On the spontaneous emission of waves by a fast shock wave in a longitudinal magnetic field

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The stability of a fast longitudinal shock wave against weak perturbations of the discontinuity surface is investigated within the framework of ideal magnetohydrodynamics. It is shown that the region of spontaneous emission of waves by the discontinuity in a plasma with an arbitrary equation of state is determined by the magnetic field, and is broader than the corresponding region in ordinary hydrodynamics. In a sufficiently strong magnetic field, the spontaneous wave emission by the discontinuity can occur in an ideal gas with a constant specific heat.

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The stability of shock waves in a medium with an arbitrary equation of state against weak perturbations of the discontinuity surface was first investigated within the framework of ordinary hydrodynamics by D'yakov,[1] Kontorovich,[2] and Iordanskiĭ.[3] D'yakov[1] showed that the shock waves are absolutely unstable when the derivative of the specific volume V with respect to the pressure P along the Hugoniot adiabat has the following values:

$$J^2(\partial V/\partial p)_H > 1+2M, \quad J^2(\partial V/\partial p)_H < -1, \tag{1}$$

where J is the mass flux through the discontinuity and M is the Mach number in the medium behind the shock wave. Spontaneous emission of acoustic waves begins in the region of neutral stability[2,3]:

$$\frac{1-M^2-\varkappa M^2}{1-M^2+\varkappa M^2} < J^2\left(\frac{\partial V}{\partial p}\right)_H < 1+2M, \tag{2}$$

where $\varkappa$ is the compression in the shock wave. The case of relativistic shock waves has been considered by Kontorovich.[4] In magnetohydrodynamics, the stability of evolutional shock waves has been investigated by Gardner and Kruskal.[5] In particular, it was shown that the region of absolute instability of a fast longitudinal shock wave is determined by the set of inequalities (1). The spontaneous emission of waves by the discontinuity was not considered.

In the present paper we investigate the region of $J^2(\partial V/\partial p)_H$ values where the discontinuity spontaneously emits waves. In the process the discontinuity surface vibrates with a constant amplitude (the imaginary part of the frequency is equal to zero: $\operatorname{Im}\omega = 0$), and excites in the plasma magnetohydrodynamic waves propagating from the shock wave.

Let us describe the plasma on both sides of the discontinuity by the equations of ideal magnetohydrodynamics[6]:

$$\begin{gathered}\frac{\partial\rho}{\partial t} + \operatorname{div}(\rho\mathbf{v}) = 0, \quad \frac{d\sigma}{dt} = 0, \\ \rho\frac{d\mathbf{v}}{dt} = -\nabla p + \frac{1}{4\pi}[\operatorname{rot}\mathbf{B}\times\mathbf{B}], \\ \frac{\partial\mathbf{B}}{\partial t} = \operatorname{rot}[\mathbf{v}\times\mathbf{B}], \quad \operatorname{div}\mathbf{B} = 0, \quad \frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{v}\nabla).\end{gathered} \tag{3}$$

Here ρ is the plasma density; $\mathbf{v}$, the flux velocity; $\mathbf{B}$, the magnetic-field intensity; σ, the specific entropy; and p, the pressure. The pressure, density, and entropy are connected by an arbitrary equation of state $\rho = \rho(\sigma, p)$.

All the quantities characterizing the plasma in front of the shock wave will be labeled below by the subscript 1. The letters without subscripts will denote the quantities behind the shock wave. The connection between the values of the hydrodynamic quantities at the discontinuity is given by the following boundary conditions[7]:

$$\{\rho(\mathbf{vf})\} = 0, \tag{4}$$

$$\left\{\rho(\mathbf{vf})\left(W + \frac{\mathbf{v}^2}{2}\right) + \frac{1}{4\pi}(\mathbf{B}[\mathbf{v}\times\mathbf{B}]\mathbf{f})\right\} = 0, \tag{5}$$

$$\left\{\left(p + \frac{\mathbf{B}^2}{8\pi}\right)\mathbf{f} + \rho(\mathbf{vf})\mathbf{v} - \frac{(\mathbf{Bf})}{4\pi}\mathbf{B}\right\} = 0, \tag{6}$$

