

# Effect of mixing of the states of conduction electrons and of impurity $d(f)$ ions on the electric conductivity

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(Submitted 30 June 1981)

Zh. Eksp. Teor. Fiz. 82, 246–253 (January 1982)

The electron spectra and the electric conductivities of a normal metal with  $d(f)$  ions and ordinary impurities and of a  $d(f)$  metal with nonresonant impurities are investigated. Conditions are obtained under which a gap can appear in the electron spectrum. It is shown that the two considered scattering mechanisms make additive contributions to the resistance. The resistance due to  $c$ - $d(f)$  scattering is a maximum when the chemical potential is closest to the  $d(f)$  level.

PACS numbers: 72.15.Eb, 71.55.Dp, 72.10.Fk

## 1. INTRODUCTION

A characteristic feature of a large number of transition ( $d$ ) and rare-earth ( $f$ ) metals and of their compounds is that they contain localized atom-like  $d(f)$  levels "imbedded" in the conduction band ( $c$ ) of the electrons. The ensuing degeneracy of the  $c$  and  $d(f)$  states can be lifted both via single-particle<sup>1-3</sup> and multiparticle<sup>4,5</sup> hybridization. This situation has been investigated in considerable detail in the case of a regular crystal. The solid-state configuration  $f^n c^a$  is transformed in this case into  $f^{n+\alpha} c^{a-\alpha}$ , and the valence of the  $d(f)$  ions becomes fractional: some of the electrons go over from one subsystem to the other. A gap appears in the spectrum of the hybridized electrons, and the density of the electronic states near the gap increases sharply.<sup>1-5</sup>

The effect of mixing of the band and atom-like states on the parameters of such a crystal is particularly strong when the chemical potential is near the  $d(f)$  level. In this case, the criteria for the appearance of a magnetically ordered state change,<sup>5</sup> the critical temperature of the superconductor may rise,<sup>6</sup> the gap in the electron spectrum becomes temperature dependent, a high-temperature electronic phase transition becomes possible,<sup>5,7</sup> and variable-valence effects appear (see the review<sup>8</sup>). The model of intermixing  $c$  and  $d(f)$  states describes well a large class of compounds based on Ce, Yb, Sm,<sup>8</sup> Nd, Pm<sup>9</sup>, as well as compounds with A-15 structure.<sup>7,10</sup>

The properties of irregular  $d(f)$  metals and their compounds in the presence of  $c$ - $d(f)$  mixing were much less investigated. The problem of the impurity  $d(f)$  ion in a normal metal was solved by Anderson.<sup>1</sup> Using the Hartree-Fock approximation for the  $d$ -states, he has shown that single-particle mixing leads to a broadening of the  $d$  level and to an increase of the state density near this level. The influence of multiparticle  $c$ - $d(f)$  mixing on the electron spectrum and on the state density was investigated in Ref. 11 both for this problem and for the problem of a normal substitutional impurity in a  $d(f)$  metal. It has been shown that at a low density of the  $d(f)$  impurities ( $c_d \ll 1$ ) the quasiparticles have a damping  $\Gamma$ , whereas the "gap" is  $\sim \Gamma c_d$  (details follow), so that the "gap" becomes washed out and the density of the ground states has a peak,

just as in the Anderson model.<sup>1</sup> In the opposite limiting case ( $1 - c_d \ll 1$ ), the quasiparticle damping is  $\sim \Gamma(1 - c_d)$ , and the gap  $\sim \Gamma$ , therefore the state density has a dip with peaks on the edges.

From among the kinetic properties of a metal with  $c$ - $d(f)$  mixing, only the resistance was investigated theoretically.<sup>12-14</sup> In Ref. 12, the coherent-potential method was used to investigate the model of a binary alloy  $A_{c_d} B_{1-c_d}$  in which each atom A contributes to the conduction band a  $d(f)$  level with energy  $\varepsilon_d^A$ , while the contribution of B is  $\varepsilon_d^B$ . The dependence of the electric resistance on the degree of non-negativity of the wave functions of the  $c$  and  $d$  electrons was considered in the Anderson non-orthogonal model in Ref. 13. A rather unusual result was obtained in Ref. 14, namely, the existence of a finite residual resistance in a regular metal. This was attributed by the authors to scattering of the conduction electrons by fluctuations of the occupation numbers of the localized electrons. This would be possible if the ground (stationary) state of the regular crystal turned out to be for some reason translationally invariant (the quasimomentum is a poor quantum number). The authors of Ref. 14, however, calculated the current by using Green's functions corresponding to a translationally invariant ground state.

