Influence of gravitation on the self-energy of charged particles

A. I. Zel'nikov and V. P. Frolov

P. N. Lebedev Physics Institute, USSR Academy of Sciences
(Submitted 12 June 1981)
Zh. Eksp. Teor. Fiz. **82**, 321–335 (February 1982)

The influence of the gravitational field of a charged black hole on the self-energy of an electric and scalar charge at rest in this field is investigated. An exact expression is obtained for the energy of the charged particle, and the transition to the limit of a homogeneous gravitational field is investigated.

PACS numbers: 95.30.Sf, 97.60.Lf

1. INTRODUCTION

The problem of the electromagnetic mass of the electron is one of the most "ancient" problems of modern theoretical physics. Even before the creation of the special theory of relativity Abraham proposed a model of a classical electron in the form of a spherically symmetric system of rigidly connected (in the sense of classical mechanics) electric charges and investigated in the framework of this model the dependence of the electromagnetic mass on the velocity. Later, Lorentz modified Abraham's theory to bring it into accord with the hypothesis advanced by Fitzgerald and himself to the effect that all rigid bodies undergo a contraction in the direction of their motion in accordance with the ratio $(1-v^2/c^2)^{1/2}:1$. The appearance of the special theory of relativity led to understanding of the fact that the inertial mass (both electromagnetic and of other origin) must depend on the velocity, the connection always having the form $m=E/c^2$, where E is the total energy of the system. A contradiction between the results of Abraham and Lorentz, which contain an extra factor $\frac{4}{3}$ compared with the general relations of relativity theory, was resolved by Laue, Poincaré, and Fermi, who showed that this factor can be explained by the contribution to the energy of nonelectromagnetic interactions, whose existence is needed to ensure equilibrium of the charges; they showed that the factor does not arise in a consistent theory.

In 1921, Fermi[1] investigated the influence of a homogeneous gravitational field on the self-energy of an electric charge. He showed that the electromagnetic interaction, which gives rise to a change $\Delta m=\Delta E_{em}/c^2$ in the inertial mass of the particle, simultaneously produces exactly the same change in its gravitational mass, in complete agreement with the equivalence principle. For an inhomogeneous gravitational field, arguments based on the equivalence principle are not valid when the system as a whole is considered, and, in general, it is to be expected that the relationship between the self-energy of a charged particle and the shift of its gravitational mass will be more complicated.

For a particle in the field of a black hole, this is indeed the case, and the corresponding corrections (in the approximation $GM/c^2r\ll 1$) were obtained in Ref. 2, in which it was also shown that the effects of the inhomogeneity of the gravitational field lead to the appearance of an additional force of repulsion in the direction of the black hole. Even earlier, Unruh[3] had shown that a similar force acts on a test charge placed within a thin hollow massive shell. Smith and Will[4] and, independently, the present authors[5] obtained an exact expression for the energy of an electric charge at rest in the field of a neutral black hole. This expression can be written in the form

$$\frac{E}{c^2}=\left(1-\frac{2GM}{c^2r_0}\right)^{1/2}\left(m_0+\frac{e^2}{2\delta c^2}+\frac{e^2w}{2c^4}\right).$$

It is interesting to note that in this case the effects of the inhomogeneity of the field reduce to the appearance of a term proportional to the 4-acceleration w of the particle. It is the presence of this term that leads to the additional gravitational force of repulsion exerted on the charged particle by the black hole; as a result, the world line of the motion of the charged particle will differ slightly from a geodesic. Smith and Will also have a detailed "local" derivation of the expression for this force in the spirit of the general treatment of DeWitt and DeWitt.[6] The aim of the present paper is to extend the previously obtained results to the case when the black hole is charged, and to investigate in detail the influence of the field of the black hole on the self-energy of a scalar charge (Sec. 2), and also to discuss the transition to the limit of a homogeneous gravitational field in the case under consideration (Sec. 3).

In the present paper, the signature of the metric and the signs in the definition of the curvature tensors are as in the book Ref. 7 and, except in the final expressions, we use a system of units throughout in which $G=c=1$.

2. SELF-ENERGY OF ELECTRIC AND SCALAR CHARGES IN THE GRAVITATIONAL FIELD OF A CHARGED BLACK HOLE

2.1. Classical charged particle in a static gravitational field

Suppose a classical particle, i.e., a system of bound electric or scalar charges, is held at rest in a static gravitational field by an appropriately chosen external force. We denote by ξ^μ the Killing vector field. Then the energy of the system is

$$E=\int_{\Sigma} T_{\mu\nu}\xi^{\mu}d\Sigma^{\nu},$$

where $T_{\mu\nu}$ is the total metric energy–momentum tensor of the system. To be specific, we shall adopt as model of the charged particle a rigid nonconducting thin sphere of mass m_0 and radius ε over whose surface there is uniformly distributed the electric (e) or scalar (g) charge. Let $l(\mathbf{x},\mathbf{y})$ be the invariant distance between the points $(t,\mathbf{x})$ and $(t,\mathbf{y})$ calculated along the geodesic joining these points; then the charge density of such a particle at rest at the point x_0 is proportional to $\delta[l(\mathbf{x},\mathbf{x}_0)-\varepsilon]$. Having in mind a subsequent transition to a point particle, we shall assume that ε is much less than the characteristic inhomogeneity scales of the gravitational field and the external electromagnetic (or scalar) field, and in the final result we shall ignore terms $O(\varepsilon)$.

The total energy E is made up of: 1) E_0, the part of the energy association with the "mechanical" mass $m_0(E_0=|\xi^2(\mathbf{x}_0)|^{1/2}m_0c^2)$; 2) E_{self}, the self-energy, i.e., the energy of the self-interaction of the charge of the particle; 3) E_{ext}, the energy of the interaction of the particle with the external field; 4) E_{int}, the energy of the additional interaction that ensures the stability of the charged particle. The introduction of this additional interaction ensures fulfillment of Lave's theorem. If we denote by ε_0 the equilibrium radius of the uncharged sphere, then

$$E_{int}(\varepsilon)=E_{int}(\varepsilon_0)+K(\varepsilon-\varepsilon_0)^2/2$$

and, choosing a sufficiently large value of the effective rigidity K, we can make the changes induced by the introduction of the charge in the value of the equilibrium radius, $\Delta\varepsilon=\varepsilon-\varepsilon_0$, and the energy, $\Delta E=E_{\text{int}}(\varepsilon)-E_{\text{int}}(\varepsilon_0)$, arbitrary small. In what follows, we shall ignore ΔE assuming that K is appropriately chosen, and, including E_0 in the constant $E_{\text{int}}(\varepsilon_0)$, we write the expression for the total energy in the form

$$E=E_0+E_{em(sc)},\quad E_{em(sc)}=E_{self}^{em(sc)}+E_{ext}^{em(sc)}. \tag{2.1}$$

