Heating and cooling of runaway electrons in inelastic scattering at high energies

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The effect of inelastic scattering at $\varepsilon \gtrsim \varepsilon_0 \gg kT$ on the heating and runaway of semiconductor (or plasma) electrons in the region $\varepsilon \leq \varepsilon_0$ is considered. It is shown that the electron flow to the origin of the energy axis, caused by inelastic scattering, leads to an increase of the number of electrons with $\varepsilon > kT$ (heating) as well as of the number of electrons with $\varepsilon < kT$ (cooling) upon increase of the field strength. In fields in which inelastic scattering exceeds quasielastic scattering, electron heating gives way to cooling. In this case the electron distribution peak is displaced toward the region of energies $\varepsilon \sim \varepsilon_c$ such that the momentum relaxation time is close to the minimum value. The field dependence of the mobility in energy scattering by deformation acoustic and optical phonons and in momentum scattering by ionized impurities is calculated for various relations between kT and ε_c (ε_c is a characteristic electron-ion interaction energy). The results are compared with the experimental data.

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I. INTRODUCTION AND FORMULATION OF THE PROBLEM

1. In a number of quasielastic mechanisms of the scattering of plasma or semiconductor electrons in an electric field E exceeding a certain characteristic value, the electron energy and the momentum are known[1-6] to increase to much higher than equilibrium values (runaway). The presence and the character of the runaway are determined by the energy dependence of the "heating function" (see Refs. 2–4)

$$\Theta(\varepsilon)=\frac{2}{3}\frac{e^2\tau(\varepsilon)\tilde{\tau}(\varepsilon)}{mT}, \tag{1}$$

where $\tau(\varepsilon)$ and $\tilde{\tau}(\varepsilon)$ are the momentum and energy relaxation times, and T is the temperature (in energy units). According to Refs. 2–4, runaway is possible if $\Theta(\varepsilon)$ does not increase more slowly than ε. In a strongly ionized plasma, where only quasielastic electron-ion and electron-electron collisions are significant, the energy of the runaway electrons increases without limit.[1] We are interested below in a situation typical of impurity semiconductors (and of a weakly ionized plasma), when the quasielastic scattering mechanisms (by impurities or phonons) that which contribute to the runaway are significant only in the energy region $\varepsilon \leq \varepsilon_0$ (first region), while at high energies, $\varepsilon > \varepsilon_0 \gg T$ (second region) inelastic-scattering mechanisms for which runaway is impossible predominate (limiting mechanisms).[2] In semiconductors, these mechanisms are impact excitation of nuclear impurities and spontaneous emission of optical phonons of frequency ω_0 (in plasma—impact excitation of the neutral atoms[7]). The values of ε_0 for these mechanisms are respectively of the order of a Rydberg or $\varepsilon_0 = \hbar\omega_0$ and amount to 10^2–10^3 K for semiconductors. In this case one also speaks of runaway, bearing in mind runaway only in the first region, i.e., up to energies $\varepsilon = \varepsilon_0$, but involving the bulk of the electrons. The problem of the distribution of the electrons and of the mean values is solved in this case by treating it as stationary,[2,4-6] and the momentum distribution function $F(\mathbf{p})$ is sought in the diffusion approximation. The electron density n is assumed in this case small, the interelectron (e–e) collisions are disregarded. This is precisely the runaway variant which we shall consider below.

2. As applied to semiconductors, the distribution function and the dependence of the mobility μ of the runaway electrons on the field were calculated by Levinson and Mazhoulite,[4-6] who considered various but only quasielastic scattering mechanisms: momentum scattering by ionized impurities (I mechanism) or by piezoacoustic (PA) phonons, and energy scattering by deformation acoustic (DA) or PA phonons

$$\tau_I(\varepsilon)\propto\varepsilon^{3/2}\Lambda^{-1}(\varepsilon),\quad \tau_{PA}(\varepsilon)\propto\varepsilon^{1/2},\quad \tilde{\tau}_{PA}(\varepsilon)\propto\varepsilon^{1/2},\quad \tilde{\tau}_{DA}(\varepsilon)\propto\varepsilon^{-1/2}$$

[$\Lambda(\varepsilon)$ is the Coulomb logarithm]. The inelastic scattering—in spontaneous emission of optical phonons—was taken into consideration only as a limiting mechanism: it was assumed that because of the short relaxation times of the electrons having $\varepsilon > \varepsilon_0$ the action of the field on them can be neglected. Therefore the symmetrical part of $F(\mathbf{p})$ ($F_0(\varepsilon)$) for electrons with $\varepsilon > \varepsilon_0$ was assumed to be in equilibrium. In Refs. 4–6, however, (just as in Ref. 2) no account was taken of the presence of an electron flux from the second region into the first as a result of emission of optical phonons or, in other words, of the energy lost to inelastic scattering.[8] The relative role of this mechanism increases with increasing electric field.[9] The distribution function and the mobility, in the case of only inelastic energy scattering at $\varepsilon > \varepsilon_0 \gg T$ and momentum scattering on account of elastic collisions at $\varepsilon < \varepsilon_0$ were obtained in Refs. 9–11, but were unrelated to runaway problems. Comparison of the results of these studies with the conclusions of Refs. 4–6 shows the need for taking into account inelastic scattering when runaway is considered. Thus, according to Refs. 4–6, when the field is increased, the maximum point of the electron distribution

$$n(\varepsilon)=\rho(\varepsilon)F_0(\varepsilon) \tag{2}$$

is first monotonically shifted to the right, toward higher energies, and then "presses" against the upper limit of the first region, and runaway terminates [$\rho(\varepsilon)\propto\varepsilon^{1/2}$ is the state density]. Most electrons have in this case

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an energy $\varepsilon \sim \varepsilon_0$ (Refs. 4–6). According to Ref. 11, in fields where inelastic energy scattering predominates while the momentum is scattered by the ions, the maximum of $n(\varepsilon)$ is found in that energy region $\varepsilon \sim \varepsilon_c \ll \varepsilon_0$ where $\tau_I(\varepsilon)$ is a minimum (for I scattering, ε_c is the characteristic energy of the electron interaction[11]). Moreover, in the same fields, for a power-law and sufficiently rapid growth of $\tau(\varepsilon)$ with ε, meaning $\tau(\varepsilon) \propto \varepsilon^\alpha$, with $\alpha > 1$, the average energy $\bar{\varepsilon}$, as shown in Ref. 10, decreases with decreasing field and may even become less than the equilibrium value. This effect was called "carrier cooling" in Ref. 10.

