Radiation of ultrarelativistic positrons moving in a single crystal near crystallographic planes

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A theoretical and experimental study of the spectral distributions of the radiation of positrons with energies of the order of several GeV is carried out under the conditions in which the angle of entry of the particles with respect to the crystallographic planes varies in the range from zero to several channeling angles. The radiation of channeled particles and particles above the barrier is taken into account, and also the departure from dipole radiation, de-channeling, and multiple scattering, and the dependence of the radiation spectrum on the form of interplanar potential is investigated. It is shown that the radiation is due to channeled particles and particles above the barrier, and that the intensity of the radiation of particles above the barrier is comparable with that of channeled particles. From comparison of theory and experiment, information is obtained on the dynamics of motion of particles in a real crystal.

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1. INTRODUCTION

A number of papers[1-7] devoted to the electromagnetic radiation by fast particles in crystals have predicted that ultrarelativistic electrons and positrons should radiate strongly as they move near crystallographic planes under channeling conditions and as they move above the barrier. This radiation is of considerable interest not only in view of the possibility of practical application due to its exceptional properties (high intensity, monochromatic nature, polarization), but also from the point of view of the general problem of interaction of particles with single crystals.

The radiation discussed was observed experimentally for positrons with energy from 4 to 14 GeV (Ref. 8) and then for electrons and positrons with energy 56 MeV.[9,10] Although the experiments confirm the main predictions of the theory, the comparison of theory and experiment requires further study. This is due to the fact that the radiation of particles in crystals depends substantially on a number of factors which characterize the dynamics of particles in real crystals.

In this article we present the results of a systematic theoretical and experimental study of the radiation of positrons with energies from 4 to 14 GeV moving near crystallographic planes in a diamond crystal 80 μm thick. In this study we made substantial use of the results of Akhiezer *et al.*,[5] who have shown that in addition to channeled particles an important contribution to the radiation is from particles above the barrier. We intend to study the influence of the following factors which determine the radiation of channeled and above-barrier particles in a crystal: 1) the nondipole nature of the radiation, 2) the dependence of the radiation on the angle of entry of the particles into the crystal with respect to the crystallographic planes, 3) the role of de-channeling and multiple scattering, 4) the dependence on the form of the interplanar potential, and 5) the dependence on the angular distribution of the particle beam in the crystal.

The article is organized as follows: in Sec. 2 we present the theory of the phenomenon and give the principal formulas which will be used later to analyze the experimental data. Our main concern here has been to obtain sufficiently simple expressions which can be used for both qualitative and quantitative analysis. For example, the influence of the nondipole nature on the radiation has been studied previously,[11,12] but the accurate formulas obtained in those studies turned out to be rather awkward, which hinders their use in the analysis. At the same time for the regions of γ-ray frequencies of interest to us in radiation of positrons with energies of several GeV it is possible to obtain in the framework of classical electrodynamics rather simple formulas which to a good approximation permit us to investigate the effect of the nondipole nature on the radiation both analytically and by means of simple numerical computer calculations. The formulas obtained in this section can be used for description of the radiation of both channeled and superbarrier particles and also of particles moving in undulators. In Sec. 3 we give a description of an experiment on the radiation of positrons moving in a diamond single crystal near crystallographic planes, and also the results of this investigation. Measurements were made for four eneriges of the positron beam (4, 6, 10, and 14 GeV) and also for various entry angles. In this experiment we achieved for the first time a beam divergence ($\Delta\theta \sim 10^{-5}$ rad) significantly smaller than the critical channeling angle. This made it possible to bring out the role of de-channeling, and also to separate the contributions of channeled and above-barrier particles.

Finally, in Sec. 4 we give a detailed comparison of theory and experiment and discuss the role of the various factors mentioned above which influence the radiation of particles in crystals. Here we have attempted to determine to what degree the experimental results permit an unambiguous discussion, and also to single out those questions to which we cannot yet give a convincing and unequivocal answer and which in our opinion require further detailed study.

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2. THEORY

In classical electrodynamics the spectral distribution of the energy radiated by a particle moving in a trajectory $\mathbf{r}(t)$ is determined[2] by the formula[13]

$$\frac{dE}{d\omega}=\frac{e^2\omega^2}{4\pi^2}\int do[\mathbf{n}\times\mathbf{I}]^2,\quad \mathbf{I}=\int_{-\infty}^{\infty}dt\,\mathbf{v}(t)\exp\{i\omega(t-\mathbf{nr}(t))\},\qquad(2.1)$$

where do is the element of solid angle, $\mathbf{n}$ is the unit vector in the direction of radiation, ω is the frequency of radiation, and $\mathbf{v}(t)=d\mathbf{r}(t)/dt$.

