Galvanomagnetic properties of two-dimensional two-component systems

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The galvanomagnetic properties of two-dimensional two-component isotropic systems located in an external transverse magnetic field are considered. The effective conductivity tensor of an arbitrary system is expressed in terms of the function f that describes the conductivity of the same system in zero magnetic field. If f as a function of two parameters (the concentration and the ratio of the conductivities of the components) is known in the entire range of variation of the parameters, then the formulas obtained in the paper furnish a complete solution to the problem of the galvanomagnetic properties of the systems considered. It is shown that, in systems that undergo the metal–dielectric phase transition, the dimension of the transition region (the smearing-out region) decreases with increasing magnetic-field intensity **H**, tending to zero as $H \to \infty$. This leads to unusual magnetic-field dependences of the elements of the effective conductivity tensor in the critical region. New—as compared to the $\mathbf{H} = 0$ case—critical exponents do not arise. It is found that the anomalous conductivity [A. M. Dykhne, Sov. Phys. JETP **32**, 348 (1971)] can exist only in the neighborhood of the critical concentration and in a finite magnetic-field range.

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1. INTRODUCTION

The study of the galvanomagnetic properties of inhomogeneous media is of considerable interest, but, as in the case of the computation of the conductivity in zero magnetic field, their theoretical investigation meets with certain mathematical difficulties. Nevertheless, a number of exact results have been obtained for two-dimensional two-component systems located in a transverse magnetic field.[1-4]

Dykhne[1] has found the effective conductivity tensor $\hat{\sigma}_e$ for arbitrary magnetic fields H in the particular case of equal component concentrations. One of the consequences of Dykhne's results[1] is the existence of an an anomalous conductivity—a change in the dependence of the diagonal element (σ_{xe}) of $\hat{\sigma}_e$ on the magnetic field at sufficiently high H values: $\sigma_{xe} \propto H^{-1}$ instead of the normal dependence $\sigma_x \propto H^{-2}$. Furthermore, Dykhne shows in his paper[1] that a medium that is slightly inhomogeneous in zero magnetic field exhibits in $H \to \infty$ certain properties of the highly inhomogeneous medium and markedly different electrical characteristics. Finally, Dykhne derives in Ref. 1 a general relation connecting the diagonal (σ_{xe}) and off-diagonal (σ_{ae}) elements of the $\hat{\sigma}_e$ tensor, and valid for arbitrary concentrations and any H. The dependence $\sigma_{xe} \propto H^{-1}$ for $H \to \infty$ has also been found by Dreĭzin and Dykhne[2] in an analysis of the perturbation-theory series in the two-dimensional case. But Dreĭzin and Dykhne do not determine in their papers[1,2] the magnetic-field and concentration ranges in which the anomalous conductivity exists.

The case of weak magnetic fields (the Hall effect) has been investigated in sufficiently great detail by Shklovskiĭ.[3] In the two-dimensional case we can, using Dykhne's general relation,[1] relate the effective Hall coefficient R_e with the effective $H = 0$ conductivity of the system. This allows us in principle to find R_e in the entire range of variation of the concentration and, in particular, study the critical behavior of the Hall coefficient.[3] The case of arbitrary magnetic fields had been briefly considered in a previous paper by the author of Ref. 4 for systems with perfectly conducting and dielectric inclusions, i.e., outside the smearing-out region.[5]

In the present paper we establish an exact relation, valid for any concentrations and arbitrary H, between the galvanomagnetic properties of a two-dimensional two-component system located in a transverse magnetic field and the properties of the same system in zero magnetic field. Specifically, the effective characteristics σ_{xe} and σ_{ae} of the medium are expressed in terms of the galvanomagnetic properties of the individual components and the function f that determines the effective conductivity of a system of the same structure in $H = 0$. If for some system f as a function of two parameters (the concentration and the ratio of the conductivities of the components) is known in the entire range of variation of the parameters, then the expressions obtained for σ_{xe} and σ_{ae} in the paper furnish a complete solution to the problem of the galvanomagnetic properties of such a system. The study of the function f is one of the most important problems of percolation theory,[5,6] and its form is known in a number in a number of limiting cases. This allows us not only to obtain all the results (pertaining to the two-dimensional case) of Refs. 1–4, but also to determine the regions of their applicability, as well as to obtain a number of new results. In particular, we are able to consider within the framework of the scaling theory[5] the galvanomagnetic properties in the entire critical region of systems that undergo the metal–dielectric phase transition.

To determine the galvanomagnetic characteristics of a two-component system, we reduce the original problem to the problem of the conductivity of a system of the same structure (we shall call it the zero system) in $H = 0$. The zero system is actually the initial system (i.e., the system in $H = 0$) with, in the general case, altered component conductivities; their ratio is a free parameter. The solution to the problem of the conductivity of the zero system is assumed to be known. Let the electric field in such a medium be $\mathbf{E}_0(r)$. The compo-

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nents of the field $\mathbf{E}(r)$ in the original system (in $H \neq 0$) are sought in the form of linear combinations of the components of the field $\mathbf{E}_0(r)$, this relation being chosen such that $\mathbf{E}(r)$ satisfies both the constant-current equations and the corresponding boundary conditions. These limitations uniquely determine the linear transformation coefficients and the ratio of the conductivities of the zero system's components, which turns out in the general case to be a function of the magnetic field. The computation of the quantities σ_{xe} and σ_{ae} with the aid of the field $\mathbf{E}(r)$ thus found allows us to express them in terms of the properties of the individual components of the original system and the function f.

The galvanomagnetic properties of the systems under consideration differ in their peculiarities. Thus, for example, in a medium with perfectly conducting impurities, when the concentration is sufficiently close to the critical value, the diagonal resistivity-tensor element (ρ_{xe}), which in a homogeneous system is H independent, exhibits a significant dependence on the magnetic field. In a system with dielectric impurities there occurs also in the vicinity of the critical concentration a drawing out of the region over which the magnetic-field dependence of the quantity σ_{xe} assumes the form $\sigma_{xe} \propto H^{-2}$ (no anomalous conductivity exists in either case). A characteristic of the $H \to \infty$ case is the fact that an isolated impurity of any conductivity make to $\hat{\sigma}_e$ the same contribution that a perfectly conducting impurity makes.

In the critical region (i.e., for $|\tau| \ll 1$, where τ characterizes the proximity to the critical point of the concentration-related transition) the galvanomagnetic properties of the system are characterized by quite a complicated dependence of σ_{xe} and σ_{ae} on the magnetic field and the concentration. In this case, since the entire concentration dependence of the quantities σ_{xe} and σ_{ae} is determined by the function f, new—in comparison with the $H = 0$ case—critical exponents do not arise. Dykhne's results[1] (with the exception of the above-mentioned general relation between σ_{xe} and σ_{ae}) are valid inside the smearing-out region Δ_H, which in the general case decreases with increasing magnetic-field intensity, tending to zero as $H \to \infty$. Consequently, the expression obtained by Dykhne[1] for $\hat{\sigma}_e$ is valid for arbitrary H only at one isolated point in concentration space: the metal-dielectric transition point. If, on the other hand, the concentration is different from the critical concentration (but the system in $H = 0$ is within the smearing-out region), then the H-region of applicability of Dykhne's results[1] is bounded from above. From this it follows, in particular, that the anomalous conductivity can, generally speaking, occur only in a finite magnetic-field range. The measurement of the galvanomagnetic characteristics of such systems in the vicinity of the transition point at two fixed concentrations corresponding to the cases $\tau > 0$ and $\tau < 0$ allows us, in principle, to investigate the entire critical region [in the (τ, h) plane, where h is the ratio of the conductivities of the components], reestablish the form of the function f in this region, and thereby verify the scaling hypothesis[5] in the two-dimensional case.

