d. Using the properties (8) of the matrices λ^a, we can easily express the contribution of these diagrams in terms of the corresponding electrodynamic constributions.[3] Using the matrix-multiplication rules, we can represent the amplitudes in the form

$$\begin{aligned}M^b&=M^c=g^2\hat{m}_sM^{(1)}J^\delta(s),\quad M^d=M^e=-g^2\hat{m}_uM^{(1)}J^\delta(u),\\ D^b&=D^c=g^2\hat{d}_sD^{(1)}J^\delta(s),\quad D^d=D^e=g^2\hat{d}_uD^{(1)}J^\delta(u),\end{aligned} \tag{26}$$

where

$$M^i=\begin{pmatrix}M_0^{\ i}\\ M_8^{\ i}\end{pmatrix},\quad D^i=\begin{pmatrix}D_{\bar{3}}^{\ i}\\ D_6^{\ i}\end{pmatrix},$$

$$\hat{m}_s=\begin{pmatrix}0 & \dfrac{N^2-1}{2N}\\ \dfrac{1}{2N} & \dfrac{N^2-2}{2N}\end{pmatrix},\quad m_u=\begin{pmatrix}0 & \dfrac{N^2-1}{2N}\\ \dfrac{1}{2N} & -\dfrac{1}{N}\end{pmatrix},$$

$$\hat{d}_s=\begin{pmatrix}\dfrac{(N-2)(N+1)}{4N} & \dfrac{N+1}{4}\\ \dfrac{N-1}{4} & \dfrac{(N+2)(N-1)}{4N}\end{pmatrix},$$

$$\hat{d}_u=\begin{pmatrix}\dfrac{(N-2)(N+1)}{4N} & -\dfrac{N+1}{4}\\ -\dfrac{N-1}{4} & \dfrac{(N+2)(N-1)}{4N}\end{pmatrix}. \tag{27}$$

$J^\delta(s)$ and $J^\delta(u)$ in (26) are integrals corresponding to the simplest Sudakov vertices

$$\begin{aligned}J^\delta(s)&=-2s\frac{(-i)}{(2\pi)^4}\\ &\times\int\frac{|s|\,d\alpha\,d\beta\,d^2k_\perp}{2(-s\alpha+k_\perp^{\ 2}+i\varepsilon)(s\beta+k_\perp^{\ 2}+i\varepsilon)(s\alpha\beta+k_\perp^{\ 2}+i\varepsilon)}\\ &=-\frac{1}{16\pi^2}\ln^2\left(-\frac{s+i\varepsilon}{\mu^2}\right)\\ J^\delta(u)&=-\frac{1}{16\pi^2}\ln^2\left(-\frac{u+i\varepsilon}{\mu^2}\right).\end{aligned} \tag{28}$$

As seen from (26)–(28), the signature degeneracy is lifted already in the single-loop approximation. The corresponding partial waves will be calculated in the following

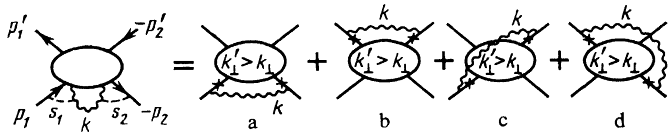

FIG. 3.

sections. We note here, however, that to obtain the scattering amplitudes in the DL approximation in our approach it suffices, as it will be made clear later, to calculate the contribution of the single-loop diagrams.

3. CONTRIBUTIONS OF DIAGRAMS WITH SOFT BREMSSTRAHLUNG MESON

In the DL approximation, the perpendicular components of the virtual momenta in the Feynman integrals corresponding to different loops can be regarded as quantities of different order of magnitude, $|k_i^\perp|\ll|k_j^\perp|$ or $|k_j^\perp|\ll|k_i^\perp|$.

