Ionization wave sustained by intense monochromatic radiation

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A stationary wave of the light-glow type propagating counter to the radiation in bulk optical breakdown of a dielectric or in microwave breakdown of a gas is studied. The structure and motion of the wave are described by the thermal conductivity equation and the Maxwell equations with account taken of the nonlinear temperature dependences of the thermal and optical characteristics of the dielectric. It is shown that in the general case the equations contain five dimensionless parameters that define the problem completely. The problem is solved numerically for various values of the parameters and the general properties of the solutions are elucidated. Some analytic expressions describing the structure of the wave and its various integral characteristics (propagation velocity, reflectance, and plasma temperature behind the wave front) are found in a number of limiting cases. The theory is compared with the experimental data [N. V. Zelikin *et al.*, Sov. Tech. Phys. Lett. **4**, 522 (1978); E. L. Klochan *et al., ibid*. **6**, 194 (1980)].

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1. INTRODUCTION

Focusing of a sufficiently powerful laser radiation into the interior of a transparent dielectric causes breakdown of the latter. Absorbing the laser radiation, the breakdown plasma heats and ionizes the adjacent layer of the dielectric and produce an ionization wave (absorption wave). The ionized dielectric screens the region of the initial breakdown, so that the ionization wave propagates mainly counter to the radiation. Under real experimental conditions, the radius of the caustic of the focusing lens greatly exceeds the width of the absorption-wave front, so that the wave front can be regarded as plane and a one-dimensional formulation of the problem is sufficient to study of the motion of the wave counter to the laser beam.

Stable propagation of the absorption wave can be maintained inside a transparent substance by radiation whose intensity is much lower than the breakdown threshold. A similar phenomenon is observed in optical and microwave breakdown of gases. The absorption wave in a condensed dielectric corresponds to the regime customarily called optical or slow burning in gas breakdown.[1] In this regime the wave-front motion is due to heat conduction. When optical burning is described, no account is taken of the energy transport connected with the mass transport, a procedure justified so long as the thermal-wave velocity is low compared with the sound velocity.

Some regularities of the propagation of absorption waves in a dielectric were investigated theoretically in Refs. 2–5. A common shortcoming of these papers is the excessive simplification of the description of the properties of the dielectric at high temperature, and the restricted region of applicability of the results. In the present article we use an approach free of the foregoing shortcomings. The radiation field is described by the complete system of Maxwell's equations, and the temperature field by the nonlinear heat-conduction equation written with allowance for the contribution of the free electrons to the energy-transport process.

The system of equations obtained as a result of this approach not only describes fully the absorption wave in condensed dielectrics, but is applicable also to the description of phenomena in optical and microwave breakdown of gases.[1] Thus, by varying the relation between the characteristic values of the constants of the problem, we can investigate, on the basis of one and the same system of equations, the absorption waves maintained by high-power monochromatic radiation of various wavelengths in substantially different media.

The conditions for the validity of this approach are a small mean free path of the particles responsible for the energy transport compared with the characteristic scale of variation of the radiation field and of the temperature, and a small absorption of the radiation on account of the photoeffect compared with the absorption by free carriers. In real cases these conditions cover the range of densities from rarefied gas to a condensed dielectric, and the electromagnetic-radiation frequency interval from the ratio band to visible light.

A unified analysis of such different situations can clarify the general laws governing the phenomenon, and this is in fact done in the present article. The results are reported in the following sequence. In Sec. 2 we describe the optical and thermophysical properties of a condensed dielectric at temperatures $T\sim 1$ eV. In Sec. 3 we derive the basic equations for the motion and structure of the absorption wave. Sections 4 and 5 devoted to numerical integration of the derived equations. In Sec. 6 are discussed the main properties of the numerical solution. In Sec. 7 and 8 we study different approximations that make an analytic solution of the problem possible, and compare the theory with experiment. The main result of the paper are briefly formulated in Sec. 9.

2. OPTICAL AND THERMOPHYSICAL PROPERTIES OF A CONDENSED DIELECTRIC

In the front of an ionization wave, the dielectric is transformed into a dense nonideal plasma. There is at

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present no rigorous quantitative theory of a nonideal plasma. To describe the properties of an ionized dielectric we shall therefore use the equations of the theory of an ideal plasma, but with the constants renormalized on account of the interparticle interaction. The plasma is assumed here to be in equilibrium and to have one temperature, since allowance for deviation from local thermodynamic equilibrium would be an exaggeration of the accuracy in this approach. Despite the simplicity of such a semiphenomenological analysis, it describes in a number of cases the properties of a nonideal plasma better than other much more complicated models.[6]

In accord with the foregoing, the real part of the dielectric constant of the plasma ε and its conductivity σ are given by the known expressions

$$\varepsilon=n_0^2-4\pi\sigma/\nu, \quad \sigma=\sigma_0(T)\nu^2/(\omega^2+\nu^2), \quad \sigma_0(T)=e^2N_e(T)/m\nu. \tag{1}$$

