Phase transitions and high-frequency magnetic phenomena in semiconductors with point defects

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It is shown that quantum virial corrections due to the interaction between electrons and point defects lead to magnetic and structural phase transitions in both Boltzmann and degenerate semiconductors. The ferro- and ferrimagnetic ordering temperatures are calculated. The conditions of appearance of a periodic superstructure in the lattice and of the related electron charge density wave are determined. Zero-angle coherent scattering ensures the possibility of propagation of high-frequency spin waves in the electron and magnetic-defect system. Spectra of the two branches of such oscillations are obtained. Two solutions for EPR and spin oscillations in an external magnetic field are considered.

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1. INTRODUCTION

The problem of electron scattering by point defects in conducting media arises usually in connection with the consideration of various kinetic properties of metals and semiconductors, for example in the calculation of the impurity resistivity. The interaction of electrons with defects, however, can also influence strongly the thermodynamics of a conducting medium. Thus, in a cold ionized gas, the virial corrections to the free energy, due to the interaction of the electrons with the neutrals, lead to possible second-order phase transitions in the system. These transitions are due to the appearance of magnetic ordering in the gas, namely a ferro- and ferrimagnetic structure, or else to the onset of a spatial periodic structure—a charge-density wave.[1] Similar effect due to the interaction between electrons and localized point defects can take place also in condensed conducting systems. The results reported below pertain mainly to semiconductors with impurities, interstices, or vacancies, although in some cases the employed calculation scheme is a good model for the description of doped metals, solid solutions, and other metallic compounds. When dealing, however, with a degenerate electron system in the presence of magnetic impurities, we shall not consider here long-range order effects connect with the Ruderman-Kittel-Kasuya-Yosida mechanism.

The calculation of the contributions made by electron–defect collisions to all the thermodynamic functions does not require in essence any special assumptions. The corresponding terms in the total free energy of the system are expressed in terms of the exact electron–defect scattering amplitude. The natural small parameters of the problem are the densities of the electrons and of the defects. Therefore, even though their interaction with one another is in no way weak, all the thermodynamic quantities can be obtained in the form of virial expansions in powers of the low densities, i.e., in the form of the thermodynamic perturbation-theory series. The amplitude of electron scattering by a paramagnetic impurity depends significantly, owing to exchange effects, on the combined spin of the electron and the defects. This manifests itself macroscopically in the fact that the magnetic part of the free energy of the system acquires a virial increment proportional to the scalar product of the magnetizations of the electronic and impurity components. At sufficiently low temperatures, the thermodynamics favors the spontaneous onset of equilibrium magnetic moments for each of the indicated components. Depending on the character of the interaction of the electron with the impurity (attraction or repulstion), the magnetic moment of the electron subsystem and the magnetic moment of the impurities have either a parallel (ferromagnetic structure) or an antiparallel (ferrimagnetic ordering) orientation relative to each other. Thus, in the presence of delocalized electrons and localized magnetic defects, direct exchange in the scattering of an electron by a defect can be the mechanism responsible for the magnetism of the system as a whole. This may be precisely the mechanism that explains the magnetic properties of narrow-gap doped semiconductors.

In analogy with magnetic phase transitions, the interaction of electrons with defects when the temperature is lowered can lead to a spontaneous change in the densities of the electrons and defects. This condensation or rarefaction of the electrons and of the defects lowers the free energy of the system. On the other hand, such a redistribution of the electrons, which violates the electroneutrality of the system in macroscopic regions, leads to the appearance of a macroscopic electric field and is accompanied by an increase of the Coulomb correlation energy of the interaction of the electrons with the ion lattice. It is these two competing mechanisms which make possible the formation in the system of a periodic spatial structure in the distribution of the densities of the electrons and defects. The principal ion lattice is then accordingly modulated, i.e., a unique superstructure is produced in a defect-containing lattice. We emphasize that this situation corresponds to the existence of a spontaneous periodic electric field inside the conductor.

The interaction of electrons with point defects causes also new nonstationary phenomena. Quantum refraction and scattering of electrons by defects introduces

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into the kinematic part of the kinetic equation a specific term that is linear in the forward-scattering amplitude and can be regarded as a unique self-consistent field of the Fermi-liquid type.[1,2] In analogy with the Landau theory,[3] the presence of such a term in the kinetic equation makes possible the propagation of collective zero-sound or spin-wave modes. We investigate in this paper, within the framework of the indicated approach, the high-frequency spin dynamics of semiconductors doped with magnetic defects. We consider the oscillations of the magnetization in a system of localized paramagnetic defects coupled to the oscillations of the electron magnetic moment.

2. INTERACTION ENERGY

We consider the effect of the interaction between electrons and point defects on the thermodynamic properties of semiconductors. If we are not interested in the disposition the defects in the crystal lattice, they can be described as stochastically distributed particles with infinitely large mass. We can use in this case the usual gas equations, and let the defect mass tend to infinity in the final expressions. We assume that the defect density N_d is small enough, $N_d r_0^3 \ll 1$, where r_0 is the radius of the interaction of the electron with the defects, so that the scattering of the electrons by each separate defect can be considered. The electron density N_e will also be assumed small: $N_e r_0^3 \ll 1$. If the electron gas is magnetically polarized, then its equilibrium density matrix $n_{\alpha\beta}^{(e)}$ is expressed in terms of a linear combination of Pauli matrices $\sigma_{\alpha\beta}$:

$$n_{\alpha\beta}^{(e)}(\mathbf{p}) = {}^1/_2 \eta_e(\mathbf{p})\delta_{\alpha\beta} + {}^1/_2 \rho_e(\mathbf{p})\sigma_{\alpha\beta}, \tag{2.1}$$

where α and β are spinor indices, p is the momentum

$$\eta_e = n_e^+ + n_e^-, \quad \rho_e = (n_e^+ - n_e^-)\mathfrak{M}_e.$$

$\mathfrak{M}_e$ is a unit vector in the direction of the magnetic moment of the electron system, and n_e^+ and n_e^- are the Fermi occupation numbers for the electrons with spins oriented parallel and antiparallel to the magnetic-polarization vector $\mathfrak{M}_e$:

$$n_e^{\pm}(\mathbf{p}) = \frac{1}{2}\left[1 - \mathrm{th}\frac{\varepsilon(\mathbf{p}) - \mu^{\pm}}{2T}\right], \quad \varepsilon = \frac{p^2}{2m_e}. \tag{2.2}$$

In (2.2), T is the temperature and the values of $\mu^{\pm}$ are determined by the normalization conditions:

$$\sum_{\mathbf{p}} n_e^{\pm}(\mathbf{p}) = N_e^{\pm}, \tag{2.3}$$

while the densities $N_e^{\pm}$ of electrons with spins directed parallel and antiparallel to $\mathfrak{M}_e$ specify the degree of polarization α_e of the electron gas:

$$N_e^+ + N_e^- = N_e, \quad N_e^+ - N_e^- = \alpha_e N_e. \tag{2.4}$$

For simplicity, we consider a crystal with cubic symmetry or an isotropic solid in which the electron energy spectrum $\varepsilon(\mathbf{p})$ has the particularly simple form (2.2) with a scalar effective mass m_e. Analogous expressions hold for a polarized system of defects with spin 1/2, namely

$$n_{\alpha\beta}^{(d)}(\mathbf{p}) = {}^1/_2 \eta_d(\mathbf{p})\delta_{\alpha\beta} + {}^1/_2 \rho_d(\mathbf{p})\sigma_{\alpha\beta},$$
$$\eta_d = n_d^+ + n_d^-, \quad \rho_d = (n_d^+ - n_d^-)\mathfrak{M}_d, \quad \sum_{\mathbf{p}} n_d^{\pm}(\mathbf{p}) = N_d^{\pm},$$
$$N_d^+ + N_d^- = N_d, \quad N_d^+ - N_d^- = \alpha_d N_d.$$

Expansion of the thermodynamic functions in powers of the small densities N_e and N_d is equivalent to functional expansion in powers of the ideal-gas density matrices (2.1) and (2.4) for electrons and defects. The contribution of the electron scattering by defects to the total free energy is then determined by the expression

$$\delta F_{int} = \sum_{\mathbf{p},\mathbf{p}'} \Phi_{\alpha\beta,\mu\nu}(\mathbf{p},\mathbf{p}')\, n_{\beta\alpha}^{(e)}(\mathbf{p})\, n_{\nu\mu}^{(d)}(\mathbf{p}'). \tag{2.5}$$

