Electron scattering by an atom in the field of resonant laser radiation

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The collision of an electron with an atom in the field of intense electromagnetic radiation that is at resonance with two atomic multiplets is investigated theoretically. Expressions are obtained for the amplitudes of the elastic and inelastic scattering with emission (absorption) of photons. The case of a ground state at resonance with a doublet is considered in detail. It is shown that photon absorption takes place predominantly in the case of resonance in inelastic transitions from a state of the lower multiplet, and photon emission takes place in transitions from a state of the upper multiplet.

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1. To describe the interaction of intense electromagnetic radiation with a plasma, particularly laser heating of a plasma, and the kinetics of the excitation of atomic states, it is necessary to know the cross sections for the elementary processes. Many methods have by now been proposed for the calculation of the cross sections for elastic and inelastic scattering of an electron by an atom in an intense electromagnetic field.

Stimulated emission and absorption of photons by an electron in scattering in an external field was investigated in the Born and in the low-frequency approximations (see the reviews[1,2]). In both approximations the contribution of the multiphoton transitions turns out to be substantial when the parameter $\gamma = F_0/F_{cr}$ becomes of the order of unity. Here F_0 is the amplitude of the electric field intensity and $F_{cr} = \hbar\omega^2 m/ep$, where ω is the frequency of the wave, $p = mv$ is of the order of the momentum transfer, and e and m are the charge and mass of the electron. If we assume as an estimate $\omega \sim 10^{15}$ sec^{-1} and $v \sim 10^8$ cm/sec, we obtain $F_{cr} \sim 6 \times 10^6$ V/cm.

The influence of the internal degrees of freedom of the target atom on the probability of single-photon transitions was accounted for, in first order in the electron-atom interaction and in the interaction with electromagnetic field, in Refs. 3 and 4. The restructuring of the atomic states by the electromagnetic field was determined by the parameter $F_0/F_a \ll 1$, where F_a is the characteristic atomic field. Therefore if $F_{cr} \ll F_a$ there is no need, when the stimulated emission and absorption by an atom is considered in the Born approximation, for taking into account consecutively in all orders the restructuring of the atomic wave functions by the field. The action of the electromagnetic field on the incident energy can be taken into account exactly, and the action on the atom need not be taken into account at all, except for a number of special cases. These include, e.g., scattering of an electron by a hydrogen-like atom in a low-frequency field,[5] when strong mixing of specifically degenerate states takes place, and also the case considered in Refs. 6 and 7, of resonance between two atomic states.

In this paper we consider the scattering of an electron by an atom in the case of resonance between two atomic multiplets.

The Schrödinger equation for an "electron plus atom" system interacting with an electromagnetic field is written in the dipole approximation in the form[1)]

$$i\partial\Psi(\mathbf{q}, \mathbf{r}, t)/\partial t = \{H_0(\mathbf{q}, \mathbf{r}, t) + V(\mathbf{q}, \mathbf{r})\}\Psi(\mathbf{q}, \mathbf{r}, t), \tag{1}$$

where $\mathbf{r}$ is the coordinate of the incident electron, $\mathbf{q} \equiv (\mathbf{q}_1, \ldots, \mathbf{q}_z)$ is the set of the coordinates of the atomic electrons, Z is the charge of the atomic nucleus,

$$V(\mathbf{q}, \mathbf{r}) = -Zr^{-1} + \sum_{\alpha=1}^{z} |\mathbf{r} - \mathbf{q}_\alpha|^{-1} \tag{2}$$

is the interaction of the incident electron with the atom,

$$H_0(\mathbf{q}, \mathbf{r}, t) = H_a(\mathbf{q}) + W(\mathbf{q}, t) + \hat{p}_r^2/2 + \mathbf{A}(t)\hat{\mathbf{p}}_r/c. \tag{3}$$

Here $H_a(\mathbf{q})$ is the Hamiltonian of the atom $\mathbf{A}(t) = \text{Re}\mathbf{A}_0 e^{i\omega t}$ is the vector potential of a plane monochromatic wave, and

$$W(\mathbf{q}, t) = \mathbf{A}(t)\sum_{\alpha=1}^{z} \hat{\mathbf{p}}_{\mathbf{q}_\alpha}/c = We^{i\omega t} + W^+e^{-i\omega t} \tag{4}$$

is the operator of the interaction of the atom with the electromagnetic wave.