Here n_0 is the reflection index of the dielectric at the frequency ω and at $N_e=0$, while $\sigma_0(T)$ is the static conductivity. The complex dielectric constant $\tilde{\varepsilon}$, the refractive index $\tilde{n}$, the extinction factor $\tilde{\varkappa}$, as well as the absorption coefficient μ are connected with ε and σ by the relations

$$\begin{gathered}\tilde{\varepsilon}=\varepsilon-i4\pi\sigma/\omega, \quad \tilde{n}=\{\varepsilon/2+[(\varepsilon/2)^2+(2\pi\sigma/\omega)^2]^{1/2}\}^{1/2},\\ \tilde{\varkappa}=\{-\varepsilon/2+[(\varepsilon/2)^2+(2\pi\sigma/\omega)^2]^{1/2}\}^{1/2},\\ \mu=4\pi\sigma/c\tilde{n}=2\omega\tilde{\varkappa}/c,\end{gathered} \tag{2}$$

where c is the speed of light in vacuum. The function $N_e(T)$ is determined by the Saha formula, which takes in the considered case of low degrees of ionization the form

$$N_e(T)\approx 7.75\cdot 10^{10}T_{\text{eV}}^{3/4}N_0^{1/2}\exp(-I/2T)\ \text{cm}^{-3}, \tag{3}$$

where N_0 is the number of particles per unit volume. Since, however, there is at present no satisfactory method of calculating the values of I and ν in a nonideal plasma, it is more convenient to determine them by comparing expressions (1)–(3) with the experimental data. Thus, for ZhS-12 glass such a comparison with the $\sigma_0(T)$ dependence measured in Refs. 7 and 8 yields $I=3.8$ eV and $\nu\sim 10^{15}$ sec^{-1}.

The transport of the absorption-wave front in the optical burning regime is due to the thermal conductivity (estimates show that the role plasma thermal absorption due to the absorption wave front plays a negligible role in our problem). In the general case, the thermal-conductivity coefficient $\varkappa$ (which must not be confused with the extinction coefficient $\tilde{\varkappa}$!) will be represented by the sum

$$\varkappa(T)=\varkappa_0+\varkappa_e(T), \tag{4}$$

where $\varkappa_{0,e}$ correspond to the contributions made to the heat flux by the "phonon" (molecular) and electronic components. The "phonon" thermal-conductivity coefficient $\varkappa_0$ depends relatively little on temperature, whereas the contribution of the electronic thermal conductivity is proportional to the number of free electrons, and accordingly $\varkappa_e(T)$ increases exponentially with increasing T, see Eq. (3).

In condensed dielectrics $\varkappa_e$ usually becomes comparable with $\varkappa_0$ at temperatures corresponding to relatively low degrees of ionization. Thus, for example, in laser glasses $\varkappa_e(T)\sim\varkappa_0$ at $T\sim 0.3$–0.5 eV, corresponding to a degree of ionization $\alpha\approx 10^{-3}$–10^{-2}.

3. ENERGY TRANSPORT

Assume for the sake of argument that the wave propagates in the positive x direction, and the laser (microwave) radiation propagates in the negative direction. In a coordinate system fixed in the absorption wave front, the heat conduction equation takes the form

$$\frac{d}{d\xi}\left(\varkappa\frac{dT}{d\xi}\right)+uC\frac{dT}{d\xi}+\frac{\sigma}{2}|E|^2=0. \tag{5}$$

Here C is the heat capacity per unit volume, $\xi=x+ut$, $E(\xi)$ is the complex amplitude of the electric field of the monochromatic radiation: $\mathscr{E}(\xi,t)=E(\xi)e^{i\omega t}$. The amplitude $E(\xi)$ and its complex conjugate $E^*(\xi)$ are determined by the wave equations

$$d^2E/d\xi^2+(\omega/c)^2\tilde{\varepsilon}E=0, \quad d^2E^*/d\xi^2+(\omega/c)^2\tilde{\varepsilon}^*E^*=0. \tag{6}$$

To solve the problem it is convenient to eliminate the electric field from the system (5) and (6). With the aid of the method used in Refs. 9 and 10, we can obtain from (6) the relations

$$\begin{gathered}\frac{d^2}{d\xi^2}|E|^2-2\left|\frac{dE}{d\xi}\right|^2+2\frac{\omega^2}{c^2}\varepsilon|E|^2=0,\\ \frac{d}{d\xi}\left(E^*\frac{dE}{d\xi}-E\frac{dE^*}{d\xi}\right)-i\frac{8\pi\sigma\omega}{c^2}|E|^2=0,\\ i\frac{4\pi\sigma\omega}{c^2}\left(E^*\frac{dE}{d\xi}-E\frac{dE^*}{d\xi}\right)+\frac{d}{d\xi}\left|\frac{dE}{d\xi}\right|^2+\frac{\omega^2}{c^2}\varepsilon\frac{d}{d\xi}|E|^2=0\end{gathered} \tag{7}$$

and next, eliminating from (7) the quantities $|dE/d\xi|^2$ and $(E^*dE/d\xi-EdE^*/d\xi)$, we obtain the equation

$$\frac{d}{d\xi}\left\{\frac{1}{\sigma}\left(\frac{c^2}{\omega^2}\frac{d^3}{d\xi^3}|E|^2+4\varepsilon\frac{d}{d\xi}|E|^2+2|E|^2\frac{d\varepsilon}{d\xi}\right)\right\}=\frac{64\pi^2\sigma}{c^2}|E|^2. \tag{8}$$

To find the functions $T(\xi)$ and $|E(\xi)|^2$ we must integrate the system of equations (5) and (8).

4. FORMULATION OF THE PROBLEM. DIMENSIONLESS VARIABLES

In the ionization-wave front, a dielectric with initial temperature T_0 is heated to a certain temperature T_m determined by the radiation intensity as well as by the properties of the medium. Without considering the cooling of the plasma behind the wave front, due to the radial thermal flux and to emission, we can write the boundary conditions for the function $T(\xi)$ in the form

$$T(-\infty)=T_m, \quad T(\infty)=T_0, \quad T'(-\infty)=T'(\infty)=0. \tag{9}$$

The prime denotes here differentiation with respect to ξ.