Summation over repeated indices is implied. Since the two-particle interaction does not depend on the spins in the nonrelativistic approximation, the interaction function $\Phi_{\alpha\beta,\mu\nu}(\mathbf{p},\mathbf{p}')$ of (2.5) does not depend on the magnetic structure of the system and coincides with its value in the absence of polarization. This is also due to the fact that in the employed approximation of the thermodynamic perturbation theory the function $\Phi_{\alpha\beta,\mu\nu}(\mathbf{p},\mathbf{p}')$ is not a functional of $n_{\alpha\beta}^{(e)}$ and $n_{\mu\nu}^{(d)}$, and by the same token does not depend on the magnetic moments of the electrons and defects. Thus, the spin dependence of the interaction function has in the exchange approximation the usual form

$$\Phi_{\alpha\beta,\mu\nu}(\mathbf{p},\mathbf{p}') = \psi(\mathbf{p},\mathbf{p}')\delta_{\alpha\beta}\delta_{\mu\nu} + \zeta(\mathbf{p},\mathbf{p}')\sigma_{\alpha\beta}\sigma_{\mu\nu}. \tag{2.6}$$

In the Boltzman region $T \gg \varepsilon_F$ ($\varepsilon_F = (2\pi^2)^{2/3}\hbar^2 N_e^{2/3}/m_e$ is the Fermi-degeneracy temperature for the electrons), calculation of the functions $\psi(\mathbf{p},\mathbf{p}')$ and $\zeta(\mathbf{p},\mathbf{p}')$, which is similar to the calculation of the second virial coefficient for gases in the quantum case,[4] leads to the result

$$\psi = {}^1/_4[3B(\Gamma_{\uparrow\uparrow}) + B(\Gamma_{\uparrow\downarrow})], \quad \zeta = {}^1/_4[B(\Gamma_{\uparrow\uparrow}) - B(\Gamma_{\uparrow\downarrow})], \tag{2.7}$$
$$B(\Gamma) = -\left[\mathrm{Re}\,\Gamma(0) + \frac{2\mu T}{\hbar}\left(\mathrm{Re}\,\Gamma\frac{\partial}{\partial q}\mathrm{Im}\,\Gamma - \mathrm{Im}\,\Gamma\frac{\partial}{\partial q}\mathrm{Re}\,\Gamma\right)\right]\frac{2\pi\hbar^2}{\mu}.$$

In (2.7), $\Gamma_{\uparrow\downarrow}$ and $\Gamma_{\uparrow\uparrow}$ are the amplitudes of the singlet and triplet scatterings, $\Gamma(0)$ is the forward-scattering amplitude, μ is the reduced mass of the colliding particles, and $2\mathbf{q} = \mathbf{p} - \mathbf{p}'$. In scattering of an electron by a neutral defect (for example, a neutral impurity level), the interaction radius r_0 is of the order of atomic dimensions, so that the condition $T \ll \hbar^2/m_e r_0^2$ is therefore always satisfied in practically the entire region of existence of the semiconductor. We have then a system of slow electrons, and it suffices to retain in expressions (2.7) only the principal term in the expansion of the scattering amplitudes $\Gamma_{\uparrow\downarrow}$ and $\Gamma_{\uparrow\uparrow}$ in powers of the small relative momentum $qr_0 \ll 1$, i.e., a constant independent of the momenta:

$$\psi = \frac{\pi\hbar^2}{2m_e}(3a_{\uparrow\uparrow} + a_{\uparrow\downarrow}), \quad \zeta = \frac{\pi\hbar^2}{2m_e}(a_{\uparrow\uparrow} - a_{\uparrow\downarrow}), \tag{2.8}$$

where $a_{\uparrow\uparrow}$ and $a_{\uparrow\downarrow}$ are the s-scattering lengths for collisions in the triplet and singlet states, and $\mu \to m_e$. Substituting (2.8) in (2.6) and (2.5), we obtain the contribution of the interaction of the electrons with the defects to the free energy of the semiconductor:

$$\delta F_{int} = \frac{\pi\hbar^2}{2m_e} N_e N_d (a_1 + a_2 \alpha_e \alpha_d \mathfrak{M}_e \mathfrak{M}_d). \tag{2.9}$$

We have introduced here the notation $a_1 \equiv 3a_{\uparrow\uparrow} + a_{\uparrow\downarrow}$ and $a_2 \equiv a_{\uparrow\uparrow} - a_{\uparrow\downarrow}$. In the scattering of an electron by a charged impurity, the role of the interaction radius is played by the impurity-potential screening parameter Λ:

$$\Lambda^2 = \frac{T}{4\pi e^2 N_e}, \tag{2.10}$$

where e is the electron charge. It is easy to verify that the criterion for slow scattering of Boltzmann electrons by defects $T \ll \hbar^2/m_e\Lambda^2$ is equivalent in this case to satisfaction of the condition

$$\hbar\Omega_L \gg T \gg \varepsilon_F, \quad \Omega_L^2 = 4\pi e^2 N_e/m_e. \tag{2.11}$$

Since $\Omega_L \propto N_e^{1/2}$ and $\varepsilon_F \propto N_e^{2/3}$, it follows that for not too high electron densities $N_e a_e^3 \ll 1$, $a_e = \hbar^2/m_e e^2$, as is the case in practically all semiconductors, there certainly exists a temperature region (2.11) in which low-energy scattering takes place and expression (2.9) is valid. We emphasize that to calculate the scattering lengths $a_{\uparrow\uparrow}$ and $a_{\uparrow\downarrow}$ one cannot use in this case perturbation theory in terms of the interaction, since the condition $T \gg m_e e^4/\hbar^2$ for the applicability of perturbation theory contradicts at $N_e a_e^3 \ll 1$ the criterion (2.11), i.e., $m_e e^4/\hbar^2 \gg \hbar\Omega_L$. Equation (2.9) contains thus the exact amplitudes of s-scattering of slow electrons by a screened Coulomb potential of a charged impurity.

The interaction energy δF_{int} of (2.9) does not depend explicitly on the temperature (only the electron and defect densities N_e and N_d depend on the temperature), and retains the form (2.9) for any degree of degeneracy of the electron gas. Expression (2.9) for an arbitrary degree of degeneracy of the electrons can be obtained also by the known renormalization method in the slow-collision limit.[5] In this case the real interaction potential is replaced by a certain arbitrary potential having the same values of $a_{\uparrow\uparrow}$ and $a_{\uparrow\downarrow}$, but which admits of the use of perturbation theory. The pseudopotential is expressed in the final results in terms of the s-scattering length. The calculation of the interaction energy by this method leads to Eq. (2.9) in first order in $q|a|/\hbar \ll 1$.

We have spoken so far only of electron interaction with point defects. Scattering of holes by defects can be described in perfect analogy. In the calculation of the thermodynamic characteristics it is necessary to take into account the contribution made by scattering of carriers of both types. To avoid excessively cumbersome equations, however, we shall consider hereafter semiconductors in which the majority carriers are, say, electrons. The results can be generalized without difficulty to the case when an appreciable number of holes is present in addition to the electrons.

3. MAGNETIC PHASES OF SEMICONDUCTOR

To investigate the thermodynamics of the magnetic phenomena in a doped semiconductor it is necessary to know the magnetic part of the free energy of the system. The principal ion lattice of the semiconductor is assumed to be nonmagnetic. Then, since the distance between defects is much larger than the lattice constant, the defects behave from the magnetic point of view like atoms of an ideal gas with a susceptibility that obeys the Curie law. The magnetic free energy, with allowance for exchange effects in the interaction (2.9) of electrons with the defects, is then

$$F_M = F_e(\alpha_e) + T[Y_d^+ + Y_d^-] + \frac{\pi\hbar^2}{2m_e} N_e N_d a_2 \alpha_e \alpha_d \mathfrak{M}_e \mathfrak{M}_d, \tag{3.1}$$

where $F_e(\alpha_e)$ is the magnetic energy of the electron system, and the following notation is introduced:

$$Y_i^{\pm} = N_i^{\pm} \ln N_i^{\pm} = \frac{N_i}{2}(1\pm\alpha_i)\ln\left[\frac{N_i}{2}(1\pm\alpha_i)\right], \quad i = e, d.$$

We have also neglected in (3.1) electron-electron exchange corrections that are small compared with (2.9) and decrease in power-law fashion with rising temperature.[4]

Minimization of the free energy with respect to the angle θ between the directions of the magnetizations of the electrons $\mathfrak{M}_e$ and of the defects $\mathfrak{M}_d$ does not lead direct to the conclusion that a ferromagnetic structure with $\theta = \pi$, i.e., with antiparallel equilibrium magnetic moments of the electrons and defects, is realized when the exchange scattering length is positive, $a_e > 0$. If the exchange scattering length is negative, $a_2 < 0$, ferromagnetic ordering with $\theta = 0$ takes place, wherein the magnetization of the electrons is parallel to the magnetic moment of the defects.