$$\{\mathbf{Bf}\} = 0, \tag{7}$$

$$\{[[\mathbf{v}\times\mathbf{B}]\times\mathbf{f}]\} = 0. \tag{8}$$

Here $\mathbf{f}$ is the unit vector along the normal to the discontinuity; W is the heat function of a unit mass; and the curly brackets denote the discontinuities of the quantities across the discontinuity surface: $\{A\} = A|_\xi - A_1|_\xi$, ξ being the radius vector of an arbitrary point of the discontinuity surface. Equations (4)–(6) describe the conservation of the mass and energy fluxes and the component of the momentum-flux tensor along the normal to the discontinuity; Eqs. (7) and (8), the discontinuities of the normal component of the magnetic-field intensity and the tangential component of the electric-field intensity. Below we shall replace Eq. (5) by the equation, which follows from the above-given system (4)–(8), of the Hugoniot adiabat:

$$\rho = \rho(p, \rho_1, p_1, \{B_\tau\}^2). \tag{9}$$

Let us choose the system of coordinates such that the unperturbed-discontinuity plane coincides with the $x = 0$ plane; then the unperturbed-flux-velocity and unperturbed-magnetic-field vectors will coincide with the x axis.

Let us represent the solution to the linearized magnetohydrodynamic equations (3) in the form of a set of plane waves with frequency ω and tangential wave-vector component $\mathbf{q} = (0, q, 0)$, leaving the discontinuity. We shall consider the direction of the group vel-

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ocity $\mathbf{v}_g$ to be the direction of propagation of the waves.[8] We also assume the shock wave to be evolutional; then the following inequalities are fulfilled[9]:

$$v_{fx1}<v_1, \quad v_{ax}<v<v_{fx}, \tag{10}$$

where v_{fx1} and v_{fx} are the components of the phase velocities of the fast magneto-acoustic waves along the x axis in the regions in front of and behind the shock wave and v_{ax} is the component of the Alfvén velocity along the x axis behind the shock wave. As a result of these inequalities, the discontinuity emits no waves into the $x<0$ region, but emits six waves into the $x>0$ region: one entropy wave, two Alfvén waves, two slow magneto-acoustic waves, and one fast magneto-acoustic wave. Another fast magneto-acoustic wave exists, but it is an incident wave. Thus, the quantities δp, $\delta\mathbf{v}$, $\delta\mathbf{B}$, and $\delta\sigma$ characterizing the perturbation of the plasma behind the shock wave can be written in the following form:

$$\delta p=\sum_l \delta A_l p_l \exp\{-i(\omega t-qy-k_{xl}x)\}, \quad \delta\mathbf{v}=\sum_l \delta A_l \mathbf{v}_l \exp\{-i(\omega t-qy-k_{xl}x)\},$$
$$\delta\mathbf{B}=\sum_l \delta A_l \mathbf{B}_l \exp\{-i(\omega t-qy-k_{xl}x)\}, \quad \delta\sigma=\sum_l \delta A_l \sigma_l \exp\{-i(\omega t-qy-k_{xl}x)\}. \tag{11}$$

Here $l=1,2,\ldots,6$ numbers the waves leaving the discontinuity and the quantities pertaining to them; the δA_l and k_{xl} are the wave amplitudes and longitudinal wave vectors; and the p_l, $\mathbf{v}_l$, $\mathbf{B}_l$, and σ_l are the quantities determining the polarization of the waves in accordance with the linearized magnetohydrodynamic equations.

Linearizing the boundary conditions (4)–(8), we represent the perturbation of the discontinuity surface in the following form:

$$R(t, y)=\eta \exp\{-i(\omega t-qy)\}. \tag{12}$$

Using the equations (11), and canceling out the factor $\exp\{-i(\omega t-qy)\}$, we obtain in the rest frame of the discontinuity the following linearized boundary conditions:

$$\sum_l \delta A_l(B_x v_{zl}-vB_{zl})=0, \quad \sum_l \delta A_l\left(v_{zl}-\frac{B_x}{4\pi J}B_{zl}\right)=0,$$
$$\sum_l \delta A_l\left(p_l\frac{M^2}{v}+\rho v_{xl}\right)=i\eta\left\{\frac{1}{v}\right\}J\omega,$$
$$\sum_l \delta A_l\left(p_l\frac{1}{v}+\rho v_{xl}\right)=i\eta\left\{\frac{1}{v}\right\}J\omega\left(1+2\frac{M^{-2}-1}{1+J^2(\partial V/\partial p)_H}\right), \tag{13}$$
$$\sum_l \delta A_l\left(v_{yl}(1-M_a^{-2})+\frac{B_x}{4\pi Jv}E_{zl}\right)=i\eta\left\{\frac{1}{v}\right\}vv_1 q, \quad \sum_l \delta A_l E_{zl}=0.$$

Here M_a is the Alfvén Mach number in the $x>0$ region and $E_{zl}=B_x v_{yl}-vB_{yl}$.