If we are interested in transitions between quasienergy states (QES) (Ref. 8) of the operator H_0 in the collision process, then the scattering problem for Eq. (1) can be posed in a stationary formulation on the basis of the QES in the same manner as was done for the problem of scattering in an external field in Ref. 9. The final expressions for the transition amplitudes are of the same form as in ordinary scattering theory, but the scalar product of the state vectors is taken to mean the expression

$$\langle\langle \Psi_1 | \Psi_2 \rangle\rangle = (\omega/2\pi)\int_0^{2\pi/\omega} dt \int d\tau \Psi_1^*(\tau, t)\Psi_2(\tau, t), \tag{5}$$

where $d\tau$ is the volume element in configuration space.

The behavior of a free electron with a time-average kinetic momentum $\mathbf{p}$ in the field of the wave is described by a QES of the form

$$\varphi_p(\mathbf{r}, t) = \exp\{i[\mathbf{p}(\mathbf{r} - \mathbf{a}(t)) - p^2 t/2]\}, \tag{6}$$

where $\mathbf{a}(t) = \mathbf{a}(t+T) = c^{-1}\int \mathbf{A}(t)dt$ is the classical dis-

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placement of the particle. The QES of the atom in the field of the wave $\varphi_{\varepsilon_i}(\mathbf{q},t)$ will be assumed approximately known. The total set of the QES of the operator H_0 (3) with corresponding quasienergies $\varepsilon=\varepsilon_i+p^2/2$ is determined by the product of the functions $\varphi_{\varepsilon_i}(\mathbf{q},t)$ and $\varphi_{\mathbf{p}}(\mathbf{r},t)$. The amplitude of scattering with transition of an atom from a state i into f with change of the electron momentum $\mathbf{p}_i$ into $\mathbf{p}_{fk}$, where

$$p_{fk}^2=p_i^2+2(\varepsilon_i-\varepsilon_f-k\omega)>0,$$

$k=0,\pm1,\ldots$, can be represented in the Born approximation, without allowance for exchange, in the form

$$f_{fi,k}=-(2\pi)^{-1}\langle\langle\varphi_{\varepsilon_f}\varphi_{\mathbf{p}_{fk}}|V|\varphi_{\varepsilon_i}\varphi_{\mathbf{p}_i}\rangle\rangle. \tag{7}$$

The differential scattering cross section is expressed in the usual manner in terms of the amplitude (7):

$$d\sigma_{fi,k}=p_{fk}p_i^{-1}|f_{fi,k}|^2d\Omega.$$

In the weak-field limit, ε_i and ε_f go over respectively into the energies of the unperturbed atomic states, and k has the meaning of the number of the photons absorbed ($k>0$) or emitted ($k>0$) by the system in the collision. Accordingly, the quantity $f_{fi,k}$ (7) will be called the amplitude for scattering with emission (absorption) of k photons.

2. To find the scattering amplitude it is necessary to construct the QES of the atom in an electromagnetic field. The sought QES satisfy the equation

$$\{H_a(\mathbf{q})+W(\mathbf{q},t)-i\partial/\partial t\}\varphi_\varepsilon(\mathbf{q},t)=0, \tag{8}$$

where the operator $W(\mathbf{q},t)$ is determined by the relation (4).