3. We investigate in this paper the change of the electron distribution $n(\varepsilon)$ and of the mobility with increasing electric field, with account taken of both the quasielastic energy scattering mechanisms that contribute to runaway in the first region, and of the extremely inelastic mechanisms in the second. It is assumed that in the first and second regions (labeled hereafter by the indices 1 and 2) the relaxation times satisfy the inequality

$$\tilde{\tau}_1 \gg \tau_1 \gg \tau_2, \tilde{\tau}_2, \tag{3}$$

with the inelastic scattering predominating in the second region. It is known that correct allowance for inelastic scattering in the runaway problem leads to results that differ qualitatively from those of Refs. 2 and 4–6. In particular, with increasing field, owing to inelastic scattering, a low-energy "tail" appears in the $n(\varepsilon)$ distribution. Therefore the maximum of $n(\varepsilon)$ moves with the field nonmonotonically—first to the right, into the region of high energies, and then, when the elastic scattering becomes significant enough, to the left, into the region of low energies, and stops in that energy region $\varepsilon \sim \varepsilon_c$ where $\tau_1(\varepsilon)$ is a minimum. If $\varepsilon_c = 0$, the maximum of $n(\varepsilon)$ "presses" against the lower limit of the first region. Motion of the maximum of $n(\varepsilon)$ to the left is accompanied [at $\tau_1(\varepsilon) \propto \varepsilon^\alpha$, $\alpha > 1$, for $\varepsilon > \varepsilon_c$] by a decrease of the random momentum and of the average energy. We shall call this motion, in analogy with Ref. 10, "cooling" of the carriers. Heating gives way to cooling, as will be shown later, at fields on the order of E_0, where

$$E_0 = \Theta^{-1/2}(T). \tag{4}$$

This is physically perfectly understandable, inasmuch as at $E = E_0$ the power drawn from the field by electrons of energy $\varepsilon = T$ becomes equal to the power lost in quasielastic collisions. Consequently, a rapid increase of the mean values with the field becomes possible only at $E \lesssim E_0$. In stronger fields, at $E > E_0$ and $\alpha > 1$, the electrons accumulate in the energy region $\varepsilon \sim \varepsilon_c \ll \varepsilon_0$, which is qualitatively at variance with the results of Refs. 4–6. It is the energy region $\varepsilon \sim \varepsilon_c$ which determines at $E > E_0$ the mean values (p_1 and τ_1) of the random momentum and of the relaxation time $\tau_I(\varepsilon)$:

$$p_1 \approx (2m\varepsilon_c)^{1/2}, \quad \tau_1 \approx \tau_I(\varepsilon_c).$$

Therefore the condition that the anisotropy of $F(\mathbf{p})$ be small, which is essential for our analysis to be valid, takes the same form at $\tau_1(\varepsilon) = \tau_I(\varepsilon)$ and $\tau_1(\varepsilon) = \tau_{PA}(\varepsilon)$:

$$E \ll E_1, \quad E_1 = (2m\varepsilon_c)^{1/2}/e\tau_1(\varepsilon_c). \tag{5}$$

We note that the field $E_1 \gg E_0$.

In connection with the cooling of the electrons, account was taken in the calculation of $n(\varepsilon)$, μ, etc. of the difference between $\tau_1(\varepsilon)$ and the idealized power-law behavior. This is of particular importance for I scattering, for which under real conditions the parameter ε_c/T varies in a wide range.[1)] We calculated the electron distribution and the field dependence of the mobility $\mu(E)$ for this mechanism at various values of the parameter ε_c/T, with account taken of the energy scattering by the DA and optical phonons. It was found that the form of $\mu(E)$ changes significantly with change of ε_c/T.

4. The situation considered is realized, for example, at $T \ll \hbar\omega_0$ in III–V semiconductors such as InSb, GaAs, etc., where the interaction of the carriers with the optical phonons is strong enough, and the momentum scattering of electrons of energy $\varepsilon < \hbar\omega_0$ is by ionized impurities. Interest in its theoretical study is stimulated by several factors. Thus, in relatively weak fields, when the energy is scattered mainly by DA and PA phonons, S-shaped current-voltage characteristics were observed[12,13] in InSb and GaAs, and were attributed to runaway.[5,6] The form of the distribution function for the same semiconductors has been extensively discussed in recent years in electric (and magnetic $\mathbf{H} \parallel \mathbf{E}$ and $\mathbf{H} \perp \mathbf{E}$) fields

$$E_1 \ll E \ll (2m\varepsilon_0)^{1/2}/\tau_2, \tag{6}$$

so strong that the electrons hardly collide on going through the first region, and the energy and momentum are scattered predominantly by optical phonons.[14] Under these conditions, according to a number of estimates (see Ref. 15 and the literature there) an inverse distribution of the carriers is possible, i.e., the condition needed for the development of active microwave devices. The calculation in the present paper permits an assessment of the realization fields values (6) in III–V semiconductors.