We consider the contributions corresponding to the restriction that the minimum transverse momentum in the diagrams of Fig. 1 has one of the bremsstrahlung mesons. We denote this momentum by k:

$$|k^\perp|\ll|k_i^\perp|. \tag{29}$$

It can then be shown (see below) that if we consider the dispersion relations with respect to the invariants

$$(p_1-k)^2\approx-s\alpha,\quad (p_2+k)^2\approx s\beta,$$

the principal contribution in the region (29) is made by the pole terms, with respect to these invariants, contained in diagrams 3a–3d, where the selected soft gluon joins the external lines (see Fig. 3). The amplitude corresponding to the internal block in Figs. 3a–3d goes on the mass shell, but the infrared cutoff parameter μ^2 in this amplitude should be replaced in accordance with (29) by $|k_\perp^2|$;

$$\mu^2\to|k_\perp^{\ 2}|. \tag{30}$$

Upon summation of the contribution of the Feynman diagrams of Fig. 1, this result seems miraculous, but is in fact a consequence of the gauge invariance of the theory. In the case of quantum electrodynamics, analogous results for the bremsstrahlung of photons, and for the DL corrections to elastic amplitudes and to the Coulomb phase of hadron processes were first proved with the aid of the dispersion method by Gribov and others.[12] A generalization of the corresponding formulas to the case of the Yang-Mills theory and to gravitation was used in Ref. 13 in a check on the gluon and graviton reggeization hypothesis.

To verify the pole dispersion equation corresponding to Fig. 3, it suffices to show that the inelastic contributions are negligibly small in the region (29) with respect to the invariants s_1 and s_2. Consider, for example, the dispersion contribution corresponding to Fig. 4, in which the initial quark breaks up under the influence of a virtual gluon with momentum k into a quark and a gluon. We estimate the corresponding inelastic amplitude T_μ. Knowledge of this amplitude is essential for the calculation of the s_1 jump with the aid of the unitarity condition. The disintegration products travel in the DL approximation along the directions of the initial quark with momentum p_1 and have by virtue of (29) trans-

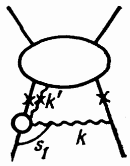

FIG. 4.

verse momenta $k_\perp'$ much larger than $k_\perp$. Substituting the propagator of the virtual gluon in the form (21), and using the gauge invariance $k_\mu T_\mu = 0$ for the amplitude T_μ, we can make the following substitution in the required matrix element $p_{2\mu} T_\mu$:

$$p_{2\mu}T_\mu \approx -sk_{\perp\mu}T_\mu/s_1, \tag{31}$$

where we have neglected the asymptotically small term $\sim s_2 p_{1\mu} T_\mu / s_1$. Obviously, $k_{1\mu}$ in the right-hand side of (31) is rendered dimensionless by quantities of the order of (or larger than) $k_\perp'$. Therefore the inelastic contribution is small compared with the pole contributions of Fig. 3a, where the virtual gluon k interacts with the quark color charge, which does not vanish as $k_\perp \to 0$. The pole dispersion representation has thus been proved.

To calculate the diagrams of Fig. 3 we can use our single-loop calculations (see Figs. 2, b–e) but in place of $M^{(1)}$ and $D^{(1)}$ we must put in (26) $M(-s/k_\perp^2)$ and $D(-s/k_\perp^2)$, where $k_\perp$ is the transverse component of the integration momenta in Eqs. (28):

$$M^{(1)}J^0(s) \to -\frac{1}{8\pi^2}\int_{\mu^2}^{|s|}\frac{d(-k_\perp^2)}{-k_\perp^2}\ln\left(-\frac{s+i\varepsilon}{-k_\perp^2}\right)M\left(\frac{s}{-k_\perp^2}\right),$$
$$D^{(1)}J^0(u) \to -\frac{1}{8\pi^2}\int_{\mu^2}^{|s|}\frac{d(-k_\perp^2)}{-k_\perp^2}\ln\left(-\frac{u+i\varepsilon}{-k_\perp^2}\right)D\left(\frac{s}{-k_\perp^2}\right). \tag{32}$$