Since the pressure, velocity, and magnetic-field-intensity perturbations are equal to zero in the entropy wave, the amplitude δA_σ of this wave does not enter into the equations (13). Furthermore, the perturbations of the velocity and magnetic field of the magneto-acoustic waves are perpendicular to the z axis; therefore, only the amplitudes, δA_{a1} and δA_{a2}, of the two Alfvén waves enter into the first two equations in (13). It is easy to see that $\delta A_{a1}=\delta A_{a2}=0$, i.e., the Alfvén waves are not emitted by a longitudinal shock wave. The remaining equations of the system (13) relate the amplitudes δA_l of the three magnetosonic waves with the amplitude η of the discontinuity-surface vibrations.

Let us introduce the polarization vectors of the magnetosonic waves:

$$\mathbf{z}_l=\left(\frac{p_l}{\rho s^2}, \frac{v_{xl}}{v}, \frac{v_{yl}}{v}, E_{zl}\frac{1}{B_x v}\right), \tag{14}$$

and write the last four equations in (13) in a matrix form:

$$\hat{B}\sum_l \delta A_l \mathbf{z}_l-\eta\beta=0. \tag{15}$$

The forms of β and $\hat{B}$ are clear from the expressions (13):

$$\beta=-i\left\{\frac{1}{v}\right\}\left(\omega, \omega\left(1+2\frac{M^{-2}-1}{1+J^2(\partial V/\partial p)_H}\right), qv_1, 0\right),$$
$$\hat{B}=\begin{pmatrix} -1 & -1 & 0 & 0 \\ -M^{-2} & -1 & 0 & 0 \\ 0 & 0 & -1+M_a^{-2} & M_a^{-2} \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{16}$$

The subscript l assumes three values corresponding to the two slow and one fast magnetosonic waves leaving the discontinuity and the vectors $\mathbf{z}_l$ are determined from the linearized system of magnetohydrodynamic equations (3), which has the form

$$\hat{N}\mathbf{z}_l-\lambda_l\hat{B}\mathbf{z}_l=0. \tag{17}$$

Here $\hat{B}$ is given by the expression (16) (in the general case the $\hat{B}$'s in (16) and (17) do not necessarily coincide); $\lambda_l=k_{xl}v$, and is determined from the dispersion equation $\det\|\hat{N}-\lambda_l\hat{B}\|=0$; and the matrix $\hat{N}$ is equal to

$$\hat{N}=\begin{pmatrix} -\omega & 0 & qv & 0 \\ 0 & -\omega & 0 & 0 \\ M^{-2} & 0 & -\omega & q^2M_a^{-2} \\ 0 & 0 & \omega & -\omega \end{pmatrix}. \tag{18}$$

The homogeneous equation (15) relates the amplitude η of the discontinuity-surface vibrations with the amplitudes δA_l of the outgoing magnetosonic waves. In order for the discontinuity to emit waves, the determinant of Eq. (15) should vanish. The thus obtained characteristic equation has quite a complicated form. It connects three different roots λ_l of the fourth-order polynomial $\det\|\hat{N}-\lambda_l\hat{B}\|=0$, and its analysis is difficult. But in the cases when there are no waves going out from the discontinuity into the region in front of the shock wave, the characteristic equation reduces to the form $\varphi(\lambda_{in})=0$, where λ_{in} is the eigenvalue of the (Erpenbeck[10]) wave incident on the discontinuity in the $x>0$ region. Indeed, let the vector $\mathbf{t}$ be perpendicular to all the $\mathbf{z}_l$ in Eq. (15). Such a vector exists (the biorthogonal-basis vector), and is determined from the equation ($\hat{E}$ is the unit matrix)

$$((\hat{B}^{-1}\hat{N})^*-\lambda_{in}^*\hat{E})\mathbf{t}=0. \tag{19}$$

Multiplying (15) by $\mathbf{c}(\lambda_{in})\equiv(\hat{B}^{-1})^*\mathbf{t}(\lambda_{in})$, we obtain the characteristic equation in the following form:

$$\beta\mathbf{c}(\lambda_{in})=0, \tag{20}$$

the vector $\mathbf{c}$ being given by the equation

$$(\hat{N}^{\cdot}-\lambda^{\cdot}_{\text{in}}\hat{B}^{\cdot})c=0. \tag{21}$$

Let us construct the matrix $\hat{D}$, the first row of which is formed by the coefficients of Eq. (20); the remaining four rows, by the coefficients of the equations (21):

$$\hat{D}=\begin{pmatrix} \omega & \omega\left(1+2\dfrac{M^{-2}-1}{1+J^2(\partial V/\partial p)_H}\right) & qv_1 & 0 \\ -\omega+\lambda^*_{\text{in}} & M^{-2}\lambda^*_{\text{in}} & M^{-2}qv & 0 \\ \lambda^*_{\text{in}} & -\omega+\lambda^*_{\text{in}} & 0 & 0 \\ qv & 0 & -\omega+\lambda^*_{\text{in}}(1-M_a^{-2}) & \omega \\ 0 & 0 & M_a^{-2}\left(\dfrac{q^2v^2}{\omega}+\lambda^*_{\text{in}}\right) & -\omega+\lambda^*_{\text{in}} \end{pmatrix}. \tag{22}$$

The simultaneous solvability of Eqs. (20) and (21) is evidently equivalent to the linear dependence of any four rows of the matrix $\hat{D}$. From the linear-dependence condition for the first four rows we obtain the characteristic equation (similar to the one obtained in Ref. 5):

$$J^2\left(\frac{\partial V}{\partial p}\right)_H=-1+\frac{2\nu(M^{-2}-1)}{M^{-2}+\varkappa(\nu^2-M^{-2}(1-\nu)^2)}, \tag{23}$$

where $\nu=(\omega-\lambda_{\text{in}})/\omega$. The characteristic equation in ordinary hydrodynamics can be reduced to the same form. The linear dependence of the last four rows of the matrix $\hat{D}$ clearly gives rise to a dispersion equation for the magnetosonic waves:

$$Q^2=\frac{(\nu^2-M^{-2}(1-\nu)^2)(\nu^2-M_a^{-2}(1-\nu)^2)}{\nu^2(M^{-2}+M_a^{-2})-M_a^{-2}M^{-2}(1-\nu)^2}, \tag{24}$$

where $Q=qv/\omega$ is the ratio of the flux velocity to the phase velocity of the discontinuity-surface perturbation (12).

The satisfaction of the system of equations (23) and (24) constitutes the condition for the emission of waves by discontinuity. Furthermore, $\operatorname{Im}\omega=0$ (neutral stability), and the group velocity $v_{gx}=d\omega/dk_{x\text{in}}$ of the fast magnetosonic wave incident on the discontinuity in the $x>0$ region should be less than zero, or $\operatorname{Im}k_{x\text{in}}<0$. In terms of the variables Q^2 and ν, the negativeness of the x component of the group velocity and the vanishing of the imaginary part of the frequency lead to the following set of relations:

$$(1-\nu)+\frac{2Q^2}{dQ^2/d\nu}<0, \tag{25}$$

$$\operatorname{Im}Q^2=0,\quad \operatorname{Re}Q^2>0, \tag{26}$$

which must be satisfied by the roots of Eqs. (23) and (24).

The relations (26) are satisfied in the segments 2, 4, and 6 of the real axis (Fig. 1). The inequality (25) is reduced by the substitution $Z=\nu^2/(1-\nu)^2$ to the form

$$2\nu\frac{dQ^2(1-\nu)^{-2}/dZ}{dQ^2/d\nu}<0 \tag{27}$$

and, as can easily be verified, is satisfied only in that part of the interval 6 (Fig. 1) where $dQ^2/d\nu<0$. The upper limit of this interval is equal to ν_6 and the lower limit ν_c is given by the equation

$$\left.\frac{dQ^2}{d\nu}\right|_{\nu=\nu_c}=0. \tag{28}$$

In the interval 6, Eq. (28) has a single root ν_c. Otherwise, the fourth-order polynomial (24) in ν would have more than four roots. Thus, the range of ν values at which the relations (24)–(26) are satisfied is

$$\nu_c<\nu<1/(1-M). \tag{29}$$

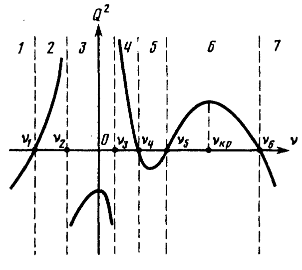

FIG. 1. Form of the dispersion dependence $\nu_1=1/(1-M_a)$, $\nu_2=[1-(M_a^2+M^2)^{1/2}]^{-1}$, $\nu_3=[1+(M_a^2+M^2)^{1/2}]^{-1}$, $\nu_4=1/(1+M_a)$, $\nu_5=1/(1+M)$, $\nu_6=1/(1-M)$.