We assume that the electromagnetic radiation is at resonance with two atomic multiplets. The corresponding atomic states are strongly mixed by the field, and the QES can be obtained in the resonance approximation generalized for the multiplets[10] that correspond to the zeroth order of perturbation theory for the QES[11,12]:

$$\varphi_\varepsilon(\mathbf{q},t)=e^{-i\varepsilon t}\Big\{\sum_{j=1}^{s_a}C_j^a\Phi_j^a(\mathbf{q})+\sum_{j=1}^{s_b}C_j^b\Phi_j^b(\mathbf{q})e^{-i\omega t}\Big\}, \tag{9}$$

where $\Phi_j^{a,b}(\mathbf{q})$ are the unperturbed atomic states,

$$H_a(\mathbf{q})\Phi_j^{a,b}(\mathbf{q})=E_j^{a,b}\Phi_j^{a,b}(\mathbf{q}),$$

s_a and s_b are the number of states in the lower and upper multiplets, respectively. The coefficients $C_j^{a,b}$ satisfy a system of equations obtained by substituting the function (9) in the Schrödinger equation (8)

$$C_i^a(\varepsilon-E_i^a)-\sum_{j=1}^{s_b}C_j^bW_{a_ib_j}=0,\quad C_j^b(\varepsilon-E_j^b+\omega)-\sum_{i=1}^{s_a}C_i^aW_{a_ib_j}^*=0, \tag{10}$$

where $i=1,\ldots,s_a$; $j=1,\ldots,s_b$. The quasienergy levels ε are determined by the roots of the secondary equation. In the general case, the system (10) determines s_a+s_b values of the quasienergy and of the corresponding QES, of which s_a functions having the form (9) and quasienergies $\varepsilon_i, i=1,\ldots,s_a$ go over, when the field is turned off, into unperturbed atomic states of the lower multiplet, and the remaining s_b functions can be conveniently written in the form

$$\varphi_{\varepsilon_j'}(\mathbf{q},t)=e^{-i\varepsilon_j't}\Big\{\sum_{j=1}^{s_a}C_j^a\Phi_j^a(\mathbf{q})e^{i\omega t}+\sum_{j=1}^{s_b}C_j^b\Phi_j^b(\mathbf{q})\Big\}, \tag{11}$$

where $\varepsilon_j'=\varepsilon_j+\omega, j=s_a+1,\ldots,s_a+s_b$, go over into the unperturbed states of the upper multiplet with energies $E_{j-s_a}^b$.

The action of the field on atomic states do not pertain to the considered multiplets can be neglected. Consequently, the corresponding QES are determined as unperturbed atomic states:

$$\varphi_{\varepsilon^c}(\mathbf{q},t)=e^{-i\varepsilon^ct}\Phi^c(\mathbf{q}), \tag{12}$$

where $\varepsilon^c=E^c$.

When the field is turned on adiabatically, the atom goes over from an unperturbed state to a QES, so that expression (7) is valid for the scattering amplitude. If the atom was in one of the states of the resonating multiplet prior to the turning on of the field, the condition for the adiabaticity of the turning of the field can be written in the form $\tau\Delta\gg1$, where τ is the turning-on time, $\Delta=|E^b-E^a-\omega|$ is the detuning from resonance, and

$$E^{a,b}=s_{a,b}^{-1}\sum_{i=1}^{s_{a,b}}E_i^{a,b}.$$

If, however, the atom was in some other state prior to the turning on of the field, then the adiabaticity condition takes the form $\tau\omega_a\gg1$, where ω_a is of the order of the frequency of the atomic transitions, and is satisfied for much smaller τ. We shall assume below the adiabaticity condition to be satisfied, and assume furthermore that spontaneous transitions during the time when the field is turned on can be neglected.

Assume that prior to the collision the atom was in a state φ_{ε_1} of the form (9). Using the explicit form of (2) of the interaction V and the function $\varphi_{\mathbf{p}}$ (6), we obtain according to (7) the following expression for the amplitude of the elastic (i.e., without change of the state of the atom) scattering with emission (absorption) of k photons:

$$\begin{aligned}f_{11,k}=-2\rho_{1k}^{-2}\Big\{Q_k(\gamma_{1k})\Big[&\sum_{i=1}^{s_a}(F_i^a(\rho_{1k})-Z)|C_{\varepsilon_1i}^a|^2\\&+\sum_{i=1}^{s_b}(F_i^b(\rho_{1k})-Z)|C_{\varepsilon_1i}^b|^2+\sum_{i\neq j}^{s_a}G_{i,j}^{a,a}(\rho_{1k})C_{\varepsilon_1i}^{a*}C_{\varepsilon_1j}^a+\sum_{i\neq j}^{s_b}G_{i,j}^{b,b}(\rho_{1k})C_{\varepsilon_1i}^{b*}C_{\varepsilon_1j}^b\Big]\\&+Q_{k+1}(\gamma_{1k})\sum_{i=1}^{s_a}\sum_{j=1}^{s_b}G_{i,j}^{a,b}(\rho_{1k})C_{\varepsilon_1i}^{a*}C_{\varepsilon_1j}^b\\&+Q_{k-1}(\gamma_{1k})\sum_{j=1}^{s_b}\sum_{i=1}^{s_a}G_{j,i}^{b,a}(\rho_{1k})C_{\varepsilon_1j}^{b*}C_{\varepsilon_1i}^a\Big\}.\end{aligned} \tag{13}$$

Here $\rho_{1k}=\mathbf{p}_{1k}-\mathbf{p}_1$, the index ε_1 indicates that the coefficients $C_j^{a,b}$ are the solution of the system of Eqs. (10) corresponding to $\varepsilon=\varepsilon_1$,

$$F_j^{a,b}(\rho)=\sum_{\alpha=1}^{z}\int d\mathbf{q}|\Phi_j^{a,b}(\mathbf{q})|^2e^{-i\rho\mathbf{q}\alpha} \tag{14}$$

are the atomic form factors, and

$$G_{i,j}^{a,b}(\rho)=\sum_{\alpha=1}^{z}\int d\mathbf{q}\Phi_i^{a*}(\mathbf{q})\Phi_j^b(\mathbf{q})e^{-i\rho\mathbf{q}\alpha},\quad Q_n(\gamma_{1k})=e^{in\varphi_0}J_n(\gamma_{1k}), \tag{15}$$

where

$$J_n(\gamma)=(\omega/2\pi)\int_0^{2\pi/\omega} dt\exp[i(\gamma\sin\omega t-n\omega t)]$$

are Bessel functions of n-th (integer) order. $Q_n(\gamma_{1k})$ has this form in the general case of elliptic wave polarization, when

$$\mathbf{a}(t)=\mathbf{a}_1\sin\omega t+\varphi\mathbf{a}_2\cos\omega t,\quad \mathbf{a}_1\mathbf{a}_2=0,$$

where $-1\leqslant\varphi\leqslant 1$ is the degree of ellipticity of the polarization,

$$|\mathbf{a}_1|=|\mathbf{a}_2|=F_0\omega^{-2},\quad \gamma_{1k}=[(\mathbf{a}_1\rho_{1k})^2+\varphi^2(\mathbf{a}_2\rho_{1k})^2]^{1/2},$$
$$\sin\varphi_0=\varphi\mathbf{a}_2\rho_{1k}\gamma_{1k}^{-1},\quad \cos\varphi_0=\mathbf{a}_1\rho_{1k}\gamma_{1k}^{-1}.$$

The same procedure is used to determine the amplitude of the scattering $f_{i1,k}$, $i\leqslant s_a$ with transition of the atom into a QES corresponding to an arbitrary state of the lower atomic multiplet. The amplitude is obtained from (13) by replacing ρ_{1k} by ρ_{ik}, γ_{1k} by γ_{ik}, and $C^{a,b}_{\varepsilon_i j}{}^*$ by $C^{a,b}_{\varepsilon_i j}{}^*$.

The scattering amplitude with transition of an atom into one of the states (11), corresponding to the upper atomic multiplet, and emission (absorption of k photons) can be represented in the form