The present paper is devoted to a study of the behavior of the resistance of a metal with  $c$ - $d(f)$  mixing in two limiting cases: low  $d(f)$ -ion impurity density in the normal metal ( $c_d \ll 1$ ) and few normal-metal impurities in a  $d(f)$  metal ( $1 - c_d \ll 1$ ). Besides the scattering of the conduction electrons by this resonant impurity (holes) we take into account also scattering by ordinary impurities. We use the same model as in Ref. 11.

## 2. HAMILTONIAN. EXPRESSION FOR THE CURRENT

We consider a system of collectivized and impurity localized electrons described by the Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2, \quad (1)$$

$$\mathcal{H}_0 = \sum_{k\sigma} \xi_k c_{k\sigma}^+ c_{k\sigma} + \omega \sum_i p_i^d X_i^{2z}, \quad (2)$$

$$\mathcal{H}_1 = \frac{1}{\sqrt{N}} \sum_{k\sigma} p_i^d (A_{i\sigma} e^{-ikr} \eta_{\sigma} c_{k\sigma}^+ X_i^{-\sigma} + A_{i\sigma} e^{ikr} \eta_{\sigma} X_i^{2-\sigma} c_{k\sigma}), \quad (3)$$

$$\mathcal{H}_2 = \frac{1}{N} \sum_{\mathbf{k}, \mathbf{k}'} p_i^n u(\mathbf{k}-\mathbf{k}') \exp\{-i(\mathbf{k}-\mathbf{k}')\mathbf{f}\} c_{\mathbf{k}\sigma}^+ c_{\mathbf{k}'\sigma}. \quad (4)$$

Here  $c_{\mathbf{k}\sigma}$  is the operator for the annihilation of a  $c$ -electron with momentum  $\mathbf{k}$ , spin  $\sigma$ , and energy  $\zeta_{\mathbf{k}} = \varepsilon_{\mathbf{k}} - \mu$ , reckoned from the chemical potential  $\mu$ ;  $X^{pq} \equiv |p\rangle\langle q|$  are the Hubbard operators that change the  $d(f)$  ion from the atomic state  $|q\rangle$  into the state  $|p\rangle$ ;  $|2\rangle$  is an  $f^{n+1}$  configuration with spin  $S=0$ , and  $|\sigma\rangle$  is a configuration with spin  $1/2$  and projection  $\sigma$ ;  $p_f^{\uparrow} = 1$  if the site  $\mathbf{f}$  is occupied by an impurity  $d(f)$  atom and  $p_f^{\uparrow} = 0$  in the opposite case;  $p_f^{\downarrow}$  is the analogous projection operator for the ordinary impurity;  $A_{f\sigma}$  is the effective mixing parameter (single-particle on account of the crystal field and multiparticle on account of the intra-atomic Coulomb and exchange interactions);  $\omega = \Omega - \mu$ , where  $\Omega$  is the energy of the transition between the states  $|2\rangle$  and  $|\sigma\rangle$ . We assume that the ion can be only in the states  $|2\rangle$  and  $|\sigma\rangle$ :

$$X_{f\sigma} + X_{f\sigma}^{-\sigma} + X_{f2} = 1. \quad (5)$$

The quantity  $u(\mathbf{k}-\mathbf{k}')$  is the Fourier transform of the potential for the scattering of the conduction electrons by the ordinary impurities.<sup>15</sup>