The derivative $J^2(\partial V/\partial p)_H$ along the shock adiabat in (23) is a monotonically increasing function of ν on the radial lines

$$|\nu|>r\equiv\left(\frac{1-\varkappa^{-1}}{1-M^2}\right)^{1/2}$$

(see Fig. 2). This region wholly contains the interval (29), since

$$\left.\frac{dQ^2}{d\nu}\right|_{\nu=r_m}>0,\quad r_m\equiv\left(\frac{1}{1-M^2}\right)^{1/2}.$$

The range of $J^2(\partial V/\partial p)_H$ values at which spontaneous emission of waves by the discontinuity occurs is given by (23) and (29):

$$-\frac{2\nu_c(1-M^{-2})}{M^2+\varkappa(\nu_c^2-M^{-2}(1-\nu_c)^2)}-1\equiv\Gamma(\nu_c)<J^2\left(\frac{\partial V}{\partial p}\right)_H<1+2M. \tag{30}$$

The upper limit of the neutral-stability region (30) for a fast longitudinal shock wave coincides with the limit (2) in the case of ordinary hydrodynamics, and does not depend on the magnetic-field strength. The lower limit of the region (30) differs from that of the corresponding hydrodynamic region (2), and is determined by the magnetic-field strength.

The root ν_c of Eq. (28) decreases monotonically with increasing magnetic-field strength. In fact, (28) is a biquadratic equation for the reciprocal magnetic Mach number M_a^{-1}:

$$M_a^{-4}(1-\nu_c)(\nu_c^2-M^{-2}(1-\nu_c)^2)+M_a^{-2}\nu_c^2(\nu_c+M^{-2}(1-\nu_c))\times(\nu_c^2-2M^{-2}(1-\nu_c)^2)+M^{-2}\nu_c^4(\nu_c+M^{-2}(1-\nu_c))=0, \tag{31}$$

whose determinant does not vanish in the interval $r_m<\nu$

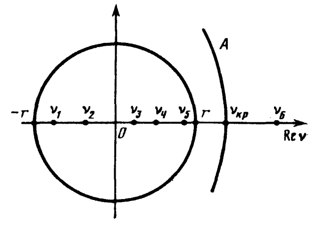

FIG. 2. Locus of the roots of the characteristic equation (23): a circle of radius $r=[(1-\varkappa^{-1})/(1-M^2)]^{1/2}$ and a pair of radial lines $\nu<-r$ and $\nu>r$.

$< 1/(1-M)$. In this case $dM_a^{-2}/d\nu_c$ does not become infinite, and $d\nu_c/dM_a^{-2}$ does not vanish; the root ν_c assumes its greatest value $\nu_{c0}=1/(1-M^2)$ in zero magnetic field, since the derivative $dQ^2/d\nu|_{\nu=\nu_{c0}} < 0$. Consequently, the function $\Gamma(\nu_c)$ decreases monotonically with increasing magnetic-field strength, i.e., the neutral-stability region broadens. The quantity $Q^2(\nu_c)$ also decreases monotonically with increasing magnetic-field strength, for

$$\frac{dQ^2}{d\alpha}=M^2\frac{\nu^2-M^{-2}(1-\nu)^2}{(\nu^2\alpha-M^{-2}(1-\nu)^2)^2}>0, \tag{32}$$

where $\alpha=M_a^2/M^2$.

