Effect of impurities on superconductors with helical ordering of localized spins

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Cooper pairing of electrons in a crystal with regularly distributed localized spins is considered. The case is studied in which exchange interaction between the electrons and localized spins leads to helical magnetic ordering of the localized spins in the superconducting state, whereas in the absence of superconductivity ferromagnetic order would occur. It is shown that scattering of conducting electrons by nonmagnetic impurities narrows the region of existence of the superconducting phase with spin ordering (*HS* phase). The narrowing, however, is not very pronounced even in dirty crystals. Thus the requirement of the purity of the crystals in which the *HS* phase may be observed is not very rigid.

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1. INTRODUCTION

In connection with experimental work on the compound $ErRh_4B_4$, Kulic, Rusinov and one of the present authors[1] considered a system of regularly distributed localized spins and conduction electrons with an exchange interaction of the spins and electrons, and a Cooper pairing of the conducting electrons. It was assumed that in the absence of superconductive pairing the indirect exchange interaction of the localized spins via the conduction electrons (RKKY interaction) would lead to ferromagnetic ordering below a Curie temperature $\theta \ll T_{c0}$, where T_{c0} is the critical temperature for superconductive pairing of the electrons in the absence of interaction between spins and electrons. To describe the system Kulic *et al.*[1] used the BCS model and the self-consistent field approximation for the magnetic impurities. It was also assumed that the electron energy spectrum was isotropic, and that there was no magnetic anisotropy or electron scattering by impurities.

Anderson and Suhl[2] showed that in the system considered ordering of the spins in the superconducting phase appears at a temperature $T_M \approx \theta$ in the form of inhomogeneous magnetic ordering with a characteristic wave vector $Q_M \approx (k_F^2 \xi_0^{-1})^{1/3}$, where ξ_0 is the superconducting correlation length. In previous work[1] the region of existence of a superconducting phase with a helicoidal type of ordering of the localized spins was found. There it was shown that for

$$\theta < \theta_{c2} \approx T_{c0}/\ln(\varepsilon_F/T_{c0})$$

the HS [Helicoidal-spin] phase persists down to zero temperature, while for systems with $\theta > \theta_{c2}$ lowering of the temperature leads to a first-order transition from the HS phase to a nonsuperconducting phase with ferromagnetic ordering of the spins (F phase). Since Ref. 1 used the simple self-consistent field approximation, which neglects the scattering of the electrons by spin excitations, the results obtained there are valid only for a system with $\theta \ll T_{c0}$ and at temperatures T which are not close to the critical magnetic point T_M. Thus, Ref. 1 found the region of existence of the HS-phase in an isotropic system without impurities in the region of the variables (T, θ) where $T \ll \theta \ll T_{c0}$ and $(\theta\varepsilon_F)^{1/2} \gg T_{c0}$.

Both impurity scattering and the effects of magnetic anisotropy have a substantial effect on the HS-phase and lead to a narrowing of its region of existence. In the present paper we investigate helicoidal ordering of the spins in a superconductor in the presence of nonmagnetic impurities and find the region of existence of the HS phase and the character of the quasiparticle spectrum in this phase as a function of the electron mean free path.

2. BASIC EQUATIONS FOR A SUPERCONDUCTOR WITH A HELICOIDAL EXCHANGE FIELD IN THE PRESENCE OF IMPURITIES

The Hamiltonian of a system of electrons in the presence of an exchange interaction with localized spins, Cooper pairing, and scattering by impurities has the form

$$\mathscr{H} = \mathscr{H}_{BCS} + \int d^3\mathbf{r}\,\psi_\alpha^+(\mathbf{r}) \Big[\sum_i \bar{I}(\mathbf{r}-\mathbf{r}_i)\sigma_{\alpha\beta}\Big]\cdot\psi_\beta(\mathbf{r}) + \sum_a \int d^3\mathbf{r}\,\psi_\alpha^+(\mathbf{r})\,V(\mathbf{r}-\mathbf{r}_a)\,\psi_\alpha(\mathbf{r}), \quad (1)$$

$$\mathscr{H}_{BCS} = \int d^3\mathbf{r}\Big[\psi_\alpha^+(\mathbf{r})\Big(-\frac{\hbar^2\nabla^2}{2m}\Big)\psi_\alpha(\mathbf{r}) + \Delta(\mathbf{r})\psi_\alpha^+(\mathbf{r})(i\sigma_y)_{\alpha\beta}\psi_\beta^+(\mathbf{r}) + \Delta^*(\mathbf{r})\psi_\alpha(\mathbf{r})(i\sigma_y)_{\alpha\beta}\psi_\beta(\mathbf{r})\Big],$$

where $\bar{I}(\mathbf{r}-\mathbf{r}_i)$ is the exchange interaction integral of the conduction electrons with spins $\mathbf{S}_i$ localized on lattice sites with coordinates $\mathbf{r}_i$, σ_i are the Pauli matrices and $V(\mathbf{r}-\mathbf{r}_i)$ is the potential of an impurity situated at the point $\mathbf{r}_a$.

The localized spins are assumed to be ordered in a helicoidal structure:

$$\langle S_{xi}\rangle = S\sigma\cos\mathbf{Q}\mathbf{r}_i, \quad \langle S_{yi}\rangle = S\sigma\sin\mathbf{Q}\mathbf{r}_i, \quad \langle S_{zi}\rangle = 0, \quad (2)$$

where the parameter $\sigma (0 \leqslant \sigma \leqslant 1)$ characterizes the helicoid amplitude and the xy plane is the easy plane of the magnetic anisotropy. Of all the Fourier components in (2), as was shown previously,[1] we need to keep only the field with wave vector $\mathbf{Q}$:

$$\mathbf{h}(\mathbf{r}) = (h\cos\mathbf{Q}\mathbf{r},\ h\sin\mathbf{Q}\mathbf{r},\ 0), \quad h = I(0)Sn\sigma,$$

where $I(0)$ is the Fourier component of the function $\bar{I}(\mathbf{r})$ with zero wave vector and n is the concentration of localized spins. The superconducting order parameter in this field is independent of coordinate.