Strictly speaking, the problem can be formulated in infinite space only under the condition $T_0=0$. Otherwise a small but finite conductivity of the dielectric at $T_0>0$ leads to total absorption of the radiation "at infinity," i.e., before it reaches the front of the ionization wave. In real experiments this difficulty does not arise because of the finite dimensions of the sample. In the numerical solution undertaken in the present paper it was convenient, rather than to introduce a finite sample length, to assume that in the Maxwell equations the plasma conductivity vanishes jumpwise at $T=T_0+\delta T$, $0<\delta T\ll T_0$. Calculations show that at sufficiently small δT the results are independent of its value.

The dissipation of the electromagnetic radiation is due in our problem to Joule losses

$$dS/d\xi=(\sigma/2)\,|E|^2. \qquad (10)$$

The boundary conditions for the Poynting vector are

$$S(-\infty)=0,\quad S(\infty)=S_0. \qquad (11)$$

Here S_0 is the absorbed part of the energy flux density of the incident electromagnetic wave. Integrating Eq. (5) with allowance for (9)–(11), we obtain the energy conservation law in the form

$$\varkappa T'+uCT+S=uCT_m=uCT_0+S_0. \qquad (12)$$

Analyzing Eqs. (1), (3)–(5), and (12) we note easily that the result of the calculation of the functions $T(\xi)$ and $S(\xi)$ depends on the choice of the concrete values of nine constants ($\omega, S_0, I, C, N_0, \nu, T_0, n_0, \varkappa_0$). To make the analysis more general, it is convenient to change to dimensionless variables, thereby reducing the number of independent parameters of the problem of five. To this end, we transform the system of Eqs. (5) and (8). We introduce the quantity

$$q(\xi)=\varkappa T'+uCT \qquad (13)$$

and represent Eq. (5) in the form

$$|E|^2=-(2/\sigma)\,q'. \qquad (14)$$

We use (14) to eliminate $|E|^2$ from (8) and integrate the obtained equation with respect to ξ from $-\infty$ to ξ with allowance for the boundary conditions

$$q(-\infty)=uCT_m,\quad q'(-\infty)=0,$$

which follow from (5), (9), and (13). As a result we obtain the equation

$$(c/\omega)^2(q'/\sigma)'''+4\varepsilon(q'/\sigma)'+2(q'/\sigma)(d\varepsilon/d\xi)=(8\pi/c)^2\sigma(q-uCT_m). \qquad (15)$$

We note that even though dimensionless variables can be introduced formally by various methods, the results become much more general if the so-called "natural" variables are used when the characteristic change of the sought functions is of the order of unity and takes place when there arguments are changed by amounts of the order of unity. In our case this result is reached for the following choice of variables. We introduce the dimensionless temperature

$$\theta=I(T_m-T)/2T_m^2.$$

Neglecting the weak T dependence of the pre-exponential factor in (3), we can then represent the plasma conductivity in the form

$$\sigma=\sigma_m e^{-f},\quad \sigma_m=\sigma(T_m),\quad f=\theta(1-2T_m\theta/I)^{-1}.$$

We introduce next the dimensionless coordinate, velocity, and energy-flux density in accordance with the relations

$$\zeta=\mu_m\xi,\quad v=\frac{C}{\mu_m\varkappa_m}u,\quad \eta=\frac{I}{2\mu_m\varkappa_m T_m^2}S, \qquad (16)$$

where

$$\varkappa_m=\varkappa(T_m),\quad \mu_m=\mu(T_m)=(2\omega/c)\tilde\varkappa_m,\quad \tilde\varkappa_m=\tilde\varkappa(T_m).$$

In the dimensionless variables, Eq. (13) takes the form

$$\eta=\chi\, d\theta/d\zeta+v\theta. \qquad (17)$$

In the derivation of this equation we used the energy conservation law (12) in the form

$$q=uCT_m-S$$

and introduced the quantity χ:

$$\chi=\varkappa/\varkappa_m=(b+e^{-f})/(b+1),\quad b=\varkappa_0/\varkappa_e(T_m). \qquad (18)$$

In terms of the new variables, Eq. (15) becomes

$$z_1^2(\eta' e^f)'''+2z_1(z_2-z_3e^{-f})(\eta' e^f)'+z_1z_3\eta' f'-\eta e^{-f}=0. \qquad (19)$$

The primes denote here differentiation with respect to ζ, and we have introduced the dimensionless parameters

$$z_1=\omega\tilde\varkappa_m^2/2\pi\sigma_m=(\mu_m\delta_m)^2,$$
$$z_2=\omega n_0^2/4\pi\sigma_m=2(k\delta_m)^2,\quad z_3=\omega/\nu, \qquad (20)$$

where $2\delta_m=c(2\pi\omega\sigma_m)^{-1/2}$ is the thickness of the skin layer at $T=T_m$, and $k\equiv n_0\omega/c$ is the wave number of the electromagnetic wave in the non-ionized dielectric.

Equations (17) and (19) determine the structure of the absorption wave front, namely the coordinate dependences of the temperature $\theta(\zeta)$ and of the radiation intensity $\eta(\zeta)$. These equations contain four independent parameters: T_m/I, b, z_2, and z_3, since the quantities $z_{1,2,3}$ are connected in accordance with (2) by the relation

$$z_1=z_3-z_2+\{1+(z_3-z_2)^2\}^{1/2}$$

and only two of them are independent.

The change of the plasma temperature in the wave front from T_0 to T_m corresponds to a change of the dimensionless quantity θ from 0 to θ_∞, where

$$\theta_\infty=I(T_m-T_0)/2T_m^2. \qquad (21)$$

When integrating the system (17), (19), the eigenvalue (the wave velocity) v is determined from the additional condition

$$\lim_{\zeta\to\infty}\theta(\zeta)=\theta_\infty. \qquad (22)$$

The quantity θ_∞ is the fifth (and last) independent parameter of the problem.