$$f_{i1,k}=-2\rho_{ik}^{-2}\Big\{Q_{k+1}(\gamma_{ik})\Big[\sum_{j=1}^{s_a}(F_j^a(\rho_{ik})-Z)C^{a*}_{\varepsilon_i j}C^a_{\varepsilon_1 j}+\sum_{j=1}^{s_b}(F_j^b(\rho_{ik})-Z)C^{b*}_{\varepsilon_i j}C^b_{\varepsilon_1 j}+\sum_{j\neq j'}^{s_a}G^{a,a}_{j,j'}(\rho_{ik})C^{a*}_{\varepsilon_i j}C^a_{\varepsilon_1 j'}$$
$$+\sum_{j\neq j'}^{s_b}G^{b,b}_{j,j'}(\rho_{ik})C^{b*}_{\varepsilon_i j}C^b_{\varepsilon_1 j'}\Big]+Q_k(\gamma_{ik})\sum_{j=1}^{s_b}\sum_{j'=1}^{s_a}G^{b,a}_{j,j'}(\rho_{ik})C^{b*}_{\varepsilon_i j}C^a_{\varepsilon_1 j'}$$
$$+Q_{k+2}(\gamma_{ik})\sum_{j=1}^{s_a}\sum_{j'=1}^{s_b}G^{a,b}_{j,j'}(\rho_{ik})C^{a*}_{\varepsilon_i j}C^b_{\varepsilon_1 j'}\Big\},\qquad (16)$$

where $\rho_{ik}=\mathbf{p}_{ik}-\mathbf{p}_1$, $p_{ik}^2=p_1^2+2(\varepsilon_1-\varepsilon_i'-k\omega)$, $s_a<i\leqslant s_a+s_b$.

We present also an expression for the amplitude of scattering with transition of the atom to the nonresonant state (12):

$$f_{c1,k}=-2\rho_{ck}^{-2}\Big\{Q_k(\gamma_{ck})\sum_{j=1}^{s_a}G_j^{c,a}(\rho_{ck})C^a_{\varepsilon_1 j}+Q_{k+1}(\gamma_{ck})\sum_{j=1}^{s_b}G_j^{c,b}(\rho_{ck})C^b_{\varepsilon_1 j}\Big\},\qquad (17)$$

where

$$G_j^{c,a}(\rho)=\sum_{\alpha=1}^{z}\int d\mathbf{q}\Phi^{c*}(\mathbf{q})\Phi_j^a(\mathbf{q})e^{-i\rho\mathbf{q}_\alpha}.$$

The coefficients $C^{a,b}_{\varepsilon_i j}$ in the formulas obtained for the scattering amplitudes have a resonant dependence on the electromagnetic-radiation frequency. Far from resonance, $|W_{a_i b_j}|\ll\Delta$, the functions $\varphi_{\varepsilon i}$ (9) $i\leqslant s_a$ go over into a lower-multiplet state unperturbed by the field, while the functions $\varphi_{\varepsilon i}$ (11), $s_a<i\leqslant s_a+s_b$, go over to the upper multiplet. This means that far from resonance the coefficients $C^{a,b}_{\varepsilon_i j}$ have the following behavior[2]:

$$C^a_{\varepsilon_i j}\to\delta_{ij},\quad C^b_{\varepsilon_i j}\to 0,\quad i\leqslant s_a;$$

$$C^a_{\varepsilon_i j}\to 0,\quad C^b_{\varepsilon_i j}\to\delta_{i,j+s_a},\quad i>s_a.$$

Near resonance, $|W_{a_i b_j}|\gtrsim\Delta$, all the coefficients $C^{a,b}_{\varepsilon_i j}$ have the same order of magnitude and the amplitudes of the processes (13), (16), and (17) differ substantially from the nonresonant ones.

In the case of a weak field, when $\gamma_{ik}\ll 1$, the Bessel functions in the equations for the scattering amplitudes can be represented in the form $J_n(\gamma)\approx(n!)^{-1}(\gamma/2)^n$ at $n\geqslant 0$, $J_{-n}=(-1)^nJ_n$ so that the resonant terms, on the one hand, violate the simple powerlaw dependence of the multiphoton-transition amplitude on the field strength, and on the other lead to a considerable increase of the amplitude. Thus, in inelastic transitions from the states of a lower resonant multiplet with photon absorption, the resonant scattering amplitude exceeds considerably the nonresonant one, since $|J_{k+1}|\gg|J_k|$ at $k\leqslant -1$ and $|J_{k+2}|\gg|J_{k+1}|$ at $k\leqslant -2$ [see Eqs. (16) and (17)]. In the case of elastic scattering, the increase of the amplitude (13) takes place near resonance both when photons are emitted and absorbed.