The general expressions for σ_{xe} and σ_{ae} allow us to consider the slightly inhomogeneous medium as well. As $H \to \infty$ such a medium possesses certain properties of systems that undergo the metal–dielectric transition. In particular, $\Delta_H \to 0$, and the anomalous conductivity can exist only at concentrations close to the critical value and in a finite magnetic-field range. The assertion that the Hall parameter has its limiting value[1] is valid under these same limitations.

In the present paper we use a macroscopic approach with local Ohm's law, so that the dimensions of the inhomogeneities are assumed to be large compared to the mean free path of the carriers. The properties of both the individual components and the system as a whole are assumed to be isotropic. The results of the present paper are directly applicable to thin two-component films, and can, with some provisos,[1] also be used to describe the transverse conductivity of a plasma in a strong magnetic field.

2. PERFECTLY CONDUCTING IMPURITIES

Let us consider a two-dimensional system [the (x, y) plane] located in an external transverse magnetic field H oriented along the normal to the (x, y) plane. Ohm's law in this case has the form

$$\mathbf{j} = \hat{\sigma}\mathbf{E}, \quad \hat{\sigma} = \begin{pmatrix} \sigma_x & \sigma_a \\ -\sigma_a & \sigma_x \end{pmatrix}. \tag{1}$$

Below it will be convenient to use the complex potential $\Phi(z)$ (here, as usual, $z = x + iy$), which can be introduced as in the $H = 0$ case (Ref. 7, §3); the derivative of it gives the components of the electric field $\mathbf{E}$:

$$\Phi'(z) = E_x - iE_y. \tag{2}$$

The components of the current density can, according to (1) and (2), be expressed in terms of $\Phi'(z)$ as follows:

$$j_x = \mathrm{Re}\{(\sigma_x + i\sigma_a)\Phi'(z)\}, \quad j_y = -\mathrm{Im}\{(\sigma_x + i\sigma_a)\Phi'(z)\}. \tag{3}$$

The equipotential surface are given as usual by the equation

$$\mathrm{Re}\,\Phi(z) = \mathrm{const}, \tag{4}$$

but the lines of flow will now be determined not by the imaginary part of $\Phi(z)$, but by the expression

$$\mathrm{Im}\{(\sigma_x + i\sigma_a)\Phi(z)\} = \mathrm{const}. \tag{5}$$

The element σ_a of the conductivity tensor drops out of the equation for the potential, and enters into the problem only through one of the boundary conditions: the continuity of the normal component of the current density.

Let us consider a system consisting of two components that differ markedly in their properties ($\sigma_2 \ll \sigma_1$). Let us, to begin with, assume that the magnetic field is equal to zero. Let the concentration p of the first component be lower than the critical concentration: $p < p_c$, $\tau = (p - p_c)/p_c < 0$. Let us also assume that the system is outside the smearing-out region,[3,5] so that the impurities of the first component can, in the first approximation, be considered to be perfectly conducting. Then we can limit ourselves to the consideration of the problem only in the region occupied by the second component and with the boundary condition $\mathbf{E}_t = 0$, where $\mathbf{E}_t$

is the tangential component of the electric-field intensity.

Let $\Phi_{20}(z)$ be the complex potential of such a problem. Since the potential is prescribed only in the region occupied by the second component, to compute the effective conductivity σ_e of the system, we cannot use the usual method[7] in which σ_e is determined as the coefficient of proportionality between $\langle \mathbf{j} \rangle$ and $\langle \mathbf{E} \rangle$. Here $\langle \ldots \rangle$ denotes averaging over the entire area of the system. Let us use another method to compute σ_e. Let the system under consideration be a square film with dimensions $2L$ ($L \to \infty$) along the x and y axes. Let us locate the coordinate origin at the center of the film. Let us assume that the mean field $\langle \mathbf{E}_0 \rangle$ is oriented along the x axis. Then, to compute σ_e, we must find the potential difference U_0 applied to the film along the x axis and the total current I_0 in the same direction. The quantity U_0 is determined with the aid of (4); I_0, with the aid of (5), in which we must set $\sigma_x = \sigma_2$ and $\sigma_a = 0$:

$$U_0 = \mathrm{Re}\,[\Phi_{20}(L+iy) - \Phi_{20}(-L+iy)],\quad I_0 = \sigma_2\,\mathrm{Im}\,[\Phi_{20}(x+iL) - \Phi_{20}(x-iL)]. \tag{6}$$

Since U_0 and I_0 clearly do not depend on y and x, we can, for example, set $x = 0$ and $y = 0$ in (6). The effective conductivity can be expressed in terms of U_0 and I_0 from (6) as follows:

$$\sigma_e = \sigma_2 f_s = I_0/U_0. \tag{7}$$

Here $f_s = f_s(\tau)$ is the function that gives the dependence of σ_e on the concentration for $\tau < 0$ outside the smearing-out region. In particular, $f_s \sim |\tau|^{-q}$ for $|\tau| \ll 1$ (Ref. 5).

Let us now place the system under consideration in a transverse magnetic field. If the condition $\hat{\sigma}_2 \ll \hat{\sigma}_1$ is still fulfilled in $H \neq 0$, then the impurities of the first component can be considered as before to be perfectly conducting impurities with the same boundary condition $\mathbf{E}_t = 0$. As mentioned above, the quantity σ_a does not enter into the equation for the potential (the Laplace equation). It also dropped out from the boundary condition. Thus, we arrive at the conclusion that, for the system under consideration, the complex potential $\Phi_2(z)$ in the magnetic field virtually coincides with $\Phi_{20}(z)$ (more exactly, this coincidence obtains for the fields $\mathbf{E}_2$ and $\mathbf{E}_{20}$):

$$\Phi_2'(z) = A\Phi_{20}'(z). \tag{8}$$

The constant A, as follows from the boundary condition $\mathbf{E}_t = 0$, is real, and will be set equal to unity below.

The complex potential (8) with the help of (4) and (5), and with allowance made for (6), allows us to express the total currents in the directions x and y and the corresponding voltage potentials in terms of I_0 and U_0:

$$I_x = \sigma_{x2} I_0/\sigma_2,\quad I_y = -\sigma_{a2} U_0;\quad U_x = U_0,\quad U_y = 0. \tag{9}$$

According to the integral Ohm law following from (1), the elements of the effective conductivity tensor $\hat{\sigma}_e$ are defined in the following manner:

$$\sigma_{xe} = (I_x U_x + I_y U_y)/(U_x^2 + U_y^2),\quad \sigma_{ae} = (I_x U_y - I_y U_x)/(U_x^2 + U_y^2), \tag{10}$$

so that from (10), (9), and (7) we finally obtain the expressions

$$\sigma_{xe} = \sigma_{x2} f_s,\quad \sigma_{ae} = \sigma_{a2}\quad (\tau < 0). \tag{11}$$

The expressions (11), which are valid for any shape and an arbitrary distribution of the perfectly conducting impurities, were derived earlier in Ref. 4 by another method. These same formulas describe the galvanomagnetic properties of systems that undergo the metal-dielectric transition in the "dielectric" ($\tau < 0$) phase outside the smearing-out region.