We note immediately that the quantities b, z_2 and z_3 can take on arbitrary positive values, whereas the ranges of variation of T_m/I and θ_∞ are limited in real cases by the conditions $T_m\ll I$ and $\theta_\infty\gg 1$. Therefore the dependence of the solution of the problem on these two parameters should be relatively weak, as is indeed confirmed by subsequent calculations (see Sec. 6).

An important characteristic of the absorption wave is also the quantity

$$\eta_\infty=\lim_{\zeta\to\infty}\eta(\zeta),$$

which has the meaning of the dimensionless intensity of the electromagnetic radiation absorbed by the wave. According to (17) and (22), we have

$$\eta_\infty = v\theta_\infty. \tag{23}$$

We determine the asymptotic form of the solution as $\xi \to -\infty$. The optical and thermophysical properties of the medium in the wave front are determined by the local value of the temperature. As $\xi \to -\infty$ the function $T(\xi) \to T_m$. Putting $T = T_m$ in (6), we obtain the asymptotic form of $E(\xi)$ as $\xi \to -\infty$:

$$E \sim \exp\left[\frac{\omega}{c}(i\tilde{n}_m + \tilde{\varkappa}_m)\xi\right], \quad EE^* \sim \exp\left(\frac{2\omega}{c}\tilde{\varkappa}_m\xi\right). \tag{24}$$

Substituting (24) in (5) and changing to the dimensionless variables, we obtain

$$\theta = C_1 e^{\zeta} \equiv e^{(\zeta-\zeta_0)} \quad \text{as} \quad \zeta \to -\infty.$$

Taking into account the translational symmetry of the problem, it is possible, without loss of generality, to assume that $\zeta_0 = 0$, i.e.,

$$\theta = e^{\zeta} \quad \text{as} \quad \zeta \to -\infty. \tag{25}$$

For the function $\eta(\zeta)$ we get from (25) and (17) the asymptotic expression

$$\eta = (1+v)e^{\zeta} \quad \text{as} \quad \zeta \to -\infty, \tag{26}$$

since $\lim_{\zeta \to -\infty} \chi = 1$.

5. REFLECTION COEFFICIENT

The gradient of the refractive index $\tilde{n}(T)$ in the ionization wave front is large, so that part of the incident radiation is reflected. Labeling the incident wave by the index 1 and the reflected by the index 2, we represent the field intensity as $\xi \to \infty$ in the form

$$E(\xi) = E_1 e^{ik\xi} + E_2 e^{-ik\xi}. \tag{27}$$

The Poynting vector corresponding to (27) is

$$S_0 = S_1 - S_2 = \frac{c\tilde{n}}{8\pi}\{|E_1|^2 - |E_2|^2\},$$

and the reflection coefficient is

$$R = S_2/S_1 = |E_2|^2/|E_1|^2. \tag{28}$$

To use (28) we must express $|E_{1,2}|^2$ in terms of the characteristics of the temperature field, in analogy with the procedure used in the derivation of (19). To this end we differentiate (27) with respect to ξ and express $E_{1,2}$ in terms of E and $dE/d\xi$. Equation (28) then takes the form

$$R = \lim_{\xi \to \infty}\{[|E|^2 - (i/k)(E^* dE/d\xi - E\, dE^*/d\xi) + k^{-2}|dE/d\xi|^2][|E|^2 + (i/k)(E^* dE/d\xi - E\, dE^*/d\xi) + k^{-2}|dE/d\xi|^2]^{-1}\}.$$

Using next (7) and changing to dimensionless variables, we obtain ultimately

$$R = (\psi_1 - \psi_2)/(\psi_1 + \psi_2),$$

$$\psi_1 = \int_{-\infty}^{\infty} (\eta e^{-f} + z_1 z_3 \eta' f')\, d\zeta, \quad \psi_2 = \eta_\infty (2z_1 z_2)^{1/2}. \tag{29}$$

6. PROPERTIES OF NUMERICAL SOLUTION

The system (17) and (19) was integrated numerically for different combinations of the independent parameters z_2, z_3, b, θ_∞ and T_m/I. The eigenvalue of the

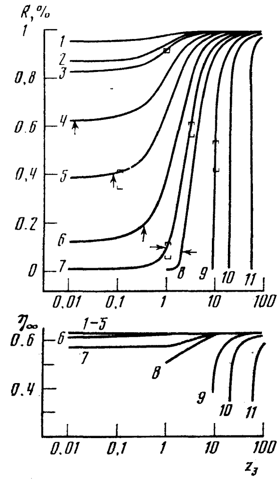

FIG. 1. Depedence on the reflection coefficient R and of the dimensionless intensity η_∞ of the electromagnetic radiation absorbed by the wave on the radiation frequency $z_3 \equiv \omega/\nu$ at $\varkappa_0/\varkappa_{em} = 0.1$; $I/T_m = 8$; $T_0/T_m = 4\times10^{-3}$. Curves 1–11 corresponds to the following values of the parameter $z_2 \equiv 2(k\delta_m)^2$: 1) $z_2 = 2\times10^{-4}$; 2) 1.6×10^{-3}; 3) 3.2×10^{-3}; 4) 2×10^{-2}; 5) 8×10^{-2}; 6) 0.32; 7) 0.98; 8) 2; 9) 8; 10) 18; 11) 50.

problem was determined from the additional condition (22). The boundary conditions for the functions $\theta(\zeta)$ and $\eta(\zeta)$ as $\zeta \to -\infty$ follow from the asymptotic forms (25) and (26). The calculation results are shown in Figs. 1–4.