2.2. Electromagnetic interaction

The original action for the electromagnetic field from the source J^{μ} in space-time with given metric $g_{\mu\nu}$ has the form

$$W[A,J]=-\frac{1}{16\pi}\int F_{\mu\nu}F^{\mu\nu}\sqrt{-g}\,d^4x+\int A_{\mu}J^{\mu}\sqrt{-g}d^4x. \tag{2.2}$$

If we write

$$T_{\mu\nu}[F_1,F_2]=\frac{1}{4\pi}\left[F_{(1)\mu\alpha}F_{(2)\nu}{}^{\alpha}-\frac{1}{4}F_{(1)\alpha\beta}F_{(2)}^{\alpha\beta}\right], \tag{2.3}$$

then the metric energy–momentum tensor for the action (2.2) is $T_{\mu\nu}[F,F]$. For the charged particle, we have

$$J^{\mu}=j^{\mu}+J^{\mu}_{ext},\quad A^{\mu}=a^{\mu}+A^{\mu}_{ext},$$

where a^{μ} is the field generated by the current j^{μ} of the particle, and A^{μ}_{ext} is the external field (of the current J^{μ}_{ext}) which maintains the charge in equilibrium in the static gravitational field. The energy E_{em} for electric charge e is defined as the difference between the total energy of the system with this charge and the energy of the system for $e=0$, and it is equal to

$$E_{em}=E_{self}^{(em)}+E_{ext}^{(em)},\quad E_{self}^{(em)}=\int_{\Sigma} T_{\mu\nu}[f,f]\xi^{\mu}d\Sigma^{\nu},$$
$$E_{ext}^{(em)}=\int_{\Sigma}(T_{\mu\nu}[f,F_{ext}]+T_{\mu\nu}[F_{ext},f])\xi^{\mu}d\Sigma^{\nu},\quad f_{\mu\nu}=2a_{[\nu,\mu]}. \tag{2.4}$$

The problem in which we are interested consists of calculating the energy E_{em} for the case when the charged particle is at rest in the field of a charged black hole of mass M and charge Q. The standard Penrose diagram for the space-time in this case is shown in Fig. 1. In region I, which is covered by the coordinates (t,r), the metric is

$$ds^2=-F(r)dt^2+F^{-1}(r)dr^2+r^2d\sigma^2,$$
$$F(r)=-\xi_{\mu}(r)\xi^{\mu}(r)=1-2M/r+Q^2/r^2, \tag{2.5}$$

where $\xi^{\mu}\partial_{\mu}=\partial_t$ is the Killing vector field, and $d\sigma^2=d\theta^2+d\varphi^2\sin^2\theta$ is the element of length on the unit sphere. In what follows, we shall regard the electric field

$$\hat{A}_{(BH)}=A_{(BH)\mu}dx^{\mu}=(Q/r^2)\,dt$$

of the black hole as part of the external field $\hat{A}_{\text{ext}}$.

The expression for the electromagnetic field produced at the point $\mathbf{x}=(r,\theta,\varphi)$ by a point charge e at rest at the point $\mathbf{x}'=(r',\theta',\varphi')$ in the gravitational field (2.4) was obtained by Leaute and Linet,[8,9] who found and corrected a small error in the expression derived earlier by Copson.[10] It has the form

$$\hat{a}(\mathbf{x})=eG_{(em)}(\mathbf{x},\mathbf{x}')\,dt,\quad G_{(em)}(\mathbf{x},\mathbf{x}')=-\frac{1}{rr'}(M+\Pi/R), \tag{2.6}$$

$$\Pi=\Pi(\mathbf{x},\mathbf{x}')=(r-M)(r'-M)-(M^2-Q^2)\lambda,$$
$$R=R(\mathbf{x},\mathbf{x}')=[(r-M)^2+(r'-M)^2$$
$$-2(r-M)(r'-M)\lambda-(M^2-Q^2)(1-\lambda^2)]^{1/2}, \tag{2.7}$$
$$\lambda=\lambda(\mathbf{x},\mathbf{x}')=\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos(\varphi-\varphi').$$

This solution, obtained in region I, can be extended by analytic continuation to the whole of the space-time.

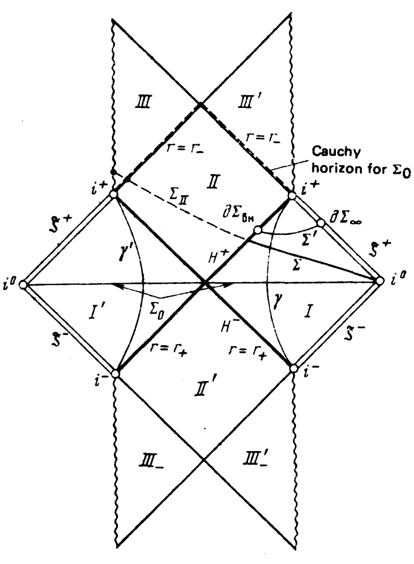

FIG. 1. Penrose diagram of the Reissner–Nordström space-time of the black hole.

However, it is then found (see Appendix 1) that the analytically continued solution has a singularity corresponding to the presence of an additional point electric charge $-e$ at rest in region I′. If such charges are absent and the charge e is the only field source, then outside the domain of influence of this charge (i.e., below the past Cauchy horizon H^-) a field must be absent: $f_{\mu\nu}=0$. The fulfillment of the vacuum Maxwell equations in the neighborhood of H^- leads in this case to the appearance of an additional field $f_{(\mathrm{sing})\mu\nu}$, whose support is concentrated on H^-. The corresponding results are given in Appendix 1.

The integration in (2.4) is over the complete spacelike surface Σ, which intersects the horizon H^+ (see Fig. 1). Note however that the integral (2.4) over the part of Σ within the event horizon is the energy of the field within the black hole, and a corresponding contribution occurs as a part in the definition of the total mass of the black hole. Therefore, being interested in what follows in calculating the energy shift in the field of a given black hole, we assume that the mass of the black hole is fixed and accordingly we shall integrate in (2.4) over the part of Σ outside the black hole. For a static[1)] field, the value of E_{em} does not change when there are deformations of Σ with fixed boundaries $\partial\Sigma_{\mathrm{BH}}\subset H^+$ and $\partial\Sigma_\infty\subset i^0$ or deformations for which $\partial\Sigma_{\mathrm{BH}}$ moves along H^+ and $\partial\Sigma_\infty$ along $\mathscr{T}^+$. In particular, the values of E_{em} calculated for the surfaces Σ and Σ' (see Fig. 1) are equal. Taking Σ to be the surface $t=\mathrm{const}$, we have[2)]

$$E_{self}^{(em)}=\frac{1}{8\pi}\int\limits_{r>r_+}F^{-1}\nabla^i a_t\nabla_i a_t\sqrt{-g}\,d^3x \tag{2.8}$$

$$=-{}^1/_2\int\limits_{r>r_+}d^3x\int\limits_{r>r_+}d^3x'j^t(\mathbf{x})\sqrt{-g(\mathbf{x})}G_{(em)}(\mathbf{x},\mathbf{x}')\sqrt{-g(\mathbf{x}')}j^t(\mathbf{x}').$$

Here, integrating by parts, we have used the field equations $\partial_i(\sqrt{-g}f^{0i})=4\pi\sqrt{-g}j^0$ and the fact that, since the potential $a_t(r_+)$ is constant on the surface of the black hole and the induced charge ΔQ of the black hole is 0, the surface integral, which is proportional to $a_t(r_+)\Delta Q$, also vanishes.