Below we shall use for making estimates the following model expressions for the quantities σ_{xi} and σ_{ai}, which are valid in the isotropic case and in the τ approximation for arbitrary H (Ref. 8):

$$\sigma_{xi} = \sigma_i/(1+\beta_i^2),\quad \sigma_{ai} = \sigma_i\beta_i/(1+\beta_i^2). \tag{12}$$

Here $\sigma_i \propto n_i\mu_i$ is the conductivity of the i-th component in $H = 0$, n_i and μ_i being the carrier concentration and mobility; $\beta_i \propto \mu_i H$. The condition of applicability of the formulas (11), $\hat{\sigma}_2 \ll \hat{\sigma}_1$, imposes limitations on the quantities n_i and μ_i. In the case in which the components have equal mobilities, i.e., in which $\mu_1 = \mu_2$ (or in which μ_1 and μ_2 are comparable in order of magnitude), the condition of applicability has the simple form $n_2 \ll n_1$ for arbitrary H. If, on the other hand, the carrier concentrations are comparable, i.e., if $n_1 \sim n_2$, but $\mu_2 \ll \mu_1$, then for $\beta_i \lesssim 1$ we have $\hat{\sigma}_2 \ll \hat{\sigma}_1$. But for $\beta_i \gg 1$

$$\sigma_{xi} \approx \sigma_i\beta_i^{-2} \propto n_i/\mu_i \text{ and } \sigma_{ai} \approx \sigma_i\beta_i^{-1} \propto n_i,$$

so that in strong fields $\sigma_{x2} \gg \sigma_{x1}$, $\sigma_{a2} \sim \sigma_{a1}$, and the condition of applicability of (11) [as well as of (20)] is no longer fulfilled. This case will be considered in Sec. 5; in Secs. 2 and 3 we shall assume that $\mu_1 \sim \mu_2$.

According to (11), the H dependence of the elements of the $\hat{\sigma}_e$ tensor for the system under consideration is the same as for $\hat{\sigma}_2$. The only difference between $\hat{\sigma}_e$ and $\hat{\sigma}_2$ is the presence of the factor $f_s = f_s(\tau)$ in the first equality in (11). This leads to the result that, for $|\tau| \ll 1$, $\sigma_{xe} \gg \sigma_{x2}$. But following from this circumstance is an unusual behavior, as H increases, of the elements of the effective resistivity tensor $\hat{\rho}_e$. Writing the tensor $\hat{\rho}$ in a form similar to $\hat{\sigma}$ from (1) (with the sign of ρ_a changed), we obtain for the elements of $\hat{\rho}_e$ with allowance for (11) the expressions

$$\rho_{xe} = \rho_{x2}(\rho_{a2}^2 + \rho_{x2}^2) f_s/(\rho_{a2}^2 + \rho_{x2}^2 f_s^2),\qquad \rho_{ae} = \rho_{a2}(\rho_{a2}^2 + \rho_{x2}^2)/(\rho_{a2}^2 + \rho_{x2}^2 f_s^2). \tag{13}$$

If we use the model expressions (12), then (13) assumes the form

$$\rho_{xe} = \rho_2(1+\beta_2^2) f_s/(f_s^2 + \beta_2^2),\quad \rho_{ae} = \rho_2\beta_2(1+\beta_2^2)/(f_s^2 + \beta_2^2), \tag{14}$$

where $\rho_2 = \sigma_2^{-1}$. We then have for the elements of $\hat{\rho}_e$ the expressions

$$\rho_{x2} = \rho_2 \propto H^0,\quad \rho_{a2} = \rho_2\beta_2 \propto H.$$

The dependence of ρ_{xe} and ρ_{ae} on H is more complicated, and here for $|\tau| \ll 1$ we can distinguish three magnetic-field regions ($f_s = |\tau|^{-q}$):

$$\beta_2 \ll 1:\quad \rho_{xe} \approx \rho_2|\tau|^q,\quad \rho_{ae} \approx \rho_2|\tau|^{2q}\beta_2; \tag{15a}$$

$$1 \ll \beta_2 \ll |\tau|^{-q}:\quad \rho_{xe} \approx \rho_2|\tau|^q\beta_2^2,\quad \rho_{ae} \approx \rho_2|\tau|^{2q}\beta_2^3; \tag{15b}$$

$$\beta_2 \gg |\tau|^{-q}:\quad \rho_{xe} \approx \rho_2|\tau|^{-q},\quad \rho_{ae} \approx \rho_2\beta_2. \tag{15c}$$

The effective Hall coefficient R_e for arbitrary H coin-

cides up to a constant factor with ρ_{ae} ($R_e = \rho_{ae}/H$), and therefore its dependence on the concentration and the magnetic field follows directly from (13)–(15).

3. DIELECTRIC IMPURITIES

Let us now consider the "metallic" phase ($\tau > 0$) outside the smearing-out region, when the impurities of the poorly conducting component can be assumed to be dielectric impurities ($\hat{\sigma}_2 = 0$). In this case the boundary condition is the equality to zero of the normal component of the current density (i.e., $j_n = 0$), which condition can, with allowance made for (3), be written in the form

$$\mathrm{Re}\{(\sigma_x + i\sigma_a)(n_x + in_y)\Phi'(z)\} = 0. \tag{16}$$

Here n is the unit vector along the normal to the boundary of an impurity. Let $\Phi_{10}(z)$ be the complex potential, satisfying the boundary condition

$$\mathrm{Re}\{(n_x + in_y)\Phi_{10}'(z)\} = 0,$$

in $H = 0$ in the region occupied by the first component. Then the derivative of the complex potential $\Phi_1(z)$, satisfying the boundary condition (16), in $H \neq 0$ in the same region can be expressed in terms of $\Phi_{10}'(z)$:

$$\Phi_1'(z) = \left(1 - i\frac{\sigma_{a1}}{\sigma_{x1}}\right)\Phi_{10}'(z). \tag{17}$$

The determination of the effective characteristics of the medium with the aid of (17) is carried out in much the same way as was done above. Thus, the potential difference and the current for $H = 0$ are given by the expressions (6) with the subscript 2 replaced by 1. For the conductivity σ_e in zero magnetic field, we have in place of (7) the expression

$$\sigma_e \equiv \sigma_1 f_d = I_0/U_0, \tag{18}$$

where $f_d = f_d(\tau)$ gives the dependence of σ_e on the concentration outside the smearing-out region. In particular, for $\tau \ll 1$, $f_d \sim \tau^t$ (Ref. 5). For the currents and the voltage potentials in $H \neq 0$, instead of (9), we obtain from (17) with allowance for (18) the expressions

$$\begin{aligned} &I_x = [(\sigma_{x1}^2 + \sigma_{a1}^2)/\sigma_{x1}]f_d U_0, \quad I_y = 0; \\ &U_x = U_0, \quad U_y = (\sigma_{a1}/\sigma_{x1})f_d U_0. \end{aligned} \tag{19}$$

Substituting (19) into (10), we finally obtain

$$\sigma_{xe} = \frac{\sigma_{x1}(\sigma_{x1}^2 + \sigma_{a1}^2)f_d}{\sigma_{x1}^2 + \sigma_{a1}^2 f_d^2}, \quad \sigma_{ae} = \frac{\sigma_{a1}(\sigma_{x1}^2 + \sigma_{a1}^2)f_d^2}{\sigma_{x1}^2 + \sigma_{a1}^2 f_d^2} \quad (\tau > 0). \tag{20}$$

The expressions (20), which are valid for an arbitrary shape, and any distribution, of the dielectric impurities, were derived earlier in Ref. 4 by another method. These same formulas describe the galvanomagnetic properties of two-dimensional systems that undergo the metal–dielectric transition in the metal ($\tau > 0$) phase outside the smearing-out region.

At low concentrations c of the second component (with the conductivity tensor $\hat{\sigma}_2$), using the method expounded in Ref. 7, we easily find that for a two-dimensional system with circular inclusions in the approximation linear in c

$$\begin{aligned} \sigma_{xe} &= \sigma_{x1}\left[1 - 2c\frac{\sigma_{x1}^2 - \sigma_{x2}^2 - (\sigma_{a1} - \sigma_{a2})^2}{(\sigma_{x1} + \sigma_{x2})^2 + (\sigma_{a1} - \sigma_{a2})^2}\right] \\ \sigma_{ae} &= \sigma_{a1}\left[1 - 4c\frac{\sigma_{x1}^2}{\sigma_{a1}}\frac{\sigma_{a1} - \sigma_{a2}}{(\sigma_{x1} + \sigma_{x2})^2 + (\sigma_{a1} - \sigma_{a2})^2}\right]. \end{aligned} \tag{21}$$

For $H = 0$ and $\hat{\sigma}_2 = 0$ we find from (21) with allowance for (18) the function f_d for $c \ll 1$:

$$f_d = 1 - 2c. \tag{22}$$

The expressions (20) with f_d given by (22) in the approximation linear in c coincide with (21) (for $\hat{\sigma}_2 = 0$). Making in (21) the substitution $\hat{\sigma}_1 \rightleftarrows \hat{\sigma}_2$, and going over to the limit $\hat{\sigma}_1 \to \infty$, we obtain (11) in the same approximation in c (in this case $f_s = 1 + 2c$).