In the Gor'kov-Nambu representation $\psi=(\psi_\uparrow^\dagger, \psi_\downarrow^\dagger, \psi_\uparrow, \psi_\downarrow)$ the Hamiltonian takes the form

$$\mathscr{H}=\frac{1}{2}\int d^3\mathbf{r}\psi_\alpha^+(\mathbf{r})\Big[\tau_z\sigma_0\Big(-\frac{\hbar^2\nabla^2}{2m}\Big)+\frac{1}{2}i\Delta\tau_+\sigma_y-\frac{1}{2}i\Delta^*\tau_-\sigma_y$$
$$+\tau_z\sigma_y h_x(\mathbf{r})-\tau_0\sigma_y h_y(\mathbf{r})+\sum_a\tau_z\sigma_0 V(\mathbf{r}-\mathbf{r}_a)\Big]\psi(\mathbf{r}), \quad (3)$$

where σ_0 and τ_0 are unit matrices I, and $\tau_\pm=(\tau_x\pm i\tau_y)/2$.

Below we shall be interested in the Green's function averaged over the impurity configuration:

$$g_{\alpha\beta}(\mathbf{r},\mathbf{r}')=-\langle T_\tau\psi_\alpha^+(\mathbf{r})\psi_\beta(\mathbf{r}')\rangle. \quad (4)$$

Carrying out the average over the impurity configuration in the usual way,[3] we get for the Fourier transform of the function $g(\mathbf{r},\mathbf{r}')$ the equation

$$g^{-1}(\mathbf{p},\mathbf{p}')=g_0^{-1}(\mathbf{p},\mathbf{p}')-\nu\int\frac{d^3\mathbf{p}}{(2\pi)^3}$$
$$\times|V(\mathbf{p}-\mathbf{p}_1)|^2\tau_z\sigma_0 g(\mathbf{p}_1+\mathbf{p}'-\mathbf{p},\mathbf{p}_1)\tau_z\sigma_0, \quad (5)$$
$$g_0^{-1}(\mathbf{p},\mathbf{p}')=[i\omega\tau_0\sigma_0-\xi\tau_z\sigma_0-{}^1/_2 i\tau_+\sigma_y\Delta+{}^1/_2 i\tau_-\sigma_y\Delta^*]$$
$$\times\delta(\mathbf{p}-\mathbf{p}')-{}^1/_2 h[(\tau_z\sigma_x+i\tau_0\sigma_y)\delta(\mathbf{p}-\mathbf{p}'+\mathbf{Q})$$
$$+(\tau_z\sigma_x-i\tau_0\sigma_y)\delta(\mathbf{p}-\mathbf{p}'-\mathbf{Q})],\quad \xi=p^2/2m-\mu.$$

We first consider the simplest case, when the impurity potential is of contact type and so $V(\mathbf{p})$ does not depend on $\mathbf{p}$. Then we shall look for the function $g^{-1}(\mathbf{p},\mathbf{p}')$ in a form analogous to that of $g_0^{-1}(\mathbf{p},\mathbf{p}')$:

$$g^{-1}(\mathbf{p},\mathbf{p}')=[i\tilde\omega\tau_0\sigma_0-\xi\tau_z\sigma_0-{}^1/_2 i\tau_+\sigma_y\tilde\Delta+{}^1/_2 i\tau_-\sigma_y\tilde\Delta^*]$$
$$\times\delta(\mathbf{p}-\mathbf{p}')-[{}^1/_4(\tau_z+\tau_0)\sigma_+\tilde h_1+{}^1/_4(\tau_z-\tau_0)\sigma_-\tilde h_2$$
$$+{}^1/_2 i(\tau_x\sigma_z+i\tau_y\sigma_0)\varkappa\tilde\Delta]\delta(\mathbf{p}-\mathbf{p}'+\mathbf{Q})-[{}^1/_4(\tau_z+\tau_0)\sigma_-\tilde h_1$$
$$+{}^1/_4(\tau_z-\tau_0)\sigma_+\tilde h_2+{}^1/_2 i(\tau_x\sigma_z-i\tau_y\sigma_0)\varkappa\tilde\Delta^*]\delta(\mathbf{p}-\mathbf{p}'-\mathbf{Q}). \quad (6)$$

Inverting Eq. (6), we find the electron Green's function:

$$g_{11}(\mathbf{p}-\mathbf{K},\mathbf{p}+\mathbf{K})=-[(i\tilde\omega-\xi_-)(\tilde\omega_1^2+\xi_+^2+\tilde h^2-\varkappa^2|\Delta|^2)$$
$$-2\varepsilon\tilde h^2-2\delta\varkappa^2|\tilde\Delta|^2+2i\varkappa|\tilde\Delta|^2\tilde h]/D,$$
$$g^{21}(\mathbf{p}+\mathbf{K},\mathbf{p}+\mathbf{K})=\{\tilde h[2i\tilde\omega\varepsilon-\tilde\omega^2+\varepsilon^2-\delta^2+|\tilde\Delta|^2(1-\varkappa^2)-\tilde h^2]$$
$$+2i\varkappa|\tilde\Delta|^2(i\tilde\omega+\varepsilon)\}/D,$$
$$g_{31}(\mathbf{p}+\mathbf{K},\mathbf{p}+\mathbf{K})=-\tilde\Delta^*\{2\tilde h(i\omega-\delta)$$
$$+i\varkappa[\tilde h^2-\tilde\omega^2-\varepsilon^2+\delta^2+2i\omega\delta+|\tilde\Delta|^2(1+\varkappa^2)]\}/D, \quad (7)$$

$$g_{41}(\mathbf{p}-\mathbf{K},\mathbf{p}+\mathbf{K})=-\tilde\Delta^*[\tilde\omega_1^2-\varkappa^2|\tilde\Delta|^2+\xi_+^2-\tilde h^2)+2\varkappa\tilde h\tilde\omega]/D,$$
$$\tilde\omega_1^2=\tilde\omega^2+|\tilde\Delta|^2,\quad \varepsilon={}^1/_2(\xi_++\xi_-),$$
$$\delta={}^1/_2(\xi_+-\xi_-),\ \xi_\pm=\xi_{\mathbf{p}\pm\mathbf{K}},\quad \mathbf{K}={}^1/_2\mathbf{Q},$$
$$D=(\tilde\omega_1^2+\varepsilon^2-\delta^2-\tilde h^2-\varkappa^2|\tilde\Delta|^2)^2+4(\delta^2+\tilde h^2)[\tilde\omega_1^2-\tilde h^2+\varkappa^2|\tilde\Delta|^2+2\tilde h\varkappa\tilde\omega].$$

Here in calculating Eq. (7) we assume $\tilde h_1=\tilde h_2=\tilde h$; the justification for this is given below.