If we write down $M(-s/k_\perp^2)$ and $D(-s/k_\perp^2)$ in the form of the Mellin transforms (12) and (13), we can evaluate the integral in (32) with respect to $k_\perp^2$ as an integral with respect to ω. (The upper limit $|s|$ must be replaced in this case by ∞.) The signature properties of the functions $M^\pm(-s/k_\perp^2)$ and $D^\pm(-s/k_\perp^2)$, which correspond to the blocks in Figs. 3a–3d, are not preserved after the integration. Indeed, let us use the equations

$$\ln\left(-\frac{s+i\varepsilon}{-k_\perp^2}\right) = \left(\ln\frac{|s|}{-k_\perp^2} - i\frac{\pi}{2}\right) - i\frac{\pi}{2}\,\mathrm{sgn}\, s,$$
$$\ln\left(-\frac{u+i\varepsilon}{-k_\perp^2}\right) = \left(\ln\frac{|s|}{-k_\perp^2} - i\frac{\pi}{2}\right) + i\frac{\pi}{2}\,\mathrm{sgn}\, s. \tag{33}$$

The term in the parentheses in the right-hand side of these equations does not change the signature of the functions M and D in (32), and its contribution to the Mellin transform can be written with logarithmic accuracy in the form

$$\left(\ln\frac{|s|}{-k_\perp^2} - i\frac{\pi}{2}\right)\int\frac{d\omega}{2\pi i}\left(\frac{s}{-k_\perp^2}\right)^\omega \xi_\omega^+ f_i^+(\omega) = \int\frac{d\omega}{2\pi i}\left(\frac{s}{-k_\perp^2}\right)^\omega \xi_\omega^+\left(-\frac{d}{d\omega}f_i^+(\omega)\right),$$
$$\left(\ln\frac{|s|}{-k_\perp^2} - i\frac{\pi}{2}\right)\int\frac{d\omega}{2\pi i}\left(\frac{s}{-k_\perp^2}\right)^\omega \xi_\omega^- f_i^-(\omega) = \int\frac{d\omega}{2\pi i}\left(\frac{s}{-k_\perp^2}\right)^\omega \xi_\omega^-\left(-\frac{1}{\omega}\frac{d}{d\omega}\omega f_i^-(\omega)\right). \tag{34}$$

The last term in the right-hand side of (33) reverses the sign of the signature of the functions M and D in (32). This term must be taken into account, with logarithmic accuracy, only when M and D have positive signature:

$$-i\frac{\pi}{2}\,\mathrm{sgn}\, s\int\frac{d\omega}{2\pi i}\left(\frac{s}{-k_\perp^2}\right)^\omega \xi_\omega^+ f_i^+(\omega) = \int\frac{d\omega}{2\pi i}\left(\frac{s}{-k_\perp^2}\right)^\omega \xi_\omega^-\frac{1}{\omega}f_i^+(\omega). \tag{35}$$

Thus, from (26) and (32)–(35) we obtain the following final result for the contribution corresponding to the case when the minimum transverse momentum in the diagrams of Fig. 1 has a bremsstrahlung gluon:

$$\begin{pmatrix} f_0^+ \\ f_8^+ \end{pmatrix}_g = 2(\hat m_s - \hat m_u)\frac{g^2}{8\pi^2\omega}\frac{d}{d\omega}\begin{pmatrix} f_0^+(\omega) \\ f_8^+(\omega) \end{pmatrix},$$
$$\begin{pmatrix} f_{\bar 3}^+ \\ f_6^+ \end{pmatrix}_g = 2(\hat d_s + \hat d_u)\frac{g^2}{8\pi^2\omega}\frac{d}{d\omega}\begin{pmatrix} f_{\bar 3}^+(\omega) \\ f_6^+(\omega) \end{pmatrix},$$
$$\begin{pmatrix} f_0^- \\ f_8^- \end{pmatrix}_g = 2(\hat m_s - \hat m_u)\frac{g^2}{8\pi^2\omega^2}\frac{d}{d\omega}\left[\omega\begin{pmatrix} f_0^-(\omega) \\ f_8^-(\omega) \end{pmatrix}\right] - 2(\hat m_s + \hat m_u)\frac{g^2}{8\pi^2\omega^2}\begin{pmatrix} f_0^+(\omega) \\ f_8^+(\omega) \end{pmatrix},$$
$$\begin{pmatrix} f_{\bar 3}^- \\ f_8^- \end{pmatrix}_g = 2(\hat d_s + \hat d_u)\frac{g^2}{8\pi^2\omega^2}\frac{d}{d\omega}\left[\omega\begin{pmatrix} f_{\bar 3}^-(\omega) \\ f_8^-(\omega) \end{pmatrix}\right] - 2(\hat d_s + \hat d_u)\frac{g^2}{8\pi^2\omega^2}\begin{pmatrix} f_3^+(\omega) \\ f_8^+(\omega) \end{pmatrix} \tag{36}$$