If for the quantities σ_{x1} and σ_{a1} we use the model formulas (12), then the expressions (20) assume the form

$$\sigma_{xe} = \sigma_1 f_d/(1 + \beta_1^2 f_d^2), \quad \sigma_{ae} = \sigma_1\beta_1 f_d^2/(1 + \beta_1^2 f_d^2). \tag{23}$$

According to (12), in the conducting (first) component the transition with respect to the magnetic field from the dependence $\sigma_{x1} \approx \sigma_1 \propto H^0$ to $\sigma_{x1} \approx \sigma_1\beta_1^{-2} \propto H^{-2}$ (and similarly from $\sigma_{a1} \approx \sigma_1\beta_1 \propto H$ to $\sigma_{a1} \approx \sigma_1\beta_1^{-1} \propto H^{-1}$) occurs at $\beta_1 \sim 1$. But for the system as a whole there occurs in the case in which $\tau \ll 1$, when $f_d \sim \tau^t$, a "drawing out" of such a transition into the region of higher magnetic fields: $\beta_1 \sim \tau^{-t} \gg 1$.

For $\beta_1 \ll \tau^{-t}$, from (23) we have

$$\sigma_{xe} \approx \sigma_1\tau^t, \quad \sigma_{ae} \approx \sigma_1\tau^{2t}\beta_1. \tag{24}$$

It follows from (24) that the effective Hall coefficient

$$R_e = H^{-1}\sigma_{ae}/\sigma_{xe}^2$$

of a two-dimensional system with dielectric inclusions does not depend on the concentration of the nonconducting component[3,9]:

$$R_e = R_1. \tag{25}$$

Outside the smearing-out region, the expression (25) is valid also in the case in which $\hat{\sigma}_2 \neq 0$ (but $\hat{\sigma}_2 \ll \hat{\sigma}_1$).

For $\beta_1 \gg \tau^{-t}$, we find from (23) that

$$\sigma_{xe} \approx \sigma_1\tau^{-t}\beta_1^{-2}, \quad \sigma_{ae} \approx \sigma_1\beta_1^{-1}. \tag{26}$$

The dependence of σ_{xe} on H in the magnetic-field range under consideration is normal: $\sigma_{xe} \propto H^{-2}$. It should, however, be noted that in this case σ_{xe} is high compared to σ_{x1}: $\sigma_{xe}/\sigma_{x1} \approx \tau^{-t} \gg 1$. According to (26), σ_{ae} for the same H coincides with σ_{a1}. It is noteworthy that the expressions (26) are similar to the formulas (11) (for high H). This, apparently, is not accidental. The boundary condition at the surface of a dielectric inclusion is the equality to zero of the normal component of the current density, i.e., $\mathbf{j}_n = 0$. For low H this corresponds to $\mathbf{E}_n \approx 0$. But for $H \to \infty$, when the diagonal element σ_x is negligible compared to σ_a, the condition $\mathbf{E}_t \approx 0$ follows from the condition $\mathbf{j}_n = 0$. Thus, in a sufficiently strong magnetic field the boundary condition for a dielectric inclusion is the same as for a perfectly conducting inclusion. This evidently explains the similarity between the formulas (26) and (11). As can be seen from (21) (see also Secs. 5 and 6), as $H \to \infty$ these conclusions are valid also for an isolated inclusion of any conductivity, except in the case of equal carrier concentrations, i.e.,

the case in which $n_1 = n_2$.

In conclusion of this section, let us give the expressions for the elements of the effective resistivity tensor, which follow from (20):

$$\rho_{xe}=\rho_{x1}f_d^{-1}, \quad \rho_{ae}=\rho_{a1}.$$

From here, using the definition of the Hall coefficient for arbitrary H ($R=\rho_a/H$), we again arrive at the result (25). Thus, the equality (25) is valid for any magnetic fields. The same conclusion can be drawn from Dykhne's general relation [see, for example, the formula (10) of Ref. 10]. Let us also note that it follows from the expression for ρ_{xe} that the magnitude of the relative magnetoresistance does not depend on the concentration of the nonconducting component (i.e., $\Delta\rho_{xe}/\rho_{xe}=\Delta\rho_{x1}/\rho_{x1}$), a fact which was observed in the modeling experiment performed by Levinshteĭn *et al.*[9]

4. THE GENERAL CASE

Using the method expounded in the preceding sections, we can also consider the galvanomagnetic properties of two-dimensional systems in the general case, in which both components have finite nonzero conductivities. This can be done by reducing the original problem to the problem of the effective conductivity of a system of the same structure, located in zero magnetic field, and whose components have some conductivities λ_1 and λ_2 (the zero system). The zero system, like the original system in H = 0, is characterized by a scalar conductivity, but the quantities λ_1 and λ_2 do not, generally speaking, coincide with the conductivities of the components of the original system in H = 0; the ratio $\lambda=\lambda_2/\lambda_1 \leqslant 1$ is a free parameter.

Let the complex potential for the zero system be equal to $\Phi_{10}(z)$ in the region occupied by the first component and $\Phi_{20}(z)$ in the region occupied by the second. Let us write the boundary conditions—the equality of the tangential components of the electric field and of the normal components of the current density—in the form (H = 0):

$$\begin{aligned}&\mathrm{Im}\{(n_x+in_y)\Phi_{10}'(z)\}=\mathrm{Im}\{(n_x+in_y)\Phi_{20}'(z)\},\\ &\mathrm{Re}\{(n_x+in_y)\Phi_{10}'(z)\}=\lambda\,\mathrm{Re}\{(n_x+in_y)\Phi_{20}'(z)\}; \quad \lambda=\lambda_2/\lambda_1.\end{aligned} \tag{27}$$

Here n is the unit vector along the normal to the interface. Let us write the effective conductivity σ_λ of the zero system for all τ in a form similar to (18):

$$\sigma_\lambda=\lambda_1 f(\tau,\lambda)=I_0/U_0. \tag{28}$$

Here, as above, U_0 and I_0 are the potential difference and the total current in the direction of the x axis. Let us write the effective conductivity of the original system in H = 0 in the form $\sigma_e=\sigma_1 f_0(\tau,h)$, where $h=\sigma_2/\sigma_1 \leqslant 1$. It is not difficult to see that $f(\tau,\lambda)$ is obtained from $f_0(\tau,h)$ by replacing h by λ, i.e., the function $f(\tau,\lambda)$ is determined by the properties of the original system in zero magnetic field.

We shall seek the derivative of the complex potential of the original system in H ≠ 0 in the first (with conductivity tensor $\hat{\sigma}_1$) and second ($\hat{\sigma}_2$) components in a form that generalizes (8) and (17):

$$\Phi_1'(z)=(A_1+iB_1)\Phi_{10}'(z), \quad \Phi_2'(z)=(A_2+iB_2)\Phi_{20}'(z). \tag{29}$$

Let us choose the real constants A_i and B_i and the parameter λ such that the boundary conditions in the original system in H ≠ 0 will be satisfied.