We minimize next the free energy (3.1) with respect to α_e and α_d. The conditions

$$\partial F_M/\partial\alpha_e = 0, \quad \partial F_M/\partial\alpha_d = 0$$

lead to a system of equations

$$\begin{aligned} &\frac{1}{N_e T}\frac{\partial F_e}{\partial \alpha_e} - \alpha_d \frac{E_2}{T}\gamma_{2d} = 0, \\ &\operatorname{arth}\alpha_d - \alpha_e \frac{E_2}{T}\gamma_{2e} = 0, \end{aligned} \tag{3.2}$$

where $E_2 = \pi\hbar^2/2m_e a_2^2$ and the "gas" parameters $\gamma_{2e} = N_e|a_2|^3$ and $\gamma_{2d} = N_d|a_2|^3$ have been introduced. The condition for the compatability of Eqs. (3.2) determines the magnetic phase-transition temperature T_c. The calculated T_c for an arbitrary degree of degeneracy of the electron gas, as well as the phenomenological conditions for the onset of magnetic instability in a semiconductor with defects, are given in the Appendix. We confine ourselves here to an investigation of the limiting case of high temperatures $T \gg \varepsilon_F$ and of the

region $T \ll \varepsilon_F$ of strong quantum degeneracy of the electrons.

As will be seen from the result, the region of applicability of the Boltzmann statistics for the calculation of the transition temperature $T_c \gg \varepsilon_F$ corresponds to the following condition imposed on the electron density:

$$N_e/N_d \ll \gamma_{2d}^2 \ll 1. \tag{3.3}$$

In this case F_e is determined by the classical expression

$$F_e = T(Y_e^+ + Y_e^-), \tag{3.4}$$

and the condition for the compatibility of (3.2) specifies the magnetic-ordering temperature:

$$T_c = E_2(\gamma_{2e}\gamma_{2d})^{1/2}. \tag{3.5}$$

At a defect density on the order of several percent, Eqs. (3.3) and (3.5) correspond to values $N_e \ll 10^{16}$ cm^{-3} and $T \ll 10^2$ K.

Near the phase-transition point $\tau \equiv (T_c - T)/T_c \ll 10^2$ K we obtain from Eqs. (3.2), taking (3.3) into account, the equilibrium values of the polarization α_e and α_d, i.e., of the order parameters:

$$\alpha_e = (6\tau)^{1/2}, \quad \alpha_d = (N_e/N_d)^{1/2}\alpha_e. \tag{3.6}$$

It is seen from (3.6) that in the Boltzmann case the electron spins are polarized much more strongly than the spins of the magnetic defects. At $\varepsilon_F \ll T \ll T_c$ the electron component is practically completely polarized:

$$\alpha_e = 1 - \exp[-2(T_c/T)^2], \tag{3.7}$$

while the degree of polarization of the impurities is still small:

$$\alpha_d = \frac{T_c}{T}\left(\frac{N_e}{N_d}\right)^{1/2} \ll 1. \tag{3.8}$$

The situation is entirely different if the phase transition takes place in the region of the Fermi degeneracy of the electron component, i.e., if $T_c \ll \varepsilon_F$. For a strongly degenerate electron gas we have

$$F_e = {}^1/_2 E_F[(1+\alpha_e)^{5/3} + (1-\alpha_e)^{5/3}], \quad E_F = {}^3/_5 \varepsilon_F N_e. \tag{3.9}$$

Substitution of (3.9) in (3.2) leads to the following expression for T_c:

$$T_c = \frac{3}{2}\frac{E_2^2}{\varepsilon_F}\gamma_{2e}\gamma_{2d}. \tag{3.10}$$

The condition $T_c \ll \varepsilon_F$ is then equivalent to an inequality inverse to (3.3):

$$N_e/N_d \gg \gamma_{2d}^2. \tag{3.11}$$

The electron density should in this case not be too high, $\gamma_{2e} \ll 1$, in order for Eqs. (2.9) and (3.2), which were derived for low-energy scattering, to remain valid. Near the temperature of the magnetic phase transition, the spontaneous magnetizations of the electrons and of the defects are given by

$$\alpha_e = \left(\frac{27}{2}\tau\right)^{1/2}\left[1 + \frac{1}{\varkappa}\right]^{-1/2}, \quad \alpha_d = (3\tau)^{1/2}[1+\varkappa]^{-1/2}, \quad \varkappa = \frac{N_d}{3N_e}\frac{T_c}{\varepsilon_F}. \tag{3.12}$$

If the electron density is high enough:

$$N_e/N_d \gg \gamma_{2d}^{3/7} \gg \gamma_{2d}^2, \tag{3.13}$$

then the parameter $\varkappa \ll 1$, and the degree of polarization of the defects greatly exceeds the degree of polarization of the electrons $\alpha_e \ll \alpha_d \ll 1$. When condition (3.13) is satisfied, the same relation between α_e and α_d holds also at $T \ll T_c$, when the defect mechanization is close to the nominal value:

$$\alpha_d = 1 - \exp(-2T_c/T), \tag{3.14}$$

and the electron gas is only insignificantly polarized:

$$\alpha_e = 3(\varkappa/2)^{1/2} \ll 1. \tag{3.15}$$

At intermediate values of the electron density

$$\gamma_{2d}^{3/7} \gg N_e/N_d \gg \gamma_{2d}^2, \tag{3.16}$$

when $\varkappa \gg 1$, the values of α_e and α_d near T_c are also determined by Eqs. (3.12), and $\alpha_d \ll \alpha_d \ll 1$. At the temperature $T^* = 2^{4/3}T_c/3$ the electron magnetic moment reaches its nominal value $\alpha_e(T^*) = 1$. Near T^* the value of α_e approaches unity in power-law fashion

$$\alpha_e = 1 - 2(\tau^*)^{3/2}, \quad \tau^* = (T - T^*)/T^* \ll 1, \tag{3.17}$$

and α_d remains small as before:

$$\alpha_d = \frac{1}{3}\left(\frac{2}{\varkappa}\right)^{1/2}\frac{T_c}{T}\alpha_e = \frac{1}{2^{3/2}\varkappa^{1/2}}(1-\tau^*) \ll 1. \tag{3.18}$$

At a defect density ~0.1%, the conditions (3.11) and $\gamma_{2e} \ll 1$ correspond to electron densities $10^{13} \ll N_e \ll 10^{22}$ cm^{-3}, with 1 K $\ll T_c \ll 10^3$ K. Equation (3.10) for the phase-transition temperature can be generalized also to include the case when account is taken of the Fermi-liquid interaction between the electrons:

$$T_c = \frac{3}{2\varepsilon_F}E_2^2\frac{1+F_1/3}{1+Z_0}\gamma_{2e}\gamma_{2d}, \tag{3.19}$$

where Z_0 and F_1 are the coefficients in the expansion of the Landau f-function[3] in Legendre polynomials. We note that it follows from (3.6)–(3.18) that $N_d\alpha_d \gg N_e\alpha_e$ in all cases.

Expressions (3.5), (3.10), and (3.19) are in essence the equations for the phase-transition temperature, since the electron and hole densities N_e and N_d, which enter in γ_{2e} and γ_{2d}, are themselves functions of temperature. It is clear, however, that Eqs. (3.5), (3.10), and (3.19) always have solutions for semiconductors with a sufficiently narrow energy gap (at a low ionization potential of definite impurity levels) which

determines the number of electrons in the conduction band.