We note that the matrices of the derivatives are diagonal by virtue of the definitions (27). Moreover, $(f_0^\pm(\omega))_g$ does not contain derivatives for the colorless channel.

4. CONTRIBUTIONS OF DIAGRAMS WITH SOFT QUARKS

One more possibility of gaining a DL contribution in the diagrams of Fig. 1 is afforded by the fact that the minimum value $|k_\perp| = \mu$ is reached in a pair of t-channel fermion lines (see Fig. 5). In this case the two blocks in Fig. 5 contain all the diagrams for the scattering amplitudes but, just as in the case of the bremsstrahlung contributions of Fig. 3, the

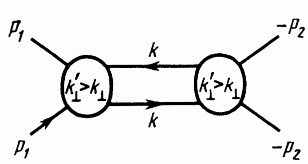

FIG. 5.

Feynman integrals with respect to $k_\perp'$ are cut off from below not by the parameter μ, but by the transverse fermion momentum $k_\perp$ (cf. (30)):

$$\mu^2 \to |k_\perp^2|. \tag{37}$$

The amplitudes for the blocks at fixed $k_\perp^2$ can be regarded as located on the mass shell ($k^2 = 0$), inasmuch as by virtue of (37) they do not contain an infrared divergence and have thresholds with respect to k^2 outside the essential integration region $k^2 \sim k_\perp^2$.

In the calculation of the dispersion diagram of Fig. 5 we can use the results obtained above for the single-loop diagram Fig. 2a [see (22), (23)]:

$$M_{0,8}\left(\frac{s}{\mu^2}\right) = \frac{-i}{(2\pi)^4}\int \frac{|s|\,d\alpha\, d\beta\, d^2k_\perp \cdot k_\perp^2}{[s\alpha\beta + k_\perp^2 + i\varepsilon]^2}\,\frac{s}{(-s\alpha)}\, M_{0,8} \times \left(\frac{-s\alpha}{-k_\perp^2}\right)\frac{1}{s\beta}\, M_{0,8}\left(\frac{s\beta}{-k_\perp^2}\right), \tag{38}$$

$$D_{\bar{3},6}\left(\frac{s}{\mu^2}\right) = \frac{-i}{(2\pi)^4}\int \frac{|s|\,d\alpha\, d\beta\, d^2k_\perp \cdot (-k_\perp^2)}{[s\alpha\beta + k_\perp^2 + i\varepsilon]^2}\,\frac{s}{(-s\alpha)}\, D_{\bar{3},6} \times \left(\frac{-s\alpha}{-k_\perp^2}\right)\frac{1}{s\beta}\, D_{\bar{3},6}\left(\frac{s\beta}{-k_\perp^2}\right),$$

where the integration is over the region

$$\infty > -k_\perp^2 > \mu^2, \quad -\infty < \alpha,\ \beta < \infty, \quad s\alpha\beta \sim k_\perp^2, \quad |s\alpha| \geqslant -k_\perp^2, \quad |s\beta| \geqslant -k_\perp^2. \tag{39}$$