II. BASIC APPROXIMATIONS AND THE KINETIC EQUATION. GENERAL PROPERTIES OF THE DISTRIBUTION

1. We consider, for the sake of argument, an n-type semiconductor with strong interaction of the electrons with the optical phonons at $\varepsilon \geqslant \hbar\omega_0$. From the relations given above for $\tau_1(\varepsilon)$ and $\tilde{\tau}_1(\varepsilon)$ it follows that the growth of $\Theta(\varepsilon)$ needed for runaway can be reached for the following combinations of the mechanisms[2)]: I/DA, I/PA, and PA/PA. As indicated in Ref. 11, the approximation of $\tau_I(\varepsilon)$ by a power law, which is valid only for electrons with $\varepsilon \gg \varepsilon_c$ in fields $E \gg E_0$, imposes stringent restrictions on ε_c ($\varepsilon_c \approx e^2/\varkappa r_c$ in the case of classical scattering and $\varepsilon_c \approx \hbar^2/2mr_c^2$ in the case of quantum scattering, where $\varkappa$ is the dielectric constant and r_c is the cutoff radius for the Coulomb scattering cross section, see Ref. 11). We shall therefore use hereafter for $\tau_I(\varepsilon)$ the expression

$$\tau_I(\varepsilon) = \frac{(2m)^{1/2}}{\pi}\left(\frac{\varkappa}{e^2}\right)^2 \frac{1}{N}\varepsilon^{3/2}\Lambda^{-1}(\varepsilon), \tag{7}$$

where N is the density of the ionized impurity; the form of the Coulomb logarithm $\Lambda(\varepsilon)$ depends on the assumed

cutoff model.[11] We note that for scattering by phonons, power-law expressions for $\tau(\varepsilon)$, $\tau_{DA}(\varepsilon)\propto\varepsilon^{-1/2}$ and $\tau_{PA}(\varepsilon)\propto\varepsilon^{1/2}$ are valid only for electrons capable of emitting an acoustic phonon, i.e., at $\varepsilon\gg ms^2$ (s is the speed of sound).[16] For electrons in InSb and GaAs we have $ms^2\sim10^{-2}$–10^{-1} K, and the corresponding energy region should not be observed in experiment. For holes in GaAs, however, ms^2 is much larger, ~1 K.

2. We consider fields bounded from above by the condition (5). In these fields the kinetic equation for the electrons in first region can be solved in the diffusion approximation. Furthermore, in these fields the length $\Delta\varepsilon$ of penetration of the electron in to the second region is small (see Refs. 9 and 14)

$$\Delta\varepsilon\sim(eE\tau_2/p_0)^{2/3}\varepsilon_0\ll\varepsilon_0, \tag{8}$$

where

$$\tau_2=\tau_2(\varepsilon_0+\Delta\varepsilon),\quad \tau_2(\varepsilon)=\tau_2(2\varepsilon_0)\,[(\varepsilon-\varepsilon_0)/\varepsilon_0]^{-1/2},$$
$$p_0=(2m\varepsilon_0)^{1/2}.$$

We shall assume below that

$$\Delta\varepsilon\ll T,\ \varepsilon_c \tag{9}$$

and that the number of electrons in the second region is negligibly small. Conditions (3), (5), (8), and (9) yield for $F_0(\varepsilon)$ a differential equation in the form of a continuity equation for the fluxes of the electron due to the field ($j_E(\varepsilon)$) and to the quasielastic collisions with the acoustic phonons ($j_{ph}(\varepsilon)$). From this equation was obtained at $\varepsilon>\Delta\varepsilon$

$$j_E(\varepsilon)-j_{ph}(\varepsilon)=j_{in}(\varepsilon)=\text{const}, \tag{10}$$

where j_{in} is flux of the electrons from the second region into the first as a result of inelastic scattering. The fluxes $j_E(\varepsilon)$ and $j_{ph}(\varepsilon)$ are given by (see Ref. 11)

$$j_E(\varepsilon)=-\frac{2}{3}\rho(\varepsilon)\frac{(eE)^2}{m}\tau_1(\varepsilon)\varepsilon\frac{dF_0}{d\varepsilon}, \tag{11}$$

$$j_{ph}(\varepsilon)=\rho(\varepsilon)\frac{\varepsilon}{\bar\tau_1(\varepsilon)}\left(F_0(\varepsilon)+T\frac{dF_0}{d\varepsilon}\right). \tag{12}$$

An explicit expression for j_{in} can be obtained only if the distribution function in the second region is known.

Equation (2) was solved in Refs. 10 and 11 under the assumption that inelastic energy scattering predominates, i.e., in fields $E\gg E_0$. In this case it can be assumed that $F_0(\varepsilon_0)=0$ on the boundary between the regions, at $\varepsilon=\varepsilon_0$. There is no need then for a direct calculation of j_{in}, which can be determined simply from the condition for the normalization of $n(\varepsilon)$. To obtain for (10) a solution valid also at $E\leq E_0$, we can no longer assume $F_0(\varepsilon_0)=0$, for in this case the term that corresponds to allowance for only the quasielastic scattering mechanism vanishes from this solution. It is therefore necessary to calculate j_{in}. Using (1) and (8) we easily obtain $F(\mathbf{p})$ at $p>p_0$ and j_{in}:

$$j_{in}=\gamma(E)F_0(\varepsilon_0)\rho(\varepsilon_0)eEp_0/m,\quad \gamma(E)\sim1. \tag{13}$$

The factor γ in j_{in} (13) depends little on the field and is determined from the condition

$$\int F(\mathbf{p})\,\mathbf{pE}\,d\Omega\equiv\gamma(E)F_0(\varepsilon_0)p_0E,\quad p=p_0,\ \mathbf{pE}\geq0,$$

where $d\Omega$ is the solid-angle element. The integration is over the "forward" (relative to the field) hemisphere of momentum space[3)] with $p=p_0$. The indeterminacy in γ is due to the fact that near the boundary between the regions, at $p\lesssim p_0$, the diffusion approximation used to calculate $F(\mathbf{p})$ turns out to be bad in fields $E\gg E_0$: the anisotropic part of $F(\mathbf{p})$ becomes of the same order as the isotropic. However, γ depends little on the degree of anisotropy of $F(\mathbf{p})$ at $p=p_0$. Thus, $\gamma=0.25$ for an almost-isotropic function $F(\mathbf{p})$ and $\gamma=1$ for maximum anisotropy.[14] We neglect below the dependence of γ on E and put $\gamma=1$.

