Emission of ion-sound waves by a Langmuir soliton moving with acceleration

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A theory is developed for the emission of ion-sound waves by a Langmuir soliton moving with acceleration. The acceleration is due to the nonuniform density distribution in the plasma.

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At the present time, the soliton is one of the most intensively studied objects in plasma theory. This is due to its value for the understanding of the dynamics of nonlinear waves. In the voluminous literature devoted to the soliton (see Refs. 1 and 2 and the literature cited therein), its fundamental property is made clear, namely, if we neglect the inertia of the ions and the amplitude of the soliton is not large,[3] then, in the interactions among themselves, the solitons, like particles, preserve their shape and velocity.

In the present work, still another property of the soliton has been observed: a soliton moving with acceleration can, like a particle, emit ion-sound waves. A soliton moving in an inhomogeneous plasma is studied in this work. It is the inhomogeneity which leads to its acceleration.[4] The spatial distribution of the radiation field is investigated for linear and quadratic profiles of the inhomogeneity. An expression is obtained for the energy loss of the soliton as a result of the emission. The problem can be of interest in the estimate of the requirements on the degree of homogeneity of the plasma for the experimental detection of the soliton and the study of its properties.

1. We limit ourselves to the consideration of the one-dimensional problem. Following Zakharov,[5] we describe the nonlinear dynamics of the plasma with the help of the time envelope E of the electric field intensity of the Langmuir oscillations E_L:

$$E_L={}^1/_2\{E(x,t)e^{-i\omega_pt}+E^*(x,t)e^{i\omega_pt}\} \tag{1}$$

and the low-frequency variation of the plasma density δn (ω_p is the plasma electron frequency). In a nonlinear plasma, the set of equations for the determination of these quantities has the form[6]

$$2i\omega_p\frac{\partial}{\partial t}E+3v_T{}^2\frac{\partial^2}{\partial x^2}E-\omega_p{}^2\frac{n(x)}{n_0}E-\omega_p{}^2\frac{\delta n}{n_0}E=0, \tag{2}$$

$$\left\{\frac{\partial^2}{\partial t^2}-c_s{}^2\frac{\partial^2}{\partial x^2}+c_s{}^2\frac{\partial}{\partial x}\frac{1}{n_0+n(x)}\frac{\partial n(x)}{\partial x}\right\}\delta n =\frac{1}{16\pi m_in_0}\frac{\partial}{\partial x}[n_0+n(x)]\frac{\partial}{\partial x}|E|^2, \tag{3}$$

where v_T is the thermal velocity of the electron, the quantity $n(x)$ characterizes the profile of the inhomogeneity of the unperturbed plasma and represents the inhomogeneous deviation of the density from its equilibrium value n_0. In obtaining Eq. (3), we have neglected the plasma drift brought about by the inhomogeneity, assuming the drift velocity to be small in comparison with the velocity of ion sound c_s and the velocity of translation of the region of localization of the Langmuir oscillations.

In the following, we shall assume $n(x)$ to be small, $n(x)\ll n_0$, which allows us to simplify Eq. (3). We introduce the dimensionless quantities

$$t'=\frac{1}{3}\frac{m_e}{m_i}\omega_pt,\quad x'=\frac{1}{3}\left(\frac{m_e}{m_i}\right)^{1/2}\frac{x}{r_D},\quad \delta n'=3\frac{m_i}{m_e}\frac{\delta n}{n_0},$$
$$n'(x)=3\frac{m_i}{m_e}\frac{n(x)}{n_0},\quad E'=\left(3\frac{m_i}{m_e}\right)^{1/2}\frac{E}{(16\pi n_0T_e)^{1/2}},\quad r_D=\frac{v_T}{\omega_p} \tag{4}$$

and we represent the system (2) and (3) in the form (for convenience, we have omitted the primes in what follows)

$$2i\frac{\partial E}{\partial t}+\frac{\partial^2}{\partial x^2}E-n(x)E-\delta nE=0, \tag{5}$$

$$\left\{\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}\right\}\delta n=\frac{\partial^2}{\partial x^2}|E|^2. \tag{6}$$

We write down the solution of Eq. (5) in the form

$$E=\mathscr{E}(x-\bar{x}(t))e^{i\varphi(x,t)}, \tag{7}$$

where $\mathscr{E}$ and φ are real quantities; $\bar{x}(t)$ is the coordinate of the center of localization of the Langmuir oscillations. The time dependence of the coordinate $x(t)$ will be determined in the following. Because of the inhomogeneity of the plasma, this dependence turns out to be nonlinear.