Obviously, the integral differs from zero only if the signatures of the two amplitudes in the integrand coincide (to check on this statement it is necessary to make the change of variables $\alpha \to -\alpha$, $\beta \to -\beta$). Next, the signature of the result of the integration coincides with the signature of each of the factors (to check on this we must substitute $s \to -s$ simultaneously with $\alpha \to -\alpha$). The two foregoing facts were established in a discussion of the enhancement problem in the Reggion diagram technique.[14]

To calculate the contribution corresponding to (38) to the t-channel partial waves, it suffices to consider the analogous integral of the degrees of $s\alpha$ and $s\beta$, multiplied by the signature factors

$$I_{\omega_1\omega_2}\left(\frac{s}{\mu^2}\right) = \frac{-i}{(2\pi)^4}\int \frac{|s|\,d\alpha\, d\beta\, d^2k_\perp \cdot k_\perp^2}{(s\alpha\beta + k_\perp^2 + i\varepsilon)^2}\,\frac{s}{(-s\alpha)} \times \left(\frac{-s\alpha}{-k_\perp^2}\right)^{\omega_1} \xi_{\omega_1}\, \frac{s}{s\beta}\left(\frac{s\beta}{-k_\perp^2}\right)^{\omega_2} \xi_{\omega_2}$$

$$= \frac{-i}{(2\pi)^4}\int \frac{dx}{x}\, d^2k_\perp \frac{k_\perp^2}{(-x + k_\perp^2 + i\varepsilon)^2} \times \left(\frac{x}{-k_\perp^2}\right)^{\omega_1}\left(\frac{s}{-k_\perp^2}\right)^{\omega_2} \xi_{\omega_1}\xi_{\omega_2}\int \frac{d\beta}{\beta}\, 2\beta^{\omega_2-\omega_1},$$

$$\left|\frac{k_\perp^2}{s}\right| < \beta < \left|\frac{x}{k_\perp^2}\right|, \quad -\infty < x < \infty, \tag{40}$$

where the change of variable $x = -s\alpha\beta$ was made. Calculating the integral with respect to β and next with respect to x, we obtain

$$I_{\omega_1\omega_2}\left(\frac{s}{\mu^2}\right) = \frac{1}{8\pi^2}\int \frac{dk_\perp^2}{-k_\perp^2(\omega_2 - \omega_1)} \times \left[\left(\frac{s}{-k_\perp^2}\right)^{\omega_2}\xi_{\omega_2} - \left(\frac{s}{-k_\perp^2}\right)^{\omega_1}\xi_{\omega_1}\right]. \tag{41}$$

Locating the contours in the integrals with respect to ω_1 and ω_2 in the corresponding Laplace transforms in such a way that, for example, $\mathrm{Re}\,\omega_2 > \mathrm{Re}\,\omega_1$, it is easy to note that the contribution of the second term in the brackets of (41) vanishes upon integration with respect to ω_2, and in the integral with respect to ω_1 in the first term there is only one pole $1/(\omega_2 - \omega_1)$ to the right of the integration contour, i.e., this integral can be evaluated by residues. We finally obtain for the partial waves corresponding to the soft pair of quarks in Fig. 5 the following expressions (cf. (25)):

$$\begin{pmatrix} f_0^\pm \\ f_8^\pm \end{pmatrix} = \frac{1}{8\pi^2\omega}\begin{pmatrix} f_0^{\pm 2}(\omega) \\ f_8^{\pm 2}(\omega) \end{pmatrix}, \quad \begin{pmatrix} f_{\bar{3}}^\pm \\ f_6^\pm \end{pmatrix} = -\frac{1}{8\pi^2\omega}\begin{pmatrix} f_{\bar{3}}^{\pm 2}(\omega) \\ f_6^{\pm 2}(\omega) \end{pmatrix}. \tag{42}$$