Substituting in (2.8) the expression for the density of a charge distributed uniformly over a sphere of radius ε with center at the point $\mathbf{x}_0=(r_0,\theta_0,\varphi_0)$,

$$j^0(\mathbf{x})=\frac{e}{4\pi\varepsilon^2\sqrt{-g(\mathbf{x})}}\delta(l(\mathbf{x},\mathbf{x}_0)-\varepsilon), \tag{2.9}$$

and the expression (2.6) for $G_{(em)}(\mathbf{x},\mathbf{x}')$, and making the necessary calculations (see Appendix 3), we obtain

$$E_{self}=\frac{e^2}{2\varepsilon}F^{1/2}(r_0)+\frac{e^2M}{2r_0^2}. \tag{2.10}$$

So far, we have considered the case when the charged particle is at rest in the field of and "eternal" black hole. However, the obtained expressions are also valid in the case when the black hole is formed in a process of gravitational collapse. The Penrose diagram for the space-time in this case is shown in Fig. 2. We choose an arbitrary spacelike surface Σ' whose boundary $\partial\Sigma'_{\mathrm{BH}}$ on the event horizon H^+ is situated outside the collapsing matter. Then it is easy to show that the minimal value of $E_{\mathrm{self}}^{(\mathrm{em})}$ for fixed position of the charge is attained for this surface when the field is static in the neighborhood of Σ', and the corresponding minimal value, which determines the self-energy of the charged particle, is obviously equal to the value $E_{\mathrm{self}}^{(\mathrm{em})}$ calculated for an eternal black hole.

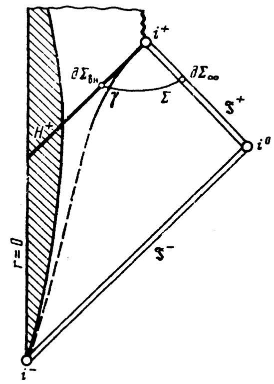

FIG. 2. Penrose diagram of the space-time of a charged black hole formed as a result of gravitational collapse.

The expression (2.4) for $E_{\mathrm{ext}}^{(\mathrm{em})}$ can be readily transformed to

$$E_{ext}^{(em)}=-\int j^t(\mathbf{x})\sqrt{-g(\mathbf{x})}A_{(ext)t}(\mathbf{x})\,d^3x=-eA_{(ext)t}(\mathbf{x}_0). \tag{2.11}$$

Thus, the total energy of the electrically charged particle at rest near the charged black hold at the point x_0 is

$$E=\tilde{m}c^2\left(1-\frac{2M}{r_0}+\frac{Q^2}{r_0^2}\right)^{1/2}-eA_{(ext)t}(r_0), \tag{2.12}$$

$$\tilde{m}=m+\Delta m,\quad m=m_0+\frac{e^2}{2\varepsilon c^2},$$
$$\Delta m=\frac{e^2M}{2r_0^2}F^{-1/2}(r_0). \tag{2.13}$$

Here, m_0 is the "mechanical" mass of the particle. For a particle far from the black hole $\Delta m\to 0$, and $\tilde{m}$ in the limit is equal to the expression m for the mass of a charged particle in flat space. The part of the energy $\Delta E=e^2M/2r_0^2$ associated with Δm is due to the rearrangement of the electric field of the charge in the gravitational field of the black hole.

Note that the appearance of this additional part of the energy is entirely due to the term M/rr' in the Green's function $G_{(em)}(\mathbf{x},\mathbf{x}')$. In accordance with Hadamard's general theory, the Green's function in 3-dimensional space has the form

$$G(\mathrm{x},\mathrm{x}')=u(\mathrm{x},\mathrm{x}')[\sigma(\mathrm{x},\mathrm{x}')]^{-1/2}+v(\mathrm{x},\mathrm{x}'),$$

where $\sigma(\mathbf{x},\mathbf{x}')$ is the square of the geodesic distance between the points $\mathbf{x}$ and $\mathbf{x}'$, and u and v are regular functions. The function u is determined uniquely but v is determined only up to a solution of the homogeneous equation and is fully determined by the choice of the boundary conditions. In the considered problem, the term M/rr' in the Green's function $G_{(em)}(\mathbf{x},\mathbf{x}')$ is the solution of the homogeneous equation. Thus, the appearance of the additional part $c^2\Delta m$ of the energy of the charge is due to the global properties of our system, namely, the boundary conditions imposed on the field on the horizon; it is not due to the local inhomogeneity of the gravitational field near the charge.

For a neutral black hole, Δm can be written in the form

$$\Delta m=\frac{e^2}{2}\frac{w}{c^4},\quad w=|w^\mu w_\mu|^{1/2}=\frac{M}{r_0^2}\left(1-\frac{2M}{r_0}\right)^{-1/2},\tag{2.14}$$

where w_μ is the 4-acceleration of the charged particle. This expression for the shift in the energy in the case of a neutral black hole agrees with the expression obtained earlier in Refs. 4 and 5.

The equilibrium position of the charged particle is determined by the condition $\partial E/\partial r_0=0$. This condition can be written in the form

$$\mu\frac{du^\alpha}{d\tau}=eF^{\alpha}_{ext\,\beta}u^\beta,\tag{2.15}$$

where u^α is the 4-velocity of the particle at rest and

$$\mu=m-e^2\frac{M}{Mr_0-Q^2}F^{1/2}(r_0).\tag{2.16}$$

Note that the "inertial" mass of the particle defined in this manner is not equal to $\tilde{m}$. The appearance in the expression (2.16) of the additional negative term is responsible for the appearance of the additional repulsive force[3]

$$\Delta f=|\Delta f^\mu\Delta f_\mu|^{1/2}=e^2M/r^3.$$

2.3. Scalar interaction

The original action for a massless scalar field has the form

$$W_{sc}[\Phi,J]=-\frac{1}{8\pi}\int_V(g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu}+\alpha R\Phi^2)\sqrt{-g}\,d^4x$$
$$+\int_V J\Phi\sqrt{-g}\,d^4x+\frac{\alpha}{4\pi}\int_{\partial V}K\Phi^2|h|^{1/2}\,d^3y,\tag{2.17}$$

where J is the source of the field. The surface term[4] ($K=\operatorname{Tr}\mathbf{K}$, K_{ij} is the extrinsic curvature, and h_{ij} is the induced metric of the boundary ∂V of V) is important for consistency of the variational procedure in the derivation of the complete system of equations for the metric $g_{\mu\nu}$ and the field Φ. For $\alpha=\frac{1}{6}$ and $J=0$, the action for the field Φ is conformally invariant. The metric energy–momentum tensor is

$$T_{\mu\nu}^{(sc)}=J\Phi g_{\mu\nu}+\frac{1}{4\pi}[\Phi_{,\mu}\Phi_{,\nu}-g_{\mu\nu}\Phi_{,\alpha}\Phi^{,\alpha}/2]$$
$$+\frac{\alpha}{4\pi}[(R_{\mu\nu}-g_{\mu\nu}R/2)\Phi^2+g_{\mu\nu}(\Phi^2)_{;\alpha}^{;\alpha}-(\Phi^2)_{;\mu\nu}].\tag{2.18}$$