The continuity of the tangential component of the electric field [the first equality in (27) with Φ_{i0} replaced by Φ_i] gives the two relations

$$A_2=A_1, \quad B_2=\lambda B_1. \tag{30}$$

The condition for the normal component of the current density in a magnetic field to be continuous can be written with allowance for (3) in the form

$$\mathrm{Re}\{(\sigma_{x1}+i\sigma_{a1})(n_x+in_y)\Phi_1'(z)\}=\mathrm{Re}\{(\sigma_{x2}+i\sigma_{a2})(n_x+in_y)\Phi_2'(z)\}. \tag{31}$$

Substituting (29) into (31), and using the equalities (27) and (30), we obtain the following homogeneous system of equations:

$$\begin{aligned}&(\sigma_{x1}-\lambda^{-1}\sigma_{x2})A_1-(\sigma_{a1}-\sigma_{a2})B_1=0,\\ &(\sigma_{a1}-\sigma_{a2})A_1+(\sigma_{x1}-\lambda\sigma_{x2})B_1=0.\end{aligned} \tag{32}$$

The condition for the system (32) to be solvable yields an equation for the determination of the parameter λ. Let us write the solution to this equation in the form

$$\lambda=\frac{1}{4\sigma_{x1}\sigma_{x2}}\{[(\sigma_{x1}+\sigma_{x2})^2+(\sigma_{a1}-\sigma_{a2})^2]^{1/2}-[(\sigma_{x1}-\sigma_{x2})^2+(\sigma_{a1}-\sigma_{a2})^2]^{1/2}\}^2. \tag{33}$$

If for the quantities σ_{xi} and σ_{ai} we use the formulas (12), then we obtain from (33) the expression

$$\lambda=\frac{1}{4\sigma_1\sigma_2}\{[(\sigma_1+\sigma_2)^2+(\sigma_1\beta_2-\sigma_2\beta_1)^2]^{1/2}-[(\sigma_1-\sigma_2)^2+(\sigma_1\beta_2-\sigma_2\beta_1)^2]^{1/2}\}^2. \tag{34}$$

The constant A_1 can, without loss of generality, be set equal to unity. Then from (29), (30), and (32) we finally obtain for the derivative of the complex potential of the original system the expressions

$$\begin{aligned}&\Phi_1'(z)=(1+iB)\Phi_{10}'(z), \quad \Phi_2'(z)=(1+i\lambda B)\Phi_{20}'(z);\\ &B=(\sigma_{a2}-\sigma_{a1})/(\sigma_{x1}-\lambda\sigma_{x2}),\end{aligned} \tag{35}$$

where the parameter λ is given by the expressions (33), (34).

The formulas (35) and (28) allow us to express the sought σ_{xe} and σ_{ae} in terms of the function f. Let us use for this purpose the method used in Secs. 2 and 3. Let us note beforehand that, to compute, for example, the potential difference, we must integrate the electric field E along a contour joining the opposite sides of the film. It is convenient to choose this contour such that it lies wholly in the region occupied by one of the components. We then obtain an expression of the type of the first formula in (6). Let us proceed in the same manner in finding the total currents. According to the percolation theory,[6] such paths, which thread the entire sample, can, in the two-dimensional case, be drawn only through the first component when $\tau > 0$ and through the second when $\tau < 0$. Therefore, in computing the voltage potentials and the currents with the aid of formulas of the type (6), we must use the complex potential $\Phi_1(z)$ when $\tau > 0$ and the potential $\Phi_2(z)$ when $\tau < 0$.

Let us first consider the metallic phase ($\tau > 0$). Using the expression for $\Phi_1'(z)$ from (35) and the relation (28), let us compute the total currents and the potential dif-

ferences:

$$I_x=(\sigma_{x1}-\sigma_{a1}B)fU_0, \quad I_y=-(\sigma_{x1}B+\sigma_{a1})U_0; \qquad U_x=U_0, \quad U_y=-BfU_0. \tag{36}$$

Substituting the expressions (36) into (10), we obtain

$$\sigma_{xe}=\sigma_{x1}f(1+B^2)/(1+B^2f^2), \quad \sigma_{ae}=\sigma_{a1}+\sigma_{x1}(1-f^2)B/(1+B^2f^2). \tag{37}$$

It can be shown that the same expressions (37) are obtained for σ_{xe} and σ_{ae} in the $\tau<0$ case, so that they are valid for arbitrary τ. Substituting into (37) the explicit form of the coefficient B from (35), and using the identity

$$(\sigma_{a1}-\sigma_{a2})^2=-\lambda^{-1}(\sigma_{x1}-\lambda\sigma_{x2})(\lambda\sigma_{x1}-\sigma_{x2}),$$

which follows from the equation for λ, we finally obtain for the elements of the effective conductivity tensor the expressions

$$\sigma_{xe}=\sigma_{x1}\sigma_{x2}(1-\lambda^2)fD^{-1}, \quad D=\lambda(1-f^2)\sigma_{x1}+(f^2-\lambda^2)\sigma_{x2}; \tag{38}$$

$$\sigma_{ae}=\sigma_{a1}-(\sigma_{a1}-\sigma_{a2})\sigma_{x1}\lambda(1-f^2)D^{-1}. \tag{39}$$

Here the parameter λ is given by the expressions (33), (34), while the function $f=f(\tau,\lambda)(\lambda\leqslant f\leqslant 1)$ is defined in (28) for all λ. The formulas (38) and (39), which are valid for any shape, and arbitrary distribution, of the inclusions, solve the formulated problem, expressing σ_{xe} and σ_{ae} in terms of the function f, i.e., in terms of the properties of the system in question in zero magnetic field. Let us note that a change in the magnetic field H leads, according to (33), to a change in the parameter λ. Consequently, the measurement of the galvanomagnetic properties of the systems under consideration at fixed τ values will, in principle, allow us to find the dependence of the function $f(\tau,\lambda)$ on λ, or, equivalently, the dependence of $f_0(\tau,h)$ on the parameter h.

Eliminating the function f from (38) and (39), we obtain Dykhne's general relation in the form given in Ref. 4. We can write the expressions (18) and (19) of Ref. 4 in a more compact and symmetric form if we introduce the off-diagonal element $\rho_a=\sigma_a/(\sigma_x^2+\sigma_a^2)$ of the resistivity tensor:

$$\rho_{ae}^{-1}d-\sigma_{ae}^{-1}b=1; \qquad d=\rho_{a1}\rho_{a2}(\sigma_{a1}-\sigma_{a2})g^{-1}, \quad b=\sigma_{a1}\sigma_{a2}(\rho_{a1}-\rho_{a2})g^{-1}, \qquad g=\sigma_{a1}\rho_{a2}-\rho_{a1}\sigma_{a2}.$$

It can also be verified that σ_{xe} and σ_{ae} from (38) and (39) satisfy the reciprocity relations of Ref. 4. The exact expression obtained in Ref. 3 (see also Ref. 10) for the effective Hall coefficient follows from (38), (39) for H $\to$ 0. Using the method expounded in Ref. 7, we easily find that, for low second-component concentrations $c \ll 1$,

$$f=1-2c(1-\lambda)/(1+\lambda).$$

The substitution of this formula into (38), (39) yields in the approximation linear in c the result (21).