In the expression for δn we isolate that part of the density perturbation which is concentrated in the region of localization of the Langmuir oscillations:

$$\delta n=\frac{\mathscr{E}^2(x-\bar{x}(t))}{\dot{\bar{x}}^2-1}+N, \tag{8}$$

where $\dot{\bar{x}}=d\bar{x}/dt$. The quantity N will characterize the perturbation of the density by the ion sound outside the soliton.

Since we are interested in the radiation of a soliton, it is natural to assume that the velocity of the source of radiation of the soliton is much less than the sound velocity:

$$\dot{\bar{x}}^2\ll 1. \tag{9}$$

Then the relation

$$\ddot{\bar{x}}\Delta l\ll 1 \tag{10}$$

is easily satisfied (or, in dimensional form, $\ddot{\bar{x}}\,\Delta l\ll c_s^2$). This condition means that the change in the velocity of

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the soliton within the time that the ion sound passes through a distance of the order of the width of the soliton Δl is small in comparison with the ion-sound velocity (we shall not be interested in the reaction of the radiation on the soliton). For this we assume that the perturbation of the density by the ion sound is not large,

$$N \ll \mathscr{E}^2. \tag{11}$$

Then, we obtain the following from Eqs. (5) and (6) with the help of Eqs. (7)–(11):

$$\frac{\partial^2}{\partial x^2}\mathscr{E} - \left\{2\frac{\partial\varphi}{\partial t} + \left(\frac{\partial\varphi}{\partial x}\right)^2 + n(x)\right\}\mathscr{E} - \delta n\mathscr{E} = 0, \tag{12}$$

$$\frac{\partial}{\partial t}\mathscr{E}^2 + \frac{\partial}{\partial x}\mathscr{E}^2\frac{\partial\varphi}{\partial x} = 0, \tag{13}$$

$$\left\{\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right\}N = -\ddot{\bar{x}}\frac{\partial}{\partial x}\mathscr{E}^2. \tag{14}$$

It follows clearly from Eq. (14) that only a soliton moving with acceleration can radiate.

An original change of variables that reduced the nonlinear Schrödinger equation for the inhomogeneous plasma to an equation for the homogeneous plasma was obtained in Ref. 4. Here we shall show that we can arrive at a similar solution by a much more lucid route.

Since the amplitude of $\mathscr{E}$ is sought in the form of a function that is dependent on the self-similar argument $\xi = x - \bar{x}(t)$, we find from (13) the following for values of $\mathscr{E}$ that vanish at infinity ($\xi \to \pm\infty$, $\mathscr{E} \to 0$)

$$\varphi = \dot{\bar{x}}(t)x + f(t), \tag{15}$$

where $f(t)$ is a function of time that is thus far arbitrary. Substituting (15) in (12), we obtain

$$\frac{\partial^2}{\partial x^2}\mathscr{E} - \{2\dot{f}(t) + 2\ddot{\bar{x}}(t)x + \dot{\bar{x}}^2(t) + n(x)\}\mathscr{E} + \mathscr{E}^3 = 0. \tag{16}$$

For the first integral of Eq. (16), we find

$$\left(\frac{\partial\mathscr{E}}{\partial x}\right)^2 - \{2\dot{f}(t) + \dot{\bar{x}}^2(t) + 2\ddot{\bar{x}}(t)x + n(x)\}\mathscr{E}^2 + \int_{-\infty}^{x} dz\left\{2\ddot{\bar{x}}(t) + \frac{\partial n}{\partial z}\right\}\mathscr{E}^2(z,t) + \frac{1}{2}\mathscr{E}^4 = 0. \tag{17}$$

It follows from (17) that

$$\int_{-\infty}^{\infty} dz\left\{2\ddot{\bar{x}}(t) + \frac{\partial n}{\partial z}\right\}\mathscr{E}^2(z,t) = 0. \tag{18}$$

If we assume that the maximum value of the amplitude $\mathscr{E} = \mathscr{E}_m$ corresponds to the point $x = \bar{x}(t)$, we can then find the equation for the function $f(t)$ from (17):

$$2\dot{f}(t) + \dot{\bar{x}}^2(t) + 2\ddot{\bar{x}}(t)\bar{x}(t) + n(\bar{x}) - \frac{1}{\mathscr{E}_m^2}\int_{-\infty}^{\bar{x}} dz\left\{2\ddot{\bar{x}}(t) + \frac{\partial n}{\partial z}\right\}\mathscr{E}^2 - \frac{1}{2}\mathscr{E}_m^2 = 0. \tag{19}$$