Going over to an integration over ε and δ instead of an integration over $\mathbf{p}$, we find to lowest order in the small parameter $\eta=2h/v_FQ$ for the case $|\tilde\Delta|^2\ll\tilde h^2$:

$$\int d\varepsilon d\delta g_{11}(\mathbf{p}-\mathbf{K})=-\frac{i\tilde\omega\pi v_FQ}{\tilde\omega_1}\{1+\eta[K(k_1)-E(k_1)]\},$$
$$\int d\varepsilon d\delta g_{21}(\mathbf{p}+\mathbf{K})=\frac{2\pi\tilde h|\tilde\Delta|^2(1-\varkappa^2)K(k_1)}{\tilde\omega_1(\tilde\omega_1^2-\varkappa^2|\tilde\Delta|^2+\tilde h^2)^{1/2}},$$
$$\int d\varepsilon d\delta g_{31}(\mathbf{p}+\mathbf{K})=-\frac{2\pi i\tilde\Delta^*\tilde\omega\tilde hK(k_1)}{\tilde\omega_1(\tilde\omega_1^2-\varkappa^2|\tilde\Delta|^2+\tilde h^2)^{1/2}}, \quad (8)$$
$$\int d\varepsilon d\delta g_{41}(\mathbf{p}-\mathbf{K})=-\frac{\pi\tilde\Delta^*v_FQ}{\tilde\omega_1}\Big\{1+\eta[K(k_1)-E(k_1)]$$
$$-\frac{\eta hK(k_1)}{(\tilde\omega_1^2-\varkappa^2|\tilde\Delta|^2+\tilde h^2)^{1/2}}\Big\},\quad k_1^2=\frac{|\tilde\Delta|^2}{\tilde\omega_1^2}\Big(1-\frac{|\tilde\Delta|^2\varkappa^2+2\tilde h\tilde\omega\varkappa}{\tilde h^2}\Big),$$

where $K(x)$ and $E(x)$ are elliptic functions.

In the opposite limit $(2\tau h)^2\ll1$ we get

$$\int d\varepsilon d\delta g_{11}(\mathbf{p}-\mathbf{K})=-\frac{i\pi\tilde\omega v_FQ}{\tilde\omega_1}\Big[1+\frac{\pi\tilde h^2|\tilde\Delta|^2}{v_FQ\tilde\omega_1^3}+\frac{\pi\tilde h^4}{v_FQ}\Big(\frac{3}{2}\frac{|\tilde\Delta|^4}{\tilde\omega_1^7}-\frac{|\tilde\Delta|^2}{\tilde\omega_1^5}\Big)\Big],$$
$$\int d\varepsilon d\delta g_{21}(\mathbf{p}+\mathbf{K})=\frac{\pi^2\tilde h|\tilde\Delta|^2(1-\varkappa^2)}{\tilde\omega_1(\tilde\omega_1^2-\varkappa^2|\tilde\Delta|^2+\tilde h^2)^{1/2}},$$
$$\int d\varepsilon d\delta g_{31}(\mathbf{p}+\mathbf{K})=-\frac{i\pi^2\tilde\Delta^*\tilde\omega\tilde h}{\tilde\omega_1(\tilde\omega_1^2-\varkappa^2|\tilde\Delta|^2+\tilde h^2)^{1/2}}, \quad (9)$$
$$\int d\varepsilon d\delta g_{41}(\mathbf{p}-\mathbf{K})=-\frac{\pi\tilde\Delta^*v_FQ}{\tilde\omega_1}\Big[1+\frac{\pi\tilde h^2}{v_FQ}\Big(\frac{|\tilde\Delta|^2}{\tilde\omega^3}-\frac{1}{\tilde\omega_1}\Big)$$
$$+\frac{\pi\tilde h^4}{v_FQ}\Big(\frac{3}{2}\frac{|\tilde\Delta|^4}{\tilde\omega_1^7}-\frac{2|\tilde\Delta|^2}{\tilde\omega_1^5}+\frac{1}{2\tilde\omega_1^3}\Big)\Big].$$

Below we show that the parameter η is in fact small throughout the whole region of existence of the HS phase. Substituting (8) and (9) in (5), we find a system of algebraic equations for the quantities $\tilde\Delta$, $\tilde\omega$, K, and h, which determines their dependence on Δ, ω, h, v_FQ, and the inverse scattering time $\tau^{-1}=4\pi\nu N(0)|V|^2$. For $|\tilde\Delta|^2\ll\tilde h^2$ we get

$$\tilde\omega=\omega+\frac{u}{2\tau(1+u^2)^{1/2}}[1+\eta(K-E)],\quad u=\tilde\omega/\tilde\Delta,$$
$$\tilde\Delta=\Delta+\frac{1}{2\tau(1+u^2)^{1/2}}\Big[1+\eta(K-E)-\frac{\eta hK}{(\tilde\omega_1^2-\varkappa^2|\tilde\Delta|^2+h^2)^{1/2}}\Big], \quad (10)$$
$$\tilde h_{1,2}=h\pm\frac{|\tilde\Delta|\tilde h(1-\varkappa^2)K}{Ql[(\tilde\omega_1^2-\varkappa^2|\tilde\Delta|^2+h^2)(1+u^2)]^{1/2}},\quad l=v_F\tau,$$
$$\varkappa=\frac{uhK}{Ql[(\tilde\omega_1^2-\varkappa^2|\tilde\Delta|^2+h^2)(1+u^2)]^{1/2}},$$
$$K=K((1+u^2)^{-1/2}),\quad E=E((1+u^2)^{-1/2}).$$

It is clear from (10) that for $u\gg e^{-Ql}$ we may neglect the quantity $\varkappa$ and the renormalization of the field h. Below we shall consider the case $Ql\gg1$ and with an accuracy up to terms of order $(Ql)^{-1}$ put $K=0$ and $\tilde h_{1,2}=h$. We choose the quantity Δ to be real, so that $\tilde\Delta$ will also be real. The condition $|\tilde\Delta|^2\ll h^2$ is equivalent to the condition $(2\tau h)^2\ll1$, and in this case we get from (10) an equation for the quantity u as a function of the variable ω/Δ:

$$\frac{\omega}{\Delta}=u\Big\{1-\frac{\eta hK}{2\tau\Delta[(\tilde\omega_1^2+h^2)(1+u^2)]^{1/2}}\Big\},\quad \tilde\omega=\Big[\Delta+\frac{1}{2\tau(1+u^2)^{1/2}}\Big]u. \quad (11)$$