An investigation<sup>12</sup> of a similar model has shown that in view of the strong localization of the  $f$  electrons, only conduction electrons contribute to the current. Using the procedure described in Ref. 15, we obtain in the Bloch-function representation the following expression for the Fourier transform of the average current:

$$j_{\alpha}(q, \nu) = -\frac{e^2}{2cNv_{\alpha}} \sum_{\mathbf{k}_1, \mathbf{k}_2} \int \frac{dE}{2\pi} \left[ \left( \frac{\partial \varepsilon_{\mathbf{k}_1}}{\partial k_{1\alpha}} + \frac{\partial \varepsilon_{\mathbf{k}_1+\mathbf{q}}}{\partial (k_1+q)_{\alpha}} \right) \times \left\langle \left\langle G_{\mathbf{k}_1, \mathbf{k}_1}(E) \left( \frac{\partial \varepsilon_{\mathbf{k}_1}}{\partial k_{1\beta}} + \frac{\partial \varepsilon_{\mathbf{k}_1+\mathbf{q}}}{\partial (k_1+q)_{\beta}} \right) G_{\mathbf{k}_1+\mathbf{q}, \mathbf{k}_1+\mathbf{q}}(E+\nu) \right\rangle \right\rangle \right] A^{\beta}(Q, \nu). \quad (6)$$

Here  $G_{\mathbf{k}_1, \mathbf{k}_2}(E) = \langle T c_{\mathbf{k}_1\sigma} c_{\mathbf{k}_2\sigma}^+ \rangle$  is the usual time-dependent Green's function<sup>15</sup>,  $A^{\beta}(Q, \nu)$  is the  $\beta$ -component of the vector potential, and

$$j_{\alpha}(q, \nu) = \iint d^3r dt j_{\alpha}(r, t) \exp\{-i(qr - \nu t)\}.$$

The double angle brackets denote configuration averaging over the impurity positions, and  $N$  is the number of unit cells with volume  $v_a$ . We consider here only the paramagnetic state and leave out therefore the spin indices of the Green's functions. We use a diagram technique<sup>15</sup> for the electron variables. An equation for the Green's function  $G_{\mathbf{k}_1, \mathbf{k}_2}$  of interest to us was obtained in Ref. 11 without allowance for the scattering of the  $c$ -electrons by the usual impurities. In our case, after excluding the  $X$  variables in the generalized Hartree-Fock approximation, which takes into account resonant scattering,<sup>5</sup> we obtain the following equation for the non-averaged Green's function:

$$G_{\mathbf{k}\mathbf{k}'}(E) = G_{\mathbf{k}\mathbf{k}'}^0(E) \delta_{\mathbf{k}\mathbf{k}'}, \quad (7)$$

$$+ G_{\mathbf{k}\mathbf{k}'}^0(E) \frac{1}{N} \sum_{\mathbf{p}} p_i^n \frac{n_i |A_i|^2}{E - \omega - i\delta \operatorname{sign} \omega} \exp\{-i(\mathbf{k}-\mathbf{p})\mathbf{f}\} G_{\mathbf{p}\mathbf{k}'}(E)$$

$$+ G_{\mathbf{k}\mathbf{k}'}^0(E) \frac{1}{N} \sum_{\mathbf{p}} p_i^n u(\mathbf{k}-\mathbf{p}) \exp\{-i(\mathbf{k}-\mathbf{p})\mathbf{f}\} G_{\mathbf{p}\mathbf{k}'}(E),$$

where

$$G_{\mathbf{k}\mathbf{k}'}^0(E) = (E - \xi_{\mathbf{k}} + i\delta \operatorname{sign} \xi_{\mathbf{k}})^{-1}, \quad A_i = A_{i\sigma} = A_{i, -\sigma},$$

$$n_i = n_{i\sigma} = n_{i, -\sigma} = \langle X_i^{\sigma\sigma} \rangle + \langle X_i^{\sigma\sigma} \rangle.$$

Expressions (6) and (7) are the basis for the calculation of the electric conductivity in the two limiting cases  $c_d \ll 1$  and  $1 - c_d \ll 1$ . The density  $c_n$  of the normal impurity will be assumed small in both cases.