Substituting (19) in (16), we obtain

$$\frac{\partial^2}{\partial x^2}\mathscr{E} - \left\{\frac{1}{2}\mathscr{E}_m^2 + 2\ddot{\bar{x}}(t)[x - \bar{x}(t)] + n(x) - n(\bar{x}) + \frac{1}{\mathscr{E}_m^2}\int_{-\infty}^{\bar{x}} dz\left(2\ddot{\bar{x}} + \frac{\partial n}{\partial z}\right)\mathscr{E}^2\right\}\mathscr{E} + \mathscr{E}^3 = 0. \tag{20}$$

In the following, we shall assume that the density of the inhomogeneous plasma changes over distances of the order of the width of the soliton. Then we can limit ourselves in the expansion

$$n(x) = n(\bar{x}) + (x - \bar{x})\frac{\partial n}{\partial x}\Big|_{\bar{x}} + \frac{1}{2}(x - \bar{x})^2\frac{\partial^2 n}{\partial x^2}\Big|_{\bar{x}} + \ldots \tag{21}$$

to only the first three terms. If the conditions

$$\mathscr{E}_m^2 \gg \Delta l^2\frac{\partial^2 n}{\partial x^2}\Big|_{\bar{x}}, \quad \frac{1}{\mathscr{E}_m^2}\int_{-\infty}^{\bar{x}} dz(z - \bar{x})\mathscr{E}^2 \tag{22}$$

are satisfied, we obtain for the amplitude of $\mathscr{E}$ the well-known equation

$$\frac{\partial^2}{\partial x^2}\mathscr{E} - \frac{1}{2}\mathscr{E}_m^2\mathscr{E} + \mathscr{E}^3 = 0, \tag{23}$$

the solution of which has the form

$$\mathscr{E} = \mathscr{E}_m \Big/ \operatorname{ch}\frac{1}{\Delta l}(x - \bar{x}(t)), \tag{24}$$

where the coordinate of the center of the soliton, according to (18) and (21), can be determined from three equations

$$\ddot{\bar{x}}(t) = -\frac{1}{2}\frac{\partial n}{\partial x}\Big|_{\bar{x}}, \tag{25}$$

while the width of the soliton is equal to $\Delta l = 2^{1/2}/\mathscr{E}_m$.

We thus obtain results that are similar to those found in Ref. 4. However, the method of calculation that we have used [with the help of Eq. (20)] enables us to show lucidly the conditions under which a Langmuir soliton of the form (24) can exist in the inhomogeneous plasma.

2. Substituting (24) and (25) in (14), we can find the explicit form of the density distribution in the emitted ion sound. We introduce the natural initial condition: at the initial instant of time, $t = 0$, let emission be absent, $N = 0$. This means that from the moment $t = 0$ the soliton begins to move with acceleration. Then the solution of Eq. (14) has the form[7]

$$N = -\frac{1}{2}\int_0^t dt'\int_{x-t+t'-\bar{x}(t')}^{x+t-t'-\bar{x}(t')} dz\,\ddot{\bar{x}}(t')\frac{\partial}{\partial z}\mathscr{E}^2(z). \tag{26}$$

Using the condition (9), and also the smallness of the change in the acceleration of the soliton within the time of passage of the ion sound across the width of the soliton,

$$\Delta l\dddot{\bar{x}} \ll \ddot{\bar{x}},$$

we find from (26)

$$N = \frac{1}{2}\mathscr{E}_m^2\Delta l\left\{\ddot{\bar{x}}(t - x + \bar{x}(t))\left[\operatorname{th}\frac{x - \bar{x}(t)}{\Delta l} - \operatorname{th}\frac{x - t - \bar{x}(0)}{\Delta l}\right] + \ddot{\bar{x}}(t + x - \bar{x}(t))\left[\operatorname{th}\frac{x - \bar{x}(t)}{\Delta l} - \operatorname{th}\frac{x + t - \bar{x}(0)}{\Delta l}\right]\right\}. \tag{27}$$

Knowing the profile of the inhomogeneity, we can, from (25) and (27), determine the spatial distribution of the perturbation of the velocity by the radiated ion sound. We note that with the help of (27) we can also find the density distribution inside the soliton. In the following, we shall assume that the soliton moves to the right, $\dot{\bar{x}} > 0$. Then the first term in the curly brackets of (27) corresponds to forward radiation and the second to backward radiation. Because of the smallness of the velocity of the soliton, the distance traversed by the radiation front is greater than the displacement of the soliton within the same interval of time. It is clear that we can speak of soliton displacement if it is greater than the width of the soliton, i.e., we can set

$$t \gg |\bar{x}(t) - \bar{x}(0)| \gg \Delta l. \qquad (28)$$

We now analyze the radiation field for specific forms of the density profile of an inhomogeneous plasma.