For a system with dielectric inclusions ($\hat{\sigma}_2\to 0, \tau>0$), $\lambda\to 0$, $f\to f_d(\tau)$, and the expressions (20) follow from (38), (39). For a medium with perfectly conducting inclusions ($\hat{\sigma}_1\to\infty, \tau<0$), $f\to\lambda f_s(\tau)$, and the passage to the limit $\hat{\sigma}_1\to\infty$ in (38), (39) leads to the formulas (11). Finally, if the concentration is equal to the critical concentration (i.e., if $\tau=0$), then for a randomly inhomogeneous medium (or, for example, for a system with a chessboard structure), $f(0,\lambda)=\lambda^{\frac{1}{2}}$ (Ref. 11), and we obtain from (38), (39) Dykhne's result[1] in the form given in Ref. 4:

$$\sigma_{xe}=(\sigma_{x1}\sigma_{x2})^{1/2}\left[1+\left(\frac{\sigma_{a1}-\sigma_{a2}}{\sigma_{x1}+\sigma_{x2}}\right)^2\right]^{1/2}, \quad \sigma_{ae}=\frac{\sigma_{x1}\sigma_{a2}+\sigma_{x2}\sigma_{a1}}{\sigma_{x1}+\sigma_{x2}} \quad (\tau=0). \tag{40}$$

One of the most interesting consequences of the result (40) is the change in the H dependence of σ_{xe} in a strong magnetic field (i.e., to anomalous conductivity[1]) as compared to the homogeneous case. It is easy to verify that $\sigma_{xe}\propto H^{-1}$ for $H\to\infty$, except in the case of equal carrier concentrations (i.e., the case in which $n_1=n_2$), when σ_{xe} has the "normal" asymptotic form: $\sigma_{xe}\propto H^{-2}$.

In conclusion of this section, let us note that the formulas (38) and (39) can be derived by the standard method, i.e., without the use of the complex potential. Let us try to express the field E in the original system ($H\neq 0$) linearly in terms of the field E^0 in the zero system:

$$E_{xi}=A_iE_{xi}^0+B_iE_{yi}^0, \quad E_{yi}=C_iE_{xi}^0+D_iE_{yi}^0, \quad i=1,\ 2. \tag{41}$$

The electric field E^0 in each component satisfies the constant-current equations $\operatorname{curl}\mathbf{E}_i^0=0, \operatorname{div}\mathbf{E}_i^0=0$. The field $\mathbf{E}_i$ should satisfy similar equations (σ_{ai} drops out of the equation $\operatorname{div}\mathbf{j}_i=0$, and enters into the problem only through the boundary condition), whence it follows that $C_i=-B_i$ and $D_i=A_i$. A comparison of (41) with (29) and (2) shows that the coefficients in (29) were chosen such that they satisfy precisely such relations. The requirement that (41) satisfy the boundary conditions leads to (30), (32), and (33). In order to compute the effective conductivity tensor $\hat{\sigma}_e$ by the standard method,[7] we must find the averaged quantities $\langle\mathbf{j}\rangle$ and $\langle\mathbf{E}\rangle$ ($\langle\mathbf{j}\rangle=\hat{\sigma}_e\langle\mathbf{E}\rangle$). Let us note that

$$\langle\mathbf{E}^0\rangle=p\langle\mathbf{E}^0\rangle_1+(1-p)\langle\mathbf{E}^0\rangle_2 \text{ and } \langle\mathbf{j}^0\rangle=\lambda_1p\langle\mathbf{E}^0\rangle_1+\lambda_2(1-p)\langle\mathbf{E}^0\rangle_2,$$

where $\langle\ldots\rangle_i$ denotes averaging over the area of the i-th component and p is the concentration of the first component. Similarly, $\langle\mathbf{j}\rangle$ and $\langle\mathbf{E}\rangle$ can be expressed in terms of the four quantities $\langle E_\alpha^0\rangle_1$ and $\langle E_\alpha^0\rangle_2$ ($\alpha=x,y$). Two of these quantities (say, $\langle E_x^0\rangle_1$ and $\langle E_y^0\rangle_1$) can be expressed with the aid of the relations

$$\langle j_\alpha^0\rangle=\sigma_\lambda\langle E_\alpha^0\rangle, \quad \sigma_\lambda=\lambda_1 f$$

in terms of $\langle E_x^0\rangle_2$, $\langle E_y^0\rangle_2$, and the function f. The elimination of the second pair ($\langle E_x^0\rangle_2$ and $\langle E_y^0\rangle_2$) leads to a relation between $\langle j_\alpha\rangle$ and $\langle E_\alpha\rangle$ of the form $\langle\mathbf{j}\rangle=\hat{\sigma}_e\langle\mathbf{E}\rangle$, i.e., yields the explicit form of σ_{xe} and σ_{ae}. The corresponding, rather tedious calculations lead to the expressions (38) and (39).

5. THE CRITICAL REGION

The general formulas (38) and (39) obtained in the preceding section allow us to fully investigate within the framework of the scaling theory[5] the galvanomagnetic properties of two-component systems in the vicinity of the metal-dielectric transition point in the critical region. In the process we determine an important characteristic: the hitherto undetermined size of the transition region (the smearing-out region), which then allows us to determine the limits of applicability of

Dykhne's results (40).

Let the components of the system under consideration have markedly different conductivities in zero magnetic field (i.e., let $h=\sigma_2/\sigma_1 \ll 1$), so that a strongly pronounced metal-dielectric phase transition occurs in the system in H = 0. The scaling theory postulates that the function $f_0(\tau, h)$, defined with the aid of the relation $\sigma_e = \sigma_1 f_0(\tau, h)$, has in the critical region ($|\tau| \ll 1, h \ll 1$) the following form[3,5]:

$$f_0(\tau, h) = h^t \psi(\tau/h^m). \tag{42}$$

Here, as above, τ characterizes the proximity to the transition point in concentration space. It is assumed that the smearing-out region is symmetric about the point $\tau = 0$, and that its dimension Δ_0 can be determined from the requirement that the argument of the function ψ be equal to unity:

$$\Delta_0 = h^m. \tag{43}$$

The function f_0 has the following asymptotic forms:

$$\Delta_0 \ll |\tau| \ll 1: \quad f_0 \sim \tau^t (\tau > 0); \quad f_0 \sim h|\tau|^{-q} \quad (\tau < 0); \tag{44a}$$

$$|\tau| \ll \Delta_0: \quad f_0 \sim h^t. \tag{44b}$$

The critical exponents t, s, q, and m are connected by the two relations[5] $q = t(1-s)/s$ and $m = s/t$, i.e., to describe the behavior of $f_0(\tau, h)$, it is sufficient to know only two of them, say, t and s. According to Ref. 11, for a randomly inhomogeneous two-dimensional medium $f_0(0, h) = h^{\frac{1}{2}}$; therefore, $s = \frac{1}{2}$ and $q = t$. In this case we have only one independent critical exponent $t \approx 1.3$ (Ref. 3). It should be pointed out that in this case the relation in (46b) is an exact equality. As noted in the preceding section, the function $f(\tau, \lambda)$ entering into the expressions (38) and (39) is obtained from $f_0(\tau, h)$ by replacing h by λ, where λ is given by (33). If $\lambda \ll 1$ (and $|\tau| \ll 1$), then all the results of the scaling theory are valid for the function $f(\tau, \lambda)$ as well.

Let us first consider the case in which the carriers have the same mobility, i.e., in which $\mu_1 = \mu_2 = \mu(\beta_1 = \beta_2 = \beta)$, but markedly different concentrations ($n_2 \ll n_1$). Then for all H we have $\hat{\sigma}_2 \ll \hat{\sigma}_1$, and from (33), (38), and (39) we obtain

$$\begin{aligned} \lambda &\approx \sigma_{x1}\sigma_{x2}/(\sigma_{x1}^2+\sigma_{a1}^2), \\ \sigma_{xe} &\approx \sigma_{x1}(\sigma_{x1}^2+\sigma_{a1}^2) f/(\sigma_{x1}^2+\sigma_{a1}^2 f^2), \\ \sigma_{ae} &\approx [\sigma_{a1}(\sigma_{x1}^2+\sigma_{a1}^2) f^2 + \sigma_{a2}\sigma_{x1}^2]/(\sigma_{x1}^2+\sigma_{a1}^2 f^2). \end{aligned} \tag{45}$$

The expression for σ_{xe} in (45) coincides in form with (20), with the important difference, however, that in (45) the function f depends not only on τ (and τ can be of either sign), but also on λ, i.e., on the magnetic field.