5. EQUATIONS FOR THE PARTIAL WAVES

Combining the contributions corresponding to the soft gluons (36) and quarks (42), as well as taking into account the Born term (20), we can write in closed form nonlinear equations for the t-channel partial waves in the DL approximation (see Fig. 6). For positive signature these equations take a particularly simple form

$$\begin{gathered} f_0^+(\omega) = a_0\frac{g^2}{\omega} + \frac{1}{8\pi^2\omega}(f_0^+(\omega))^2, \\ f_i^+(\omega) = a_i\frac{g^2}{\omega} + b_i\frac{g^2}{\pi^2\omega}\frac{d}{d\omega}f_i^+(\omega) + c_i\frac{1}{8\pi^2\omega}(f_i^+(\omega))^2, \\ f_i^+(\omega)\,|_{\omega\to\infty} = a_i g^2/\omega, \end{gathered} \tag{43}$$

where

$$\begin{aligned} &a_0 = (N^2-1)/2N; \quad a_8 = -1/2N; \quad b_8 = N/8, \quad c_8 = +1; \\ &a_{\bar{3}} = -(N+1)/2N, \quad b_{\bar{3}} = (N+1)(N-2)/8N, \quad c_{\bar{3}} = -1; \\ &a_6 = (N-1)/2N, \quad b_6 = (N-1)(N+2)/8N, \quad c_6 = -1. \end{aligned} \tag{44}$$

In the case of negative signature, we obtain from (36) and (42) more complicated equations, which contain by way of inhomogeneities solutions of the equations (43):

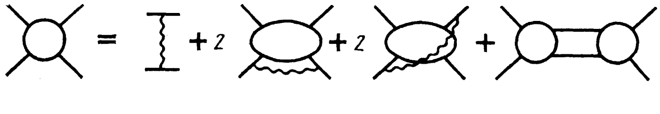

FIG. 6.

$$f_0^-(\omega)=a_0\frac{g^2}{\omega}-\frac{N^2-1}{N}\frac{g^2}{4\pi^2\omega^2}f_8^+(\omega)+\frac{1}{8\pi^2\omega}(f_0^-(\omega))^2$$

$$f_8^-(\omega)=a_8\frac{g^2}{\omega}-\frac{N^2-4}{2N}\frac{g^2}{4\pi^2\omega^2}f_8^+(\omega)-\frac{1}{N}\frac{g^2}{4\pi^2\omega^2}f_0^+(\omega)$$
$$+b_8\frac{g^2}{\pi^2\omega^2}\frac{d}{d\omega}(\omega f_8^-(\omega))+\frac{1}{8\pi^2\omega}(f_8^-(\omega))^2,$$
$$f_{\bar 3}^-(\omega)=a_{\bar 3}\frac{g^2}{\omega}-\frac{N+1}{2}\frac{g^2}{4\pi^2\omega^2}f_6^+(\omega)\tag{45}$$
$$+b_{\bar 3}\frac{g^2}{\pi^2\omega^2}\frac{d}{d\omega}(\omega f_{\bar 3}^-(\omega))-\frac{1}{8\pi^2\omega}(f_{\bar 3}^-(\omega))^2,$$
$$f_6^-(\omega)=a_6\frac{g^2}{\omega}-\frac{N-1}{2}\frac{g^2}{4\pi^2\omega^2}f_{\bar 3}^+(\omega)$$
$$+b_6\frac{g^2}{\pi^2\omega^2}\frac{d}{d\omega}(\omega f_6^-(\omega))-\frac{1}{8\pi^2\omega}(f_6^-(\omega))^2,$$

where the coefficients a_i and b_i were defined above [see (44)].