a) For a linear profile $n(x) = 2\alpha x$, the motion of the center of the soliton is described according to (25) by the formula

$$\bar{x}(t) = \bar{x}(0) + wt - {}^1/_2 \alpha t^2,$$

where $\bar{x}(0)$ is the initial value of the coordinate, w is the initial velocity, and the accelerator is constant and equals $\ddot{\bar{x}} = -\alpha$, i.e., if $\alpha > 0$, then the soliton is slowed. We note that the restriction (22) is lifted for a linear profile of the inhomogeneity, since $\partial^2 n(x)/\partial x^2 = 0$.

We obtain from (27) the following expression for the radiation field:

$$N(x,t) = 2^{-1/2}\alpha\mathscr{E}_m\left\{\text{th}\frac{t+x-\bar{x}(0)}{\Delta l} - \text{th}\frac{t-x+x(0)}{\Delta l} - 2\,\text{th}\frac{x-\bar{x}(t)}{\Delta l}\right\}. \qquad (29)$$

The qualitative spatial distribution of the density, corresponding to (29), is shown in Fig. 1.

The region occupied by the radiation field in front of the soliton is narrower than the region behind it. Here the perturbations of the density in these regions have different signs. The change in the sign takes place in regions of localization of the soliton at a given instant of time. These regions broaden rapidly; the boundaries of the regions move away from the soliton with the velocity of ion sound. Ahead of the soliton, the perturbation reaches its maximum value

$$N = -2^{1/2}\alpha\mathscr{E}_m$$

at the point x_m, which is located outside the soliton, $|\bar{x}(t) - x_m| \gg \Delta l$. At the point $x_1 = t + \bar{x}(0)$, the perturbation is equal to

$$N = -2^{-1/2}\alpha\mathscr{E}_m.$$

The perturbation falls off rapidly to the right of this point. According to (29), the perturbation in this region is equal to

$$N = -2^{-1/2}\alpha\mathscr{E}_m\{1 - \text{th}(z/\Delta l)\}, \quad z = x - t - \bar{x}(0) \geqslant 0, \qquad (30)$$

and consequently the perturbation falls off exponentially at large z ($z \gg \Delta l$). We note that the falloff takes place more rapidly the smaller the value of the soliton width Δl. In the limit $\Delta l \to 0$, the radiation front takes the form of a sharp boundary. This was to be expected. Actually, we assign a width Δl to a soliton of the form (24). However, there are in fact no sharp boundaries and the soliton extends over all space, although it falls

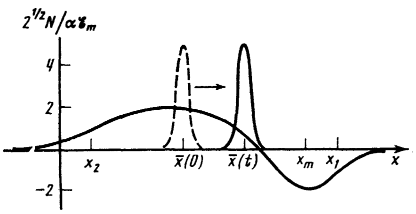

FIG. 1. Radiation field in the case $n(x) = 2\alpha x$. The arrow indicates the direction of motion of the soliton.

off exponentially outside the width Δl. Therefore, the radiation front emitted from the soliton is also smeared out with the same characteristic width Δl. Only the radiation front of a source with sharp boundaries has a vanishingly small width. In the case $\Delta l \to 0$, the radiation field to the right of the point $x_1 = t + \bar{x}(0)$ should be absent, since the radiation front reaches exactly this point within the time t measured from the beginning of action of the source.

Similarly, behind the soliton, to the left of the point $x_2 = \bar{x}(0) - t$, the perturbation also falls off rapidly. In this region, it is described by the formula

$$N = 2^{-1/2}\alpha\mathscr{E}_m\{1 - \text{th}(y/\Delta l)\}, \quad y = \bar{x}(0) - t - x \geqslant 0.$$

At this point $x_2 = \bar{x}(0) - t$ itself, the perturbation is equal to $N = 2^{-1/2}\alpha\mathscr{E}_m$. The maximum value of the perturbation behind the soliton is $N = 2^{1/2}\alpha\mathscr{E}_m$ is reached near the point $x(0)$.