4. STRUCTURAL PHASE TRANSITION

We are not interested in the specific structure of the undistorted crystal lattice of the semiconductor. We therefore describe it simply by specifying the ion density, i.e., the average number N_i of ions per unit volume. The free energy F per unit volume of the semiconductor is a function of the temperature T and of the densities N_e, N_i, and N_d. If the particles were to have no charge, the thermodynamic conditions for the stability of the system with respect to stratification (i.e., to the onset of spontaneous δN_e, δN_i, and δN_d) would reduce simply to the system of equations

$$\partial F/\partial N_k=\xi_k(T, N_i, N_d, N_e)=0, \quad k=e, i, d,$$

which indicate that the free energy is a minimum. Since, however, the lattice ions and the electrons are charged (we assume hereafter for the sake of argument that the defects are neutral), this stratification, which violates the macroscopic homogeneity of the system, is accompanied also by the onset of a macroscopic electric field. The corresponding equilibrium conditions, in which the appearing external electric field must be taken into account, should therefore be supplemented by the self-consistent Poisson equation that describes this field. In the approximation linear in δN_k we obtain

$$\begin{aligned}
&\frac{\partial \xi_e}{\partial N_e}\delta N_e+\frac{\partial \xi_e}{\partial N_i}\delta N_i+\frac{\partial \xi_e}{\partial N_d}\delta N_d=e\varphi,\\
&\frac{\partial \xi_i}{\partial N_e}\delta N_e+\frac{\partial \xi_i}{\partial N_i}\delta N_i+\frac{\partial \xi_i}{\partial N_d}\delta N_d=-e\varphi,\\
&\frac{\partial \xi_d}{\partial N_e}\delta N_e+\frac{\partial \xi_d}{\partial N_i}\delta N_i+\frac{\partial \xi_d}{\partial N_d}\delta N_d=0, \quad \Delta\varphi+4\pi e(\delta N_i-\delta N_e)=0,
\end{aligned} \tag{4.1}$$

where φ is the potential of the electric field, and $\mp e$ stands for the charges of the electron and ion. When Eqs. (4.1) are used to describe a fully ionized gas ($N_d=0$), the usual formulas for the Debye screening are obtained automatically.

Eliminating δN_e, δN_i, and δN_d from the first three equations of (4.1) and substituting the obtained expressions in the Poisson equation, we get

$$\Delta\varphi-4\pi e^2D(A_n^k)\varphi=0, \quad D(A_n^k)=\frac{A_d^d(A_e^e+A_i^i+2A_i^e)-(A_d^e+A_d^i)^2}{\det\|A_n^k\|}, \tag{4.2}$$

where we have introduced the symmetric 3×3 matrix $\|A_n^k\|$ with elements

$$A_n^k=A_k^n=\partial^2F/\partial N_n\partial N_k, \quad k=e, i, d.$$

The quantity $4\pi e^2D(A_n^k)$ determines the character of the screening of the electrostatic field in a semiconductor with defects. At $D(A_n^k)<0$ Eq. (4.2) has an oscillating solution with wavelength $\lambda_s=\pi^{1/2}/e|D|^{1/2}$, which corresponds to a periodic spatial distribution of the electrons, ions, and defects (and of the electric field) inside the semiconductor, i.e., to the appearance, in the electron gas, of a charge-density wave and of a matching superstructure in the crystal lattice with defects. The temperature values at which $D(A_n^k)=0$ determine the points of the structural phase transition. In principle, of course, it is possible for the function $D(T)$ to be only the tangent to the temperature axis at one or several points. At these points we then have $D(A_n^k)=0$, and at all the remaining temperatures $D(A_n^k)>0$ and no structural phase transition takes place. The points at which $D(A_n^k)=0$ correspond to a pure Coulomb solution of Eq. (4.2) for the potential φ.

The use of expression (4.2) for a weakly ionized gas leads to the results of Ref. 1 for the temperature of the structural transition and for the period of the charge-density wave. In our case, the addition made to A_d^e by the interaction of the electrons with the defects is also equal to $\pi\hbar^2a_1/2m_e$, but the calculation of all the elements of the matrix $\|A_n^k\|$ calls already for invoking concrete model concepts concerning the thermodynamics of the crystal lattice and of the electrons, on which we shall not dwell at present. We emphasize that it is meaningful to speak of a charged density wave only when the wave-length λ_s of the periodic superstructure is macroscopic in scale:

$$\lambda_sN_e^{1/3}\gg1, \quad \lambda_sN_d^{1/3}\gg1, \quad \lambda_sN_i^{1/3}\gg1.$$

5. KINETIC EQUATION

We do not consider in this paper dissipative relaxation processes connected with scattering of the electrons by the defects. We therefore confine ourselves to simple gaskinetic estimates, in the τ approximation, of the kinetic-equation collision terms that are quadratic in the scattering amplitude. On the other hand, quantum refraction effects (without dissipation!) lead to the appearance, in the kinetmatic part of the Boltzmann equation, of an additional term that is linear in the zero-angle scattering amplitude. To calculate this essentially quantum correction, we use the renormalization method employed in Ref. 6 to determine the coefficient of refraction of a particle beam by a rarefied medium. We introduce a certain effective potential of the electron–defect interaction:

$$u_{\alpha\beta,\,\mu\nu}(r)=u_1(r)\,\delta_{\alpha\beta}\delta_{\mu\nu}+u_2(r)\,\sigma_{\alpha\beta}\sigma_{\mu\nu},$$

which satisfies the condition $|u|\ll\hbar^2/\mu r_0^2$ for the applicability of perturbation theory, and define it in such a way that in the Born approximation it yields the true value of the zero-angle scattering amplitude $\Gamma_{\alpha\beta,\,\mu\nu}(0)$, i.e.,

$$\int u_{\alpha\beta,\mu\nu}dV=-\frac{2\pi\hbar^2}{\mu}\Gamma_{\alpha\beta,\mu\nu}(0). \tag{5.1}$$

The effective Hamiltonian of the electron system is in this case (it is assumed that the main contribution is made by electron scattering by defects)

$$\hat{H}=\sum_{i,k}\Big(\varepsilon_{ik}+\sum_{r,s}u_{ks}{}^{ir}\hat{b}_r^+\hat{b}_s\Big)\,\hat{a}_i^+\hat{a}_k, \tag{5.2}$$

where ε_{ik} is the single-electron energy spectrum, $u_{ks}^{ir} = \langle ir|u|ks\rangle$ is the corresponding matrix element, $\hat{a}_i^+$, $\hat{a}_k$ and $\hat{b}_r^+$, $\hat{b}_s$ are the Fermi creation and annihilation operators with the usual commutation relations

$$[\hat{a}_i,\ \hat{a}_k^+] = \delta_{ik}, \quad [\hat{b}_i,\ \hat{b}_k^+] = \delta_{ik}. \tag{5.3}$$

We define the single-particle density matrices of the electrons $f_{ik}^{(e)}$ and of the defects $f_{rs}^{(d)}$ by the relations

$$f_{ik}^{(e)} = \langle \hat{a}_k^+ \hat{a}_i \rangle, \quad f_{rs}^{(d)} = \langle \hat{b}_s^+ \hat{b}_r \rangle. \tag{5.4}$$

The kinetic equation for $f_{ik}^{(e)}$ is obtained by averaging the Liouville equation for the operator $f_{ij}^{(e)} = \hat{a}_k^+ \hat{a}_i$:

$$\frac{\partial f_{ik}^{(e)}}{\partial t} = \frac{i}{\hbar} \langle [\hat{H}, \hat{f}_{ik}^{(e)}] \rangle. \tag{5.5}$$

In Eq. (5.5) we left out of the total Hamiltonian of the system those terms this correspond to defects and commute with $f_{ik}^{(e)}$. In the first approximation in u we obtain from (5.5), with the aid of (5.2), (5.3), and Wick's theorem

$$\frac{\partial f_{ik}^{(e)}}{\partial t} = \frac{i}{\hbar} \sum_m (\tilde{\varepsilon}_{mi} f_{km}^{(e)} - \tilde{\varepsilon}_{km} f_{mi}^{(e)}) = \frac{i}{\hbar} [\hat{f}_{ik}^{(e)}, \hat{\tilde{\varepsilon}}_{ik}],$$
$$\tilde{\varepsilon}_{ik} = \varepsilon_{ik} + \sum_{r,s} u_{ks}^{ir} f_{rs}^{(d)}. \tag{5.6}$$