### 3. LOW DENSITY OF THE $d(f)$ IMPURITIES

To carry out the configuration averaging we use the diagram technique of Ref. 15. Averaging over a physically small volume with uniform distribution of the impurities restores the translational invariance

$$\langle\langle G_{\mathbf{k}_1, \mathbf{k}_2}(E) \rangle\rangle = G_{\mathbf{k}_1, \mathbf{k}_2}(E) \delta_{\mathbf{k}_1, \mathbf{k}_2}.$$

Iterating (6) and (7), we average each term of the resultant series in accordance with the following rules:

$$\left\langle \left\langle \frac{1}{N} \sum_{\mathbf{f}} z_i p_i^n \exp\{-i(\mathbf{k}-\mathbf{k}')\mathbf{f}\} \right\rangle \right\rangle = c_d z \delta_{\mathbf{k}\mathbf{k}'},$$

$$\left\langle \left\langle \frac{1}{N} \sum_{\mathbf{f}} p_i^n \exp\{-i(\mathbf{k}-\mathbf{k}')\mathbf{f}\} \right\rangle \right\rangle = c_n \delta_{\mathbf{k}\mathbf{k}'},$$

and set the analytic expressions in correspondence with the diagrams in the usual manner<sup>15</sup>—a solid line means  $G_{\mathbf{k}}^0(E)$ , a cross means the potential of the normal impurity  $p_i^n u(\mathbf{k}-\mathbf{k}') \exp\{-i(\mathbf{k}-\mathbf{k}')\mathbf{f}\}$ , a triangle is the bare  $c-d(f)$  interaction

$$v_i^{\sigma}(\mathbf{k}-\mathbf{k}', E) = p_i^n n_i |A_i|^2 \exp\{-i(\mathbf{k}-\mathbf{k}')\mathbf{f}\} / (E - \omega + i\delta),$$

and a dashed line joins like nodes. Account must be taken in the averaging of the fact that  $p_i^n p_i^n = 0$ , since it is assumed that one site contain either a matrix atom or a normal impurity or else a  $d(f)$  impurity.

The choice of the diagrams is similar to that in Refs. 11 and 15, and the equation for  $G_{\mathbf{k}}(E) = \langle\langle G_{\mathbf{k}\mathbf{k}}(E) \rangle\rangle$  takes in our case the form

$$\text{---} = \text{---} + \text{---} \triangle \text{---} + \text{---} \times \text{---}, \quad (8a)$$

$$\text{---} \triangle \text{---} = \text{---} \triangle \text{---} + \text{---} \triangle \text{---} \triangle \text{---}. \quad (8b)$$

Solving this equation we obtain

$$G_{\mathbf{k}}(E) = \left[ E - \xi_{\mathbf{k}} + \frac{i}{2\tau_n} \operatorname{sign} E - \frac{\Delta^2 c_d}{E - \omega + i\Gamma \operatorname{sign} E} \right]^{-1}, \quad (9)$$

where

$$\Gamma = \pi g_0 \Delta^2, \quad \Delta^2 = n |A|^2, \quad \tau_n^{-1} = c_n g_0 \int d\Omega |u(\theta)|^2 / 4\pi$$

is the relaxation time due to the normal impurities,<sup>15</sup> and  $g_0$  is the density of the electronic states on the Fermi surface. The poles of the function (9) determine the spectrum and the damping of the electrons. Depending on the relation between the parameters  $\Gamma$ ,  $\tau_n^{-1}$ , and  $\Delta^2 c_d$ , two situations are possible:

$$A. \Delta_0^2 = \left( \Gamma - \frac{1}{2\tau_n} \right)^2 - 4\Delta^2 c_d < 0,$$

$$E_{\mathbf{k}}^{\pm, 2} = \frac{1}{2} \left[ \omega + \xi_{\mathbf{k}} \pm v_{\mathbf{k}} - \frac{i}{2} \operatorname{sign} E \left( \Gamma + \frac{1}{2\tau_n} \pm v_{\mathbf{k}} \operatorname{sign} B_{\mathbf{k}} \right) \right]; \quad (10a)$$

B.  $\Delta_0^2 > 0$ ,

$$E_k^{1,2} = \frac{1}{2} [\omega + \xi_k \pm v_+ \text{sign}(\omega - \xi_k)] - \frac{i}{2} \text{sign} E \left[ \Gamma + \frac{1}{2\tau_n} \pm v_- \text{sign}(\omega - \xi_k) \text{sign} B_k \right],$$