This relation, together with Eq. (9), allows us to find the self-consistency condition which determines the dependence of the parameter Δ on Δ_0 and τ^{-1}, where Δ_0 is the superconducting gap in the absence of localized spins: that is, for $h=0$ we have

$$\Delta=\frac{\lambda}{N(0)}T\sum_\omega\int\frac{d^3\mathbf{p}}{(2\pi)^3}g_{41}(\omega,\mathbf{p}), \quad (12)$$

where λ is the dimensionless electron interaction constant. In the case $(2\tau h)^2\gg1$ relation (10) takes the form

$$\tilde\omega=\omega+\frac{u}{2\tau(1+u^2)^{1/2}}\Big[1+\frac{\pi h\eta|\tilde\Delta|^2}{2\tilde\omega_1^3}\Big],$$
$$\tilde\Delta=\Delta+\frac{1}{2\tau(1+u^2)^{1/2}}\Big[1+\frac{\pi h\eta}{2}\Big(\frac{|\tilde\Delta|^2}{\tilde\omega_1^3}-\frac{1}{\tilde\omega_1}\Big)\Big], \quad (13)$$

and to within terms of higher order smallness in the parameter $(2\tau h)^2$ we obtain for u as a function of ω/Δ the equation

$$\frac{\omega}{\Delta}=u\Big[1-\frac{\pi\eta h}{4\Delta(1+u^2)^{1/2}}\Big]. \quad (14)$$

3. DETERMINATION OF THE FREE ENERGY FUNCTIONAL FOR THE HS PHASE

Since we are interested in temperatures $T < \theta \ll T_{c0}$, in calculating the right-hand side of Eq. (12) we can replace the sum over ω by an integration. As a result we get for the case $\tau h \gg 1$

$$\frac{1}{\lambda}=\int_{u_0}^{\omega_D/\Delta}\frac{du}{(1+u^2)^{1/2}}+\eta\int_{u_0}^{\infty}\frac{du}{(1+u^2)^{1/2}}\left[K-E-\frac{\eta h K}{(\tilde{\omega}_1^2+h^2)^{1/2}}\right]$$

$$-\frac{\eta}{2\tau\Delta}\int_{u_0}^{\infty}\frac{du(K-E)h}{(1+u^2)(\tilde{\omega}_1^2+h^2)^{1/2}}. \qquad (15)$$

Here the upper limit of integration is set at infinity, provided the integral converges. The quantity u_0 is determined by the condition that the expression in curly brackets on the right-hand side of (11) tends to zero: u_0 satisfies the equation

$$\eta k_1^0 K(k_1^0)/2\tau\Delta=1, \quad k_1^0=(1+u^2)^{-1/2}. \qquad (16)$$

Going over in (15) to the new integration variable $k_1 =(1+u^2)^{-1/2}$, $k_1'=(1-k_1^2)^{1/2}$ and using the integral representation for K and E, we get from (15)

$$\ln\frac{\Delta_0}{\Delta}=\operatorname{arch}\frac{1}{k_1^0}+\eta\left(\ln\frac{8h}{\Delta e}+\frac{C}{2\tau\Delta}\right)\arcsin k_1^0-\frac{a\eta}{2\tau\Delta}k_1^0[1-(k_1^0)^2]^{1/2},$$

$$a\approx\frac{\pi^2}{16}, \quad C=\frac{1}{4\pi}\left[4K^2\left(\frac{1}{\sqrt{2}}\right)-\pi^2\Big/K^2\left(\frac{1}{\sqrt{2}}\right)\right]\approx 0.9. \qquad (17)$$

Below we shall show that k_1^0 is very close to unity, so we neglect the first and last terms on the right-hand side of (17) and replace arc sin k_1^0 by $\pi/2$. In the case $(2\tau h)^2 \ll 1$ similar but simpler calculations lead to the expression

$$\ln\frac{\Delta_0}{\Delta}=\frac{\pi^2h^2}{4v_FQ\Delta}+\frac{2\pi\tau h^2}{v_FQ}\ln\frac{1}{4\tau\Delta e^{1/2}}-\frac{\pi^2h^2}{16v_FQ\Delta}(2\tau h)^2 \qquad (18)$$

under the condition that $\pi^2 h^2/v_F Q\Delta \ll 1$. This condition guarantees the positivity of the expression in square brackets on the right-hand side of (14), and we shall see below that it is satisfied in the region of existence of the HS phase.

The self-consistency equations (17) and (18) allow us to find the free energy functional $F(\sigma, \mathbf{Q}, \Delta)$ (see Ref. 1). This functional is determined by the condition that minimization of it with respect to Δ gives the self-consistency equation (12), while minimization with respect to σ gives the self-consistency equation for the magnetic order parameter. The equilibrium value of $\mathbf{Q}$ is also determined from the condition that the functional F should be a minimum, and the minimum value of $F(\sigma, \mathbf{Q}, \Delta)$ gives the free energy of the HS phase.

According to previous work[1] for $l \gg \xi_0$ helicoidal ordering of the spins with wave vector $Q_M=(\pi^2\Delta_0/4a^2v_F)^{1/2}$ appears at the point $T_M=\theta(1-3a^2Q_M^2)\approx\theta$. Here a is a quantity with the dimensions of length, of the order of magnitude k_F^{-1}. In Ref. 1 it was shown that impurity scattering does not change the values of Q_M and T_M it is necessary to take into account critical magnetic fluctuations, and the effects of these can change the results of the self-consistent field approximation, which was used to describe the magnetic system. In this paper we consider the behavior of the system far from T_M, where these fluctuations are small. For this purpose it is adequate to know the functional F in the region of parameters where $h \gg \Delta$. The free energy functional in the absence of impurity scattering was obtained in Ref. 1; here we find it taking scattering into account.

Integrating (17) and (18) with respect to Δ, we find

$$F(\sigma, Q, \Delta)=F_s(\Delta)+F_M(\sigma, Q, T)+F_{int}(\sigma, Q, \Delta),$$

$$F_s(\Delta)=-\frac{1}{2}N(0)\Delta^2\ln\frac{e\Delta_0^2}{\Delta^2}, \quad \theta=\frac{2(S+1)h_0^2N(0)b^2}{3nS}, \qquad (19)$$

$$h=h_0\sigma,$$

$$F_M(\sigma, Q, T)=-\frac{1}{2}N(0)b^2h^2\left[1-a^2Q^2-\frac{2(S+1)T}{3S\theta\sigma^2}\int_0^{\sigma}b_s(x)\,dx\right],$$