In standard manner (see, e.g., Ref. 7) in (5.6) from $f_{p_1\alpha;\, p'\beta}^{(e)}$ to the Wigner momentum distribution function

$$n_{\alpha\beta}^{(e)}(\mathbf{p}) = \sum_{\mathbf{k}} e^{-i\mathbf{k}\mathbf{r}} f_{\mathbf{p}-\hbar\mathbf{k}/2,\alpha;\ \mathbf{p}+\hbar\mathbf{k}/2,\beta}^{(e)}, \tag{5.7}$$

we obtain the kinetic equation for the polarization matrix of the electron density $n_{\alpha\beta}^{(e)}(\mathbf{p})$, which takes in the weakly inhomogeneous case the usual quasiclassical form:

$$\frac{\partial n_{\alpha\beta}^{(e)}}{\partial t} + \frac{1}{2}\left(\frac{\partial \varepsilon_{\alpha\gamma}}{\partial \mathbf{p}} \nabla n_{\gamma\beta}^{(e)} + \nabla n_{\alpha\gamma}^{(e)} \frac{\partial \varepsilon_{\gamma\beta}}{\partial \mathbf{p}}\right) - \frac{1}{2}\left(\nabla \varepsilon_{\alpha\gamma} \frac{\partial n_{\gamma\beta}^{(e)}}{\partial \mathbf{p}} + \frac{\partial n_{\alpha\gamma}^{(e)}}{\partial \mathbf{p}} \nabla \varepsilon_{\gamma\beta}\right) + \frac{i}{\hbar}[\hat{\varepsilon}, \hat{n}]_{\alpha\beta} = I_{\alpha\beta}, \tag{5.8}$$

where $[\hat{\varepsilon}, \hat{n}]_{\alpha\beta}$ is a commutator in spin space, and the renormalized excitation energy is expressed with the aid of (5.1) in terms of the forward-scattering amplitude

$$\varepsilon_{\alpha\beta}(\mathbf{p}) - \varepsilon_{\alpha\beta}^{(0)}(\mathbf{p}) = -\frac{2\pi\hbar^2}{\mu} \sum_{\mathbf{p}'} \operatorname{Re} \Gamma_{\alpha\beta,\mu\nu}(0)\, n_{\nu\mu}^{(d)}(\mathbf{p}')$$
$$= -\frac{2\pi\hbar^2}{m_e} N_{\nu\mu}^{(d)} \operatorname{Re} \Gamma_{\alpha\beta,\mu\nu}(0). \tag{5.9}$$

In (5.9), $\varepsilon_{\alpha\beta}^{(0)}(\mathbf{p})$ is the unrenormalized single-electron energy, $n_{\alpha\beta}^{(d)}$ is the polarization density matrix of the defects. The collision integral $I_{\alpha\beta}$ contains terms quadratic in u, which we have left out when deriving the kinetic part of Eq. (5.8), and a term linear in u, which is proportional to $\operatorname{Im} \Gamma_{\alpha\beta,\mu\nu}(0)$, and which is the principal term in the expansion of $I_{\alpha\beta}$ in the gradients of the distribution function. Using the optical theorem

$$\operatorname{Im} \Gamma_{\alpha\beta,\mu\nu}(0) = \frac{|\mathbf{p}-\mathbf{p}'|}{4\pi\hbar} \sigma_{\alpha\beta,\mu\nu},$$
$$\sigma_{\alpha\beta,\mu\nu} = \frac{3\sigma_{\uparrow\uparrow} + \sigma_{\uparrow\downarrow}}{4} \delta_{\alpha\beta}\delta_{\mu\nu} + \frac{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}}{4} \sigma_{\alpha\beta}\sigma_{\mu\nu}, \tag{5.10}$$

where $\sigma_{\alpha\beta,\mu\nu}$ is the total scattering cross section, we obtain the gradient correction to the local collision integral:

$$I_{\alpha\beta}^{(1)} = -\frac{\hbar}{2\mu} \nabla n_{\alpha\gamma} \frac{\partial}{\partial \mathbf{p}} \sum_{\mathbf{p}'} |\mathbf{p}-\mathbf{p}'| \sigma_{\gamma\beta,\mu\nu} n_{\nu\mu}^{(d)}(\mathbf{p}')$$
$$= -\frac{\hbar}{2m_e} N_{\nu\mu}^{(d)} \nabla n_{\alpha\gamma}^{(e)} \frac{\partial}{\partial \mathbf{p}} (p\sigma_{\alpha\beta,\mu\nu}). \tag{5.11}$$

In the gaskinetic approximation,

$$I^{(1)} \propto (\delta n^{(e)}) N^{(d)} v_{Te} \sigma k \lambda_e,$$

where v_{Te} is the thermal velocity, λ_e is the deBroglie wavelength of the electron, and k is the wave vector of the inhomogeneity.

For the scattering of long-wave electrons $|a_{\uparrow\uparrow}|/\lambda_e \ll 1$ and $|a_{\uparrow\downarrow}|/\lambda_e \ll 1$, the real gradient term in the left-hand side of the kinetic equation (5.8),

$$\frac{2\pi\hbar^2}{m_e} \frac{\partial n_{\alpha\gamma}^{(e)}}{\partial \mathbf{p}} a_{\gamma\beta,\mu\nu} \nabla N_{\nu\mu}^{(d)},$$

which describes a quantum self-consistent interaction of the Fermi-liquid type, greatly exceeds the imaginary nonlocal correction (5.11) in the collision integral

$$I_{\alpha\beta}^{(1)} = -\frac{\hbar}{2m_e} \sigma_{\alpha\beta,\mu\nu} N_{\nu\mu}^{(d)} \frac{\mathbf{p}}{p} \nabla n_{\alpha\gamma}^{(e)},$$

and allowance for the collective effects connected with the zero-angle electron scattering is not an exaggeration of the accuracy. Thus, for the interaction of slow electrons with defects, in the principal approximation in r_0/λ_e, the Fermi-liquid theory scheme can be used[3], namely, the excitation energy is defined as a variational derivative $\varepsilon_{\alpha\beta} = \delta E/\delta n_{\alpha\beta}$ (E is the total energy of the system) and $\varepsilon_{\alpha\beta}$ is then substituted in the Boltzmann quasiclassical equation.

6. SPIN WAVES

We shall use the collisionless kinetic equation (5.8) to study the high-frequency oscillations of the magnetic moment in a semiconductor. Just as before, we consider a cubic lattice, in which the unrenormalized energy spectrum of the electron has the simple form $\varepsilon_{\alpha\beta}^{(0)} = (p^2/2m_e)\delta_{\alpha\beta}$. To describe the magnetic dynamics of the defects we likewise use Eq. (5.8) and the relations (2.1)–(2.4), and go in the final results to the limit of an infinitely large defect mass $m_d \to \infty$. We assume that the small perturbations of the polarization density matrices

$$\delta n_{\alpha\beta}^{(e)} = \nu_e \sigma_{\alpha\beta}, \quad \delta n_{\alpha\beta}^{(d)} = \nu_d \sigma_{\alpha\beta}$$

relative to the equilibrium values (2.1) and (2.2) vary

in the spin-wave field in proportion to $\exp(i\omega t - i\mathbf{k}\cdot\mathbf{r})$, where ω and $\mathbf{k}$ are the frequency and wave vector of the oscillations. Taking the foregoing into account, the linearized kinetic equations for the electrons and defects take the form

$$\begin{aligned}(\omega-\mathbf{kv}_e)\mathbf{v}_e+\mathbf{kv}_e\left(\frac{\partial n_e^+}{\partial\varepsilon}+\frac{\partial n_e^-}{\partial\varepsilon}\right)Z_0\sum_{\mathbf{p}'}\mathbf{v}_d(\mathbf{p}')-i\frac{2Z_0}{\hbar}\left[\rho_e\sum_{\mathbf{p}'}\mathbf{v}_d(\mathbf{p}')\right]\\ +i\frac{2Z_0}{\hbar}\alpha_d N_d[\mathfrak{M}_d\times\mathbf{v}_e(\mathbf{p})]+i\frac{Z_0}{\hbar}\alpha_d N_d[\mathfrak{M}_d\times\rho_e]=0,\\ (\omega-\mathbf{kv}_d)\mathbf{v}_d+\mathbf{kv}_d\left(\frac{\partial n_d^+}{\partial\varepsilon}+\frac{\partial n_d^-}{\partial\varepsilon}\right)Z_0\sum_{\mathbf{p}'}\mathbf{v}_e(\mathbf{p}')-i\frac{2Z_0}{\hbar}\left[\rho_d\sum_{\mathbf{p}'}\mathbf{v}_e(\mathbf{p}')\right]\\ +i\frac{2Z_0}{\hbar}\alpha_e N_e[\mathfrak{M}_e\times\mathbf{v}_d(\mathbf{p})]+i\frac{Z_0}{\hbar}\alpha_e N_e[\mathfrak{M}_e\times\rho_d]=0,\end{aligned}\tag{6.1}$$