Let us consider the possibility of a neutral stability in the case of complex k_{x1n}. The inequality (25) is then replaced by the following inequality:

$$\operatorname{Im}[(1-\nu)/Q]<0. \tag{33}$$

The domain of complex solutions of Eq. (23) is the circle $|\nu|<r_m$ (Fig. 2). The curve (24) consists of the real-axis segments $\nu_1<\nu<\nu_2$, $\nu_3<\nu<\nu_4$, and $\nu_5<\nu<\nu_6$ and the curve A passing through the point ν_c and extending to infinity. Assuming that $\nu=r_m e^{i\psi}$, we obtain from (24) the equation

$$\begin{aligned}&\operatorname{Im} Q^2|(M_a^{-2}+M^{-2})\nu^2-M_a^{-2}M^{-2}(1-\nu)^2|^2\\ &=2r_m(r_m-\cos\psi)\sin^2\psi(1+r_m^2(M_a^2-1))^2+((M_a^2-1)r_m^2\cos\psi\\ &+2r_m-\cos\psi)^2+M^2(M_a^2-1)(1+r_m^2(M_a^2-1))=0,\end{aligned} \tag{34}$$

which cannot be satisfied; consequently, the curve A does not intersect the boundary of the region $|\nu|<r_m$, i.e., a neutral stability does not occur in the case of complex k_{x1n}, and we arrive at the expression (30).

In ordinary hydrodynamics the boundary of the neutral-stability region and the derivative along the shock adiabat depend in the following manner on the compression in the shock wave:

$$\Gamma(\varkappa)=\frac{\varkappa(\gamma-1)-(\gamma+1)}{\varkappa(\gamma+3)-(\gamma+1)}, \tag{35}$$

$$J^2\left(\frac{\partial V}{\partial p}\right)_H=\frac{\varkappa(\gamma-1)-(\gamma+1)}{2\varkappa}. \tag{36}$$

The curve $\Gamma(\varkappa)$ lies above the $J^2(\partial V/\partial p)_H$ curve (Fig. 3), and spontaneous wave emission does not occur. In the magnetohydrodynamics of a fast longitudinal shock

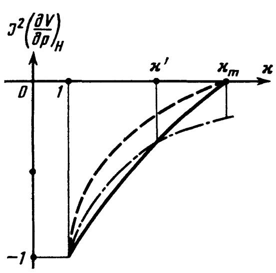

FIG. 3. Spontaneous emission in an ideal gas with constant specific heat. The continuous curve is a plot of the derivative along the Hugoniot adiabat; the dashed curve, the lower boundary of the region of spontaneous emission of waves by a discontinuity in ordinary hydrodynamics ($M_a^{-1}=0$); the dot-dash curve, the lower boundary of the region of spontaneous emission of waves by the discontinuity in magnetohydrodynamics (M_a^{-1} is fixed)[30]; $\varkappa_m=(\gamma+1)/(\gamma-1)$.

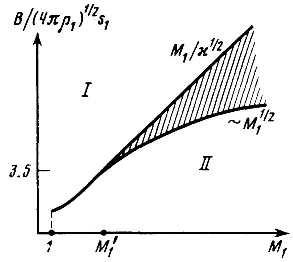

FIG. 4. Region of spontaneous emission of waves by a discontinuity in an ideal gas with constant specific heat (the dashed region). In the region I the shock wave is nonevolutionary; in the region II it is absolutely stable; s_1 is the velocity of sound in the medium in front of the shock wave.

wave, the form of the derivative along the shock adiabat is given by the expression (36). In the limit of a strong shock wave, i.e., for $\varkappa\to(\gamma+1)/(\gamma-1)$, the expression for the boundary $\Gamma(\nu_c)$ of the neutral-stability region assumes the form

$$\Gamma(\nu_c)=-\frac{(\nu_c-2/(\gamma+1))(\nu_c-2\gamma/(\gamma+1))}{(\nu_c-2/(\gamma+1))(\nu_c-2\gamma/(\gamma+1))-2\nu_c(\gamma-1)/(\gamma+1)}. \tag{37}$$

Since in the limit of a strong shock wave $\nu_{c0}=2\gamma/(1+\gamma)$, we have (for a fixed M_a) $\nu_c<2\gamma/(\gamma+1)$ and $\Gamma(\nu_c)<0$. The derivative along the Hugoniot adiabat then vanishes; thus, the inequality (30) is satisfied, i.e., a fast longitudinal shock wave of sufficiently high intensity in an ideal gas with constant specific heat spontaneously emits magnetosonic waves.

Numerical estimates show that, in a monatomic gas, and for $M_c^{-1}\leq 1$, the discontinuity begins to spontaneously emit waves when the Mach number M_1' in the medium in front of the shock wave is of the order of 7. The qualitative behavior of the boundary of the neutral-stability region for a fast shock wave in an ideal gas with constant specific heat is depicted in Fig. 4.

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