The self-energy $E_{self}^{(sc)}$ of a charge at rest at the point $\mathbf{x}_0$ with density

$$j(\mathbf{x})=g\frac{(-g_{tt}(\mathbf{x}))^{1/2}}{4\pi\varepsilon^2(-g(\mathbf{x}))^{1/2}}\delta[l(\mathbf{x},\mathbf{x}_0)-\varepsilon]$$

is

$$E_{self}^{(sc)}=\int_\Sigma T_{\mu\nu}^{(sc)}\xi^\mu\,d\Sigma^\nu=I_1+\alpha I_2+(1/4-\alpha)I_3.\tag{2.19}$$

Here

$$I_1=-1/2\int\varphi j\sqrt{-g}\,d^3x=-1/2\int j(\mathbf{x})\sqrt{-g(\mathbf{x})}$$
$$\times G_{(sc)}(\mathbf{x},\mathbf{x}')j(\mathbf{x}')\sqrt{-g(\mathbf{x}')}\,d^3x\,d^3x'=-\frac{g^2}{2\varepsilon}F^{1/2}(r_0)+O(\varepsilon),$$
$$I_2=\frac{1}{4\pi}\int[(\varphi^2)_{;t}^{;t}-R_t^t\varphi^2]\sqrt{-g}\,d^3x=-g\frac{(M^2-Q^2)^{1/2}}{r_0^2}+O(\varepsilon),\tag{2.20}$$
$$I_3=\frac{1}{4\pi}\int d\Omega[r^2-2Mr+Q^2]\partial_r(\varphi^2)]|_{r=r_+}^{r=\infty}=0$$

[$G_{(sc)}(\mathbf{x},\mathbf{x}')=1/R(\mathbf{x},\mathbf{x}')$ is the static scalar Green's function obtained by Linet[5] in Ref. 13]. Substituting (2.20) in (2.19), we finally obtain

$$E_{self}^{(sc)}=-\frac{g^2}{2\varepsilon}F^{1/2}(r_0)-\alpha g^2\frac{\sqrt{M^2-Q^2}}{r_0^2}=\tilde{m}c^2F^{1/2}(r_0),\tag{2.21}$$

$$\tilde{m}=m+\Delta m,\quad m=m_0-\frac{g^2}{2\varepsilon c^2},$$
$$\Delta m=-\alpha g^2\frac{(M^2-Q^2)^{1/2}}{r_0^2}F^{-1/2}(r_0).\tag{2.22}$$

For an uncharged black hole, Δm has a form analogous to (2.14):

$$\Delta m=-\alpha g^2w/c^4,\tag{2.23}$$

where w is the modulus of the 4-acceleration of the charge. For $\alpha=0$, the corresponding correction is absent.

Comparing the equation of motion of a particle with mass μ and scalar charge g in the external field Φ_{ext},

$$\frac{d}{d\tau}[(\mu-g\Phi_{ext})u^\alpha]=g\Phi_{ext}^{,\alpha},$$

with the equilibrium condition

$$\frac{\partial}{\partial r_0}[E_{self}^{(sc)}+E_{ext}^{(sc)}]=0,$$

we obtain

$$\mu=m_0-\frac{g^2}{2\varepsilon}+\Delta\mu,\quad\Delta\mu=2\alpha g^2\frac{(M^2-Q^2)^{1/2}}{Mr_0-Q^2}F^{1/2}(r_0).\tag{2.24}$$

As in the case of the electromagnetic field, μ is not, in general, equal to $E^{(sc)}/c^2$. The case $\alpha=0$ is an exception. It is easy to show that the expression $E^{(sc)}\equiv E_{self}^{(sc)}+E_{ext}^{(sc)}$ for the total energy is related the action (2.17) as follows:

$$W_{sc}|_{t_1}^{t_2}=-E^{(sc)}(t_2-t_1),$$

where $W_{sc}|_{t_1}^{t_2}$ is the action calculated for a 4-volume V lying outside the black hold and contained between the surface $t=t_1$ and $t=t_2$. Note that in the absence of the surface term in (2.17) this equation is violated.

3. UNIFORMLY ACCELERATED MOTION

3.1. Transition to the limit of a homogeneous gravitational field

In this section, we shall discuss the energy shift of a charged particle in a homogeneous gravitational field. For the transition to the limit of a homogeneous field, we proceed as follows. In the Schwarzschild metric

$$ds^2=-\left(1-\frac{2M}{r}\right)dt^2+\left(1-\frac{2M}{r}\right)^{-1}dr^2+r^2(d\theta^2+\sin^2\theta\,d\varphi^2)\tag{3.1}$$

we go over to new coordinates (τ, z, ρ, φ) by means of the substitution

$$\tau=\frac{t}{4Mw},\quad z=\mathscr{Z}(r)=\frac{r^2}{M}\left(1-\frac{2M}{r}\right)^{1/2},$$
$$\rho=4M\tan\frac{\theta}{2},\quad\varphi=\varphi;\tag{3.2}$$

$$ds^2=-\frac{z^2w^2}{(r/2M)^4}d\tau^2+\frac{dz^2}{[1+(z^2/r^2)(2M/r)]^2}+\left(\frac{r}{2M}\right)^2\frac{d\rho^2+\rho^2\,d\varphi^2}{1+(\rho/4M)^2}.\tag{3.3}$$

Here, $r=\mathscr{F}^{-1}(z)$ and w is an arbitrary constant. The coordinate z is introduced in such a way that z^{-1} is the 4-acceleration of the particle at rest at the point r. If now for fixed (τ, z, ρ, φ) we let M tend to infinity, the expression (3.3) goes over into the Rindler metric

$$ds^2=-z^2w^2d\tau^2+dz^2+d\rho^2+\rho^2d\varphi^2, \tag{3.4}$$

which describes a static homogeneous gravitational field.[6] The coordinate τ is normalized in such a way as to make it equal to the proper time for a particle at rest at the point $z=w^{-1}$.

In the described limiting process, the increase in the mass M is accompanied by an increase in the radius of the black hole's surface, so that in the limit the horizons $H^{\pm}$ are transformed into null hyperplanes (the horizons $z=0$ of the Rindler space). The invariant distance to the horizon of a particle having acceleration $1/z$ tends to the finite value z, and the maximal value of the curvature invariant $|R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}|^{1/2}=4\sqrt{3}M/r^3$, which characterizes the inhomogeneity of the gravitational field, tends to zero everywhere outside the black hole.