In above-barrier motion and in motion of particles in the crystal in the planar channeling regime it is convenient to separate the longitudinal and transverse motions:

$$\mathbf{r}_\parallel(t+T)=\mathbf{r}_\parallel(t)+\mathbf{v}_\parallel T,\quad \mathbf{r}_\perp(t+T)=\mathbf{r}_\perp(t),\qquad(2.2)$$

where T is the period of oscillation. Here

$$\mathbf{v}(t+T)=\mathbf{v}(t).\qquad(2.3)$$

In the limit in which the number of oscillations of a particle in the crystal is large ($N\to\infty$), the quantity $[\mathbf{n}\times\mathbf{I}]^2$ in Eq. (2.1) can be represented in the form

$$[\mathbf{n}\times\mathbf{I}]^2\to N\frac{2\pi}{T}[\mathbf{n}\times\mathbf{I}_T]^2\sum_n\delta(q-\omega_0 n),\qquad(2.4)$$

$$\omega_0=2\pi/T,\quad q=\omega(1-\mathbf{nv}_\parallel),\quad n=0,\pm1,\pm2,\ldots,$$
$$\mathbf{I}_T=\int_0^T dt\,\mathbf{v}(t)\exp\{i\omega(t-\mathbf{nr}(t))\}.\qquad(2.5)$$

The characteristic angles of scattering and radiation of relativistic particles in a crystal are small, and therefore in Eqs. (2.1) and (2.4) we can carry out an expansion in these angles. To first order we find the following experession for the spectral distribution of the energy radiated by a particle per unit path length:

$$\frac{d^2E}{dld\omega}=\frac{e^2\omega_0^2\omega}{(2\pi)^3}\sum_n\int d\varphi|\mathbf{I}_\perp-\mathbf{n}_\perp I_\parallel|^2,\qquad(2.6)$$

where the summation over the harmonics is carried out with the restriction

$$n\geqslant\Omega(1+\gamma^2\overline{v_\perp^2}),\qquad(2.7)$$

$I_\parallel$ and $\mathbf{I}_\perp$ are the longitudinal and transverse components of the vector $\mathbf{I}_T$, φ is the azimuthal angle, $\Omega=\omega/\omega_m$, $\omega_m=2\gamma^2\omega_0$, $\gamma=\varepsilon/m$ is the Lorentz factor of the positron, and

$$\overline{v_\perp^2}=\frac{1}{T}\int_0^T dt v_\perp^2(t).$$

The quantity $\beta=\gamma(\overline{v_\perp^2})^{1/2}$ is a measure of the nondipole nature of the radiation. This quantity is the ratio between the characteristic angle of scattering of the particle in the crystal and the characteristic angle of radiation of a relativistic particle.

We shall consider several limiting cases of Eq. (2.6). In the dipole approximation, i.e., for $\beta\ll1$, Eq. (2.6) can be represented in the form[2]

$$\frac{d^2E}{dld\omega}=\frac{e^2}{(2\pi)^2}\sum_n\left[1-2\frac{\Omega}{n}\left(1-\frac{\Omega}{n}\right)\right]\frac{\omega|\mathbf{w}_n|^2}{\omega_0^2n^2},\qquad(2.8)$$

where

$$\mathbf{w}_n=\int_0^T dt\,\dot{\mathbf{v}}_\perp(t)\exp(in\omega_0t)$$

is the Fourier harmonic of the acceleration and $n\geqslant\Omega$. This important limiting case of Eq. (2.8) is very significant, since in the dipole approximation the formulas for the radiation are greatly simplified and this permits study of the influence of a large number of different factors (multiple scattering, dependence on the point of entry of the particle into the crystal, the spread of the particle in angle, and so forth) on the radiation of channeled and above-barrier particles. In addition, as we shall show below, the formulas of the dipole approximation can be used for description of the radiation spectrum even with a nondipole parameter of the order of unity in the region of γ-ray frequencies where the maximum of the radiation occurs.

It follows from Eq. (28) that the shape of the radiation spectrum of an individual particle in the dipole approximation is universal for periodic motion in the transverse direction and does not depend on the form of the potential. The maximum of the radiation occurs at $\omega=2\gamma^2\omega_0$, i.e., at $\Omega=1$.