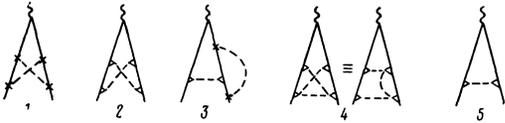
where

$$v_{\pm}^2 = \frac{1}{2} [(A_k^2 + B_k^2)^{1/2} \pm A_k], \quad A_k = (\omega - \xi_k)^2 - \Delta_0^2, \\ B_k = 2(\omega - \xi_k) \left( \Gamma - \frac{1}{2\tau_n} \right). \quad (10b)$$

Plots of  $E_k^{1,2}$  are shown in Fig. 1. We emphasize that the presence of a gap in the spectrum in case A does not mean at all that a gap exists in the state density. Indeed, as follows from (10a), the gap width is  $\sim \Delta^2 g_0 c_d$ , whereas the damping of the excitation at  $k = k_\alpha$  is  $(\Gamma + \tau_n^{-1})/2$ . Since we always have  $\Delta^2 g_0 c_d < \frac{1}{2}(\Gamma + \tau_n^{-1})$ , no gap can appear in the state density.

We proceed to an analysis of the iteration series for the vertex part, which we shall define as the quantity in the double angle brackets of Eq. (6). We denote the corresponding vector (the result of the averaging) by  $\Pi_{k,E}^{k+q,E+\nu}$ .

It is easy to verify with diagrams of the form 1, 2, 3



and all other diagrams with intersecting dashed lines contain integration in a wide range of momenta (just as in the case of a normal metal<sup>15</sup>) and are small compared with the ladder diagrams. Diagrams of type 4 are taken into account automatically by using the effective interaction (8b). The result is the following integral equation for the vertex:

$$\text{Diagram 1} = \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4}, \quad (11)$$

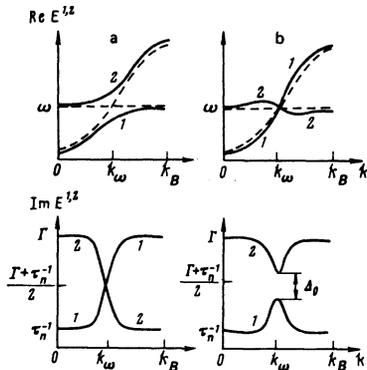


FIG. 1. Spectrum and damping of the conduction electrons for two cases: a)  $\Delta_0^2 < 0$ ; b)  $\Delta_0^2 > 0$ . The situation  $\Gamma > \tau_n^{-1}$  is presented. In the opposite case  $\Gamma < \tau_n^{-1}$  curves 1 and 2 of  $\text{Im} E^{1,2}$  change places.

or in analytic form

$$\Pi_{k_1, \sigma_1}^{k_1+q, \sigma_1+\nu} = \Pi_{k_1, \sigma_1}^{(0)k_1+q, \sigma_1+\nu} + G_{k_1}(\omega_1) \frac{c_n}{N} \sum_{k_2} |u(k_1 - k_2)|^2 \Pi_{k_2, \sigma_2}^{k_2+q, \sigma_2+\nu} G_{k_2+q}(\omega_2 + \nu) \\ + G_{k_1}(\omega_1) \frac{c_d}{N} \sum_{k_2} \left( \frac{n|A|^2}{E - \omega + i\Gamma \text{sign} E} \right)^2 \Pi_{k_2, \sigma_2}^{k_2+q, \sigma_2+\nu} G_{k_2+q}(\omega_2 + \nu), \quad (11')$$

where

$$\Pi_{k_1, \sigma_1}^{(0)k_1+q, \sigma_1+\nu} = G_{k_1}(\omega_1) \left( \frac{\partial \epsilon_{k_1}}{\partial k_1} + \frac{\partial \epsilon_{k_1+q}}{\partial (k_1+q)} \right) G_{k_1+q}(\omega_1 + \nu). \quad (12)$$

We shall be interested below in the static electric conductivity in the homogeneous case:

$$\sigma^{ab}(0, 0) = \lim_{\nu \rightarrow 0} \sigma^{ab}(q=0, \nu).$$

As seen from (6), it is connected with the vector by the relation

$$(A^b(q=0, \nu) = (c/iv)E^b(q=0, \nu)): \\ \sigma^{ab}(0, \nu) = -\frac{e^2}{iv\nu_a} \frac{1}{N} \sum_{k_1} \int \frac{dE}{4\pi} \left( \frac{\partial \epsilon_{k_1}}{\partial k_{1\alpha}} \right) (\Pi^b)_{k_1, E}^{k_1, E+\nu}. \quad (13)$$