where $\mathbf{v}_e=\mathbf{p}/m_e$, $\mathbf{v}_d=\mathbf{p}/m_d$, and $Z_0=\pi\hbar^2a_2/\mu$. In a magnetically ordered semiconductor, the vectors $\mathfrak{M}_e$ and $\mathfrak{M}_d$ are collinear or anticollinear (see Sec. 2). The expressions obtained below are valid also at temperatures above the magnetic phase transition if the polarization of the spins of the electrons and defects is reached by different dynamic methods (injection of polarized particles, optical pumping) and $\mathfrak{M}_e\cdot\mathfrak{M}_d=\pm1$. Equations (6.1) must be supplemented by Maxwell's equations, but we confine ourselves here to an investigation of transverse oscillations, which do not involve Maxwell's equations. We choose the z axis in the $\mathfrak{M}_e$ direction. In this situation Eq. (6.1) become much simpler:

$$\begin{aligned}(\omega\mp\Omega_{int}^{(d)}-\mathbf{kv}_e)\mathbf{v}_e^++\mathbf{kv}_e\left(\frac{\partial n_e^+}{\partial\varepsilon}+\frac{\partial n_e^-}{\partial\varepsilon}\right)Z_0\sum_{\mathbf{p}'}\mathbf{v}_d^+(\mathbf{p}')\\ +\frac{2Z_0}{\hbar}\rho_e\sum_{\mathbf{p}'}\mathbf{v}_d^+(\mathbf{p}')=0,\\ (\omega-\Omega_{int}^{(e)}-\mathbf{kv}_d)\mathbf{v}_d^++\mathbf{kv}_d\left(\frac{\partial n_d^+}{\partial\varepsilon}+\frac{\partial n_d^-}{\partial\varepsilon}\right)Z_0\sum_{\mathbf{p}'}\mathbf{v}_e^+(\mathbf{p}')\\ \pm\frac{2Z_0}{\hbar}\rho_d\sum_{\mathbf{p}'}\mathbf{v}_e^+(\mathbf{p}')=0.\end{aligned}\tag{6.2}$$

Here $\Omega_{int}^{(i)}=(2Z_0/\hbar)\alpha_i N_i$, $i=e$ or d, the upper signs correspond to the ferromagnetic structure $\mathfrak{M}_e\cdot\mathfrak{M}_d=1$, and the lower to the ferrimagnetic order $\mathfrak{M}_e\cdot\mathfrak{M}_d=-1$, and the circular components

$$v_{ex}\pm iv_{ey}=v_e^{\pm},\quad v_{dx}\pm iv_{dy}=v_d^{\pm}$$

were introduced. The equations for ν_e^- and ν_d^- are obtained from (6.2) by making the substitutions $\omega\to-\omega$ and $\mathbf{k}\to-\mathbf{k}$. Integrating (6.2) with respect to $d\mathbf{p}$, we obtain a system of equations whose compatibility condition determines the dispersion law of the magnetization oscillations:

$$1-Z_0^2[R_e(\omega\mp\Omega_{int}^{(d)})+Q_e(\omega\mp\Omega_{int}^{(d)})][R_d(\omega-\Omega_{int}^{(e)})\pm Q_d(\omega-\Omega_{int}^{(e)})]=0,\tag{6.3}$$

where the functions $R_{e,d}(x)$ and $Q_{e,d}(x)$ are given by

$$\begin{aligned}R_i(x)&=\sum_{\mathbf{p}}\frac{\mathbf{kv}_i(\partial n_i^+/\partial\varepsilon+\partial n_i^-/\partial\varepsilon)}{x-\mathbf{kv}_i},\\ Q_i(x)&=\frac{2}{\hbar}\sum_{\mathbf{p}}\frac{n_i^+-n_i^-}{x-\mathbf{kv}_i};\quad i=e,d.\end{aligned}\tag{6.4}$$

Within the accuracy limits, $n_i^{\pm}$ in (6.1)–(6.4) should be taken to mean the occupation numbers of the ideal gas (2.2) neglecting the interaction of the electrons with the defects.

At high temperatures, in the Boltzmann region $T\gg\varepsilon_F$ we have for the functions $r_i(x)$ and $Q_i(x)$

$$\begin{aligned}R_i(x)&=\frac{N_i}{T}\left[1+F\left(\frac{x}{2^{1/2}kv_{Ti}}\right)\right],\\ Q_i(x)&=-\frac{\Omega_{int}^{(i)}}{xZ_0}F\left(\frac{x}{2^{1/2}kv_{Ti}}\right);\quad i=e,d,\end{aligned}\tag{6.5}$$

where $v_{Ti}^2=T/m_i$ is the thermal velocity and the function $F(y)$ is expressed in terms of the probability integral:

$$F(y)=\frac{y}{\pi^{1/2}}\int_{-\infty}^{\infty}\frac{e^{-z^2}dz}{z-y},\quad \frac{\pi^{1/2}}{2y}[F(y^{1/2})+F(-y^{1/2})]=\frac{\pi e^y}{iy^{1/2}}\operatorname{Erfc}(iy^{1/2}).\tag{6.6}$$

With the aid of the asymptotic representation of the function $F(y)$ (see, e.g., Ref. 8)

$$F(y)=-1-\frac{1}{2y^2}-\frac{3}{4y^4}+\ldots+i\pi^{1/2}ye^{-y^2},\quad y\gg1,$$

we obtain the long-wave ($kv_{Te}\ll\Omega_{int}^{(d)}$, $kv_{Td}\ll\Omega_{int}^{(e)}$) solutions of the dispersion equation (6.3):

$$\omega=-\frac{\Omega_{int}^{(e)}\Omega_{int}^{(d)}}{\Omega_{int}^{(d)}\pm\Omega_{int}^{(e)}}\left(\frac{k^2v_{Te}^2}{\Omega_{int}^{(d)2}}+\frac{k^2v_{Td}^2}{\Omega_{int}^{(e)2}}\right),\tag{6.7}$$

$$\omega=\Omega_{int}^{(e)}\pm\Omega_{int}^{(d)}+\frac{\Omega_{int}^{(e)}\Omega_{int}^{(d)}}{\Omega_{int}^{(d)}\pm\Omega_{int}^{(e)}}\left(\frac{k^2v_{Te}^2}{\Omega_{int}^{(e)2}}+\frac{k^2v_{Td}^2}{\Omega_{int}^{(d)2}}\right),\tag{6.8}$$

and in this case the collisionless damping is exponentially small. We emphasize that in this case, even in the exchange approximation, without allowance for the weak relativistic interactions, a high-frequency spin wave with a gaplike dispersion law (6.8) (an "optical" magnon of sorts) exists besides the traditional gapless branch (6.7).

The results, in the form (6.7) and (6.8), can be used to describe the oscillations of the magnetic moment in a weakly ionized gas.[1] In our case of localized defects, it is necessary to go in (6.7) and (6.8) to the limit $m_d\to\infty$:

$$\omega=-\frac{N_e\alpha_e}{N_d\alpha_d\pm N_e\alpha_e}\frac{k^2v_{Te}^2}{\Omega_{int}^{(d)}},\quad \Omega_{int}^{(d)}=\frac{2\pi\hbar a_2}{m_e}N_d\alpha_d,\tag{6.9}$$

$$\omega=\Omega_{int}^{(e)}\pm\Omega_{int}^{(d)}+\frac{N_d\alpha_d}{N_d\alpha_d\pm N_e\alpha_e}\frac{k^2v_{Te}^2}{\Omega_{int}^{(e)}},\quad \Omega_{int}^{(e)}=\frac{2\pi\hbar a_2}{m_e}N_e\alpha_e.\tag{6.10}$$

With the aid of Eqs. (6.7), (6.8) and (6.9), (6.10) it is easy to verify that the spectra of the gap and Goldstone spin modes always diverge in different directions and never cross. In the Boltzmann region, the condition $\omega\tau\gg1$ that the relaxation absorption be small calls for an appreciable spin polarization. Inasmuch as in a magnetically ordered Boltzmann semiconductor, as follows from (3.3) and (3.6)-(3.8), we have $N_e\alpha_e\ll N_d\alpha_d$, a wave-vector region

$$\Omega_{int}^{(d)}/v_{Te}\gg k\gg(\tau\partial\omega/\partial k)^{-1},\tag{6.11}$$

in which both the collision and the Čerenkov absorption are small exists, and observation of a weakly damped spin wave is possible only for the gap mode (6.10). The existence of the region (6.11) is ensured by satisfaction of the inequality $1 \gg \tau \gg (\gamma_{2e}\gamma_{2d})^{1/2}$ near T_c and of the obvious relation $(T_v/T)^2 \gg |a_2|/\lambda_e$ at $\varepsilon_F \ll T \ll T_c$. At a defect density of several percent, the wavelength of the weakly damped oscillations of the magnetization should exceed 10^{-4}–10^{-3} cm. The existence of a gapless spin wave calls for a stronger magnetic polarization of the system, which can be reached both by dynamic-polarization methods and by turning on a sufficiently strong external magnetic field (see below).