In determination of the radiation spectrum of a beam of particles it is necessary to sum the spectra radiated by particles moving along various trajectories. If all particles oscillate in the transverse direction with the same frequency ω_0 (which occurs for channeled positrons if the interplanar potential is parabolic), then according to Eq. (2.8) the maxima of the radiation of all particles occur at the same frequency. Accordingly, we should observe a sharp peak in the total-radiation spectrum and the entire spectrum should coincide in shape with the radiation spectrum of a single particle. However, if the oscillation frequencies of particles moving along different paths are different, then the maxima of the radiation of different particles will occur at different frequencies, which will lead to a smearing of the peak and to a decrease of its height.

Let us consider now the departure from dipole radiation. For channeled particles $(\overline{v_\perp^2})^{1/2}\sim\theta_c$, where $\theta_c=(2U_0/\varepsilon)^{1/2}$ is the critical angle of planar channeling[14,15] (U_0 is the maximum of the interplanar potential). Consequently, $\beta\sim\gamma\theta_c\propto\varepsilon^{1/2}$. Thus, with increase of the energy the departure from dipole radiation increases.

Note that it is necessary to distinguish the cases of moderate ($\beta\sim1$) and strong ($\beta\gg1$) departure from dipole radiation. For 4–14 GeV positrons moving in diamond the case of moderately nondipole radiation occurs, and therefore we shall consider below only this case.

When the departure from dipole radiation is taken into account, the general formulas for the radiation become considerably more complicated. However, in the case of moderate nondipole nature in the low-frequency region the main effect of the nondipole nature can be easily demonstrated analytically. The point is that the integrands in the expressions for $\mathbf{I}_\perp$ and $I_\parallel$ involve not the quantity β itself, but the quantity $\beta\omega/\omega_m$, and therefore for sufficiently low frequencies (up to the maximum of the radiation of the first harmonic) the dipole approximation remains applicable. The restriction (2.7), however, is important in summation of the contributions of individual particles to the radiation. For the first harmonic, which gives the main contribution in the region of the maximum for $\beta \lesssim 1$, this limitation has the form

$$\omega \leqslant 2\gamma^2\omega_0(\varepsilon_\perp)/[1+\gamma^2\overline{v_\perp^2}(\varepsilon_\perp)], \quad (2.9)$$

where $\varepsilon_\perp$ is the transverse energy of the particle.[14,15] Since the main contribution to the radiation is given by particles with $\varepsilon_\perp \approx U_0$, the frequency of the radiation at the maximum is

$$\omega_{max}=2\gamma^2\omega_0(U_0)/[1+\gamma^2\overline{v_\perp^2}(U_0)]. \quad (2.10)$$

From this formula it follows that with increase of the energy the dependence of ω_{max} on ε changes: from $\omega_{max} \propto \varepsilon^{3/2}$ to $\omega_{max} \propto \varepsilon^{1/2}$.[16,17] However, this is valid only in the case of moderate nondipole nature, in which the first harmonic provides the dominant contribution to the radiation of channeled particles (in the region of the maximum of the radiation spectrum). On further increase of the energy ε we go over to the case of strong nondipole nature ($\gamma\theta_c \gg 1$). Here the relative contribution of the first harmonic decreases, and the radiation spectrum (in the region of the maximum) will be determined by the sum of a large number of harmonics, as in the spectrum of synchrotron radiation, and the $\omega_{max} \propto \varepsilon^{1/2}$ dependence necessarily goes over to the $\omega_{max} \propto \varepsilon^2$ dependence.[18,19]

Thus, we see that the main effect of a moderate departure from dipole radiation consists of a cutoff of the radiation spectrum beginning at the frequency ω_{max}, and for $\omega < \omega_{max}$ the spectrum remains practically unchanged in comparison with the dipole approximation (the results of a numerical calculation carried out by computer are given in Sec. 4 and confirm this conclusion). Since the spectral density of the radiation near the peak is a rapidly rising function of ω, the cutoff mentioned leads to a significant decrease of the intensity of radiation at the maximum (in comparison with the dipole approximation). As a consequence of this, in the practical use of this radiation it is desirable to work far beyond the limits of applicability of the dipole approximation, since in this case there is a substantial decrease of the intensity of radiation at the maximum and of the monochromaticity. For this reason the electron accelerators which exist at the present time with energies of one or a few GeV are most appropriate for study of the radiation discussed, and also for study of the promise for its practical utilization.