The reflectivity of the wave R and the intensity of the absorbed radiation η_∞ depend on the parameter z_2 and z_3 more strongly than on b, T_m/I and θ_∞. Figure 1 shows therefore plots of $R(z_3)$ and $\eta_\infty(z_3)$ obtained for 11 numerical values of the parameter z_2 and for parameter values typical of the considered problem, $b = 0.1$, $I/T_m = 8$, and $\theta_\infty = 3.084$. The five pairs of square brackets indicate the intervals of the variation of R and η_∞ when b, θ_∞, and I/T_m are varied in the intervals $10^{-2} \leq b \leq 10^2$ and $4 \leq \theta_\infty < I/2T_m \leq 10$ for five typical combinations of the parameters $z_{2,3}$. For the same five

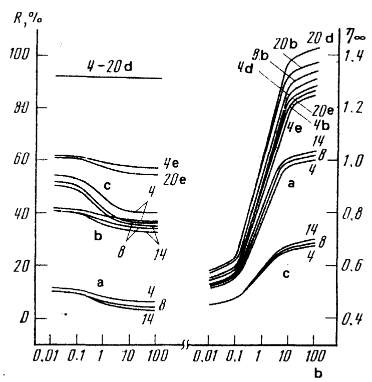

FIG. 2. The same quantities as in Fig. 1, as functions of the ratio $b \equiv \varkappa_0/\varkappa_{em}$ of the maximum values of the coefficient of the "phonon" and electronic thermal conductivities. The values of I/T_m are marked on the curves, $T_0 = 0$. The calculations were performed for five typical combinations of the parameters $z_2 \equiv 2(k\delta_m)^2$ and $z_3 = \omega/\nu$; a) 0.98 and 0.1; b) 0.08 and 0.1; c) 8 and 10; d) 0.98 and 3; e) 3.2×10^{-3} and 1.

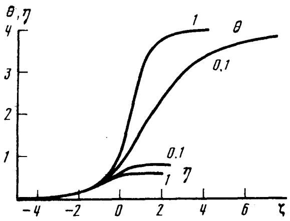

FIG. 3. Structure of ionization-wave front. Dependence of the dimensionless radiation of the plasma temperature from its maximum value θ and of the dimensionless intensity of the electromagnetic radiation η on the dimensionless coordinate ζ at two values of the parameter $b \equiv \varkappa_0/\varkappa_{em}$ ($b = 1$ and 0.1). $2(k\delta_m)^2 = 0.89$; $\omega = \nu$; $I/T_m = 8$; $T_0/T_m = 4\times 10^{-3}$.

combinations of $z_{2,3}$, Fig. 2 shows plots of $R(b)$ and $\eta_\infty(b)$ at certain values of I/T_m and $\theta_\infty \approx I/2T_m$. The wave velocity $v = \eta_\infty/\theta_\infty$, see (23).

In the geometric-optics approximation, the instant of reflection corresponds to the value $T_m = T^*$, where T^* is determined from the condition $\varepsilon(T^*) = 0$ (see Sec. 7 below). It follows from (1) and (20) that $z_3 = z_2$ at $T_m = T^*$. For a number of curves on Fig. 1, the values of $z_3 = z_2$ are marked by arrows. In this case $\varepsilon(T_m) > 0$ ($\varepsilon(T_m) < 0$) if $z_3 < z_2$ ($z_3 > z_2$).

Knowing the optical and thermophysical quantities that characterize the concrete substance ($I, n_0, \nu, C, N_0, T_0, \varkappa_0$) and the radiation frequency ω, we can use the plots 1 and 2 to interpret the experimentally measured functions $u(S_1)$, $T_m(S_1)$, and $R(S_1)$. In practice, it is convenient to use for plots 1 and 2 the following procedure: assign T_m several values whose realization is possible in the considered problem; from the known properties of the medium, the temperature T_m, and the radiation frequency ω we calculate the values of the parameters z_2, z_3, b, θ_∞, and I/T_m [Eqs. (20), (21)]; the obtained dimensionless parameters are used to determine η_∞, R, and v; the transition to dimensional functions $S_1(T_m)$, $u(T_m)$, and $R(T_m)$ is in accord with Eqs. (16).

Typical structures of the ionization wave are shown in Fig. 3. At $\zeta < -1$ the profiles of $\theta(\zeta)$ and $\eta(\zeta)$ take the forms (25) and (26), i.e., $\theta(\zeta)$ is independent of the

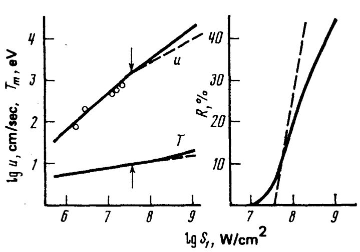

FIG. 4. Dependence of the ionization-wave velocity, plasma temperature behind the front, and the reflectivity of the front on the neodymium-laser radiation intensity. Glass ZhS-12. The dashed lines show the results of the calculation performed in the geometric-optical approximation. The solid lines are the results of the solution of the complete system of Maxwell's equations. The points are experimental.[7,8]

parameters of the problem, and $\eta(\zeta)$ depends on them weakly, since usually

$$v = \eta_\infty/\theta_\infty \sim T_m^2/I(T_m - T_0) \sim T_m/I \ll 1.$$

The smallness of the v causes the profile of $\eta(\zeta)$ calculated for the case $v = 0$ to differ little from the true profile not only at $\zeta < -1$, but in the entire range of variation of ζ. The same properties are possessed also by the plot of $\theta(\zeta)$, but not in the entire range of variation of ζ, only in the region where there is a substantial energy release, i.e., a substantial change in the profile of $\eta(\zeta)$.