In the case of strong quantum degeneracy of the electrons $T \ll \varepsilon_F$, the functions $R_e(x)$ and $Q_e(x)$ in the disperson equation (6.3) take the form

$$R_e(x) = -g_+ w(s_+) - g_- w(s_-), \quad s_\pm = x/kv_\pm,$$
$$Q_e(x) = \frac{3N_e\alpha_e}{\hbar x} + \frac{3N_e^+}{\hbar x}(1-s_+^2)w(s_+) - \frac{3N_e^-}{\hbar x}(1-s_-)w(s_-), \tag{6.12}$$
$$w(s) = \frac{s}{2}\ln\frac{s+1}{s-1} - 1,$$

where $g_\pm = 3N_e^\pm/m_e v_\pm^2$ are the state density on the separated Fermi surface corresponding to electrons with different spin orientations, and $v_\pm$ are the limiting Fermi velocities. Substituting in (6.3) the functions R_e and Q_e from (6.12) for the electrons and expressions (6.5) for the defects, we obtain in second order in $kv_\pm/\Omega_{int}^d \ll 1$ the magnon spectra ($m_d \to \infty$)

$$\omega = -\frac{\Omega_{int}^{(e)}}{\Omega_{int}^{(d)} \pm \Omega_{int}^{(e)}}\frac{2}{3}\frac{E_+ - E_-}{N_e\alpha_e}\frac{k^2}{m_e\Omega_{int}^{(d)}},$$
$$E_\pm = \frac{3}{10}(6\pi^2)^{2/3}\frac{\hbar^2}{m_e}(N_e^\pm)^{2/3}, \tag{6.13}$$

$$\omega = \Omega_{int}^{(e)} \pm \Omega_{int}^{(d)} + \frac{\Omega_{int}^{(d)}}{\Omega_{int}^{(d)} \pm \Omega_{int}^{(e)}}\frac{2}{3}\frac{E_+ - E_-}{N_e\alpha_e}\frac{k^2}{m_e\Omega_{int}^{(e)}}. \tag{6.14}$$

In a spontaneously magnetized semiconductor we always have $N_d\alpha_d \gg N_e\alpha_e$, so that Eqs. (6.13) and (6.14) become even simpler. Just as in the Boltzmann region, the condition (6.11) can be satisfied here only for the gap mode (6.14) at $\tau \gg \gamma_{2e}^{4/3}$ if $\varkappa \ll 1$ [see (3.12)] and at $\tau \gg \gamma_{ed}^2$ if $\varkappa \gg 1$.

7. SPIN WAVES IN A MAGNETIC FIELD

We consider also the important case when the polarization of the electron and the defect spins is produced by turning on an external magnetic field **H**. In this case it is necessary to add to the kinetic equation a term that takes into account the Lorentz force acting on the charged particle in the magnetic field, as well as a term due to the increment $\beta\sigma_{\alpha\beta}\cdot\mathbf{H}$ to the single-particle energy. We consider for the sake of argument a semiconductor at $T > T_c$ with neutral magnetic defects (the assumption that the localized defect is neutral is not restrictive, since its mass is assumed to be infinitely large in any case). In the s-wave approximation, the collisionless kinetic equations for transverse oscillations of the magnetic moment take the following form:

$$(\omega+\omega_0-\Omega_{int}^{(d)}-\mathbf{kv}_e)\nu_e^+ + i\frac{e}{c}[\mathbf{v}_e\times\mathbf{H}]\frac{\partial}{\partial\mathbf{p}}\nu_e^+ + \frac{2Z_0}{\hbar}\rho_e\sum_{\mathbf{p}'}\nu_d^+(\mathbf{p}') + \mathbf{kv}_e\left(\frac{\partial n_e^+}{\partial\varepsilon} + \frac{\partial n_e^-}{\partial\varepsilon}\right)Z_0\sum_{\mathbf{p}'}\nu_d^+(\mathbf{p}') = 0, \tag{7.1}$$

$$(\omega+\omega_0-\Omega_{int}^{(e)}-\mathbf{kv}_d)\nu_d^+ + \mathbf{kv}_d\left(\frac{\partial n_d^+}{\partial\varepsilon} + \frac{\partial n_d^-}{\partial\varepsilon}\right)Z_0\sum_{\mathbf{p}'}\nu_e^+(\mathbf{p}') + \frac{2Z_0}{\hbar}\rho_d\sum_{\mathbf{p}'}\nu_d^+(\mathbf{p}') = 0, \tag{7.2}$$

where $\omega_0 = 2\beta H/\hbar$ is the electron-spin free-precession frequency. Integrating (7.2), we obtain

$$\sum_{\mathbf{p}}\nu_d^+(\mathbf{p}) = -Z_0[R_d(\omega+\omega_0-\Omega_{int}^{(e)}) + Q_d(\omega+\omega_0-\Omega_{int}^{(e)})]\sum_{\mathbf{p}}\nu_e^+(\mathbf{p}) \tag{7.3}$$

and substituting next (7.3) and (7.1) we obtain an integro-differential equation for ν_e^+, which takes in cylindrical coordinates in **p**-space the form

$$i(\omega+\omega_0-\Omega_{int}^{(d)}-k_\perp v_{\perp e}\cos\varphi + k_z v_{ze})\nu_e^+ + \omega_{Le}\frac{\partial\nu_e^+}{\partial\varphi} = iZ_0^2\left[\frac{2\rho_e}{\hbar} + \mathbf{kv}_e\left(\frac{\partial n_e^+}{\partial\varepsilon} + \frac{\partial n_e^-}{\partial\varepsilon}\right)\right][R_d(\omega+\omega_0-\Omega_{int}^{(e)}) + Q_d(\omega+\omega_0-\Omega_{int}^{(e)})]\sum_{\mathbf{p}'}\nu_e^+(\mathbf{p}'), \tag{7.4}$$

where $\mathbf{k}\cdot\mathbf{v}_e = k_z v_{ze} + k_\perp v_{\perp e}\cos\varphi$ and $\omega_{Le} = eH/m_e c$ is the Larmor frequency, which depends in contrast to ω_0 on the effective electron mass m_e. To solve (7.4) we use a method similar to that employed in Refs. 8 and 9. We introduce the notation

$$\frac{\omega+\omega_0-\Omega_{int}^{(d)}-k_z v_{ze}}{\omega_{Le}} = \delta, \quad -\frac{k_\perp v_{\perp e}}{\omega_{Le}} = \gamma, \tag{7.5}$$
$$Q(\varphi) = \frac{iZ_0^2}{\omega_{Le}}[R_d(\omega+\omega_0-\Omega_{int}^{(e)}) + Q_d(\omega+\omega_0-\Omega_{int}^{(e)})]\left[\frac{2\rho_e}{\hbar} + \mathbf{kv}_e\left(\frac{\partial n_e^+}{\partial\varepsilon} + \frac{\partial n_e^-}{\partial\varepsilon}\right)\right]\sum_{\mathbf{p}}\nu_e^{(+)}(\mathbf{p}').$$

The change of variable $\nu_e^+ = e^{-i\gamma\sin\varphi}g$ reduces (7.4) to the form

$$\partial g/\partial\varphi + i\delta g = e^{i\gamma\sin\varphi}Q(\varphi). \tag{7.6}$$