3. EXPERIMENT

In this section we set forth the results of an experiment[3)] intended for observation and study of the properties of the electromagnetic radiation of relativistic positrons moving at a small angle to crystallographic planes. The experiment was carried out in the positron beam of the SLAC linear accelerator in the USA at energies of 4, 6, 10, and 14 GeV. Preliminary results of the experiment have been published elsewhere.[8] A diagram of the experimental arrangement is shown in Fig. 1.

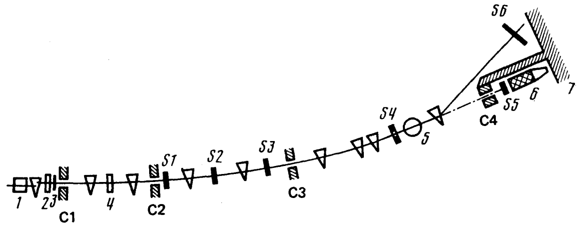

FIG. 1. Diagram of experiment. 1—pulsed magnet, 2—tungsten target, 3 and 4—secondary-emission monitors, 5—goniometer, 6—NaI(Tl) spectrometer, 7—lead shield, K1–K4—collimators, ▽—deflecting magnets, S1–S6—scintillation counters.

A beam of electrons from the linear accelerator with energy 22 GeV hit a tungsten target which served as the source of secondary positrons. The positron beam was shaped by a system of collimators, bending magnets, and quadrupole lenses and directed to a goniometer on which was mounted a diamond crystal 80 μm thick. Additional adjustment of the beam in the horizontal and vertical planes was accomplished by means of correcting magnets. Control and monitoring of the beam during the adjustment of the beam line was carried out by means of scintillation counters with remote control. Directly in front of the crystal was placed a scintillation veto counter with a center opening 2 mm in diameter, which served to exclude the effect of the beam halo and possible deviations of the positron beam from its central position. The number of positrons recorded by the veto counter varied from 1 to 5% during the experiment, depending on the quality of the electron-beam shaping at the exit from the accelerator. A temperature-sensitive probe and a secondary-emission monitor with an opening in the center mounted on the positron converter, were used to monitor the beam shaping. The maximum of the temperature-sensitive probe readings and the minimum of the current monitor as functions of the field of the correcting magnets at the accelerator output served as an indication of optimal beam shaping at the entrance to the beam line. The shaping of the beam in the line itself was accomplished in such a way that at the crystal location an approximately paralled beam of positrons was provided. To monitor the beam divergence, measurements were made of its size ahead and behind the crystal location. During the experiment the divergence of the positron beam did not exceed 10^{-5} rad. After passing through the crystal the positrons were deflected by a magnetic spectrometer and recorded by a scintillation counter, the dimensions of which were chosen to be sufficiently large to measure essentially the entire spectrum of scattered positrons.

The photon flux from the crystal was measured by a total-absorption counter employing a NaI(Tl) crystal 20 radiation lengths thick. Calibration of the NaI(Tl) detector in the low-energy region was accomplished by means of radioactive sources: ^{137}Cs ($E_\gamma = 0.661$ MeV), ^{60}Co ($E_\gamma = 1.17$ and 1.33 MeV), Pu-Be ($E_\gamma = 4.4$ MeV), and Cm-^{13}C ($E_\gamma = 6.13$ MeV). In the high-energy region the calibration was carried out by measurement of the spectra of positrons with energies 4, 6, 8, 10, and 14 GeV by the NaI(Tl) detector for various voltages on the photomultiplier. The correctness of the calibration was checked in an experiment in which the bremsstrahlung cross section from an amorphous target was measured and checked against the Bethe-Heitler cross section. In addition, calibration measurements were made of the spectra of bremsstrahlung from a carbon target (the equivalent of the diamond) and with disorientation of a crystalline target, and also of the coherent bremsstrahlung spectrum for a diamond crystal. The measured values of the cross sections for these processes agree with the theoretically predicted values.[20]

The experiment utilized a goniometer whose vertical rotation axis was perpendicular to the (011) plane of the crystal, and the (100) plane was set perpendicular to the positron beam. The orientation of the diamond crystal was based on the well known effects of coherent bremsstrahlung of electrons in crystalline targets.[20] The accuracy of orientation of the crystal was 1.15 $\times 10^{-5}$ rad. The path from the crystal to the NaI(Tl) detector was traversed by the photons in a vacuum chamber. In front of the NaI(Tl) counter was placed a photon collimator which provided photon angular collimation $2 \cdot 10^{-3}$ rad. The possible background of charged particles arising in the collimator walls was excluded by a scintillation counter placed in front of the NaI(Tl) detector.