The unit of length was chosen so as to be μ_m^{-1} (16), so that practically total absorption of the radiation takes place at a distance $\Delta\zeta_1 \approx 3$ from the start of the noticeable energy release. This result is a direct consequence of the correct choice of the "natural" dimensionless variable, and make it possible to draw the profile of $\eta(\zeta)$ with high accuracy, using the asymptotic form (26), the value of η_∞, and the general form of the curve (Fig. 3).

Light absorption begins only after the dielectric is substantially heated. Therefore, the "temperature" width $\Delta\zeta_2$ of the front is somewhat larger than the "absorption" width. In dimensional variables $L_T \sim \varkappa_m/Cu$, so that $\Delta\zeta_2 \sim \mu_m\varkappa_m/Cu$. In analogy with the $\eta(\zeta)$ profile, we can use the asymptotic form (25), the value θ_∞, the general form of the curve (Fig. 3), and the known fact front width $\Delta\zeta_2$ to draw with good accuracy the profile of $\theta(\zeta)$. Changing next to dimensional quantities, we obtain the profiles of $T(\xi)$ and $S(\xi)$.

Great interest attaches usually to a comparison of the numerical results with different approximate analytic solutions of the problem. We proceed now to construct such solutions.

7. GEOMETRICAL-OPTICS APPROXIMATION

The energy-transport equations investigated in the preceding sections are valid at arbitrary values of the optical parameters of the plasma, and are therefore always applicable within the framework of the considered problem. In some limiting cases, however, these equations can be greatly simplified. The foregoing pertains particularly to the geometrical-optics approximation.

From the analysis of the preceding section it follows that the characteristic scale over which the profile $S(\xi)$ changes in the absorption wave is the quantity μ_m^{-1}. Therefore the geometrical-optics approximation is applicable if the following inequality is satisfied

$$\mu_m\lambda = 2\pi(2z_1/z_2)^{1/2} \ll 1$$

and $\hat{\varepsilon}(T)$ differs from zero everywhere in the front of the wave (here $\lambda \equiv 2\pi/k$ is the wavelength of the radiation in the non-ionized dielectric). In view of the monotonicity of the $T(\xi)$ profile, the last condition reduces in fact to a restriction on T_m, and this, with (1) and (20) taken into account, leads to the inequality $z_2 \gg z_3$. In this case the value of the Poynting vector of the electromagnetic wave satisfies the equation

$$dS/d\xi = \mu(T)S. \tag{30}$$

We express now S in terms of T and T' in accordance with relations (12) and substitute in (30). As a result we arrive at the equation

$$\frac{d}{d\xi}\left(\varkappa\frac{dT}{d\xi}\right)-\mu\varkappa\frac{dT}{d\xi}=-uC\left[\mu(T_m-T)+\frac{dT}{d\xi}\right]. \tag{31}$$

Equation (31) admits of further simplification. It was noted above that in the region of the main energy release the wave structure can be described neglecting the finite wave propagation velocity, i.e., the right-hand side of (31) can be replaced by zero. As a result we arrive at an equation that can be integrated in quadratures and takes, with allowance for the boundary conditions (9), the form[1)]

$$\xi=\int\limits_{T_m}^{T}\varkappa\left(\int\limits^{T'}\varkappa\mu\,dT''\right)^{-1}dT', \quad T\sim T_m. \tag{32}$$

The relation (32) determines the structure of the wave, i.e., the dependence of $T(\xi)$ in the region of the main energy release at arbitrary $\varkappa(T)$ and $\mu(T)$ dependences.

The structure of the wave in the region $T\sim T_0$ can be easily obtained from Eq. (5) if it is recognized that at $T\sim T_0$ practically no energy is released by the wave, so that the term $(\sigma/2)|E|^2$ in the left-hand side of (5) can be left out. Then

$$\xi=-\int\limits^{T}\frac{\varkappa\,dT'}{uC(T'-T_0)}, \quad T_0\leqslant T\leqslant T_m\left(1-\frac{T_m}{I}\right). \tag{33}$$

In accord with the foregoing, the asymptotic relation (33) cannot be satisfied at $u=0$. To determine u we note that in the considered approximation, in the entire-energy-release region, expression (12) degenerates into the relation $S+\varkappa dT/d\xi=0$. From this, taking (32) into account, we arrive at the known relation[1]

$$S_0\approx-\varkappa\frac{dT}{d\xi}\bigg|_{T\sim T_m(1-T_m/I)}\approx\int\limits_{T_0}^{T_m}\varkappa\mu\,dT \tag{34}$$

[we have used the very strong $\mu(T)$ dependence and extended the integration in the right-hand side of (34) over the entire range of T].

Expressions (32)–(34), together with the energy conservation law

$$uC(T_m-T_0)=S_0, \tag{35}$$

which follows from (12), solve completely the posed problem, i.e., they determine in implicit form the velocity of the ionization wave and its structure as functions of the intensity of the incident radiation and of the thermophysical characteristics of the dielectric.

In some limiting cases, these dependences can be written in explicit form. In particular, in the case of negligibly small electronic thermal conductivity ($b\gg1$) the wave structure in the region of the substantial change of the intensity of the incident radiation takes in terms of the dimensionless variables (16) and (18) the form

$$\theta=\ln(1+e^{\zeta}), \quad \eta=(1+e^{-\zeta})^{-1}$$

(we have used the condition $f\approx\theta$).