Substituting (11)–(13) in (10) we obtain

$$F_0(\varepsilon)=C\exp[J(\varepsilon,\varepsilon_0)]\left\{1+\frac{2eE}{(2m)^{1/2}}\frac{\varepsilon_0}{T}\right.$$
$$\left.\times\int_\varepsilon^\infty\frac{\bar\tau_1(\varepsilon')\,d\varepsilon'}{\varepsilon'^{1/2}[\Theta(\varepsilon')E^2+1]}\exp[-J(\varepsilon',\varepsilon_0)]\right\}, \tag{14}$$

where $C=F_0(\varepsilon_0)$ is a normalization constant and

$$J(\varepsilon,\varepsilon_0)=\int_\varepsilon^{\varepsilon_0}\frac{d\varepsilon'}{T[\Theta(\varepsilon')E^2+1]}. \tag{15}$$

The second term in (14) corresponds to allowance for inelastic scattering. Neglecting this term, we obtain for $F_0(\varepsilon)$ and expression that coincides with those given in Refs. 4–6.

3. We analyze now the behavior of $F_0(\varepsilon)$ and $F(\varepsilon)$ for different scattering mechanisms, assuming so far for simplicity that the relaxation times are power-law functions

$$\tau_1(\varepsilon)\propto\varepsilon^\alpha,\quad \bar\tau_1(\varepsilon)\propto\varepsilon^\beta. \tag{16}$$

We introduce the notation

$$x=\frac{\varepsilon}{T},\quad u=\frac{\varepsilon_0}{T},\quad b=\left(\frac{3}{2}\frac{\bar\tau_1(T)}{\tau_1(T)}\right)^{1/2},\quad p=\left(\frac{E}{E_0}\right)^2. \tag{17}$$

We then obtain for $F_0(x)$ at $0\leq x\leq u$

$$F_0(x)=C\exp[J(x,u)]\left\{1+bup^{1/2}\int_x^u\frac{x'^\beta\exp[-J(x',u)]\,dx'}{x'^{1/2}[px'^{\alpha+\beta}+1]}\right\}. \tag{18}$$

We consider first $F_0(x)$ in field so strong that

$$E_0\ll E\ll E_1, \tag{19}$$

i.e., $p\gg1$. In this case the energy losses to acoustic phonons can be neglected. Recognizing that $J(\varepsilon,\varepsilon_0)\ll1$ at $p\gg1$ and neglecting the unity term in the curley brackets of (18), we obtain

$$F_0(x)\approx\text{const}\int_x^u\frac{x'^\beta dx'}{x'^{1/2}[px'^{\alpha+\beta}+1]}. \tag{20}$$

As already mentioned, runaway is possible at $\alpha+\beta>1$. For these scattering mechanisms $F_0(x)$ is determined for $x\gg x_p=(1/p)^{1/(\alpha+\beta)}$ only by the momentum relaxation time and takes the form

$$F_0(x)\propto(x^{-(\alpha+1/2)}-u^{-(\alpha+1/2)}),\quad \alpha\neq-1/2, \tag{21a}$$

$$F_0(x)\propto\ln(u/x),\quad \alpha=-1/2. \tag{21b}$$

For the known runaway mechanisms, $\alpha>0$. In this case $n(x)$ is a function that decreases at $x>x_p$. Therefore the maximum point x_m for the largest value of the elec-

tron distribution satisfies the condition

$$x_m \ll x_p = (1/p)^{1/(\alpha+\beta)}. \quad (22)$$

Thus, for all the scattering mechanisms that contribute to runaway, the point at which the electron distribution in the fields (19) has a maximum shifts with increasing field towards energies lower than the equilibrium value. At $\alpha > 1$, an analogous conclusion can be drawn with respect to the point where the function $t(x) \equiv xn(x)$, which determines the average energy, has a maximum.

We consider now two scattering-mechanism examples at which there is no runaway: 1) $\tau_1(\varepsilon) \propto \varepsilon^{1/2}$, $\tilde{\tau}_1(\varepsilon) \propto \varepsilon^{-1/2}$ (momentum scattering by dipoles, energy scattering by *DA* phonons). In this case, in fields (19), the distribution has again a maximum as $x \to 0$; the largest contrition to the average energy is given by electrons with $x \sim u$; 2) $\tau_1(\varepsilon) \propto \varepsilon^{-1/2}$, $\tilde{\tau}_1(\varepsilon) \propto \varepsilon^{-1/2}$ (scattering by *DA* phonons). In this case $n(x)$ is a maximum at $x \sim p$, i.e., the maximum shifts towards higher energies with changing field.

These examples demonstrate a property that can be proved also in the general case: in the presence of inelastic scattering, the position of the maximum of $n(x)$ in fields (19) is determined only by the form of $\tau_1(x)$; at $\tau_1(x) \propto x^\alpha$ it occurs at $\alpha > 0$ for energies lower than the equilibrium value, and at $\alpha < 0$ for energies higher than equilibrium. In other words, heating in the fields (19) is possible only for scattering mechanisms for which there is no runaway; for scattering mechanisms for which runaway takes place in fields $p < 1$, cooling takes place at $p \gg 1$, i.e., after the end of the runaway. The physical cause of the cooling in fields (19) is essentially the same that led to the strong dependence of the mean values on ε_c/T and p in the case of *I* scattering (see Ref. 11). Namely, when $\tau_1(\varepsilon)$ increases with energy the field exerts the weakest action on the electrons near the "source" at $\varepsilon = 0$, where the relaxation time $\tau_1(\varepsilon)$ is small. It is necessary in this case to take into account the absorption of the acoustic phonons by the electrons, which is characterized by a time $\tilde{\tau}_1(\varepsilon) \propto \varepsilon^\beta$; this mechanism corresponds formally to unity in the denominator of the integrand of (20). However, this absorption is significant only at $\varepsilon \lesssim T(1/p)^{1/(\alpha+\beta)} < T$ [see (22)]. We can investigate similarly the distribution function and $n(\varepsilon)$ for a relaxation time $\tau_1(\varepsilon)$ that increases with energy at $\varepsilon > \varepsilon_c$ and is constant or decreases at $\varepsilon < \varepsilon_c$. In this case the distribution takes on the largest value at $\varepsilon \lesssim \varepsilon_c$.