According to (45), the parameter λ is small, so that, when $|\tau| \ll 1$, we can use the scaling-theory results (42)–(44) for the function f. Let us, to begin with, determine the dimension Δ_H of the smearing-out region in a magnetic field. The quantity Δ_H is obtained from (43) by replacing h by λ: $\Delta_H = \lambda^m$. Substituting the expression for λ from (45), and using the model formulas (12), we obtain for Δ_H the expression

$$\Delta_H = \Delta_0 (1+\beta^2)^{-m}, \tag{46}$$

where Δ_0 and m are the same quantities figuring in (43). Thus, the dimension of the smearing-out region essentially depends on the magnetic field. As H is increased, the quantity Δ_H decreases, tending to zero as $H \to \infty$. Consequently, if the system in H = 0 is within the smearing-out region (i.e., if $|\tau| \ll \Delta_0, \tau \neq 0$), then there arises in sufficiently high fields a situation in which $|\tau| \gg \Delta_H$, i.e., in which the system "leaves" the transition region. It is the narrowing of the smearing-out region with increasing H that determines all the peculiarities of the galvanomagnetic properties of systems that undergo the metal-dielectric transition in the vicinity of the point $\tau = 0$.

The expressions (40) are obtained from (38) and (39) in the case in which $f \approx \lambda^{\frac{1}{2}}$, which can be done when $|\tau| \ll \Delta_H$. From this it follows that Dykhne's results (40) are applicable in the entire range of magnetic fields only at the isolated point $\tau = 0$. If, on the other hand, $\tau \neq 0$ (but $|\tau| \ll \Delta_0$), then the (magnetic-field) region of applicability of the expressions (40) is bounded from above: $\beta^{2m} \ll \Delta_0/|\tau|$. When $|\tau| \gtrsim \Delta_0$ the formulas (40) are inapplicable; the conductivity is not anomalous in this concentration region.

Let us now investigate the dependence of σ_{xe} and σ_{ae} on H in the critical region. According to the foregoing, if the system in zero magnetic field is in a state lying outside the smearing-out region (i.e., if $|\tau| \gg \Delta_0$), then for all $H \neq 0$ we shall have $|\tau| \gg \Delta_H$. In this case the galvanomagnetic properties of the system located in a magnetic field of arbitrary intensity H are described by the expressions (11) ($\tau < 0$) or (20) ($\tau > 0$), and are considered in detail in Secs. 2 and 3. Of greatest interest are the properties of the system located in H = 0, and in a state lying inside the smearing-out region ($|\tau| \ll \Delta_0, \tau \neq 0$). In this case, according to (45), when $\tau > 0$ we can distinguish four magnetic-field regions [we use for σ_{xi} and σ_{ai} the model formulas (12) with $\beta_1 = \beta_2 = \beta$]:

$$\beta \ll 1: \qquad \sigma_{xe} \approx (\sigma_1\sigma_2)^{1/2}, \quad \sigma_{ae} \approx 2\sigma_2\beta; \tag{47a}$$

$$1 \ll \beta \ll \tau^{-t}(\sigma_2/\sigma_1)^{1/2}: \qquad \sigma_{xe} \approx (\sigma_1\sigma_2)^{1/2}\beta^{-1}, \quad \sigma_{ae} \approx 2\sigma_2\beta^{-1}; \tag{47b}$$

$$\tau^{-t}(\sigma_2/\sigma_1)^{1/2} \ll \beta \ll \tau^{-t}: \qquad \sigma_{xe} \approx \sigma_1\tau^t, \quad \sigma_{ae} \approx \sigma_1\tau^{2t}\beta; \tag{47c}$$

$$\beta \gg \tau^{-t}: \qquad \sigma_{xe} \approx \sigma_1\tau^{-t}\beta^{-2}, \quad \sigma_{ae} \approx \sigma_1\beta^{-1}. \tag{47d}$$

The stepwise dependence of σ_{xe} on β (i.e., on H) and the "double-humped" dependence for σ_{ae} are noteworthy. The anomalous conductivity $\sigma_{xe} \propto \beta^{-1} \propto H^{-1}$ occurs in the magnetic-field range indicated in (47b).

The results (47) have a simple meaning. The expressions (47a) and (47b) correspond to the case in which the system is in a state lying inside the smearing-out region ($\tau \ll \Delta_H$), and can be derived from (40). The formulas (47c) and (47d), on the other hand, correspond to the case in which $\tau \gg \Delta_H$, when the system has already "left" the smearing-out region, and coincide (when $\beta_1 = \beta$) with (24) and (26). Similarly, we can distinguish three magnetic-field regions when $\tau < 0$. In the first two regions σ_{xe} and σ_{ae} are given by the formulas (47a) and (47b) (with τ replaced by $|\tau|$); in the third, in which $\beta \gg |\tau|^{-t}(\sigma_2/\sigma_1)^{\frac{1}{2}}$, by the expressions (11).

Let us now consider the case in which the carriers have markedly different mobilities, but are of nearly equal concentrations, i.e., in which $\mu_2 \ll \mu_1$, but $\delta = (n_1 - n_2)/n_1 \ll 1$; for definiteness we set $n_1 > n_2$, so that $\delta > 0$. When $\mu_2 \ll \mu_1$ and $\delta \ll 1$, in weak magnetic fields $\hat{\sigma}_2$

$\ll \hat{\sigma}_1$, while in strong fields $\sigma_{x2} \gg \sigma_{x1}$ and $\sigma_{a1} \approx \sigma_{a2}$, so that the results obtained in Secs. 2 and 3 are directly applicable here. In the case under consideration we obtain from (34) for the parameter λ the expression

$$\lambda \approx (\sigma_2/\sigma_1)(1+\beta_2^2\delta^2)^{-1}, \quad \beta_2 = (\mu_2/\mu_1)\beta_1 \approx (\sigma_2/\sigma_1)\beta_1. \tag{48}$$

It follows from (48) that in sufficiently strong magnetic fields, i.e., for $\beta_2\delta \gg 1$, the quantity $\Delta_H \ll \Delta_0$, and, as in the $\mu_1 = \mu_2$ case, the system, which in $H=0$ is in a state lying inside the smearing-out region, leaves this region when $H \to \infty$.

When $\tau > 0$, $\tau \ll \Delta_0$, we can again distinguish four magnetic-field regions in the H dependence of σ_{xe}:

$$\beta_1 \ll (\sigma_1/\sigma_2)^{1/2}: \quad \sigma_{xe} \approx (\sigma_1\sigma_2)^{1/2}; \tag{49a}$$

$$(\sigma_1/\sigma_2)^{1/2} \ll \beta_1 \ll (\sigma_1/\sigma_2)\delta^{-1}: \quad \sigma_{xe} \approx (\sigma_1\sigma_2)^{1/2}(\sigma_1/\sigma_2)\beta_1^{-2}; \tag{49b}$$

$$(\sigma_1/\sigma_2)\delta^{-1} \ll \beta_1 \ll (\sigma_1/\sigma_2)^{1/2}\tau^{-1}\delta^{-1}: \quad \sigma_{xe} \approx (\sigma_1\sigma_2)^{1/2}\beta_1^{-1}\delta; \tag{49c}$$

$$\beta_1 \gg (\sigma_1/\sigma_2)^{1/2}\tau^{-1}\delta^{-1}: \quad \sigma_{xe} \approx \sigma_1\tau^{-1}\beta_1^{-2}. \tag{49d}$$

The quantity σ_{ae} under the same conditions varies in the following manner:

$$\beta_1 \ll (\sigma_1/\sigma_2)^{1/2}: \quad \sigma_{ae} \approx \sigma_2\beta_1; \tag{50a}$$

$$\beta_1 \gg (\sigma_1/\sigma_2)^{1/2}: \quad \sigma_{ae} \approx \sigma_1/\beta_1. \tag{50b}$$

The expressions (49a)–(49c) and (50a) correspond to $\tau \ll \Delta_H$, and can be derived from (40), while (49d) and (50b) correspond to $\tau \gg \Delta_H$, and coincide in form with (47d). The anomalous conductivity occurs in the magnetic-field region indicated in (49c).