If $l \gg \xi_h \gg Q^{-1}$ and $\eta/2\tau\Delta \leqslant 1$, then we have

$$F_{int}(\sigma, Q, \Delta)=\frac{N(0)\pi h\Delta^2}{v_FQ}\left[\ln\frac{8h}{\Delta e^{1/2}}+\frac{C}{\tau\Delta}\right], \qquad (19')$$

whereas if $\xi_h \gg l \gg Q^{-1}$, $l \ll \xi_0$ and $\pi h^2/v_F Q\Delta \leqslant 1$, then we get

$$F_{int}(\sigma, Q, \Delta)=\frac{N(0)\pi^2h^2\Delta}{2v_FQ}\left[1-(\tau h)^2+\frac{4\tau\Delta}{\pi}\ln\frac{1}{4\tau\Delta}\right], \qquad (19'')$$

where $b_s(x)$ is the reciprocal Brillouin function and $\xi_h = v_F/h$. The functional F_M describes the magnetic part of the system, F_s its purely superconducting part and F_{int} the interaction between these subsystems. The parameter $b^2=1+\beta_e'$, where the quantity β_e' is of order unity, takes account of the short-range part of the indirect exchange interaction of the spins. Taking the direct exchange interaction of the spins into account renormalizes the parameter a^2 and gives a correction β_d to the parameter b^2. The short-range part of the magnetic interaction of the spins also renormalizes the parameter a^2 and gives a correction β_m' to the parameter b^2. (The signs of β_m' and β_e' can in principle be either positive or negative, depending on the lattice type and the structure of the energy bands).

The long-range part of the magnetic interaction selects the direction of $\mathbf{Q}$, with the energy minimum attained for a transverse helicoid, i.e., for $Q_x=Q_y=0$ and $Q_z=Q\neq 0$. For this direction of $\mathbf{Q}$, the long-range part of the magnetic interaction leads to the addition to b^2 of the term

$$\beta_m=2\pi M^2/N(0)h^2,$$

where $\mathbf{M}$ is the magnetic moment per unit volume. In previous work[4,5] it has been shown that the contribution of the magnetic dipole interaction in F_{int} is negligibly small in real systems. Thus, in type-II superconductors with an indirect RKKY exchange interaction comparable in magnitude to the dipole interaction, the term F_{int} is primarily determined by the exchange interaction. However, in principle, for type-I superconductors we cannot exclude the situation where the main contribution comes from the magnetic dipole interaction considered in Refs. 6–9, or where the two interactions make an approximately equal contribution to the formation of the HS phase.[10] As regards real compounds of the type $ErRh_4B_4$, the expressions (19) allow us to find all the characteristics of the HS-phase, provided we can neglect the magnetic anisotropy within the easy plane. In the functional (19) we also neglected the magnetic exchange scattering of electrons; this is in fact small provided $\theta_e \ll T_{c0}$, where $\theta_e = \hbar^2 N(0)/n$ is the characteristic energy parameter of the RKKY interaction.

The functional (19) was found for the case of scattering of the electrons by point impurities. In the Appendix it is shown that all the expressions in (19) are preserved also the case of symmetric scattering of the electrons by the impurities, if we take τ to be given by

$$\tau^{-1} = vN(0)\int |V(p-p_1)|^2 d\Omega_1.$$

Knowledge of the free energy functional allows us to determine the equilibrium values of h, Δ and $\mathbf{Q}$ as functions of temperature and to find the phase boundary between the HS phase and the normal ferromagnetic (F) phase.

4. TRANSITION FROM THE HELICOIDAL SUPERCONDUCTING PHASE TO THE NORMAL FERROMAGNETIC F PHASE

Along the first-order phase transition line from the HS superconducting phase to the F phase the equilibrium values of the free energy in the two phases are equal, while up to terms of order $\Delta\eta^2/\hbar^2$ the equilibrium value of σ in the HS phase is given by the same expression as in the ferromagnetic phase. Thus, at the first-order phase transition only the direction of the spins is changed. From Eq. (19) we find an equation for the equilibrium parameters of the HS-phase on the phase boundary. For $(2\tau h)^2 \gg 1$ we have

$$\ln\frac{\Delta_0}{\Delta_{c2}} = 1 - 3\left(4 + \frac{2\tau\Delta_{c2}}{C}\ln\frac{8h_0\sigma_{c2}}{\Delta_{c2}e^{1/2}}\right)^{-1},$$

$$Q_{c2} = \frac{\pi h_0\sigma_{c2}}{v_F}\left(\ln\frac{8h_0\sigma_{c2}}{\Delta_{c2}e^{1/2}} + \frac{2C}{\tau\Delta_{c2}}\right), \tag{20}$$

$$\left(\frac{1}{\sigma_{c2}}\ln\frac{\Delta_0^2 e}{\Delta_{c2}^2}\right)^{1/2} = \frac{\pi h_0^2 ab\sigma_{c2}^2}{v_F\Delta_{c2}}\left(\ln\frac{8h_0\sigma_{c2}}{\Delta_{c2}e^{1/2}} + \frac{2C}{\tau\Delta_{c2}}\right), \quad \frac{3S\theta\sigma_{c2}}{T_{c2}(S+1)} = b_S(\sigma_{c2}).$$

In the case $l \ll \xi_0$ these equations give $\Delta_{c2} = \Delta_0 e^{-1/4}$.

For $(2\tau h)^2 \ll 1$ the parameters Δ_{c2}, σ_{c2} and Q_{c2} are determined on the phase boundary by the equations

$$\Delta_{c2} = \Delta_0 e^{-1/4}, \quad Q_{c2} = \frac{\pi^2 h_0^2\sigma_{c2}^2}{v_F\Delta_{c2}}, \quad \frac{2\pi^2 h_0^3\sigma_{c2}^3 ab}{v_F\Delta_{c2}} = 1,$$

$$\frac{3S\theta\sigma_{c2}}{T_{c2}(S+1)} = b_S(\sigma_{c2}). \tag{21}$$

Using Eqs. (20) and (21), we find the first-order phase transition line in explicit form

$$T_{c2} = \frac{3S(\alpha T_{c0}\theta Y)^{1/2}}{(S+1)b_S[(\alpha T_{c0}Y/\theta)^{1/2}]}, \quad \alpha = \frac{2^{3/2}(S+1)N(0)v_F\Delta_0 b}{3S\pi enaT_{c0}},$$

$$Y = \begin{cases} (\ln\beta)^{-1}, & l \gg \xi_0, \quad \beta = 2^{13/2}v_F/\pi ab\Delta_0 \approx \varepsilon_F/\Delta_0, \\ e^{1/2}\tau\Delta_0/8C, & \xi_0 \gg l \gg \xi_h, \\ 2^{-3/2}(\Delta_0 ae^2/\pi v_F)^{1/3} \approx (\Delta_0/\varepsilon_F)^{1/3}, & \xi_h \gg l \gg Q^{-1}, \end{cases} \tag{22}$$

where the parameter α is of order unity. The curve $T_{c2}(\theta)$ intersects the axis $T=0$ at the point θ_{c2} defined by the relation $\theta_{c2}/T_{c0} = \alpha Y$; the parameter θ_{c2} decreases with decreasing l, but not very strongly, in fact when l decreases from values such that $l \gg \xi_0$ to values such that $\xi_h \gg l \gg Q^{-1}$ the parameter θ_{c2} decreases by no more than a factor of 2 or 3 [from $T_{c0}/\ln(\varepsilon_F/\Delta_0)$ to $T_{c0}(\Delta_0/\varepsilon_F)^{1/2}$].