For the transition to the limit of a homogeneous gravitational field in the expressions (2.7) and $G_{(sc)}=R^{-1}$, which describe the field of the test electric and scalar charges (for $Q=0$), we take the axis $\rho=0$ passing through the point at which the charge is situated. Then, bearing in mind that

$$\begin{aligned}
&\lambda=\cos\theta=[1-(\rho/4M)^2]/[1+(\rho/4M)^2],\\
&R^2=(r-M)^2+(r'-M)^2-2(r-M)(r'-M)\lambda-M^2(1-\lambda^2)\\
&\quad=\frac{1}{64M^2}\{[(z^2+z'^2+\rho^2)^2-4z^2z'^2]+O(M^{-2})\},\\
&\Pi=(r-M)(r'-M)-M^2\lambda={}^1/_8(z^2+z'^2+\rho^2)+O(M^{-2})
\end{aligned} \tag{3.5}$$

we obtain up to omitted terms of order $O(M^{-2})$ expressions for the field at the point (z, ρ, φ) produced by a point charge at the point $(z', 0, 0)$:

$$\hat{a}=a_\mu dx^\mu=-ew\left[1+\frac{z^2+z'^2+\rho^2}{[(z^2+z'^2+\rho^2)^2-4z^2z'^2]^{1/2}}\right]d\tau, \tag{3.6}$$

$$\varphi=g\frac{2z'}{[(z^2+z'^2+\rho^2)^2-4z^2z'^2]^{1/2}}. \tag{3.7}$$

If we set $z'=w^{-1}$, then the expression (3.6) differs from the expression (III.3) given by Boulware[17] by the gauge term $-ewd\tau$.

3.2. The "self-energy" of a uniformly accelerated charged particle

By means of the coordinate transformation $T=z\sinh(w\tau)$, $Z=z\cosh(w\tau)$, $X=\rho\cos\varphi$, $Y=\rho\sin\varphi$, the metric (3.4) is reduced to the standard expression for the metric of flat space:

$$ds^2=-dT^2+dX^2+dY^2+dZ^2. \tag{3.8}$$

The line z, ρ, $\varphi=$const is then the world line of an observer moving with constant 4-acceleration $1/z$ along the z axis, and $wz\tau$ is his proper time. The coordinates (r, z, ρ, φ) cover only part (region I in Fig. 3) of the complete Minkowski space. The complete Minkowski space arises in a natural manner on the transition to the limit $M\to\infty$ from the Kruskal metric, and the expressions (A1.6) and (A2.5) go over into the expressions for the electromagnetic and scalar fields from a point charge moving with constant acceleration[7] equal to w:

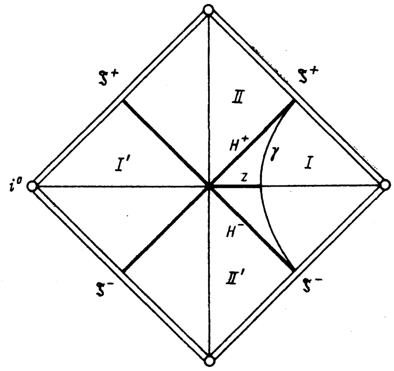

FIG. 3. Penrose diagram of Rindler space-time.

$$\begin{aligned}
\hat{\tilde{a}}=\tilde{a}_\mu dx^\mu=&-\frac{e}{2}\theta(V)\left\{\frac{\rho^2-UV-U'V'}{[(\rho^2-UV-U'V')^2-4UVU'V']^{1/2}}-1\right\}\\
&\times\left(\frac{dV}{V}-\frac{dU}{U}\right)-2e\theta(V)\frac{\rho\,d\rho}{\rho^2-U'V'},\\
\tilde{\varphi}=g(-U'V')^{1/2}&\frac{2\theta(V)}{[(\rho^2-UV-U'V')^2-4UVU'V']^{1/2}},\\
U=T-Z,\quad V=T+Z,&\quad \rho^2=|X-X'|^2+|Y-Y'|^2.
\end{aligned} \tag{3.9}$$

Here, $\tilde{a}_\mu$ is augmented by a term of the form $\partial_\mu(h)\theta(V)$, which leads to a singularity of the form $\delta(V)$ in $\tilde{f}$. The need for this term was noted by Bondi and Gold.[18]

In the limit $M\to\infty$, the Killing vector field $\xi^\mu\partial_\mu=4Mw\partial_t$ goes over into the Killing vector field $\xi^{(R)\mu}\partial_\mu=\partial_\tau$ in the Rindler space. Accordingly, the expression for the energy of the charged particle in the field of the black hole must go over into an integral of the motion of the form

$${}^RE=\int_{z>0}T_\tau^\tau w\rho z\,dz\,d\rho\,d\varphi. \tag{3.10}$$

Using the expressions (3.6) and (3.7) and integrating (3.10) over the region $z>0$, we obtain[8)]

$${}^RE_{self}^{(em)}=e^2/2\varepsilon-e^2w/2,\quad {}^RE_{self}^{(sc)}=-g^2/2\varepsilon-\alpha g^2w. \tag{3.11}$$

Comparison of (3.11) with the expressions (2.14) and (2.23) draws attention to the circumstance that the term in ${}^RE_{self}$ proportional to the acceleration is equal to $c^2\Delta m^{sc}$ for a scalar field whereas there is a difference in the sign for the electromagnetic field. The reason for this difference is that the integral which determines $E_{self}/(1-2M/r_0)^{1/2}$ does not have the property of uniform convergence and therefore, in particular

$$\lim_{M\to\infty}(E_{self}/(1-2M/r_0)^{1/2})$$

may differ (and, as we have seen, does indeed differ for an electric charge) from the value of the integral for ${}^RE_{self}$, in which the passage to the limit $M\to\infty$ is made under the integral sign.

The expression ${}^RE_{self}^{(em)}$ is equal to the expression for the self-energy of a uniformly accelerated charge obtained earlier by Ritus[19] and investigated in detail by him in Ref. 20. It should be noted however that for a uniformly accelerated particle, in either a scalar or electromagnetic field, the inertial mass μ determined by means of the equations of motion does not contain corrections that depend on the acceleration. The proof of this assertion for an electric charge obtained by Fermi[1] can be readily transferred to the case of a scalar charge.

We thank V. L. Ginzburg, V. I. Ritus, and W. Unruh for stimulating discussions and valuable comments.

APPENDIX 1

In this Appendix, we give the main results relating to the properties of the electromagnetic field of a point charge e at the point $r=r_0$, $\theta=0$ outside a Reissner–Nordström black hole with mass M and charge Q.