We discuss now the analytic properties of the solutions of the obtained equations in the ω plane. The equations for the channel with colorless quantum numbers are purely algebraic, so that the solution can be written in explicit form:

$$f_0^+(\omega)=4\pi^2\omega\{1-[1-g^2(N^2-1)/4\pi^2N\omega^2]^{1/2}\},$$

$$f_0^-(\omega)=4\pi^2\omega\left\{1-\left[1-\frac{g^2(N^2-1)}{4\pi^2N\omega^2}\left(1-\frac{1}{2\pi^2\omega}f_8^+(\omega)\right)\right]^{1/2}\right\}.\tag{46}$$

Thus, $f_0^+(\omega)$ contains a square-root branch point at

$$\omega=\omega_0^+=[g^2(N^2-1)/4\pi^2N]^{1/2}.$$

It is important to note that the partial wave with negative signature, which makes in the DL region (4) a small contribution because of the suppression on account of the signature factor $\xi^-(\omega)\sim i\omega\sim g$, has a singularity $\omega=\omega_0^-$ in the ω plane to the right of the point $\omega=\omega_0^+$. Indeed, at sufficiently large N the function $f_8^+(\omega)$ can be replaced by its Born term, therefore,

$$\omega_0^-\approx\omega_0^+(1+1/2N^2).\tag{47}$$

It can be verified that the difference $\omega_0^- - \omega_0^+$ remains positive for all the admissible values $N=2,3,\ldots$. We thus obtain a very interesting physical result, namely that the negative signature, initially suppressed by the numerical smallness of the signature factor, becomes dominant in the colorless channel when the energy is increased. A similar result holds also for other channels. To check on this fact, let us find the explicit solution of Eqs. (43) for f_i^+, $i\neq 0$. These are Riccati equations.[15] By standard procedures they reduce to a linear Schrödinger equation, and it turns out that the potential in this linear equation is harmonic in the variable ω, i.e., its solution is a parabolic-cylinder function.[15] Thus, for partial waves with positive signature in nonsinglet channels we obtain the following result (cf. Ref. 3):

$$f_i^+(\omega)=a_i\frac{g^2}{p_i}\frac{d}{d\omega}\ln\left(\exp\left[\frac{1}{2}\left(\frac{\omega}{\omega_i}\right)^2\right]D_{p_i}\left(\frac{\omega}{\omega_i}\right)\right);$$
$$p_i=\frac{a_i}{8b_i}c_i,\quad \omega_i^2=\frac{g^2}{\pi^2}b_i\tag{48}$$

The singularities of $f_i^+(\omega)$ are poles of first order, located at the zeros of the function $D_p(x)$. At $N=2$, the equations for $f_{\bar 3}^{\pm}(\omega)$ and $f_6^{\pm}(\omega)$ coincide with the equations for $f_0^{\pm}(\omega)$ and $f_8^{\pm}(\omega)$, since the isotopic quantum numbers in the channels D and M are identical. This means that $f_{\bar 3}^+(\omega)$ has at $N=2$ singularities to the right of the $\mathrm{Re}\,\omega=0$ axis, but $f_{\bar 3}^-=f_0^-$ has singularities in the ω plane even farther to the right (see (46)). At $N\geqslant 3$ the zeros of the function $D_p(x)$, which coincide with the poles of $f_i^+(\omega)$, are located to the right of the imaginary axis,[15] whereas the $f_i^-(\omega)$ have at zero singularities of the form

$$f_i^-(\omega)\sim 1/\omega\ln\omega,\tag{49}$$

i.e., they dominate in the asymptotic relation. Indeed, the singular behavior of (49) agrees, as can be easily verified, with Eqs. (45).

To illustrate the appearance of the singularity (49) in negative signature, we consider the backward e^+e^- scattering in quantum electrodynamics, where the equations have the same form as the equations for $f_{\bar 3}^-$ and f_6^- [cf. (45)]:

$$f^-(\omega)=\frac{g^2}{\omega}+\frac{g^2}{2\pi^2\omega^2}\frac{d}{d\omega}(\omega f^-(\omega))-\frac{1}{8\pi^2\omega}(f^-(\omega))^2.\tag{50}$$

By changing from the Riccati equation (50) to the Schrödinger equation we easily obtain its explicit solution

$$f^-(\omega)=\frac{g^2}{p}\frac{d}{d\omega}\ln\psi\left(\frac{\omega}{\omega_0}\right);\quad p=-\frac{1}{4},\quad \omega_0=\left(\frac{g^2}{2\pi^2}\right)^{1/2};$$
$$\psi(l)=\int_0^\infty\frac{dx}{x}\left(\frac{1+x}{x}\right)^{p/2}\exp\left(-x\frac{l^2}{2}\right),\quad l=\omega\left(\frac{2\alpha}{\pi}\right)^{-1/2}.\tag{51}$$

This solutions agrees with the result obtained in Ref. 5 by the traditional methods, and contains a singularity of the type (49) at $\omega=0$.