The effective mixing interaction is local and does not depend on the angles. Therefore the integration of the vector  $\Pi_{k_2}^{k_1}$  over the angles at  $q=0$  in the last term of (11) causes the latter to vanish. Thus, in the homogeneous case the contribution of the state mixing to the vertex (11) is not trivial. Equation (11) is solved next in the standard manner.<sup>15</sup> The difference lies in the form of the Green's function  $G_k(\omega)$ : in the case of a normal metal it is necessary to put  $c_d=0$  in (9). Solving (11) and substituting the result in (13), we obtain

$$\sigma^{aa}(0, 0) = -\lim_{\nu \rightarrow 0} \frac{2e^2}{iv\nu_a N} \sum_{k_1} \int \frac{dE}{2\pi} \left( \frac{\partial \epsilon_{k_1}}{\partial k_{1\alpha}} \right)^2 G_{k_1}(E) G_{k_1}(E+\nu) \frac{\tau_d^{-1} + \tau_n^{-1}}{\tau_d^{-1} + \tau_r^{-1}}. \quad (14)$$

Here  $\tau_d$  is the relaxation time due to the scattering of the  $c$ -electrons by the resonant  $d(f)$ -ion impurities

$$\tau_d^{-1} = c_d \frac{2\Gamma\Delta^2}{\omega^2 + \Gamma^2}, \quad (15)$$

and we have introduced the transport relaxation time

$$\tau_r^{-1} = c_n g_0 \int d\Omega |u(\theta)|^2 (1 - \cos \theta) / 4\pi.$$

Calculating the sum over  $k_1$  in (14) we get ultimately

$$\sigma^{aa}(0, 0) = 2 \frac{e^2}{v_a} g_0 \left[ \int \frac{d\Omega}{4\pi} \left( \frac{\partial \epsilon_{k_1}}{\partial k_{1\alpha}} \right)^2 \left( \frac{1}{\tau_d} + \frac{1}{\tau_r} \right)^{-1} \right]_0, \quad (16)$$

where the zero subscript means that the quantities are calculated on the Fermi surface

$$E=0 \text{ or } \xi_f = \Delta^2 c_d \omega / (\omega^2 + \Gamma^2).$$

Equation (16) shows that, since the potential of the mixing interaction has no angular dependence the contributions to the resistance from both mechanisms are additive. It is seen also that the increment (15) to the resistance has a resonant character: it is a maximum when the chemical potential coincides with  $d(f)$  level ( $\omega=0$ ) and decreases like  $\omega^{-2}$  with increasing distance from the chemical potential. If it is assumed that the density of the  $d(f)$ -ion impurities is  $c_d=0.01$  and the state density on the Fermi surface is  $g_0=0.1 \text{ eV}^{-1}$ ,

then we get from (15)  $\tau_d^{-1} 10^{14} \text{ sec}^{-1}$  at  $\omega = 0$ , and the mechanism of the resonant scattering will in practice always predominate.

#### 4. LOW HOLE DENSITY

In this case we introduce for the analysis new projection operators  $h_f = 1 - p_f$  and substitute them in the Hamiltonian (1)–(3) and in the equation for the non-averaged function  $G_{\mathbf{k}\mathbf{k}'}(E)$ . We then obtain in place of (7)

$$G_{\mathbf{k}\mathbf{k}'}(E) = \bar{G}_{\mathbf{k}\mathbf{k}'}^{(0)}(E) \delta_{\mathbf{k}\mathbf{k}'} - \bar{G}_{\mathbf{k}\mathbf{k}'}^{(0)}(E) \frac{1}{N} \sum_{\mathbf{p}} \frac{h_{\mathbf{p}} n_{\mathbf{p}} |A_{\mathbf{p}}|^2}{E - \omega + i\delta} \exp\{-i(\mathbf{k}-\mathbf{p})\mathbf{f}\} G_{\mathbf{p}\mathbf{k}'}(E) + \bar{G}_{\mathbf{k}\mathbf{k}'}^{(0)}(E) \frac{1}{N} \sum_{\mathbf{p}} p_{\mathbf{p}} n_{\mathbf{p}} (\mathbf{k}-\mathbf{p}) \exp\{-i(\mathbf{k}-\mathbf{p})\mathbf{f}\} G_{\mathbf{p}\mathbf{k}'}(E), \quad (17)$$