4. A simple analysis of the distribution function and of the form of $n(\varepsilon)$ is possible only in fields (19), when inelastic energy scattering dominates. In weaker fields, at $p \lesssim 1$ (the only case when runaway is possible) it is necessary to take into account both the elastic and inelastic energy scattering. In this case $F_0(\varepsilon)$ (14) can be determined as a rule only by numerical methods. We have therefore calculated the electron distribution $n(\varepsilon)$, the function $t(\varepsilon) = \varepsilon n(\varepsilon)$, and the field dependence of the mobility $\mu(E)$ in fields bounded from above by the condition (5), but only for one combination of the quasielastic mechanisms, namely *I/DA*. We took into account here also the inelastic energy scattering. These scattering mechanisms can be vital for compensated doped semiconductors, and also for a weakly ionized plasma, where the quasielastic scattering is due to collisions between the electrons and the neutral atoms, and the momentum is scattered by the ions.

III. ELECTRON DISTRIBUTION FOR MOMENTUM SCATTERING BY IONS AND ENERGY SCATTERING BY ACOUSTIC AND OPTICAL PHONONS

1. The momentum relaxation time for scattering by ions is given by Eq. (7); we use in this case for the Coulomb logarithm $\Lambda(\varepsilon)$ an expression obtained in the case of an abrupt cutoff of the potential at a distance $r = \frac{1}{2}N^{-1/2}$ from the ion (the Conwell-Weisskopf model, see Ref. 17):

$$\Lambda(\varepsilon) = \ln[1 + (\varepsilon/\varepsilon_c)^2]. \quad (23)$$

This form of $\Lambda(\varepsilon)$ corresponds to the correct behavior of the scattering cross section and of $\tau_I(\varepsilon)$ (7) both at $\varepsilon \gg \varepsilon_c$ and in the case of scattering of "slow" particles with $\varepsilon \ll \varepsilon_c$ (see also Ref. 11). We note that if we use for $\tau_I(\varepsilon)$ an equation of the form

$$\tau_I(\varepsilon) = \frac{\tau(\varepsilon_c)}{1+d}\left[\left(\frac{\varepsilon}{\varepsilon_c}\right)^{-1/2} + d\left(\frac{\varepsilon}{\varepsilon_c}\right)^{3/2}\right], \quad d \sim 1,$$

we obtain for the position of the maximum of $n(\varepsilon)$ and for the mobility in fields (19) expressions close to (7) and (23) those in the case of $\tau_I(\varepsilon)$. We note also that the analytic expression obtained in Refs. 9 and 11 with the aid of the model (23) for the mobility of nonequilibrium electrons in fields (19) agrees well with the experimental data.[18] We calculated $F_0(\varepsilon)$, $n(\varepsilon)$, and $\mu(E)$ with a computer. The parameters b and T/ε_c were chosen close to the real ones for the electrons in InSb and GaAs. The values of $\tau_{DA}(T)$ were taken from the experimentally determined mobility for *DA* scattering.[19,20] In addition, for comparison with the results of Refs. 4 and 6, the calculations were performed also for the case of an idealized model with Coulomb scattering, in which $\Lambda(\varepsilon)$ is constant. Some of the calculation results are shown in Figs. 1–3. All the curves of Figs. 1 and 2 are given for the same "total" electron density

$$n = \int n(\varepsilon)\, d\varepsilon = \text{const}.$$

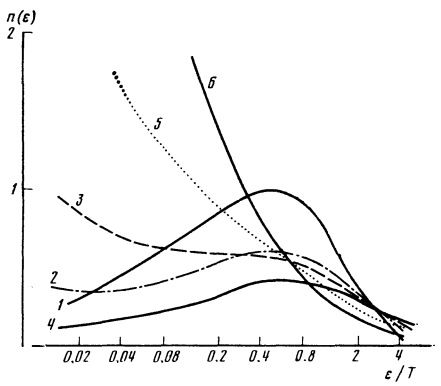

FIG. 1. Electron distribution in various electric fields for the idealized model $\tau_1(\varepsilon)$ $(T/\varepsilon_c \to \infty)$, $u = 70$, $b \approx 4.5 \cdot 10^3$; curve 1) $p = 0$, 2) $p = 0.25$; 3) $p = 0.33$; 4) $p = 0.33$ $(j_{in} = 0)$; 5) $p = 0.5$; 6) $p = 1$.

2. The effect of the field on the electron distribution $n(\varepsilon)$ for the idealized model[4] $\Lambda(\varepsilon)$ = const is shown in Fig. 1. It is seen that with increasing E the maximum of the distribution (ε_m) first shifts towards higher energies from its equilibrium position $\varepsilon_m(E=0) = 0.5T$, and the number of electrons in it decreases to zero. At the same time $n(\varepsilon)$ increases both on the right side of the equilibrium maximum (this constitutes heating and runaway), and on the left, at $\varepsilon \ll T$. The increase of $n(\varepsilon)$ at $\varepsilon < T$ is due to the dumping of the electrons into the region $\varepsilon \sim \Delta\varepsilon \ll T$ as a result of inelastic scattering. Its influence on the form of $n(\varepsilon)$ can be seen from a comparison of curve 4, plotted for the case $j_{in}(\varepsilon) = 0$, with curve 3. It can be concluded from the data of Fig. 1 that heating and cooling at $\Lambda(\varepsilon)$ = const take place simultaneously. It follows therefore that the distribution function differs substantially from a Maxwellian with effective temperature $T^* > T$. Therefore the use of the latter in the interpretation of the experimental data[21] can lead to errors.