In concluding this section, we note that from the explicit formulas (46) and (48) follows satisfaction of the property (19) which guarantees the absence of nonanlytic terms in the ω plane.

6. DISCUSSION OF RESULTS

We have thus obtained in this paper, in the DL approximation, nonlinear equations for the t-channel partial waves of the scattering of quarks on the mass shell, and investigated the analytical properties of their solutions in the ω plane.

The derivation of equations for the diquark system in the t-channel is of interest for the discussion of the analytic properties of a partial wave in the case of baryon exchange, where the singularities of $f_{\bar 3}(\omega)$ and $f_6(\omega)$ can generate singularities in the colorless channel by the very same mechanism that is responsible in the hadron world for the appearance of Mandelstam branch cuts.

Indeed, exchange of a diquark state having the quantum numbers $\bar 3$ of the antiquark according to the $SU(3)$ color group with a reggeized quark leads to singularities of $f_j(u)$ in the baryon colorless channel u, in the same manner by which a pomeron singularity results from summation of Mandelstam branch cuts due to exchange of two reggeized gluons.[11]

In exactly the same way, knowledge of the partial wave $f_6(\omega)$ is necessary for the calculation of the branch cut connected with the diquark, quark, and gluon exchange. In the colorless meson channel, a similar mechanism leads to a branch cut in the j plane on account of the exchange of an octet state [the partial wave $f_8(\omega)$] and the reggeized gluon.

The method developed in this paper can also be successfully used to calculate gluon (or photon) scattering amplitudes with quark-anti-quark state exchange in the t-channel. Notice should be taken of the possible appearance for these processes, in the j plane, of new pole singularities corresponding to quark-quark scattering. It is easy to generalize the formulas derived above for the case $\mu^2 < -t$, and also include the phenomenological region of small $k_\perp \sim \Lambda$, which is responsible for the onset of Regge trajectories corresponding to the mesons $\pi, \rho, \omega, \ldots$. It is possible to take consistently into account, at least near the threshold $t = 4m_q^2$ for the production of massive quarks, the "Coulomb singly-logarithmic terms" (see Ref. 6) that describe the Regge trajectories in the nonrelativistic approximation. Great interest attaches also to a refinement of the derived equations by taking more accurate account of the asymptotic freedom in QCD.

In quantum electrodynamics, the method proposed above makes it possible to obtain the known results[3,5] in a most economical manner [see (48) and (51)]. In the case of the forward $e^+e^- \to \mu^+\mu^-$ annihilation (the charge does not change direction) our equations for t-channel partial waves with negative signature $f_0^-(\omega)$ reduce to algebraic ones and their solution is elementary [cf. (46)]:

$$f_0^\pm(\omega) = \frac{4\pi\alpha}{\omega} f^\pm(l), \quad l = \omega\left(\frac{2\alpha}{\pi}\right)^{-1/2},$$

$$f^-(l) = 2l[l - (l^2 - 1 + l^{-2} f^+(l))^{1/2}], \quad f^+(l) = 2l/[l + (l^2-1)^{1/2}]. \tag{52}$$

Besides the singularity at $l = 1$, the partial wave $f^-(l)$ has in the l plane a pair of immobile square-root branch points at $l \approx 0.9343 \pm i0.4804$, which lead to scattering-amplitude oscillations superimposed on their monotonic growth. This is a new result. The usual method based on the solution of the Bethe-Salpeter equations did not make it possible to find $f^-(l)$.

Thus, the approach developed in the present article is applicable to all gauge theories (including gravitation[13]).

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