For $\theta < \theta_{c2}$ the HS phase survives right down to zero temperature, whereas for $\theta > \theta_{c2}$ there is a first-order phase transition to the F phase. From (22) it is clear that decreasing l leads to a shrinking of the interval of existence of the HS phase. In the region where it is small compared to θ we have

$$\frac{T_M - T_{c2}}{\theta} = \frac{T_{c0}}{\theta}\alpha g(S)Y, \quad g(S) = \frac{9S(S^2+S+1/2)}{5(S+1)^2}. \tag{23}$$

On the HS-F phase boundary the superconducting order parameter, which is Δ_0/e for $l \gg \xi_0$ and $\Delta_0 e^{-1/4}$ for $l \ll \xi_0$, vanishes discontinuously. With decreasing l the magnitude of the wave vector Q_{c2} changes slightly from the value

$$Q_{c2} = Q_M(16/\pi^2\beta)^{1/4}(\ln\beta)^{1/4} < Q_M$$

for $l \gg \xi_0$ to $Q_{c2} = Q_M e^{-1/2}$ for $Q_M^{-1} \ll l \ll \xi_h$. Thus, in the HS phase the parameters Δ and Q change more weakly with decreasing temperature the greater the impurity concentration.

The limit of supercooling of the HS phase is obtained from Eq. (22) by multiplying the quantity α by $(27/8e)^{1/2} \approx 1.1$ for $l \gg \xi_0$, by $3^{3/2}/e \approx 1.7$ for $\xi_h \ll l \ll \xi_0$ and by $3e^{-2/3} \approx 1.4$ for $Q_M^{-1} \ll l \ll \xi_h$ (see Ref. 1). The limit of superheating of the F phase in the absence of domain-wall effects (i.e., for $L \gg \xi_0$ where L is the domain width) and without taking account of the exchange scattering of electrons by critical magnetic fluctuations is determined by the condition

$$\sigma = \frac{\Delta_0}{2h_0}\left[1 + \left(\frac{2\pi M_0\Delta_0}{H_{c2}^*(0)h_0}\right)^2\right]^{1/2}, \quad \frac{3S\theta\sigma b_S(\sigma)}{(S+1)} = T_{c2}^{(u)}, \tag{24}$$

where M_0 is the magnitude of the magnetic moment per unit volume at $T=0$ and $H_{c2}^*(0)$ is the critical orbital magnetic field at $T=0$, which increases as l^{-1} for $l \ll \xi_0$. Thus in general the limit of superheating $T_{c2}^{(u)}$ will depend on l: however, this dependence vanishes if the condition

$$(2\pi M_0\Delta_0/H_{c2}^*(0)h_0)^2 \ll 1$$

is met. In real superconductors of the $ErRh_4B_4$ type the condition $4\pi M_0 \lesssim H_{c2}^*(0)$ is satisfied. Then for $h_0 \gg \Delta_0$ the limit of superheating of the F phase is determined by the exchange interaction and does not depend on l.

Knowing the equilibrium parameters of the HS phase, we can check that the conditions assumed in obtaining the functional (19) are indeed fulfilled. In the case $(2\tau h)^2 \ll 1$ we assumed that $\pi h^2/v_F Q \lesssim 1$; on the T_{c2} line we get from Eq. (21) the result $\pi h^2/v_F Q\Delta = 1/\pi$, and in the rest of the HS phase the value of this parameter is less than on the T_{c2} line. In the region $(2\tau h)^2 \gg 1$ we took $\eta/2\tau\Delta \lesssim 1$ and $1 - k_0 \ll 1$. On the T_{c2} line we obtain

$$\frac{\eta}{2\tau\Delta} = \left[\pi\tau\Delta\left(\ln\frac{8h}{\Delta e^{1/2}} + \frac{2C}{\tau\Delta}\right)\right]^{-1} < \frac{1}{2\pi C}.$$

It then follows from (16) that $1 - k_1^0 < 8e^{-4\pi C}$, which justifies the simplification made in the expression (17). It also follows from Eqs. (20) and (21) that the condition $\eta \ll 1$ is satisfied. In the derivation of the functional (19) we neglected the exchange scattering of the electrons, taking $\theta \ll T_{c0}$. This condition is the better fulfilled in the HS phase the smaller l.

5. THE ONE-ELECTRON EXCITATION SPECTRUM IN THE HS PHASE

In the absence of impurities, i.e., for $l \gg \xi_0$, the energy gap in the quasiparticle spectrum vanishes on a belt on the Fermi surface perpendicular to the vector

Q, in an angular interval of order η (see Ref. 1). This effect is connected with the fact that electrons traveling perpendicular to the vector **Q** feel a strong exchange field $h > \Delta$ which is constant in space. Impurity scattering changes the quasiparticle spectrum in the HS phase substantially, since such scattering changes the direction of motion of the electrons. In the case $(2\tau h)^2 \gg 1$ we get from Eq. (8)

$$\rho(E) = \frac{N(0)}{\pi v_F Q} \operatorname{Im} \int d\varepsilon\, d\delta\, g_{11}(\mathbf{p}-\mathbf{K})$$

$$= \operatorname{Im} \frac{iu}{(1+u^2)^{1/2}} \{1 + \eta [K((1+u^2)^{-1/2}) - E((1+u^2)^{-1/2})]\}, \qquad (25)$$

where $u(E)$ is defined as the analytic continuation of the function $u(\omega)$ defined by expression (11) to the imaginary half-axis $\omega = -iE$.

For $E \to 0$ we get from (11) and (25)

$$\rho(0) = u_0 [1 + \eta \ln (4/u_0)],$$

where u_0 is defined by Eq. (16). Thus the density of states for $E=0$, though very small (note that $u_0 < 4e^{-2\pi C}$), is nonzero.