An appropriate coincidence-anticoincidence technique was used to select useful events and to provide control of the multichannel analyzer which served to record the spectra of γ rays detected by the NaI(Tl) counter. Recording of the signal of the NaI(Tl) detector was accomplished beginning 2 μsec after the accelerator pulse. Appropriate electronics selected possible background events which appeared during the accelerator pulse. Cases of coincidence of the veto counter and the NaI(Tl) detector and also of the appearance of two pulses in the scintillation counter beyond the magnetic spectrometer or in the NaI(Tl) counter during the accelerator pulse blocked the signal from the γ-ray counter from reaching the analyzer input. The measurements were made with an average intensity of no more than one positron in the accelerator pulse. All of the information obtained was read by a ZSI-11 microcomputer and recorded on floppy disks. Operative control during the accumulation of information was accomplished by presenting the obtained data on a screen in tabular and graphical form. The information was then transferred from the floppy disks to magnetic tape and processed to obtain the final data.

Two series of measurements were made in the experiment. The first series consisted of a determination of the spectral distributions of the radiation of positrons with energies 4, 6, 10, and 14 GeV which entered the diamond crystal parallel to the (110) plane. In the second series we measured the radiation spectra of positrons with energy 10 GeV as a function of the angle θ_0 of entry of the particles relative to the (110)

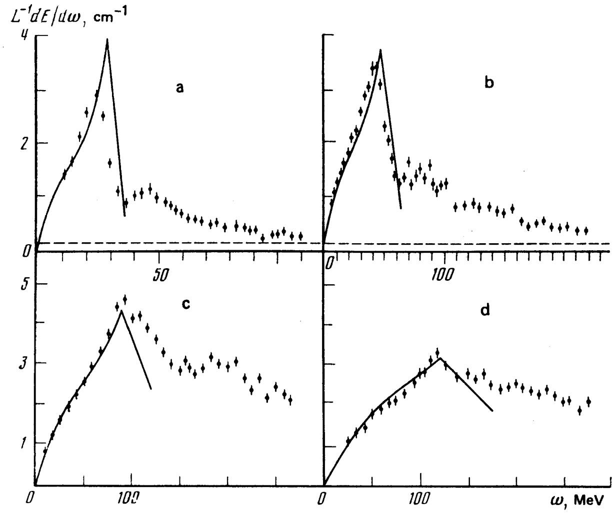

FIG. 2. Spectral distributions of the radiation of positrons entering a diamond crystal along the (110) crystallographic plane. a—$\varepsilon = 4$ GeV, b—$\varepsilon = 6$ GeV, c—$\varepsilon = 10$ GeV, d—$\varepsilon = 14$ GeV.

crystallographic plane. The results of the experiments are presented in Figs. 2 and 3, where the ordinate shows the intensity of radiation $d^2E/dld\omega$ per unit length per positron. The dashed lines in Fig. 2 correspond to the Bethe-Heitler spectrum[21]:

$$\frac{d^2E_{B-H}}{dld\omega} = \frac{16e^6Z(Z+1)n}{3m^2} \ln 183\, Z^{-1/3},$$

where $e^2/\hbar c = \frac{1}{137}$, $Z|e|$ is the charge of the nucleus and n is the density of atoms of the material.

4. ANALYSIS OF THE EXPERIMENT

In the analysis of the experiment we first calculated the spectra on the assumption that all particles are moving under conditions of channeling. In the calcula-

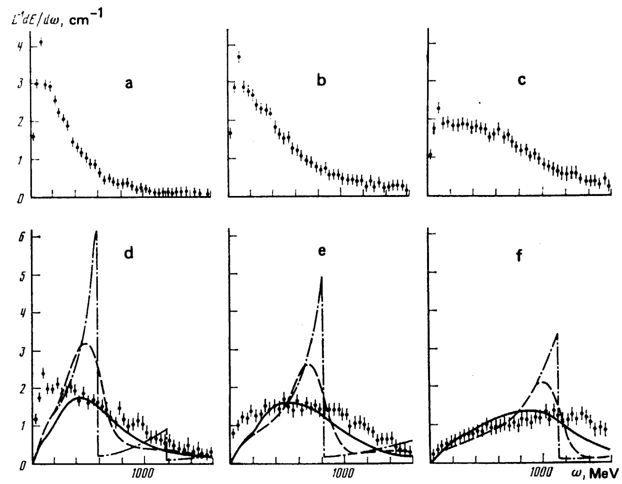

FIG. 3. Spectral distributions of the radiation of positrons with $\varepsilon = 10$ GeV for various values of the angle θ_0 between the incident beam and the crystallographic plane. a—$\theta_0 = 2.2 \cdot 10^{-5}$, b—$\theta_0 = 4.4 \cdot 10^{-5}$, c—$\theta_0 = 6.6 \cdot 10^{-5}$, d—$\theta_0 = 8.8 \cdot 10^{-5}$, e—$\theta_0 = 11.5 \cdot 10^{-5}$, f—$\theta_0 = 15.5 \cdot 10^{-5}$ rad. The significance of the individual curves is explained in the text.