The increase of the amplitude of scattering with emission (absorption) of photons in a weak field near resonance can be explained using simple qualitative considerations. In a weak field, the probabilities of direct absorption (emission) of a photon by an electron in a collision are low. In the case of resonance, the photon absorption (emission) can be represented as a cascade process: a transition of an atom from one resonant state to another with absorption or emission of photons are subsequent exchange of energy with the electron, or emission of photons followed by exchange of energy with electron, absorption (emission) of photons by the electron. In such a cascade transition the electron absorbs (emits) directly a smaller number of photons, and the transition probability increases correspondingly. Thus, in elastic scattering with absorption of photons the atom absorbs resonantly a photon and transfers the excitation energy to the electron, thus returning to the initial state of the lower multiplet. This transition is described by the second term of (13), which in the case of resonance exceeds substantially the remaining terms. When photons are emitted in the elastic collision process, one photon is resonantly emitted by the atom previously excited by an excited electron into the upper multiplet state [see the third term in (13)]. In inelastic scattering with transition of the atom into a nonresonant state and with absorption of photons, one photon is resonantly absorbed by the atom, after which a transition of the atom to the final state takes place, accompanied by a change of energy with the electron [see the second term in (17)]. Finally, in scattering with excitation of the state of the upper multiplet and with absorption of a photon, the atom absorbs resonantly a photon [see the first term of (16)]. On the other hand, if more than one photon is produced in the collision, the atom transfers the energy to the electron after the resonant absorption and goes over to a lower-multiplet state, from which it is resonantly excited by absorbing another photon. Such a transition is described by the last term of (16).

3. In the case of resonance between two states, the equations obtained for the transition amplitudes in the preceding section coincide with the results of Refs. 6 and 7. We consider here in greater detail the scatter-

ing of an electron by an atom under conditions of resonance between the ground state with energy E^a and the doublet E_1^b, E_2^b.

The system of Eqs. (10) for the coefficients of the linear superposition of the atomic state that are not perturbed by the field takes the form

$$C_1^a(\varepsilon-E^a)-C_1^bW_1-C_2^bW_2=0, \quad C_i^b(\varepsilon-\varepsilon_i^b)-C_1^aW_i^*=0, \qquad (18)$$

where $W_i=W_{a_1b_i}$, $\varepsilon_i^b=E_i^b-\omega$, $i=1,2$. The roots of the corresponding secular equation

$$(\varepsilon-E^a)(\varepsilon-\varepsilon_1^b)(\varepsilon-\varepsilon_2^b)-|W_1|^2(\varepsilon-\varepsilon_2^b)-|W_2|^2(\varepsilon-\varepsilon_1^b)=0 \qquad (19)$$

can be obtained from the Cardano formulas

$$\varepsilon_i=y_i-a/3,$$

where

$$y_{1,2}=-(A+B)/2\pm i(A-B)\cdot 3^{1/2}/2, \; y_3=A+B,$$
$$A=(u+v^{1/2})^{1/3}, \quad B=(u-v^{1/2})^{1/3}, \quad AB=(a^2/3-b)/3,$$
$$u=ab/6-c/2-(a/3)^3, \quad v=u^2+(b-a^2/3)^3/27,$$

a, b, and c are the coefficients of the powers of ε in the cubic Eq. (19):

$$a=-(E^a+\varepsilon_1^b+\varepsilon_2^b), \quad b=E^a(\varepsilon_1^b+\varepsilon_2^b)+\varepsilon_1^b\varepsilon_2^b-|W_1|^2-|W_2|^2,$$
$$c=\varepsilon_1^b|W_2|^2+\varepsilon_2^b|W_1|^2-E^a\varepsilon_1^b\varepsilon_2^b.$$