The condition (11), when the conclusions given above are valid, has the following form for a linear profile of the inhomogeneity:

$$\alpha \ll \mathscr{E}_m,$$

or in dimensional form

$$\frac{\mathscr{E}_m}{(16\pi n_0 T_e)^{1/2}} \gg \frac{m_i}{m_e}\frac{r_D}{L},$$

where L is the characteristic length of the inhomogeneity.

b) The case of a quadratic inhomogeneity $n(x) = \alpha^2 x^2$ is of interest. Here, according to (25), the soliton executes harmonic oscillations about the point $x = 0$ without changing its shape. Choosing the initial condition in suitable fashion, we can rewrite the law of motion in the form

$$\bar{x}(t) = -(w/\alpha)\cos\alpha t, \qquad (31)$$

i.e., at the instant of time $t = 0$ the soliton is located at the left turning point $\bar{x}(0) = -w/\alpha$.

Substituting (31) in (27), we obtain the following relation for the density perturbation in the field of ion sound:

$$N(x,t) = 2^{-1/2}\alpha w\mathscr{E}_m\left\{\cos\alpha(t-x)\left[\text{th}\frac{x-\bar{x}(t)}{\Delta l} - \text{th}\frac{x-t-\bar{x}(0)}{\Delta l}\right] - \cos\alpha(t+x)\left[\text{th}\frac{x+t-\bar{x}(0)}{\Delta l} - \text{th}\frac{x-\bar{x}(t)}{\Delta l}\right]\right\}. \qquad (32)$$

Thus, just as in the case of a linear profile, two wavefronts leave the soliton in different directions; however, in the given case the distribution of the perturbations in space has a periodic structure.

As an illustration, we consider the distribution at the time $t = (4k+1)\pi/2\alpha$ (k is an integer), when the soliton reaches the point of equilibrium, moving to the right, $\dot{\bar{x}} > 0$. From Eq. (32) we get

$$N(x,t) = 2^{-1/2}\alpha w\mathscr{E}_m \sin\alpha x\left\{\text{th}\frac{\alpha x + (4k+1)\pi/2 + w}{\alpha\Delta l} - \text{th}\frac{\alpha x - (4k+1)\pi/2 + w}{\alpha\Delta l}\right\}. \qquad (33)$$

The principal features of this distribution can be traced qualitatively in Fig. 2.

In the range of values of the coordinate

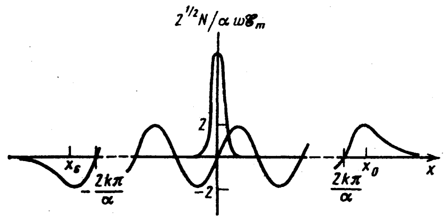

FIG. 2. Radiation field in the case $n(x)=\alpha^2x^2$ at the instant $t=(4k+1)\pi/2\alpha$.

$$-2k\pi/\alpha\leqslant x\leqslant 2k\pi/\alpha$$

the density perturbation has a purely periodic character, by virtue of the condition (28), and has the constant amplitude

$$N=2^{1/2}\alpha w\mathscr{E}_m\sin\alpha x.$$

In the region of positive $x>0$, the maximum farthest from the soliton, equal to $N=2^{1/2}\alpha w\mathscr{E}_m$ is achieved at the point x_0, located in the interval

$$(4k+{}^1/_2)\pi/2\alpha<x_0<(4k+1)\pi/2\alpha-w/\alpha.$$

To the right of the point $(4k+1)\pi/2-w/\alpha$ the perturbation of the density is described by the formula

$$N\approx 2^{1/2}\alpha\mathscr{E}_m\cos\alpha s\{1-\operatorname{th}(s/\Delta l)\},\qquad s=x-(4k+1)\pi/2\alpha+w/\alpha\geqslant 0.$$

The perturbation thus diminishes with increase of s without having a chance to oscillate, since

$$\alpha\Delta l\ll 1 \tag{34}$$

by virtue of the smoothness of the profile of the inhomogeneity [see Eq. (21)]. The decrease in the perturbation takes place more rapidly the smaller the width of the soliton.

In the region of negative values of the coordinate, $x<0$, the extremal value of the perturbation farthest from the soliton, equal to $N=-2^{-1/2}\alpha w\mathscr{E}_m$ is achieved near the point.

$$x_s=-(4k+1)\pi/2\alpha.$$

For even smaller values of the coordinate, $x<x_s$, the perturbation is described by the formula

$$N=-2^{-1/2}\alpha w\mathscr{E}_m\cos\alpha q\{1-\operatorname{th}(q/\Delta l)\},\qquad q=-x-(4k+1)\pi/2\alpha-w\alpha\geqslant 0,$$

which also indicates a rapid decrease in the perturbation with increasing distance from the soliton.