The case of equal carrier concentrations, i.e., the case in which $n_1 = n_2$, or $\delta = 0$, is a distinct one. According to (48), in this case the parameter λ (and, consequently, Δ_H) does not depend on the magnetic field, and the system is in a state lying either inside or outside the smearing-out region for all H. The anomalous conductivity does not occur in this case.

In conclusion of this section, let us note that the measurement of the galvanomagnetic properties of the systems under consideration in the vicinity of the transition point (i.e., in the region $|\tau| \ll \Delta_0$, $\tau \neq 0$) at two fixed concentrations corresponding to the cases $\tau > 0$ and $\tau < 0$ allows us to investigate the entire critical region, and thereby find the function ψ from (42). On the other hand, the results of such measurements can be used to verify the scaling hypothesis[5] in the two-dimensional case.

6. THE SLIGHTLY INHOMOGENEOUS MEDIUM

If the properties of the components are close to each other, then the function $f_0(\tau, h)$ can be found, using the method expounded in Ref. 7, for arbitrary concentration c up to $(1-h)^2 \ll 1$ inclusively:

$$f_0 = 1-(1-p)(1-h)-p(1-p)(1-h)^2/2. \tag{51}$$

Here p is the concentration of the first component. The parameter λ in this case can, according to (33), be written in the form

$$\lambda \approx 1-\sigma_{x1}^{-1}[(\sigma_{x1}-\sigma_{x2})^2+(\sigma_{a1}-\sigma_{a2})^2]^{1/2}. \tag{52}$$

The expression (52) has been written up to terms of first order in $(\hat{\sigma}_1 - \hat{\sigma}_2)$, which is sufficient for our purposes. The substitution of (51) (with h replaced by λ) and (52) into (38) and (39) yields the expressions

$$\begin{aligned}\sigma_{xe} &\approx \sigma_{x1}\left[1-(1-p)\frac{\sigma_{x1}-\sigma_{x2}}{\sigma_{x1}}-\frac{1}{2}p(1-p)\frac{(\sigma_{x1}-\sigma_{x2})^2-(\sigma_{a1}-\sigma_{a2})^2}{\sigma_{x1}^2}\right],\\ \sigma_{ae} &\approx \sigma_{a1}\left[1-(1-p)\frac{\sigma_{a1}-\sigma_{a2}}{\sigma_{a1}}-p(1-p)\frac{(\sigma_{x1}-\sigma_{x2})(\sigma_{a1}-\sigma_{a2})}{\sigma_{x1}\sigma_{a1}}\right].\end{aligned} \tag{53}$$

The formulas (53) can also be derived directly by the method expounded in Ref. 7.

In sufficiently strong magnetic fields the last term in the square brackets in the expression (53) for σ_{xe} is, generally speaking, proportional to H^2, and increases without restriction as $H \to \infty$. This means that the next terms of the expansion become fairly large, and we should sum an infinite series.[1,2] Dykhne's result[1] [see (40)], which is valid for $p = \frac{1}{2}$ ($\tau = 0$), and the analysis of the perturbation theory series[2] give $\sigma_{xe} \propto H^{-1}$, i.e., an anomalous conductivity. Here it is not clear whether the anomalous conductivity exists in the case in which $p \neq \frac{1}{2}$. It can be asserted that it does not occur at low concentrations of one of the components. Thus, if the inclusions have a circular shape, then the summation of the perturbation theory series in the approximation linear in the concentration $c = 1-p \ll 1$ of the second component will give for σ_{xe} and σ_{ae} the expressions (21). It can be seen from (21) that the correction term that arises in σ_{xe} for $H \to \infty$ is finite and small ($\sim c \ll 1$), so that the anomalous conductivity does not occur.

For the analysis of the behavior of σ_{xe} in strong fields in the case of arbitrary p values, let us use the expression (38). To begin with, let us note that the correction term in (52) also increases without restriction as $H \to \infty$, and the expansion (52) becomes inapplicable, so that for λ we must use the expression (33) or (34). Let us consider the case in which

$$\mu_1 = \mu_2, \quad \delta = (n_1-n_2)/n_1 \ll 1, \quad \delta > 0.$$

Then, when $\beta\delta \ll 1$, the parameter λ is close to unity and the formulas (52) and (53) are valid. But for $\beta\delta \gg 1$ we find from (34) that

$$\lambda \approx (\beta\delta)^{-2}, \tag{54}$$

i.e., $\lambda \ll 1$. It is not difficult to see from (38) that the anomalous conductivity arises only when $f \approx \lambda^{\frac{1}{2}}$, which is possible only when $|\tau| \ll 1$. Under these conditions $f \ll 1$, and from (38) we have

$$\sigma_{xe}/\sigma_{x1} \approx f/(\lambda+f^2), \quad (\beta\delta \gg 1, \ |\tau| \ll 1). \tag{55}$$

In the expression (55) $\sigma_{x1} \approx \sigma_{x2} \approx \sigma_0\beta^{-2}$, where $\sigma_0 \approx \sigma_1 \approx \sigma_2$, while the function f, as for systems that undergo the metal-dielectric transition, is given by the formulas (42), (44) with h replaced by λ. From (55), (54), and (44) we obtain

$$\delta^{-1} \ll \beta \ll |\tau|^{-t}\delta^{-1}: \quad \sigma_{xe} \approx {}^1/_2\sigma_0\beta^{-1}\delta; \tag{56a}$$

$$\beta \gg |\tau|^{-t}\delta^{-1}: \quad \sigma_{xe} \approx \sigma_0|\tau|^{-t}\beta^{-2}. \tag{56b}$$

The anomalous conductivity exists in the magnetic-field region indicated in (56a). Notice that the concentration region where the anomalous conductivity occurs is broader in the case of a slightly inhomogeneous medium ($|\tau| \ll 1$) than in the case of systems that undergo the metal-dielectric transition ($|\tau|^t \ll (\sigma_2/\sigma_1)^{\frac{1}{2}} \ll 1$; $t \approx 1, 3$).

The quantity σ_{ae} does not possess such anomalies, and when the small corrections $\sim\delta \ll 1$ are neglected, we have $\sigma_{ae} \approx \sigma_{a1} \approx \sigma_{a2}$ for all H. Consequently, the saturation of the effective Hall parameter $\beta_e = \sigma_{ae}/\sigma_{xe}$, predicted in Ref. 1 for $\tau = 0$, occurs in the case in which $\tau \neq 0$ in the magnetic-field region $\delta^{-1} \ll \beta \ll |\tau|^{-t}\delta^{-1}$. In the case of equal carrier concentrations, i.e., for $n_1 = n_2$, the parameter $\lambda = \sigma_2/\sigma_1$ does not depend on H, and σ_{xe} does not exhibit anomalous properties.

Thus, it follows from the results of the present and the preceding sections that the anomalous conductivity predicted in Ref. 1 can exist only in systems with compositions close to the critical composition ($p \approx p_c$) and (for $p \neq p_c$) in a finite magnetic-field range. The situation most favorable for the observation of the anomalous conductivity is the one in which $(\mu_1 - \mu_2)/\mu_1 \sim 1$ and $(n_1 - n_2)/n_1 \sim 1$. We have in this case the broadest concentration range $|\tau| \ll 1$ and the magnetic fields are not too high: $1 \ll \beta \ll |\tau|^{-t}$.

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