For $1 \gg E/\Delta \gg u_0$ and $\eta/2\tau\Delta \ll 1$ we get

$$\rho(E) = \frac{\eta\pi N(0) E}{2\Delta} \left(1 + \frac{1}{2\tau\Delta}\right) \qquad (26)$$

i.e., the density of quasiparticles for small $E (\ll \Delta)$ is considerably increased in comparison with the pure HS phase, if $\xi_0 \gg l \gg \xi_h$. This effect is explained by the fact that impurity scattering leads to all electrons on the Fermi surface spending a part of their time $\tau \gg 1/h$ moving perpendicular to the vector Q and feeling the effect of the strong exchange field h.

The situation is quite different in the case $(2\tau h)^2 \ll 1$, [when the duration of motion perpendicular to **Q** is already too small for the electrons to experience this field effectively. For $(2\tau h)^2 \ll 1$] we get from (9) and (14) formula (25), where we must put $\eta = 0$. The density of states per unit energy is the same as in a superconductor with magnetic impurities and a reciprocal lifetime for magnetic scattering of $\tau^{-1} = \pi\eta h$.

Thus, in the case $(2\tau h)^2 \ll 1$ impurity scattering leads to an energy gap in the quasiparticle spectrum of the HS phase, although this gap is less than the superconducting order parameter. In general the density of states for $E \ll \Delta$ in the HS phase first increases with decreasing l so long as $l \gg \xi_h$, then decreases and finally vanishes in the limit $l \ll \xi_h$. So from the point of view of the investigation of gapless superconductivity most interest would attach to specimens with a mean free path which satisfies the inequalities $\xi_0 \gg l \gg \xi_h$.

6. CONCLUSIONS AND QUALITATIVE INTERPRETATION OF THE EXPERIMENTAL DATA ON $ErRh_4B_4$

We now summarize our fundamental results on phase transitions in a superconductor with $T_{c1} \gg \theta_e$, $\beta_m \approx 1$ and weak anisotropy in the easy plane.

1. As the temperature is decreased, at [the point $T_M \approx \theta$ in the superconducting phase there appears] helicoidal ordering of the spins in a second-order transition. The magnitude of the wave vector Q_M at the transition point is practically independent of mean free path provided $Q_M l \gg 1$.

2. On cooling of the superconducting phase with helicoidal spin ordering, this HS phase persists to zero temperature, if $\theta < \theta_{c2}$, or goes over by a first-order transition into a normal ferromagnetic phase at a temperature T_{c2}, if $\theta > \theta_{c2}$. The parameters θ_{c2} and T_{c2} depend on l, but not very strongly, and in practice $T_M - T_{c2}$ and θ_{c2} change by at most a factor of 2 or 3 when l changes from $l \gg \xi_0$ to $Q_M^{-1} \ll l \ll \xi_h$. The supercooling temperature $T_{c2}^{(d)}$ differs from T_{c2} by 10 to 70 per cent, depending on l.

3. For $\theta > \theta_{c2}$ the ferromagnetic phase is stable below T_{c2}, but for $\theta < \theta_{c2}$ it is metastable, and the HS phase may be converted into this phase by the application and subsequent removal of a magnetic field. When superheated the F phase stays metastable right up to a temperature $T_{c2}^{(u)} \approx T_M$ and in principle could be converted by superheating directly into the nonmagnetic S phase, avoiding the HS phase.

4. In the HS phase the wave vector Q is practically independent of temperature, the variation being smaller the smaller the quantity l. The amplitude of helicoidal ordering in the HS phase varies exactly as it would vary in a ferromagnetic phase in the absence of superconductivity. The discontinuity of the amplitude at the point T_{c2} is very small; even at this point only the direction of the spins changes and the latent heat evolved is of the order of the superconducting energy.

At present there are no data on the magnitude of the magnetic anisotropy in the easy plane for crystals of $ErRh_4B_4$. For sufficiently strong anisotropy a domain type of structure will be realized in $ErRh_4B_4$. The presence of such a structure changes the value of F_{int} only weakly from that obtained from the exactly soluble model with helicoidal spin ordering, but the functional F_M of the magnetic subsystem is modified considerably more strongly. In this case our qualitative conclusions remain in force but the quantitative results are changed. Within the framework of this picture we may understand all the peculiarities of the behavior of $ErRh_4B_4$ observed experimentally in Refs. 11 and 12.

APPENDIX

For a spherically symmetric impurity scattering potential we have $V \equiv V(|\mathbf{p}-\mathbf{p}'|)$. We rewrite Eq. (5) by inserting on the right-hand side terms which cancel one another:

$$g^{-1}(\mathbf{p},\mathbf{p}') = g_0^{-1}(\mathbf{p},\mathbf{p}') - \frac{\nu N(0)}{v_F Q} \int d\varepsilon_1\, d\delta_1 \, |V(\mathbf{p}-\mathbf{p}_1)|^2 \tau_z \sigma_0 \{ g(\mathbf{p}_1+\mathbf{p}'-\mathbf{p}, \mathbf{p}_1)$$
$$- g(\mathbf{p}_1+\mathbf{p}'-\mathbf{p}, \mathbf{p}_1)|_{h=0} \} \tau_z \sigma_0 - \frac{\nu N(0)}{v_F Q} \int d\varepsilon_1\, d\delta_1 \, |V(\mathbf{p}-\mathbf{p}_1)|^2$$
$$\times \tau_z \sigma_0 \{ g(\mathbf{p}_1+\mathbf{p}'-\mathbf{p}, \mathbf{p}_1)|_{h=0} - g_0{}^0(\mathbf{p}_1+\mathbf{p}'-\mathbf{p}, \mathbf{p}_1)|_{h=0} \} \tau_z \sigma_0$$
$$- \frac{\nu N(0)}{v_F Q} \int d\varepsilon_1\, d\delta_1 \, |V(\mathbf{p}-\mathbf{p}_1)|^2 \tau_z \sigma_0 g_0{}^0 (\mathbf{p}_1+\mathbf{p}'-\mathbf{p}, \mathbf{p}_1)|_{h=0} \tau_z \sigma_0. \qquad (A.1)$$

We shall seek the function $g^{-1}(p, p')$ in the form (6) with $\tilde{\omega} \equiv \tilde{\omega}(\delta)$ and $\tilde{\Delta} \equiv \tilde{\Delta}(\delta)$; $g_0^0(p, p') = g(p, p')$ when $\tilde{\omega}(\delta)$ is replaced by $\tilde{\omega}(0)$ and $\tilde{\Delta}(\delta)$ by $\tilde{\Delta}(0)$. The functional dependence of $\tilde{\omega}(\delta)$ and $\tilde{\Delta}(\delta)$ on δ is determined by the second and third terms on the right-hand side of (A.1), which are proportional to the small parameter $\eta=2h/v_FQ$.