tions we used two approximations of the interplanar potential: a parabolic potential

$$U(x)=4U_0x^2/d^2,\quad |x|\leqslant d/2$$

and a linear (sawtooth) potential

$$U(x)=2U_0|x|/d,\quad |x|\leqslant d/2,$$

where x is the coordinate orthogonal to the crystallographic planes and d is the distance between planes. It turned out, first, that for $\theta_0=0$ the experimental spectra in the region of the radiation maximum were a factor of 1.5–2 below the theoretical spectra and, second, the calculated spectra for the different approximations of the potential were practically identical for frequencies $\omega\lesssim\omega_{max}$. These circumstances indicate the necessity of taking into account the radiation not only of channeled particles, but also of superbarrier particles. Accordingly, we calculated the radiation spectra of above-barrier particles for the different approximations of the interplanar potential and for various distributions of the superbarrier particles in angle. The calculations were made with a rather simple model which permits not only a good description of the present experiment but also extraction of indirect information on the dynamics of the motion of ultrarelativistic positrons in a real crystal. The model contains two parameters: the fraction of above-barrier particles $(1-\eta)$ and the width of their angular distribution $\bar{\theta}(L)$, which can be determined independently from the condition of best agreement of the theoretical results with experiment.

1. Radiation spectra of channeled particles

To obtain the radiation spectra of channeled positrons the spectral distributions for the individual particles (2.6) must be averaged over the points of entry of the particles into the crystal[5]:

$$\frac{d^2\bar{E}_c}{dl\,d\omega}=\frac{1}{d}\int\limits_{-d/2}^{d/2}dx_0\frac{d^2E_c(x_0)}{dl\,d\omega}. \qquad (4.1)$$

Here, if the particle trajectory $x(t)$ along the x axis, which is orthogonal to the crystal planes, is known, determination of the radiation spectrum (4.1) reduces to calculation of a triple integral over x_0, φ, and t and can be carried out by computer.

It is convenient to separate explicitly all dimensional parameters, writing Eq. (4.1) in the form

$$\begin{gathered}d^2\bar{E}_c/dl\,d\omega=Af_c(\alpha,\zeta),\\ A=32e^2U_0^2/\pi^4m^2\theta_cd,\\ \alpha=\gamma\theta_c,\quad \zeta=\omega/\omega^*,\quad \omega^*=4\gamma^2\theta_c/d\end{gathered} \qquad (4.2)$$

(for a parabolic potential $\zeta=\Omega$). The representation (4.2) is convenient for bringing out the role of the different parameters. In addition, this representation permits one to determine the relation between the results of the dipole approximation $(\alpha\to 0)$ and higher approximations.

For light elements the interplanar potential can be approximated with high accuracy by a parabolic function.[15] The results of calculation of the function $f_c(\alpha,\zeta)$ in this very simple case are given in Fig. 4. Curves 1 and 2 correspond to positrons with $\varepsilon=6$ GeV

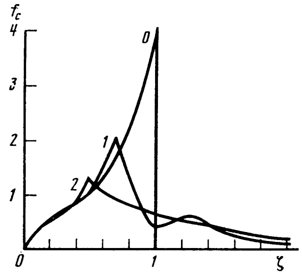

FIG. 4. Radiation spectra of channeled particles.

$(\alpha=0.96)$ and $\varepsilon=14$ GeV $(\alpha=1.47)$ entering the diamond crystal along the (110) crystallographic plane. The value of U_0 in the calculations was taken equal to 20 eV, and for this value of U_0 a good approximation of the interplanar continuous potential by the parabolic function is achieved at distances of the order of the Thomas-Fermi radius R from the planes which give the main contribution to the radiation [in particular, for the Molière continuous interplanar potential[15] $U_M(x)$ we have

$$4U_0\frac{(d/2-R)^2}{d^2}=U_M\left(\frac{d}{2}-R\right)$$

for $U_0\approx 20$ eV]. Curve 0 in Fig. 4 corresponds to the result of the dipole approximation.[4)]