For the parabolic profile of the inhomogeneity, the condition (11) [in dimensional quantities] takes the form

$$\frac{\mathscr{E}_m}{(16\pi n_0T_e)^{1/2}}\gg\left(\frac{m_i}{m_e}\right)^{1/2}\frac{r_D}{L}\frac{v}{c_s},$$

where v is the velocity of the soliton, and from (22) we get the inequality

$$\mathscr{E}_m^2/16\pi n_0T_e\gg r_D/L.$$

These inequalities can be satisfied also at small values of the amplitude of the soliton.

3. In the calculations given above, we have neglected the radiation recoil, as can be done at $\mathscr{E}_m^2\ll 1$.[6] This condition is implied everywhere above. The condition (11) guarantees the smallness of the energy of the sound waves in comparison with the energy of the soliton. It is of interest to obtain an expression for the rate of energy loss by the soliton. We begin with the consideration of the conservation laws for the system (5) and (6).[3,8]

The total number of plasmons is conserved in the radiation process. This follows immediately from (5):

$$\int_{-\infty}^{\infty}dx|E|^2=\text{const}.$$

For the change in the momentum of the system "soliton + sound wave," we obtain the expression

$$\frac{\partial}{\partial t}\int dx\left\{i\left(E\frac{\partial E^*}{\partial x}-E^*\frac{\partial E}{\partial x}\right)+\delta nv\right\}=-\int dx|E|^2\frac{\partial n}{\partial x}. \tag{35}$$

As was to be expected, the momentum of the system is not conserved. The term on the right side of Eq. (35) represents the force acting on the system and due to the inhomogeneity of the plasma. Neglecting the radiation recoil, and using the relations (8), (11), and (34), we can easily obtain the equation of motion for the soliton (25) from (35).

The energy conservation law has the form[8]

$$\frac{\partial}{\partial t}Q=-\frac{\partial}{\partial x}\left\{\frac{i}{2}\left(\frac{\partial E}{\partial x}\frac{\partial}{\partial x}\frac{\partial E^*}{\partial x}-\frac{\partial E^*}{\partial x}\frac{\partial}{\partial x}\frac{\partial E}{\partial x}\right)\right.$$
$$\left.+\frac{i}{2}(n(x)+\delta n)\left(E\frac{\partial E^*}{\partial x}-E^*\frac{\partial E}{\partial x}\right)+\delta nv+v|E|^2\right\}, \tag{36}$$

where

$$Q=\frac{\partial E}{\partial x}\frac{\partial E^*}{\partial x}+n(x)|E|^2+\delta n|E|^2+\frac{v^2}{2}+\frac{\delta n^2}{2},\quad \frac{\partial}{\partial t}\delta n=-\frac{\partial v}{\partial x}.$$

If we determine the total energy of the system "soliton + sound," (i.e., the integral over the volume whose boundaries the front of the radiation has not yet crossed, and within which all the energy of the radiation is concentrated), then the total energy will be conserved and there will be no energy flux across the boundary of the chosen region of integration. We therefore carry out the integration of Eq. (36) over a volume whose boundaries are far from the initial position of the center of the soliton $\bar{x}(0)$ at a distance Δl which satisfies the condition

$$t\gg\Delta L\gg|\bar{x}(t)-\bar{x}(0)|+\Delta l,\ \Delta l. \tag{37}$$

Using (7), (8) and (24), we can find the total energy flux P through the bounding planes which pass through points $\bar{x}(0)-\Delta L$ and $\bar{x}(0)+\Delta L$:

$$\frac{\partial W}{\partial t}=-Nu|_{\bar{x}(0)-\Delta L}^{\bar{x}(0)+\Delta L}=P,\quad W=\int_{\bar{x}(0)-\Delta L}^{\bar{x}(0)+L}Qdx, \tag{38}$$

where the velocity u of the particles in the sound wave is determined from the equation

$$\partial u/\partial x=-\partial N/\partial t.$$

If we assume that the distance from the bounding planes to the radiation front is greater than the width of the soliton, $t-\Delta L\gg\Delta l$, then we obtain the following expression for the energy flux, with the help of (25) and (27):

$$P=\frac{1}{2}\mathscr{E}_m^2\left(\frac{\partial n}{\partial x}\right)^2\Big|_{\bar{x}(t-\Delta L)}.$$

Consequently, the energy flux is proportional to the square of the density gradient of the inhomogeneous plasma.

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