In the opposite limiting case of large electronic thermal conductivity ($b\ll1$) we have

$$\theta=\ln[\operatorname{cth}(-\zeta/2)], \quad \eta=[2\operatorname{ch}^2(\zeta/2)]^{-1}, \quad \zeta\leqslant0.$$

We note that in this case the wave terminates at $\zeta=0$, as is usually the case for purely nonlinear thermal conductivity.[11] The "tongue" that projects into the region $0\leqslant\zeta<\infty$ appears only in the next order in b, i.e., when account is taken of the small but finite linear component in the thermal-conductivity coefficient.

With increasing radiation intensity, the value of T_m increases, and consequently the value of ε behind the wave front decreases, see (1) and (3). Strictly speaking, at too small a value of ε, the geometrical-optics approximation no longer holds. However, a comparison with numerical calculations shows that at $\mu_m\lambda\ll1$ this approximation can be formally extended not only to the region of small values of ε, but also to the case $\varepsilon(T_m)<0$. The solution of the problem is determined here as before by relations (32)–(35), but the quantity S_0, just as in Sec. 5, must be taken to mean the difference between the incident radiation flux S_1 and the one reflected from the front S_2. The reflection coefficient is determined in this case by the expression

$$R=\exp\left[-2\int\limits_{T_0}^{T^*}\mu(dT/d\xi)^{-1}\,dT\right], \tag{36}$$

where T^* is obtained from the condition $\varepsilon(T^*)=0$.

An investigation of the explicit dependences of u, T_m, and R on S leads in the considered case to formulas that practically coincide with the results of Ref. 3, and will therefore not be presented here. We indicate only that the quantity T_m depends on the radiation intensity logarithmically: in the optical buring regime the radiation energy is consumed mainly in forward propulsion of the absorption wave, i.e., in ionization of the "cold" dielectric, and not in raising the already ionized material to a higher temperature. The velocity of the non-reflecting absorption wave ($S_1\approx S_0\ll S^*$, where S^* is that value of the intensity at which $T_m=T^*$) turns out to be, apart from logarithmic corrections, $\sim S_1$. On the other hand, in the region of strong reflection ($S^*\ll S_0\ll S_{1,2}$) we have with the same accuracy $u\sim S^{1/2}$.

By way of example, Fig. 4 shows plots of u, T_m, and R against the radiation intensity ($\lambda=1.06$ μm) for ZhS-12 glass, calculated on the basis of a solution of the complete system of Maxwell's equations, as well as the same dependences but obtained from the geometrical-optics approximation (the material constant used in the calculations correspond to the data of Refs. 7 and 8). The intensity $S_1=S^*$ is marked on the plots by arrows. It is seen that, in accord with the statements made above, at $S_1\leqslant S^*$ the plots obtained in the geometric-optics approximation practically coincide with the result of the solution of Maxwell's equations. At $S_1>S^*$ the use of geometrical optics leads to somewhat smaller values of u and T_m than would follow from the solution of Maxwell's equations, this being due to the underestimate, in the geometrical-optics approximation, of the contribution made to the plasma heating by the radiation penetrating beyond the plane $\varepsilon(T)=0$.

When R is determined in the range $0.1 \leq R \leq 0.9$, geometrical optics yields an absolute error not exceeding 20%. At not too high radiation intensities, however, R itself is quite small, so such a comparatively small optical error can lead to an appreciable relative error, see Fig. 4.

On the whole, however, with exception of the noted circumstance, the geometrical-optics approximation describes satisfactorily the ionization wave in condensed dielectrics at all values of the laser-radiation intensity of practical interest. This conclusion (if the condition $\mu_m\lambda \ll 1$ is satisfied) is quite general and is confirmed by numerical calculations performed for other sets of independent parameters that characterize the problem.

Figure 4 shows also results of a measurement of the dependence of u on S_1 (Refs. 7 and 8). Good agreement is seen between theory and experiment. No reflection of the laser radiation from the ionization wave front was registered in the cited papers (the experimental accuracy was not worse than 4%); this also agrees with our calculations, since all the experimental points on Fig. 4 are located to the left or the region where there is reflection of any significance.

8. OTHER APPROXIMATIONS. THE FRESNEL FORMULAS

It was shown in the preceding section that the geometrical-optics approximation leads to the largest error in the value of R. The cause of this error is not some special roughness of the geometrical-optics approximation, but the smallness of the reflection coefficient itself in the radiation-intensity region of physical interest, a coefficient that must be determined with high accuracy. It is therefore advisable to present, besides (36), some other approximate formulas for the reflection coefficient, independent of the validity of the geometrical-optics approximation.

We turn to expression (29). Recognizing that $f \approx \theta$ and that in the calculation of the integral that determines ψ_1 we can put, in accord with the statements made above, $\eta \approx \chi d\theta/d\xi$ in the entire energy-release region, we can obtain in a number of cases an expression for R in closed form. In particular,

$$R = \frac{(b+{}^1/_2)/(b+1) - \eta_\infty (2z_1z_2)^{1/2}}{(b+{}^1/_2)/(b+1) + \eta_\infty (2z_1z_2)^{1/2}} \quad \text{if} \quad \omega \ll \nu, \tag{37}$$

$$R = \frac{(1+\eta_\infty^2 z_1z_3/2) - \eta_\infty (2z_1z_2)^{1/2}}{(1+\eta_\infty^2 z_1z_3/2) + \eta_\infty (2z_1z_2)^{1/2}} \quad \text{if} \quad b \gg 1. \tag{38}$$