In region I (see Fig. 1) in the coordinates (t, r, θ, φ) we have

$$\hat{a}=a_\mu dx^\mu=-\frac{e}{rr'}(M+\Pi/R)\,dt, \tag{A1.1}$$

where Π and R are defined by (2.7′). The nonvanishing components of the field $f_{\mu\nu}$ are

$$f_{rt}=\frac{e}{r^2r_0}\left\{M+\frac{\Pi}{R}\right\}-\frac{er_0}{rR^3}F(r_0)\{r_0-M-(r-M)\cos\theta\},$$
$$f_{\theta t}=\frac{er_0r}{R^3}F(r)F(r_0)\sin\theta. \tag{A1.2}$$

The horizons $H^\pm$ are equipotential surfaces and the potential on them is

$$\lim_{r\to r_+} a_t=-e/r_0.$$

The nonvanishing components of the energy–momentum tensor $T^\mu_\nu=T^\mu_\nu[f,f]$ are

$$T_t^{\,t}=-T_\varphi^{\,\varphi}=-\frac{1}{8\pi}\left[(f_{rt})^2+\frac{1}{r^2F(r)}(f_{\theta t})^2\right], \tag{A1.3}$$
$$T_r^{\,r}=-T_\theta^{\,\theta}=-\frac{1}{8\pi}\left[(f_{rt})^2-\frac{1}{r^2F(r)}(f_{\theta t})^2\right],\quad T_\theta^{\,r}=-\frac{1}{4\pi}f_{rt}f_{\theta t}.$$

Here and in what follows, $F(r)=1-2M/r+Q^2/r^2$.

To obtain the analytic continuation of this solution in regions I, II, I′, II′ (see Fig. 1), we introduce coordinates (u, v, θ, φ) regular in these regions and associated in region I with the coordinates (r, t, θ, φ) by the relations

$$u=-\exp\left\{-\frac{r_+-r_-}{2r_+^2}(t-r^*)\right\},\quad v=\exp\left\{\frac{r_+-r_-}{2r_+^2}(t+r^*)\right\}, \tag{A1.4}$$
$$\theta=\theta,\quad \varphi=\varphi;$$
$$r^*=r+\frac{r_+^2}{r_+-r_-}\ln\left|\frac{r}{r_+}-1\right|-\frac{r_-^2}{r_+-r_-}\ln\left|\frac{r}{r_-}-1\right|,$$
$$r_\pm=M\pm(M^2-Q^2)^{1/2}.$$

For $-\infty<u,\ v<\infty$, $(\theta,\varphi)\in S^2$ these coordinates cover the entire region I, II, I′, II′, and the analytic continuation of the Reissner–Nordström metric in them has the form

$$ds^2=-2B\,du\,dv+r^2d\sigma^2, \tag{A1.5}$$
$$B=\frac{2r_+^4}{r^2}\frac{(r-r_+)(r-r_-)}{(r_+-r_-)^2}\exp\left\{-r^*\frac{r_+-r_-}{r_+^2}\right\}.$$

[Note that the transformations (A1.4) differ from the standard transformations given in Ref. 7 by the additional factor 2 in the argument of the exponentials.] The nonvanishing components of the analytically continued solution (A1.1), (A1.2) in these coordinates are

$$\hat{a}=-\frac{e}{rr'}\left(M+\frac{\Pi}{R}\right)\left(\frac{dv}{v}-\frac{du}{u}\right)\frac{r_+^2}{r_+-r_-},$$
$$f^{uv}=\frac{e}{Br^2r_0}\left\{M+\frac{\Pi}{R}-\frac{r}{R^3}[r_0-M-(r-M)\cos\theta](r_0^2-2Mr_0+Q^2)\right\},$$
$$f^{v\theta}=\frac{e}{2rr_0}\frac{\sin\theta}{R^3}\frac{r_+-r_-}{r_+^2}(r_0^2-2Mr_0+Q^2)v, \tag{A1.6}$$
$$f^{u\theta}=-\frac{e}{2rr_0}\frac{\sin\theta}{R^3}\frac{r_+-r_-}{r_+^2}(r_0^2-2Mr_0+Q^2)u.$$

Note that this solution is invariant under the transformation $(u\to-u,\ v\to-v)$, which maps region I onto region I′, and therefore, besides the singularity corresponding to the world line γ of the charge, it also contains a singularity on the line γ' corresponding to the additional charge $-e$ in the region I′. If additional charges are absent, then in the regions I′ and II′, which lie outside the domain of influence of the charge e, the field must be absent. In this case, the corresponding solution can be obtained in the form

$$\tilde{f}_{\mu\nu}=f_{\mu\nu}^{(reg)}+f_{\mu\nu}^{(sing)},\quad f_{\mu\nu}^{(reg)}=f_{\mu\nu}\theta(v),\quad f_{\mu\nu}^{(sing)}=\psi_{\mu\nu}\delta(v). \tag{A1.7}$$

When $\tilde{f}_{\mu\nu}$ is substituted in the Maxwell equations, we obtain the following equations for $\psi_{\mu\nu}$:

$$\psi^{\mu v}=0,\quad \frac{1}{\sqrt{-g}}(\psi^{\mu\nu}\sqrt{-g})_{,\nu}=-f^{\mu v}|_{v=0}. \tag{A1.8}$$

The solution of these equations restricted to H^- is

$$\psi_{v\theta}=-e\sin\theta\frac{(M^2-Q^2)^{1/2}}{r_0}\frac{M+(M^2-Q^2)^{1/2}}{r_0-M-(M^2-Q^2)^{1/2}\cos\theta}, \tag{A1.9}$$

the remaining components vanishing. The presence of the term with $\delta(v)$ in (A1.7) ensures fulfillment of the vacuum Maxwell equations in the neighborhood[9)] of H^-.

The energy–momentum tensor is

$$T^{\mu\nu}=T^{\mu\nu}[\tilde{f},\tilde{f}]=T^{\mu\nu}_{(sing)}+T^{\mu\nu}[f,f]\theta(v).$$

The tensor $T^{\mu\nu}[f,f]$ is obtained by analytic continuation of (A1.3), and the nonvanishing components of $T^{\mu\nu}_{(sing)}$ are

$$T^{uu}_{(sing)}=\frac{e^2(r_+-r_-)\sin^2\theta}{8\pi Br_+^2r_0^2R^2}\left\{\frac{r_+^2}{B}\delta^2(v)-\delta(v)\theta(v)\frac{r_0^2-2Mr_0+Q^2}{R^2}u\right\},$$
$$T^{u\theta}_{(sing)}=\frac{e^2(r_+-r_-)}{8\pi Br_+^2r_0^2}\frac{\sin^2\theta}{R^2}\delta(v)\theta(v)\left\{\frac{r_+-r_-}{2R}\sin\theta-2\frac{\cos\theta}{\sin\theta}\right\}, \tag{A1.10}$$
$$R|_{H^-}=|r_0-M-(M^2-Q^2)^{1/2}\cos\theta|.$$