From the system (18) and from the normalization condition

$$|C_1^a|^2+|C_1^b|^2+|C_2^b|^2=1 \qquad (20)$$

we obtain the coefficients $C^a_{\varepsilon_f 1}, C^b_{\varepsilon_f 1}, C^b_{\varepsilon_f 2}$:

$$C^b_{\varepsilon_f j}=C^a_{\varepsilon_f 1}W_j^*(\varepsilon_f-\varepsilon_j^b)^{-1}, \quad C^a_{\varepsilon_f 1}=[1+|W_1|^2(\varepsilon_f-\varepsilon_1^b)^{-2}+|W_2|^2(\varepsilon_f-\varepsilon_2^b)^{-2}]^{-1/2}.$$

In the particular case $E_1^b=E_2^b=E^b$ (the formulas that follow are valid also in the case of a "strong" field, $|W_i|\gg E_2^b-E_1^b$) the expressions for the coefficients and for the values of the quasienergy are simplified. We obtain directly from (19) the values of the quasienergy

$$\varepsilon_{1,2}=(E^a+E^b-\omega\pm\Omega)/2, \quad \varepsilon_3=E^b-\omega, \qquad (21)$$

where $\Omega=[\delta^2+4(|W_1|^2+|W_2|^2)]^{1/2}$, $\delta=E^b-E^a-\omega$. For the coefficients, taking the normalization (20) into account, we obtain the following expressions ($i,j=1,2$):

$$C^a_{\varepsilon_f 1}=[\Omega+(-1)^i\delta]^{1/2}(2\Omega)^{-1/2}, \quad C^b_{\varepsilon_f j}=(-1)^{i+1}2W_j^*[2\Omega(\Omega+(-1)^i\delta)]^{-1/2}, \qquad (22)$$
$$C^a_{\varepsilon_{3'}1}=0, \quad C^b_{\varepsilon_{3'}1}=W_2(|W_1|^2+|W_2|^2)^{-1/2}, \quad C^b_{\varepsilon_{3'}2}=-W_1(|W_1|^2+|W_2|^2)^{-1/2}.$$

As follows from (21) and (22), the function $\varphi_{\varepsilon'_3}$ (11) is a linear superposition of the unperturbed states of the upper degenerate level, $\varepsilon'_3=E^b$. In the case $\delta<0$ the QES φ_{ε_1} (9) goes over, when the field is turned off, into a lower unperturbed state of the atom, while φ'_{ε_2} goes over into a superposition of upper states. If the detuning δ is of the opposite sign, the functions φ'_{ε_1} and φ_{ε_2} go over into a superposition of upper states and into a lower state of the atom when the field is turned off. Assuming for the sake of argument $\delta<0$, so that when the field is turned on adiabatically the atom goes over from the ground state into the QES φ_{ε_1}, we obtain, say, for the amplitude of elastic scattering with emission (absorption) of a photons (13) the expression

$$f_{11,k}=-2\rho_{1k}^{-2}\Big\{Q_k(\gamma_{1k})\Big[F_1^a(\rho_{1k})(1+|\delta|/\Omega)/2-Z$$
$$+2[\Omega(\Omega+|\delta|)]^{-1}\Big[\sum_{j=1}^{2}F_j^b(\rho_{1k})|W_j|^2+\sum_{i\neq j}^{2}G_{ij}^{b,b}(\rho_{1k})W_iW_j^*\Big]\Big]$$
$$+\Omega^{-1}\Big[Q_{k+1}(\gamma_{1k})\sum_{j=1}^{2}G_{1,j}^{a,b}(\rho_{1k})W_j^*+Q_{k-1}(\gamma_{1k})\sum_{j=1}^{2}G_{j,1}^{b,a}(\rho_{1k})W_j\Big]\Big\}, \qquad (23)$$

where the form factors F and the functions G are defined by (14) and (15). Equation (13) determines explicitly the resonant behavior of the scattering amplitude $f_{11,k}$. We note that the simplest example of the considered type is scattering of an electron by a hydrogen atom under conditions of resonance between the ground $1S$ and the $2P$ levels in a field of elliptic polarization, when the ground and $2P$ states are mixed, with orbital-momentum projection ±1 on the field propagation direction.