We see that the dipole approximation is applicable for the description of the radiation up to a frequency ω_{max} corresponding to the maximum of the radiation of the first harmonic [for a parabolic potential, according to Eq. (2.12) of Ref. 5, $\omega_{max}=\omega^*/(1+\alpha^2/2)$]. This important result can be obtained also analytically directly from Eq. (2.6) if we expand the latter in the parameter $\alpha^2\xi^2\zeta/2$. In the first approximation we find that for $\theta_0=0$

$$\frac{d^2\bar{E}_c}{dl\,d\omega}=\frac{4e^2U_0^2}{m^2\theta_cd}\zeta[1-2\zeta(1-\zeta)]\int\limits_0^1 d\xi\xi^2\tau\left(1-\zeta\left(1+\frac{\alpha^2\xi^2}{2}\right)\right), \qquad (4.3)$$

where $\xi=2x_0/d$ and $\tau(y)$ is a step function [$\tau(y)=1$ if $y\geqslant 0$, $\tau(y)=0$ if $y<0$]. This simple formula describes with high accuracy (see Fig. 4) the radiation of channeled positrons in the frequency region $\omega\lesssim\omega_{max}$.

A comparison of the calculated channeled-positron spectra with experiment at $\theta_0=0$ shows that, first, in the low-frequency region $\omega\lesssim\omega_{max}$ the calculated curves are similar in shape to the experimental curves but exceed them in absolute value by 1.5–2 times and, second, in the high-frequency region $\omega>\omega_{max}$ the calculated spectra fall off with increase of ω more rapidly than the experimental spectra. These circumstances indicate that in the experiment discussed even with incidence of the particles on the crystal almost parallel to the crystallographic planes (with the angle $\theta_0\lesssim 10^{-5}$ rad) in addition to channeled particles there is a significant fraction of above-barrier particles which contribute substantially to the radiation in the frequency region $\omega>\omega_{max}$.[5)]

Note that the calculation of the intensity of radiation of channeled positrons in the case of a parabolic potential shows that the maximum intensity of the radiation should be reached for $\theta_c > \theta_0 \neq 0$. This increase of the radiation is due to the increase of the population of states with higher transverse energy for $\theta_c > \theta_0 \neq 0$. In the experiment discussed, however, the dependence of the radiation spectra on the angle θ_0 was traced and the increase mentioned above in the radiation was not observed (see Fig. 3). This circumstance indicates that in an experiment the imperfections in the crystal lattice have an important influence on the radiation.

2. Radiation spectra of above-barrier particles

Above-barrier particles can result from de-channeling upon incidence of the beam along crystallographic planes and also on entry of the particles into the crystal at an angle θ_0 to the planes. For $\theta_0 > \theta_c$ all positrons entering the crystal are above-barrier.

Turning to calculation of the radiation of above-barrier positrons, we note that at all energies considered it is possible for analysis of the radiation spectra of above-barrier particles to use the formulas of the dipole approximation, a circumstance which is due to a certain straightening of the trajectories of above-barrier particles in comparison with the trajectories of channeled particles (the angle at which a channeled particle is deflected with respect to the velocity vector of its longitudinal motion is equal in order of magnitude to $\vartheta \sim \theta_c$; on the other hand, for a superbarrier particle $\vartheta \lesssim \theta_c^3/\theta_0^2$).

We shall first consider the radiation of an above-barrier positron moving in a crystal at a fixed angle θ_0 to the crystallographic planes. For a parabolic distribution of the interplanar potential the radiation spectrum of an above-barrier positron in the dipole approximation has the form

$$\frac{d^2E_u}{dl\,d\omega} = \frac{16e^2U_0^2}{m^2\theta_c d}\zeta\varkappa^2 \sum_{n=1}\left[1 - \frac{2\zeta\varkappa}{n\pi}\left(1 - \frac{\zeta\varkappa}{n\pi}\right)\right]\frac{\tau(n\pi - \zeta\varkappa)}{[(n\pi)^2 - \varkappa^2]^2}, \quad (4.4)$$

where $\varkappa = \arcsin \theta_c/\theta_0$ [this formula follows from Eqs. (2.7) and (2.16) of Ref. 5 if as $U(x)$ we use $U(x) = 4U_0^2x^2/d^2$].

Equation (4.4) shows that the maximum of the spectral distribution of the radiation of an above-barrier positron occurs at $\omega = \pi\omega^*/\varkappa \geqslant 2\omega^*$ and that the magnitude of the radiation intensity at the maximum is equal in order of magnitude to the intensity of radiation at the maximum of a channeled particle.