We now describe briefly the variation, with the field, of the function $t(\varepsilon) = \varepsilon n(\varepsilon)$, which determines the average energy. In contrast to $n(\varepsilon)$ and $t(\varepsilon)$, there is always a maximum in this field region. This maximum initially, up to $p \leq 0.5$, shifts with the field to the right, and then at $p > 0.5$ to the left. Thus, the decrease of the average electron energy with the field begins already at $p \sim 0.5$.

3. We consider now $n(\varepsilon)$ for the case of a "real scattering law," i.e., with allowance for the $\Lambda(\varepsilon)$ dependence (23). From qualitative considerations and from the examples given above it is clear that there is no runaway at all at $\tau_I(\varepsilon)$ and $\Lambda(\varepsilon)$ [Eqs. (7) and (23)] and at $\varepsilon_c > T$ at small degrees of heating, while for most electrons $\tau_I(\varepsilon)$ is a decreasing function of the energy. Runaway can come into play only in fields such that a noticeable fraction of electrons with $\varepsilon > \varepsilon_c$ appears. For the case $T < \varepsilon_c$ we introduce the characteristic field

$$E_0' \equiv \left(\frac{3}{2}\frac{m\varepsilon_c}{e^2\tau_I(\varepsilon_c)\bar{\tau}(\varepsilon_c)}\right)^{1/2} \approx E_0\frac{\varepsilon_c}{T}. \qquad (24)$$

It has the same physical meaning as E_0, but as applied to electrons with energy $\varepsilon = \varepsilon_c$, for which $\tau_I(\varepsilon)$ is close to the minimum. It is clear from qualitative considerations that the cooling and heating effects should become significant at $E > E_0'$. We note that according to Ref. 9 elastic scattering predominates at $E \gg E_0' \ln(\varepsilon_0/\varepsilon_c)^2$.

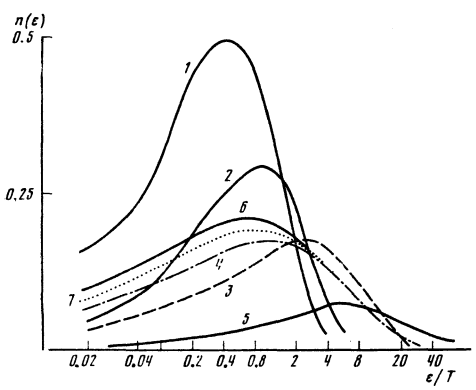

FIG. 2. Electron distribution in various electric fields for a "real" scattering law (7), (23): $\varepsilon_c = 2.5T$, $u = 70$, $b \approx 4.5 \cdot 10^3$; curves 1) $p = 0$; 2) $p = 1$; 3) $p = 10$; 4) $p = 20$; 5) $p = 20$ ($j_{in} = 0$); 6) $p = 30$; 7) $p = 50$.

The influence of the electric field on the electron distribution at $T < \varepsilon_c$ is illustrated in Fig. 2 for $\varepsilon_c = 2.5T$, $u = 70$, and $b \approx 4.5 \cdot 10^3$ (these are the parameters for the electrons in InSb at $T = 4$ K and $N = 10^{15}$ cm^{-3}); the change of the parameter b at $b \gg u$ hardly affects the form of $n(\varepsilon)$. In this case, in contrast to the data of Fig. 1, $n(\varepsilon)$ has a maximum which shifts initially to the right with increasing field, corresponding to heating, and then to the left and "stops" (cf. curves 4, 6, and 7 of Fig. 2). Thus, "cooling" of the electrons with increasing field takes place also at $T < \varepsilon_c$. Comparison of curves 4 and 5 demonstrates the influence of inelastic scattering on the form of $n(\varepsilon)$. We note that (in contrast to the idealized model), at $T < \varepsilon_c$ cooling takes place in stronger fields than heating, and the position of the maximum ε_m is determined both by the field and by the value of ε_c. At $p \gg 1$ there is no longer a dependence of ε_m on p (Fig. 2); in this case, according to Ref. 11, $\varepsilon_m \approx 0.25\varepsilon_c$.

The changes of the function $t(\varepsilon)$ with changing field are briefly the following. In the entire range of fields this function has a sharp maximum which (in contrast to the case $\varepsilon_c = 0$) moves with the field only to the right and stops at the same fields as the maximum $n(\varepsilon)$. It is located, however, at an energy that exceeds ε_m by more than one order of magnitude. This agrees with the results of Ref. 11 that in the presence of only inelastic scattering the values of ε and ε_m differ parametrically by a factor $\delta(\varepsilon_0/\varepsilon_c)^{1/2} \gg 1$, where $\delta > 1$ is a numerical factor [see Eq. (20) and the calculation of ε_m in Ref. 11].

IV. FIELD DEPENDENCE OF THE MOBILITY

1. In this section we present part of the results of our calculations of the dependence of the relative mobility $\mu' = \mu(E)/\mu(0)$ on the electric field in the case of a "real" law of momentum scattering by ionized impurities [see (27) and (23)], and of energy scattering by *DA* and optical phonons.

For convenience in comparing the results with the available data by others, we shall designate as the first and second sections [of the $\mu(E)$] dependence those field ranges where energy scattering by acoustic or optical phonons predominates, respectively. The expression for $\mu(E)$ on the second section [the range (19)] were obtained in Refs. 9 and 10 without allowance for e–e collisions, and in Ref. 22 by assuming that e–e collisions lead to a Maxwellian distribution function in the first region. In Ref. 21 it was assumed in the calculation of the mean values of the first and second field sections that $F_0(\varepsilon)$ is a Maxwellian function at all energies.[5)]

2. The calculations of the dependence of $\mu'(E)$ on the dimensionless field E/E_0 were carried out for different values of the parameters T/ε_c, $\hbar\omega_0/T$ and b [see (18)], close to the real values for the electrons in InSb and

GaAs at $T \sim 4-10$ K. In addition, the calculation was carried out for several arbitrary values of the parameters T/ε_c and b.