For $T_{\mu\nu}$ we have for $u=0$ (on H^+)

$$\xi^\mu T_\mu^{\,u}|_{u=0}=T_t^{\,u}|_{u=0}=0. \tag{A1.11}$$

APPENDIX 2

Suppose a scalar charge g is at rest in the gravitational field of a charged black hole. The solution to the equation

$$\varphi_{;\mu}^{\;;\mu}=-4\pi j \tag{A2.1}$$

in the metric (2.6) with point source

$$j=gF^{1/2}\frac{\delta(r-r_0)\delta(\cos\theta-1)}{2\pi r^2}$$

was found in Ref. 13 and has the form

$$\varphi=g\left(1-\frac{2M}{r_0}+\frac{Q^2}{r_0^2}\right)^{1/2}\frac{1}{R}. \tag{A2.2}$$

As in the electromagnetic case, we shall seek the complete solution of Eq. (A2.1) in the form

$$\tilde{\varphi}=\varphi_{(reg)}+\varphi_{(sing)},\quad \varphi_{(reg)}=\varphi\theta(v),\quad \varphi_{(sing)}=\psi\delta(v). \tag{A2.3}$$

Substituting $\tilde{\varphi}$ in (A2.1), we obtain equations for ψ

$$(1-\lambda^2)\frac{\partial^2\psi}{\partial\lambda^2}-2\lambda\frac{\partial\psi}{\partial\lambda}-\frac{r_+-r_-}{r_+}\psi=0,\qquad \frac{\partial\psi}{\partial u}=0,\quad \lambda=\cos\theta. \tag{A2.4}$$

If the black hole is not maximally charged, the only bounded solution is $\psi=0$. (For an extremal black hole, $\psi=$const is also a solution, but this solution is not determined by the exterior scalar field and therefore bears no relation to the charge g.)

Thus, $\varphi_{(sing)}$ is zero and

$$\tilde{\varphi}=\varphi\theta(v). \tag{A2.5}$$

However, the energy–momentum tensor will contain a singular part:

$$T^{(sc)}_{\mu\nu}=T^{(sc)}_{\mu\nu(reg)}+T^{(sc)}_{\mu\nu(sing)}. \tag{A2.6}$$

The tensor $T^{(sc)}_{(\mu\nu\,sing)}$ has only two nonvanishing components:

$$T^{(sc)}_{vv(sing)}=\frac{1}{\pi}\left(\frac{1}{4}-\alpha\right)\theta(v)\delta(v)(\varphi^2)_{,v}+\varphi^2\left\{\frac{1}{4\pi}\delta^2(v)-\frac{\alpha}{4\pi}\left[\delta^2(v)+\theta(v)\delta'(v)-\frac{1}{B}\frac{\partial B}{\partial v}\theta(v)\delta(v)\right]\right\},$$
$$T^{(sc)}_{v\theta(sing)}=\frac{1}{2\pi}\left(\frac{1}{4}-\alpha\right)\theta(v)\delta(v)(\varphi^2)_{,\theta}. \tag{A2.7}$$

APPENDIX 3

To calculate the integrals (2.8) and (2.20) in the case in which are interested when the charge is uniformly distributed over the surface of a rigid sphere of radius ε with center at the point $r=r_0$, $\theta=0$, we introduce the new coordinates (x, y, z) and coordinates (η, ω, Φ) associated with the coordinates (r, θ, φ) by the relations

$$\begin{aligned}&x=r\sin\theta\cos\varphi; && x=\eta\sin\omega\cos\Phi;\\ &y=r\sin\theta\sin\varphi; && y=\eta\sin\omega\sin\Phi;\\ &dz^2+\beta z^2=r\cos\theta-r_0+\beta r^2\sin^2\theta; && z=\eta\cos\omega,\end{aligned} \tag{A3.1}$$

where for the considered Reissner–Nordström metric

$$\alpha=\left(1-\frac{2M}{r_0}+\frac{Q^2}{r_0^2}\right)^{1/2},\quad \beta=\frac{Mr_0-Q^2}{2r_0^3}. \tag{A3.2}$$

In these coordinates, in the neighborhood of the point r_0 the metric (2.5) can be written in the form

$$\begin{aligned}ds^2&=-F[r(x,y,z)]dt^2+(dx^2+dy^2+dz^2)(1+O(x^2+y^2+z^2))\\ &=-F[r(\eta,\omega,\Phi)]dt^2+[d\eta^2+\eta^2(d\omega^2+\sin^2\omega d\Phi^2)](1+O(\eta^2)).\end{aligned} \tag{A3.3}$$

In these coordinates, the invariant δ function has the simple form

$$\delta(l(x,x_0)-\varepsilon)=\delta(\eta-\varepsilon) \tag{A3.4}$$

and the integration in (2.8) and (2.20) reduces to calculation of the integrals with respect to the angular variables for fixed value $\eta=\varepsilon$. The expansions to second order in ε of the quantities in these integrals needed for their calculation have the form

$$\begin{aligned}&R=\alpha r_0[2(1-\Lambda)]^{1/2}\left[1+\varepsilon\frac{\beta}{2\alpha}(\cos\omega+\cos\omega')\right],\\ &\Lambda=\cos\omega\cos\omega'+\sin\omega\sin\omega'\cos(\Phi-\Phi'),\\ &r=r_0\{1+\varepsilon\alpha\cos\omega+{}^1/_2\varepsilon^2[\alpha^2+(\beta-\alpha^2)\cos^2\omega]\},\\ &\Pi=(r-M)(r'-M)-M^2\lambda=r_0^2\alpha^2+(r_0^2-Mr_0)(\cos\omega+\cos\omega')\alpha\varepsilon,\\ &rr'=r_0^2\{1+\varepsilon\alpha(\cos\omega+\cos\omega')\\ &\quad+\varepsilon^2[\alpha^2\cos\omega\cos\omega'+{}^1/_2[\alpha^2+(\beta-\alpha^2)(\cos^2\omega+\cos^2\omega')]]\}.\end{aligned} \tag{A3.5}$$

Substituting these expansions in the expressions (2.8) and (2.20) for the self-energy, we obtain

$$\begin{aligned}&E^{(em)}_{self}=\frac{e^2}{2r_0}\left[\frac{\alpha}{\varepsilon}J_1+\frac{M}{r_0}J_2+\frac{\beta}{2}J_3+O(\varepsilon)\right],\\ &I_1=-\frac{g^2}{2r_0}\left[\frac{\alpha}{\varepsilon}J_1+\frac{\beta}{2}J_3+O(\varepsilon)\right];\end{aligned} \tag{A3.6}$$

$$\begin{aligned}&J_1=\frac{1}{(4\pi)^2}\int\frac{d\Omega d\Omega'}{[2(1-\Lambda)]^{1/2}}=1,\\ &J_2=\frac{1}{(4\pi)^2}\int d\Omega d\Omega'=1,\\ &J_3=\frac{1}{(4\pi)^2}\int\frac{\cos\omega+\cos\omega'}{[2(1-\Lambda)]^{1/2}}d\Omega\, d\Omega'=0,\\ &d\Omega=\sin\omega d\omega d\Phi.\end{aligned} \tag{A3.7}$$

From this the results (2.10) and (2.20) given in the main text follow.