4. We discuss now some generalizations and consequences of the results.

We have confined ourselves above to electron scattering by an atom in the case of one-photon resonance between two multiplets. In the case of two-photon resonance, and also when account is taken of the contribution of the nonresonant levels in one-photon resonance, the system (10) is somewhat modified. It contains composite matrix elements of second order, which determine the restructuring of the levels of each multiplet, and in the case of two-photon resonance, in addition, the matrix elements $W_{a_ib_j}$ are replaced by corresponding composite matrix elements of second order. Such a system of equations was obtained in Ref. 12 in connection with an investigation of the restructuring of atomic multiplets in two-photon resonance. For alkali-metal atoms the composite matrix elements can be calculated by the model-potential method.[13] A more significant difference of the case of two-photon resonance is the change in the indices of the functions Q_k in Eqs. (13), (16), and (17) for the scattering amplitudes, namely, $k+1$ is replaced by $k+2$, and $k+2$ by $k+4$.

In the case of resonance of multiplicity $m>2$, the off-diagonal composite matrix elements which couple the resonant states are of the order of $(F_0/F_a)^m$ in the field, and the composite matrix elements were coupled with states of a given multiplet r of the order of $(f_0/f_a)^2$. At small detunings, $|E^b-E^a-m\omega|\leq(F_0/F_a)^2$, the contribution of the resonant terms to the scattering amplitude turns out to be substantial only in a sufficiently weak field, $F_0/F_{cr}\leq(F_{cr}/F_a)^{m/2-1}$, for transitions with emission (absorption) of $k\geq m$ photons. The nonresonant terms are in this case of the order of $(F_0/F_{cr})^k$, and the main contribution is made by the resonant terms $\sim(F_0/F_{cr})^{k-m}(F_0/F_a)^{m-2}$ [for inelastic scattering with transition into an upper multiplet state at $k\geq 2m$ the main contribution is made by terms $\sim(F_0/F_{cr})^{k-2m}(F_0/F_a)^{2m-4}$].

We have obtained the amplitudes of scattering with transition from the state φ_{ε_1}, corresponding to the lower atomic multiplet. The expressions for the amplitudes of scattering with transition from states of the upper multiplet have the same structure, except that the

functions Q_k have different indices: $k \pm 1$ is replaced by $k \mp 1$ and $k+2$ by $k-2$. If the atom was in some nonresonant state prior to the collision, the amplitude of scattering with transition to another nonresonant state does not have a resonant part in the Born approximation, and the scattering amplitude with transition into one of the states of the resonating multiplets has a structure of the form (17).

Analysis of the resonance between multiplets makes it possible to take into account the strong influence of the radiation polarization on the mixing of the atomic states, and consequently on the scattering amplitude. Thus, for the resonant $1S$ and $2P$ levels of hydrogen in a field with linear or circular polarization, only two atomic states with definite projections of the orbital momentum will be mixed, in contrast to the usual case of elliptic polarization of the field, when the ground and $2P$ states are mixed with projections of the angular momentum ± 1 on the field propagation direction.

It was noted in the analysis of the resonance that the amplitude of the inelastic scattering of an electron by an atom, with absorption of photons in a weak field, increases strongly if the atom is in one of the lower multiplet states prior to the application of the field. Consequently, in the resonance region, at relatively low field intensity, $F_0 \ll F_{cr}$, the radiation is predominantly absorbed in the collision process in this case.

On the other hand, if the atom was excited into one of the upper multiplet states prior to the application of the field, it is obvious that under these conditions predominant emission of photons should take place in the course of the collision. This conclusion follows directly on the change, indicated above, of the form of the inelastic scattering amplitudes for this case.

[1)] We use here the atomic system of units $\hbar = m = e = 1$.

[2)] If some of the multiplet levels are degenerate, the corresponding unperturbed states of the atoms are not uniquely defined. The coefficients $C^a_{\tilde{\epsilon}_i j}$, $i \leqslant s_a$, and $C^b_{\tilde{\epsilon}_i j}$, $i > s_a$ will behave in the indicated manner if the unperturbed states are so chosen that the QES go over into these states when the field is turned off.

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