Solving this equation with the aid of a Fourier expansion with respect to the variable φ:

$$g = \sum_{m=-\infty}^{\infty} g_m(p_\perp, p_z)e^{im\varphi}, \tag{7.7}$$

we obtain

$$\nu_e^+ = e^{-i\gamma\sin\varphi}\sum_{m=-\infty}^{\infty}\frac{Q_m}{i(m+\delta)}e^{im\varphi}, \tag{7.8}$$

where Q_m are the coefficients in the Fourier expansion of the function $e^{i\gamma\sin\varphi}Q(\varphi)$. Integrating (7.8) with respect to $d\mathbf{p}/(2\pi\hbar)^3$ and recognizing that the equilibrium distribution functions $n_e^\pm$ do not depend on φ, we obtain ultimately the sought dispersion equation

$$1=Z_0^2[R_d(\omega+\omega_0-\Omega_{int}^{(e)})+Q_d(\omega+\omega_0-\Omega_{int}^{(e)})]$$

$$\times\int\frac{p_\perp dp_\perp dp_z 2\pi}{(2\pi\hbar)^3\omega_{Le}}\sum_{m=-\infty}^{\infty}\frac{1}{m+\delta}\left\{J_m^2(\gamma)\left[\frac{2\rho_e}{\hbar}+k_z v_{ze}\left(\frac{\partial n_e^+}{\partial\varepsilon}+\frac{\partial n_e^-}{\partial\varepsilon}\right)\right]\right.$$

$$\left.+\frac{1}{2}[J_{m+1}(\gamma)+J_{m-1}(\gamma)]k_\perp v_{\perp e}\left(\frac{\partial n_e^+}{\partial\varepsilon}+\frac{\partial n_e^-}{\partial\varepsilon}\right)\right\},\qquad(7.9)$$

where $J_m(\gamma)$ is a Bessel function. Equation (7.9) can be used to describe the transverse oscillations of the magnetization in a cold plasma[1] in the presence of an external magnetic field. In the case of localized defects we have

$$R_d=0,\quad Q_d(x)=\Omega_{int}^{(d)}/Z_0 x.$$

Equation (7.9) becomes substantially simpler near the cyclotron resonances

$$|k_z|v_{Te}\ll\omega_{Le},\quad|\omega+\omega_0-\Omega_{int}^{(d)}-l\omega_{Le}|\ll\omega_{Le},\quad l=0,\pm1\ \pm2,\ldots.$$

Under these conditions, to determine the oscillation frequency in the vicinity of the l-th resonance it suffices to retain in the right-hand side of (7.9) only one term of the sum with $m=l$.

The solution of (7.9) at $\mathbf{k}=0$ corresponds to electron spin resonances (ESR) in a semiconductor with magnetic defects. At $\mathbf{k}=0$ only the term with $m=0$ remains in the right-hand side of (7.9), all the remaining terms vanish, and the dispersion equation becomes

$$(\omega+\omega_0-\Omega_{int}^{(e)})(\omega+\omega_0-\Omega_{int}^{(d)})=\Omega_{int}^{(e)}\Omega_{int}^{(d)}.\qquad(7.10)$$

Equation (7.10), which determines the ESR frequencies, has two solutions:

$$\omega=-\omega_0=-2\beta H/\hbar,\qquad(7.11)$$

$$\omega=-\omega_0+\Omega_{int}^{(e)}+\Omega_{int}^{(d)}=-\frac{2\beta H}{\hbar}+\frac{2\pi\hbar a_2}{m_e}(N_e\alpha_e+N_d\alpha_d),\qquad(7.12)$$

with $\alpha_d=\tanh(\beta H/T)$ and $\alpha_e=\alpha_d$ in the Boltzmann case $T\gg\varepsilon_F$, while in the Fermi-degeneracy region the value of α_e is given by

$$(1+\alpha_e)^{2/3}-(1-\alpha_e)^{2/3}=2\beta H/\varepsilon_F.\qquad(7.13)$$

Thus, there are two ESR frequencies whose values can differ strongly in sufficiently strong magnetic fields and at not too low a density of the defects (or electrons). We emphasize that the appearance of the second frequency is of pure exchange origin.

To avoid cumbersome expressions, we confine ourselves here to spin waves propagating strictly parallel to the external magnetic field, i.e., at $k_\perp=0$. In this case, too, only the term with $m=0$ remains in (7.9), which is then equivalent to Eq. (6.3) in which ω is replaced everywhere by $\omega+\omega_0$. Accordingly, the magnon spectrum is determined in this case by expressions (6.9) and (6.10) or by (6.13) and (6.14), to which it is necessary to add also the energy gap $(-\omega_0)$ due to the interaction of the spins of the free Fermi particles with the magnetic field. For the region (6.11) to exist, a sufficiently strong magnetic field is needed, to ensure the necessary degree of spin polarization. Thus, for example, to observe the weakly damped branches (6.9) and (6.10) at $N_e\sim N_d$ the magnetic field must be such that

$$\text{th}\,(\beta H/T)\gg|a_2|/\lambda_e,$$

which is reached at $H\sim100$ kOe already at high temperatures $T\ll10^3$ K.

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APPENDIX

From the phenomenological point of view, the question of the possibility of a magnetic phase transition in a system of electrons and defects reduces to a test of the stability of the quadratic form obtained when the total free energy is expanded in powers of the small magnetic moments of the electrons $\mathbf{M}_e$ and of the defects $\mathbf{M}_d$:

$$F=\text{const}+\frac{M_e^2}{2\chi_e}+\frac{M_d^2}{2\chi_d}+\frac{\mathbf{M}_e\mathbf{M}_d}{\chi_{int}},\qquad(A.1)$$

where χ_e and χ_d are the paramagnetic susceptibilities of the independent subsystems of electrons and defects, and χ_{int} is the susceptibility increment due to the interaction of the subsystems. Indeed, when an external magnetic field H is applied, the increment to the magnetic free energy due to the interaction is

$$\delta F=\tfrac{1}{2}\chi_{int}H^2.\qquad(A.2)$$

On the other hand, for example in the Boltzmann case, the calculation of δF by means of Eqs. (2.5), (2.6)–(2.8) leads to the result

$$\delta F=\text{th}^2\frac{\beta H}{T}\sum_{\mathbf{p},\mathbf{p}'}\zeta(\mathbf{p},\mathbf{p}')n_e(\mathbf{p})n_d(\mathbf{p}').\qquad(A.3)$$

For the localized defects $n_d(p)=(2\pi\hbar)^3N_d\delta(\mathbf{p})$ and

$$\delta F=N_d\,\text{th}^2\frac{\beta H}{T}\sum_{\mathbf{p}}\zeta(\mathbf{p},0)n_e(\mathbf{p}),\qquad(A.4)$$

or in the s-wave approximation in the principal order in H

$$\delta F=\left(\frac{\beta H}{T}\right)^2\frac{\pi\hbar^2a_2}{2m_e}N_dN_e,\quad\chi_{int}=2\frac{\beta^2}{T^2}E_2(\gamma_{2e}\gamma_{2d})^{1/2}.\qquad(A.5)$$

In similar fashion an expression is obtained for χ_{int} also in the quantum degeneracy region. Depending on the sign of χ_{int}, the expansion (A.1) corresponds to ferromagnetism $\mathbf{M}_e\uparrow\uparrow\mathbf{M}_d$ (at $\chi_{int}<0$) or else to a ferremagnetic structure $\mathbf{M}_e\uparrow\downarrow\mathbf{M}_d$ (at $\chi_{int}>0$). The compatibility of the equilibrium conditions $\partial F/\partial M_e=0$ and

$\partial F/\partial M_d = 0$ specifies an equation that determines the phase-transition temperature T_c:

$$\chi_{int}^2 = \chi_e \chi_d, \tag{A.6}$$

where χ_{int} is given by (A.2)–(A.5), $\chi_d = \beta^2 N_d/T$, χ_e is determined by the usual equations for an ideal Fermi gas[10]:

$$\chi = \frac{\beta^2}{T} N_e \frac{F_{1/2}'(\mu/T)}{F_{1/2}(\mu/T)}, \quad F_{1/2}(z) = \frac{2}{\pi^{1/2}} \int_0^\infty \frac{x^2 dx}{e^{x-z}+1}$$

and μ is the chemical potential.

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