[1]Note that for fixed surface Σ $(\partial\Sigma_{BH}\subset H^+)$, $(\partial\Sigma_\infty\subset i^0)$ and given position of the charge on it the minimal value of $E^{(em)}_{self}$ is attained in the case when $f_{\mu\nu}$ is static in the neighborhood of Σ. It is for this reason that the energy of this lowest energy state is appropriately regarded as the self-energy. Note also that the integral $E^{(em)}_{self}$ in (2.4) calculated over any spacelike surface Σ lying in the region II (see Fig. 1) with boundaries on $r=r_-$ and $r=r_+$ for the analytic continuation (A1.6) of the static solution is equal to zero. Therefore, in particular, for a Schwarzschild black hole ($r_-=0$) the value of $E^{(em)}_{self}$ is the same if Σ is a complete surface or a part of it lying outside the black hole. For a charged black hole, the analogous integral over the part of Σ in the region III between $r=r_-$ and $r=0$ is positive and diverges as $r\to 0$. Note, however, that the field in region III is not uniquely determined by the Cauchy data in regions I′ and I, since the choice of the boundary conditions at the singularity at $r=0$ is arbitrary. In particular, this circumstance can be used to make the corresponding integral vanish.

[2]Note that if in the expression (2.4) that determines $E^{(em)}_{self}$ we take as Σ the surface Σ_0 (see Fig. 1) containing besides the part $t=$ const, $r>r_+$ its continuation in I′ and as $\hat{f}$ we use $\tilde{f}$ (A1.7), then the terms containing $f_{(sing)}$ lead to the appearance in the integrand of quantities proportional to $v\delta^2(v)$. It is easy to show that the corresponding integral, whose definition must be augmented, is equal to the integral

$$\Delta E=\int T_{\mu\nu}[\tilde{f},\tilde{f}]\xi^\mu d\Sigma^\nu$$

calculated over the part of H^+ containing $H^+\cap H^-$ and describes the flux of the electromagnetic field into the black hole when the charge e is carried up to it. The quantity ΔE, which depends on the details of this process, determines the corresponding change in the mass of the black hole. The substitution $\tilde{f}\to\tilde{f}_{(ring)}$ and fixing $\Delta E=0$ corresponds to the formulation of the problem that we have adopted in which the mass of the black hole is assumed given.

[3]The equivalence principle is discussed in this connection

in Ref. 11.

4)With regard to the appearance of a similar surface term in the action for a purely gravitational field, see, for example, Ref. 12.

5)We should like to take this opportunity of thanking Linet for pointing out to us that the expression for $C_{(sc)}(\mathbf{x}, \mathbf{x}')$ obtained in our Ref. 14 can be transformed to the simple form $1/R(\mathbf{x}, \mathbf{x}')$ obtained by Linet in Ref. 13.

6)A discussion of the metric (3.4) in connection with the equivalence principle and of the equivalence principle in connection with the question of the radiation of a uniformly accelerated electron can be found in Refs. 15–17.

7)Note that these continuations of the solutions (3.6) and (3.7), which are "static" in Rindler coordinates, describes in region II (Fig. 3) a radiation field (see, for example, Ref. 17). Similarly, for an observer falling into the black hole (in region II in Fig. 1) the analytic continuations (A1.6) and (A2.5) of the solutions static outside the hole are a radiation field.

8)The normalization of the Killing vector field is chosen in such a way that the terms $\sim\varepsilon^{-1}$ are equal to the expression for the self-energy of the particle at rest.

9)Similar questions relating to the definition of the field within a black hole produced by an electric charge (in connection with a discussion of Jean's hypothesis) were considered independently by Demianski and Novikov in their Ref. 22, which appeared after our Ref. 21.

1E. Fermi, Nuovo Cimento 22, 176 (1921); [included in the Russian translation of Vol. 1 of Fermi's Scientific Works published by Nauka, Moscow (1971), p. 54].

2A. Vilenkin, Phys. Rev. D 20, 373 (1979).

3W. G. Unruh, Proc. R. Soc. London Sect. A348, 447 (1976).

4A. G. Smith and C. M. Will, Phys. Rev. D 22, 1276 (1980).

5V. P. Frolov and A. I. Zel'nikov, in: Abstracts of Contributed Papers, Ninth International Conference GRG, Vol. 3, (1980), p. 555; A. I. Zel'nikov and V. P. Frolov, Trudy III Mezhdunarodnogo seminara po problemam fiziki vysokikh énergiĭ i kvantovoĭ teorii polya (Proc. Third. Intern. Seminar on Problems of High Energy Physics and Quantum Field Theory), Protvino (1980).

6C. M. DeWitt and B. S. DeWitt, Physics (Long Island City, N.Y.) 1, 3 (1964).

7C. W. Misner, K. S. Thorne, and J. A. Wheeler, Gravitation, W. H. Freeman, San Francisco (1973) [Russian translation published by Mir, Moscow (1977)].

8B. Leaute and B. Linet, Phys. Lett. A58, 5 (1976).

9B. Linet, J. Phys. A 9, 1081 (1976).

10E. T. Copson, Proc. R. Soc. London Sect. A118, 184 (1928).

11B. Leaute and B. Linet, "Principle of equivalence and electromagnetism," Preprint, Inst. Henri Poincaré (1980).

12G. W. Gibbons and S. W. Hawking, Phys. Rev. 15, 2752 (1977).

13B. Linet, C. R. Acad. Sci. A284, 215 (1977).

14V. P. Frolov and A. I. Zel'nikov, J. Phys. A 13, L345 (1980).

15T. Fulton and F. Rohrlich, Ann. Phys. (N.Y.) 9, 499 (1960).

16V. L. Ginzburg, Usp. Fiz. Nauk 98, 569 (1969) [Sov. Phys. Usp. 12, 565 (1970)].

17D. G. Boulware, Ann. Phys. (N.Y.) 124, 169 (1980).

18H. Bondi and T. Gold, Proc. R. Soc. London Sect. A229, 416 (1955).

19V. I. Ritus, Zh. Eksp. Teor. Fiz. 75, 1560 (1978) [Sov. Phys. JETP 48, 788 (1978)].

20V. I. Ritus, Zh. Eksp. Teor. Fiz. 80, 1288 (1981) [Sov. Phys. JETP 53, 659 (1981)].

21V. P. Frolov and A. I. Zel'nikov, "On the influence of a gravitational field on the self-energy of charged particles," Preprint, University of British Columbia, Vancouver, B. C., April (1981).

22M. Demianski and I. D. Novikov, "Electric charge in the Kruskal space-time and the Jeans conjecture," Preprint No. 653, Space Research Institute, USSR Academy of Sciences (1981).

Translated by Julian B. Barbour