where the bare Green's function is now

$$\bar{G}_{\mathbf{k}\mathbf{k}'}^{(0)}(E) = \left[ E - \xi_{\mathbf{k}} + i\delta \text{sign } \xi_{\mathbf{k}} - \frac{\Delta^2}{E - \omega + i\delta \text{sign } \omega} \right]^{-1}.$$

The equation for the average Green's function takes in this case also the form (8), from which we get

$$G_{\mathbf{k}}(E) = \left[ E - \xi_{\mathbf{k}} - \frac{\Delta^2}{E - \omega} + c_n \Delta^2 \frac{E - \omega}{(E - \omega)^2 + \Gamma^2} + i \text{sign } E \left( \frac{1}{2\tau_n} + \frac{1}{2\tau_h(E)} \right) \right]^{-1}, \quad (18)$$

where

$$\frac{1}{\tau_h(E)} = c_h \frac{2\Gamma\Delta^2}{(E - \omega)^2 + \Gamma^2}. \quad (19)$$

In the limiting case when there are no normal impurities,  $\tau_n \rightarrow \infty$ , this expression coincides with that obtained in Ref. 11. The excitation spectrum determined from (18) consists of two subbands separated by a gap. This gap can appear only when  $\Delta \gg \tau_n^{-1} + \tau_h^{-1}$ .

Equation (11) for the vertex part remains in force in the considered limiting case. A calculation perfectly analogous to (13) yields

$$\sigma^{aa}(0, 0) = 2 \frac{e^2}{v_a} g_0 \left[ \int \frac{d\Omega}{4\pi} \left( \frac{\partial \epsilon_{\mathbf{k}}}{\partial k_a} \right)_0^2 \right] \left[ \frac{1}{\tau_n(0)} + \frac{1}{\tau_{ir}} \right]^{-1}. \quad (20)$$

All the quantities in this equation are calculated at the point  $E = 0$ :

$$\xi_{\mathbf{p}} = \Delta^2 / \omega - c_n \Delta^2 \omega / (\omega^2 + \Gamma^2). \quad (21)$$

As follows from (20), in this limiting case, too, the resistances due to the two mechanisms are additive. The resonance relaxation time that enters in (20) has outwardly the same form as at  $c_d \ll 1$  [(see (15) and (19)]. The difference is that the minimum value of  $\omega$  in (19) and (20) cannot reach zero and is equal to the half-width of the "indirect" gap  $\Delta^2/w$  ( $w$  is the width of the  $c$ -band). At  $\omega \gg \Delta^2/w$  the principal mechanism contributing to the resistance is scattering by normal impurities. With decreasing  $\omega$  [when the chemical potential approaches the  $d(f)$ -ion transition energy] the resonance scattering mechanism can become dominant, and this takes place against the background of a general increase of

the resistance on account of the decrease of the state density when the edges of the gap are approached.

It must be noted that if the chemical potential is located directly near the gap, i. e., when the lower subband is almost completely filled or the upper one is completely empty (the Fermi momentum is small), diagrams of the type 1, 2, and 3 and others with crossing of dashed lines no longer contain small quantities, since the requirement that all the electron momenta be close to the Fermi surface does not restrict the range of integration over the angles. The same can be stated also concerning the choice of corrections to the Green's function. Therefore in the narrow region  $\omega < c_n w + \Delta^2/w$  our results are only qualitative.

In conclusion, we note the following. The described abrupt change of the resistance for a normal metal with  $d(f)$  impurities and for a normal impurity in a  $d(f)$  metal should be expected primarily in substances with variable valence under any action that changes the relative position of the chemical potential and the  $d(f)$  level, e. g., when compounds based on Ce and Sm are alloyed.<sup>8</sup> The same effects can apparently take place when high pressure is applied to a  $d(f)$  metal.

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Translated by J. G. Adashko