Figure 3 shows several typical $\mu'(E)$ plots that illustrate the main features of the behavior of the mobility for the considered scattering mechanisms. The parameters of curves 1, 2, and 4 correspond to electrons in InSb at $T = 4$ K (1, 2) and $T = 10$ K (4) and to values of N equal to 10^{15} cm^{-3} (1), 10^{14} cm^{-3} (2), and 10^{12} cm^{-3} (4). Curve 4 should be regarded as a "model" curve: in the presently available InSb the total impurity density is not less than 10^{14} cm^{-3}. The parameters of curve 3 correspond to the electrons in GaAs at $T = 4$ K and $N = 10^{14}$ cm^{-3}. For comparison, the same figure shows plots of $\mu'(E)$ (curves 5, 6) for an idealized law of scattering by ionized impurities ($T/\varepsilon_c \to \infty$).

It is seen from the presented curves that at $T < \varepsilon_c$ the $\mu'(E)$ dependence in fields corresponding to energy scattering by optical phonons saturates (the second quasiohmic section[9-11]); the value of μ' on the second section is then larger the larger T/ε_c (at constant u). Accordingly, the law governing the increase of the mobility in the first section, where energy scattering by DA phonons predominates, is also determined by the value T/ε_c: when N decreases the $\mu'(E)$ dependence becomes stronger (cf. curves 1 and 2).

At $T > \varepsilon_c$, the increase of $\mu'(E)$ on the first section turns out to be stronger than at $T < \varepsilon_c$ (cf. curves 1 and 6, 3 and 5); after reaching the maximum corresponding to energy scattering by optical phonons, μ' decreases with increasing E (see curves 4–6). This is the consequence of the cooling effect. In the case $\varepsilon_c \to 0$, cooling leads to a monotonic decrease of the mobility with the field (curves 5 and 6). Interest attaches to the behavior of $\mu'(E)$ at small ($\varepsilon_c \ll T$) but finite values of ε_c (see curve 4). In this case the inelastic scattering also leads to cooling and to a decrease of μ'. In contrast to the case $\varepsilon_c = 0$, however, the cooling is limited: when the maximum of $n(\varepsilon)$ approaches ε_c, further cooling stops. This explains the saturation of $\mu'(E)$.

3. Let us dwell briefly on the decrease of μ' in weak fields, $E < E_0$ (see curves 1 and 3). This decrease is small and is observed only for the case $T < \varepsilon_c$. The physical reason is apparently that in the assumed model

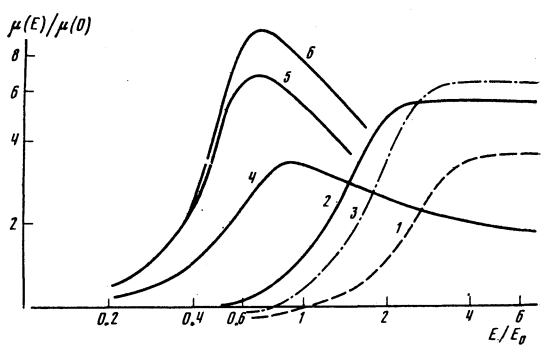

FIG. 3. Dependence of the mobility on the field at different values of the parameters T/ε_c and $u = \hbar\omega_0/T$; $T/\varepsilon_c = 0.4$ (curve 1); 0.86 (2), 0.65 (3), 9.5 (4), ∞ (5, 6); $u = 70$ (1, 2, 6), 97 (3, 5), 29 (4).

of $\tau_I(\varepsilon)$ [Eqs. (7), (23)] in the case of weak heating, so long as most electrons are in the energy range $\varepsilon < \varepsilon_c$, the average relaxation time $\overline{\tau}_I$ should decrease with increasing heating. With further heating, the maximum shifts into the region $\varepsilon \gtrsim \varepsilon_c$, and both $\overline{\tau_I(\varepsilon)}$ and $\mu'(E)$ increases with the field.

V. CONCLUSIONS. COMPARISON WITH EXPERIMENT

1. It is seen from the foregoing that inelastic scattering of high-energy electrons with $\varepsilon \gtrsim \varepsilon_0 \gg T$ leads to several effects. The electrons land in the region of low energies $\varepsilon \sim \Delta\varepsilon \sim T$, and in the case of the nonequilibrium electrons the form of $\tau_1(\varepsilon)$ at $\varepsilon \sim \Delta\varepsilon \approx T$ becomes important. In the case when $\tau_1(\varepsilon)$ increases with energy at $\varepsilon > \varepsilon_c$, inelastic scattering leads to a shift of the maximum of $n(\varepsilon)$ to the left, i.e., to cooling of the electrons. Therefore, for all real quasielastic scattering mechanisms, electron runaway can occur only in a narrow range of fields and should give way to their cooling. The cooling is stronger the smaller ε_c and the stronger the runaway. As a result of the cooling, most electrons are in the energy region $\varepsilon \sim \varepsilon_c \ll \varepsilon_0$ [at $\tau_1(\varepsilon) \propto \varepsilon^\alpha$, $\alpha > 1$]. Therefore the law governing the variation of the mean values (for example of μ) with changing field is determined, other conditions being equal, by the parameter T/ε_c (Fig. 3). This must be kept in mind when comparing the calculated and experimental $\mu(E)$ dependences.