The regions of applicability of (37) and (38) are not connected with conditions $\mu_m\lambda \gg 1$ and $\mu_m\lambda \ll 1$, and can be close to either large or small values of this parameter. At $\mu_m\lambda \gg 1$, however, the scale over which the structure of the absorption wave changes is small compared with λ. In this case the reflection of the electromagnetic wave from the front of the ionization wave proceed as from a plasma having a temperature $T = T_m$ and an abrupt boundary, i.e., the usual Fresnel formulas are valid:

$$R = \frac{(\tilde{n}_m - n_0)^2 + \tilde{\varkappa}_m^2}{(\tilde{n}_m + n_0)^2 + \tilde{\varkappa}_m^2}$$
$$= \left[\frac{2z_2}{z_1}\left\{\left(1 + \frac{z_1 - 2z_3}{2z_2}\right)^{1/2} - 1\right\}^2 + 1\right]\left[\frac{2z_2}{z_1}\left\{\left(1 + \frac{z_1 - 2z_3}{2z_2}\right)^{1/2} + 1\right\}^2 + 1\right]^{-1}. \tag{39}$$

Inasmuch as under real conditions, in the case $\mu_m\lambda \gg 1$, one of the formulas (37) or (39) becomes applicable simultaneously with (39), we can recommend the following method of determining the integrated characteristics of the absorption wave. We determine $z_{1,2,3}$ and θ_∞ from the known constants of the dielectric and the known electromagnetic-wave frequency, specifying some value of T_m [see (20) and (21)]. Next, formula (39) is used to determine R. From a comparison of (39) with (37) and (38) we obtain η_∞ from the known R. Finally, knowing η_∞, we determine v from the relation $\eta_\infty = v\theta_\infty$.

9. DISCUSSION OF RESULTS

We formulate here the main result of our investigation of an equilibrium stationary wave of the optical burning type.

When expressed in the "natural" dimensionless variables (16), the ionization-wave problem is completely defined by the five dimensionless parameters b, $z_{2,3}$, T_m/I, and θ_∞. The dependence of the integrated characteristics and the structure of the wave in the region of the main energy release on T_m/I and θ_∞ is relatively weak.

For a rough determination of the integrated characteristics of the ionization wave we can use the relation $\eta_\infty = v\theta_\infty$ and the fact that, in the "natural" dimensionless variables chosen by us, the quantity η_∞ is always close to unity. Hence, changing to dimensional relations, we obtain

$$\frac{IS_0}{2\mu_m\varkappa_m T_m^2} \sim 1, \quad \frac{uC}{\mu_m\varkappa_m}\frac{I(T_m - T_0)}{2T_m^2} \sim 1. \tag{40}$$

In the case $\mu_m\lambda \ll 1$ (geometrical optics), a radiation reflection of any significance from the ionization-wave front is produced only if $T_m > T^*$, where T^* is determined from condition $\varepsilon(T^*) = 0$. Therefore, at $T_m \leq T^*$, the intensity S_0 in (40) actually coincides with the intensity S_1 of the incident radiation. At $T_m > T^*$ we have $S_0 = S_1(1 - R)$, where R is given by Eq. (36).

In the opposite limiting case, $\mu_m\lambda \gg 1$, the Fresnel formulas can be used and R is determined by Eq. (39).

A somewhat more exact method of determining the integrated characteristics and the structure of the wave is described in Secs. 6–8. To obtain detailed information on the absorption wave one can use Figs. 1–4, which shows plots that cover the entire real range of variation of the parameters of the problem.

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[1)]Since the eigenvalue u of the problem is replaced here by $u=0$, we cannot stipulate satisfaction of all the boundary conditions (9), and since the main energy release is concentrated in a small vicinity $T=T_m$, it is necessary to choose from among the four boundary conditions (9) the two posed at $\xi \to -\infty$.

[1]Yu. P. Raizer, Laser-Induced Discharge Phenomena, Consultants Bureau, 1974.

[2]P. S. Kondratenko and B. I. Makshantsev, Zh. Eksp. Teor. Fiz. **66**, 1734 (1974) [Sov. Phys. JETP **39**, 851 (1974)].

[3]I. I. Tribel'skiĭ, Fiz. Tverd. Tela (Leningrad) **18**, 1347 (1976) [Sov. Phys. Solid State **18**, 775 (1976)].

[4]I. E. Poyurovskaya, *ibid.* **19**, 2876 (1977) [**19**, 1685 (1977)].

[5]I. E. Poyurovskaya and V. I. Fisher, Abstracts, 4th All-Union Conf. on Nonresonant Interaction of Optical Radiation with Matter, Leningrad, State Opt. Inst. Press, 1978.

[6]V. A. Alekseev and A. A. Vedenov, Usp. Fiz. Nauk **102**, 665 (1970) [Sov. Phys. Usp. **13**, 830 (1971)].

[7]N. V. Zelikin, N. E. Kask, V. V. Radchenko, G. M. Fedorov, O. V. Fedorovich, and D. B. Chopornyak, Pis'ma Zh. Tekh. Fiz. **4**, 1796 (1978) [Sov. Tech. Phys. Lett. **4**, 522 (1978)].

[8]E. L. Klochan, S. P. Popov, and G. M. Fedorov, *ibid.* **6**, 453 (1980) [**6**, 194 (1980)].

[9]B. E. Meĭerovich and L. P. Pitaevskiĭ, Zh. Eksp. Teor. Fiz. **61**, 235 (1971) [Sov. Phys. JETP **34**, 121 (1972)].

[10]B. E. Meĭerovich, *ibid.* **61**, 1891 (1971) [**34**, 1006 (1972)].

[11]Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and of High Temperature Hydrodynamic Phenomena, Academic, 1966, 1967.

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