We linearize (A.1) in $\omega_1(\delta)\equiv\tilde{\omega}(\delta)-\tilde{\omega}(0)\sim\eta\tilde{\omega}(0)$ and $\tilde{\Delta}_1(\delta)\equiv\tilde{\Delta}(\delta)-\tilde{\Delta}(0)\sim\eta\tilde{\Delta}(0)$. For $(2\tau h)^2\gg1$ we have

$$\tilde{\omega}(0)=\omega+\frac{\eta u(K-E)}{2\tau(0)(1+u^2)^{1/2}}+\frac{u}{2\tau(1+u^2)^{1/2}}$$

$$\tilde{\Delta}(0)=\Delta+\frac{\eta}{2\tau(0)(1+u^2)^{1/2}}\left\{K-E-\frac{hK}{\tilde{\Delta}(0)[1+u^2+h^2/\tilde{\Delta}^2(0)]^{1/2}}\right\}+\frac{1}{2\tau(1+u^2)^{1/2}},$$
$$u=\tilde{\omega}(0)/\tilde{\Delta}(0),\quad K\equiv K((1+u^2)^{-1/2}),\quad E\equiv E((1+u^2)^{-1/2}). \tag{A.2}$$

The equations for $\omega_1(\delta)$ and $\Delta_1(\delta)$ have the form

$$\omega_1(\delta)=\frac{\nu N(0)}{v_FQ(1+u^2)^{3/2}}\int|V(\mathbf{p}-\mathbf{p}_1')|^2\,d\varphi_1\,d\delta_1[\omega_1(\delta_1)-u\Delta_1(\delta_1)]$$
$$+\frac{\eta u}{(1+u^2)^{1/2}}\left[\frac{1}{2\tau(\delta)}-\frac{1}{2\tau(0)}\right](K-E),\quad \delta_0=\frac{v_FQ}{2},$$
$$\Delta_1(\delta)=-\frac{\nu N(0)u}{v_FQ(1+u^2)^{3/2}}\int|V(\mathbf{p}-\mathbf{p}_1)|^2d\varphi_1\,d\delta_1[\omega_1(\delta_1)-u\Delta_1(\delta_1)]$$
$$+\frac{\eta}{(1+u^2)^{1/2}}\left[\frac{1}{2\tau(\delta)}-\frac{1}{2\tau(0)}\right]\left(K-E-\frac{hK}{\Delta(0)(1+u^2+h^2/\tilde{\Delta}^2(0))^{1/2}}\right), \tag{A.3}$$
$$\frac{1}{2\tau(\delta)}=2\nu N(0)\int_0^{2\pi}|V(\mathbf{p}-\mathbf{p}_1)|^2\,d\varphi_1,\quad \frac{1}{2\tau}=\nu N(0)$$
$$\times\int|V(p-p_1)|^2\,d\Omega_1=\frac{1}{v_FQ}\int_{-\delta_0}^{+\delta_0}\frac{d\delta}{2\tau(\delta)}.$$

We seek the solution of (A.3) in the form

$$\Delta_1(\delta)=\frac{\eta}{(1+u^2)^{1/2}}\left\{\left[\frac{1}{2\tau(\delta)}-\frac{1}{2\tau(0)}\right]\left(K-E-\frac{hK}{\tilde{\Delta}(0)(1+u^2+h^2/\tilde{\Delta}^2(0))^{1/2}}\right)\right.$$
$$\left.-\frac{uhK\nu N(0)}{(1+u^2)^{1/2}(1+u^2+h^2/\tilde{\Delta}^2(0))^{1/2}\tilde{\Delta}^3(0)v_FQ}\int|V(\mathbf{p}-\mathbf{p}_1)|^2d\varphi_1\,d\delta_1\,v(\delta_1)\right\},$$
$$\omega_1(\delta)-u\Delta_1(\delta)=\frac{\eta uhKv(\delta)}{(1+u^2)^{1/2}\tilde{\Delta}(0)(1+u^2+h^2/\tilde{\Delta}^2(0))^{1/2}}. \tag{A.4}$$

Here $v(\delta)$ satisfies the equation

$$v(\delta)-\frac{\nu N(0)}{v_FQ\tilde{\Delta}(0)(1+u^2)^{1/2}}\int|V(\mathbf{p}-\mathbf{p}_1)|^2\,d\varphi_1\,d\delta_1\,v(\delta_1)=\frac{1}{2\tau(\delta)}-\frac{1}{2\tau(0)}. \tag{A.5}$$

Integrating (A.5) with respect to δ, we see that the following integral relation is valid:

$$\frac{1}{v_FQ}\int_{-\delta_0}^{+\delta_0}v(\delta)\,d\delta\left(1-\frac{1}{2\tau\tilde{\Delta}(0)(1+u^2)^{1/2}}\right)=\frac{1}{2\tau}-\frac{1}{2\tau(0)}. \tag{A.6}$$

Here we used the definition of τ^{-1} from (A.3).

The self-consistency equation (12), linearized in $\omega_1(\delta)$ and $\Delta_1(\delta)$, has the form

$$\frac{1}{\lambda}=\int_0^{\omega_D}\frac{d\omega}{\Delta(1+u^2)^{1/2}}\left\{1+\eta\left[K-E-\frac{hK}{\tilde{\Delta}(0)[1+u^2+h^2/\tilde{\Delta}^2(0)]^{1/2}}\right]\right.$$
$$\left.\times\left[1+\frac{u^2}{(1+u^2)^{3/2}v_FQ\tilde{\Delta}(0)}\int_{-\delta_0}^{+\delta_0}v(\delta)\,d\delta\right]\right\}. \tag{A.7}$$

From (A.2) we find

$$\frac{\omega}{\Delta}=u\left(1-\frac{\eta hK}{2\tau(0)\Delta\tilde{\Delta}(0)[1+u^2+h^2/\tilde{\Delta}^2(0)]^{1/2}}\right). \tag{A.8}$$

The quantity u_0 is determined by the condition that the right-hand side of (A.8) should be zero:

$$\eta k_1^0K(k_1^0)/2\tau(0)\Delta=1,\quad k_1^0=(1+u_0^2)^{-1/2}. \tag{A.9}$$

Substituting (A.6) and (A.7) in (A.8) and integrating with respect to u, we again obtain Eq. (16), with τ^{-1} defined by (A.3). Here Δ depends on $\tau(0)$ only through the quantity k_1^0 defined in (A.9). In the case $(2\tau h)^2\ll1$ similar calculations lead to expression (18) with the parameter τ^{-1} defined in (A.3).

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