2. The accumulation of the electrons in the energy region $\varepsilon \sim \varepsilon_c$ leads to a number of consequences. The first concerns the condition for low anisotropy of $F(\mathbf{p})$, i.e., the upper limit of the second section—the field E_1. As already mentioned, following cooling to energies $\varepsilon \sim \varepsilon_c$ it is necessary to take τ_1 in Eq. (5) to mean $\tau_1(\varepsilon \sim \varepsilon_c)$, and the momentum p_1 to mean $(2m\varepsilon_c)^{1/2}$. At $\tau_1(\varepsilon) = \tau_I(\varepsilon)$ we have then

$$E_1 \approx (2m\varepsilon_c)^{1/2}/e\tau_I(\varepsilon_c) \propto N^{3/4}. \tag{25}$$

We note that, without allowance for the cooling, a much smaller value was obtained for E_1 in Ref. 9:

$$(2m\varepsilon_0)^{1/2}/e\tau_I(\varepsilon_0) \sim (\varepsilon_c/\varepsilon_0)E_1.$$

The lower limit of the second section at $T < \varepsilon_c$ is determined by the field E_0' (24). In this case $E_0' \propto N^{1/2}$. Thus, the ratio of the limiting fields of the second section E_1/E_0' depends little on the impurity density:

$$E_1/E_0' \propto N^{1/4}, \quad T < \varepsilon_c. \tag{26}$$

This result agrees qualitatively with measurements of the mobility of nonequilibrium electrons in different n-InSb samples with $\varepsilon_c > T$ (Ref. 18): when N was increased from 10^{14} cm^{-3} to 10^{15} cm^{-3}, the fields corresponding to the start and end of the second section increased by several times, but their ratio remained practically constant (2 ± 0.2).

The second consequence is more significant. It concerns the condition (6) for the realization of collisionless motion of the carriers (at $H = 0$) in the III–V semiconductors InSb and GaAs, and by the same token the possibility of using them for active microwave devices. Substituting in (6) the expression for E_1 and the values of $\tau_I(\varepsilon_c)$ and ε_c from (7) and (23), we find that the range

of fields (6) is realized if the following inequality is satisfied with large margin:

$$\pi(e^2/\varkappa)N^{1/3} \ll p_0/\tau_2(2\hbar\omega_0). \qquad (27)$$

Using the known values of p_0 and $\tau_2(2\hbar\omega_0)$ for the electrons in InSb and GaAs (Ref. 6), we find that (27) is satisfied at

$$N \ll 6\cdot 10^{14}\ \text{cm}^{-3}\ \ (\text{InSb}), \quad N \ll 2\cdot 10^{16}\ \text{cm}^{-3}\ \ (\text{GaAs}). \qquad (28)$$

Thus, the field range (6) cannot be realized in the presently available InSb with $N \gtrsim 10^{14}$ cm^{-3}. Consequently, the saturation of the drift velocity v_{dr} corresponding to this field range should likewise not be observed. This conclusion agrees with the experimental data of Ref. 8: the drift velocity in n-InSb with $N \sim (1-2)\cdot 10^{14}$ cm^{-3} already exceeded at $E \gg E_1$ the value corresponding to saturation, $v_{dr.sat} = p_0/2m$, but continued to increase slowly with the field. In the n-GaAs available at the present time, $N \lesssim 10^{15}$ cm^{-3} and the collisionless regime can be realized in accordance with (28). We note that saturation of the drift velocity in n-GaAs with $N \sim 1.3\cdot 10^{15}$ cm^{-3} was observed in Ref. 23.

3. We now compare the $\mu'(E)$ dependences shown in Fig. 3 with the experimental data. The section where μ' is independent of E, corresponding to a stationary maximum of $n(\varepsilon)$ (see Fig. 2) was observed in Ref. 18 in n-InSb samples with electron density $n \geq 10^{13}$ cm^{-3} and $\varepsilon_c > T$. The experimental mobilities in this section agree well with the calculated ones (see Refs. 18 and 11). The rapid increase of the average relaxation time $\overline{\tau}_I \propto \mu$ with the field, $\overline{\tau}_I \propto E^{0.6-0.8}$, which is close to the $\mu'(E)$ dependence on the first section of Fig. 3, was observed in Ref. 24 in experiments on cyclotron resonance of photoexcited electrons of p-InSb with $N \approx (1.5-5)\cdot 10^{14}$ cm^{-3}. The electron density was $\sim 10^6$–10^9 cm^{-3}, and collisions between electrons can be neglected.[24] A close field dependence of the conductivity, $\sigma \propto E^{0.6}$, was observed in Ref. 25 in samples of strongly compensated n-InSb with $n \sim 8\cdot 10^{12}$ cm^{-3}. We note that in n-InSb samples with $n > 5\cdot 10^{13}$ cm^{-3} the $\mu(E)$ dependence is weaker in the first section than the calculated (Fig. 3) $\mu(E) \propto E^{0.3}$ (Ref. 18). This difference can be due to the fact that at $n > 5\cdot 10^{13}$ cm^{-3} and in electric fields corresponding to the first section, hitherto unaccounted electron transfer into the second section as a result of e–e collisions comes into play and increases the relative role of the inelastic energy scattering.[6]

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[1)] The question of the validity of the kinetic equation at $\varepsilon_c/T \gtrsim 1$ was discussed in Ref. 11.

[2)] In momentum scattering by dipoles, τ_D takes the same form as in PA scattering; the first index characterizes the momentum scattering mechanism, and the second the energy scattering.

[3)] The flux due to the absorption of acoustic phonons by electrons with $\varepsilon \approx \varepsilon_0$ in fields $E \sim E_0$ is relatively small,

$$\sim (T/\varepsilon_0)^{1/2} j_{\text{in}} \text{ for } PA/PA, \quad \sim (T/\varepsilon_0)^{1/2} (\tau_I(\varepsilon_0)/\tau_{DA}(\varepsilon_0))^{1/2} j_{\text{in}} \text{ for } I/DA,$$
$$\sim (T/\varepsilon_0)(\tau_I(\varepsilon_0)/\tau_{PA}(\varepsilon_0))^{1/2} \text{ for } I/PA,$$

[4)] According to Ref. 11, in a strong field at $p \gg 1$ the approximation $\Lambda(\varepsilon) = \text{const}$ for the I/DA scattering mechanism is valid at $\varepsilon_c \ll T/p$.

[5)] In this case, however, the conditions for the validity of the Maxwellian distribution for $\varepsilon > \varepsilon_0$ were not considered (see the discussions in Ref. 6).

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