The above-barrier positrons pass close to the nuclei of the lattice atoms. Here as the result of multiple scattering by the atoms there is a redistribution of these particles in angle. An important change of the distribution function $g(\theta, l)$ of positrons in angle at depth l at high energies occurs in lengths greater than that traveled by a particle in a period of one oscillation, and therefore the radiation spectrum of a beam of positrons which has passed through the crystal will be determined by the formula

$$\frac{d\bar{E}_u}{d\omega} = \int_0^L dl \int d\theta\, g(\theta, l)\,\frac{d^2E_u}{dl\,d\omega}, \quad (4.5)$$

where L is the thickness of the crystal.

We shall show the radiation spectrum of above-barrier positrons (4.5) depends substantially on the value of the mean angle of multiple scattering of the particles by the crystal. For this purpose let us consider the simplest case, in which the distribution of particles in angle at depth l is rectangular[6]:

$$g(\theta, l) = \frac{1}{2\bar{\theta}(l)}\tau(\bar{\theta}(l) - |\theta_0 - \theta|),$$

and assume that the mean square scattering angle of particles in the crystal changes with l according to the same law as the mean square scattering angle of particles in an amorphous medium, i.e., $\bar{\theta}(l) \propto l^{1/2}$. The radiation spectrum of superbarrier positrons in this case for various values of the angles θ_0 and $\bar{\theta}(L)$ is given in Fig. 3.[7] The curves shown refer to positrons with $\varepsilon = 10$ GeV moving in a diamond crystal near the (110) crystallographic plane. The dot-dash curves refer to the case in which $\bar{\theta}(L) = 0$, i.e., when there is no multiple scattering of the particle; the dashed curves correspond to $\bar{\theta}(L) = 0.5\theta_c$; the solid curves are for $\bar{\theta}(L) = 1.5\theta_c$.

These results show that the intensity of radiation at the maximum of the radiation spectrum depends substantially on $\bar{\theta}(L)$. This is due to the fact that for various values of the angle θ the maxima of the spectral distributions of the radiation of above-barrier positrons (4.4) occur at different frequencies.

3. Comparison of theory with experiment and discussion of results

The spectral distribution of the radiation of positrons moving in a crystal near the crystallographic planes, with inclusion of both channeled and above-barrier particles, can be represented in the form

$$\frac{d\bar{E}}{d\omega} = \eta\frac{d\bar{E}_c}{d\omega} + (1 - \eta)\frac{d\bar{E}_u}{d\omega}, \quad (4.6)$$

where η is the fraction of channeled particles ($\eta \leqslant 1$); $d\bar{E}_c/d\omega$ and $d\bar{E}_u/d\omega$ are the spectral distributions of radiation corresponding to channeled and above-barrier particles (the bar indicates averaging over the distribution of positrons in angle in the crystal).

Before proceeding to compare the results of theory and experiment, we note that the dynamics of the motion of relativistic particles in real crystals, with allowance for various imperfections of the crystal lattice (mosaic structure, presence of extended defects, surface imperfections), multiple scattering of particles by the atoms of the crystal, and the phenomenon of quasichanneling, has not been studied either theoretically or experimentally up to the present time. Nevertheless, as has been shown above, in a number of cases the factors enumerated have a substantial influence on the radiation of particles moving in a crystal. For this reason special interest attaches to the indirect information on the dynamics of particles in a crystal which can be obtained from comparison of theory and experiment, and also by the subsequent verification of this information by other methods.

For $\theta_0 < \theta_c$ the main contribution to the radiation in the low-frequency region ($\omega < \omega_{max}$) is from channeled particles, and therefore the value of η can be found from comparison of theory and experiment in this frequency region. In the range of positron energies considered, the radiation of channeled positrons for $\theta_0 = 0$ and small values of ω is described with high accuracy by Eq. (4.3). Comparing this spectral distribution of radiation with the experimental results, we find that for ε equal to 4, 6, 10, and 14 GeV the values of η are respectively 0.7, 0.7, 0.8, and 0.6. The spectral distributions of the radiation of channeled positrons corresponding to these values of η are shown in Fig. 2 by the solid curves.

We see that, even for $\theta_0 = 0$, in addition to channeled particles there is a rather large group of de-channeled above-barrier particles. The appearance of this group of particles in the experiment discussed can be due to an imperfection of the crystal surface, to mosaic structure of the lattice, or to the presence of extended defects. De-channeled positrons give an important contribution to the radiation in the high-frequency region $\omega > \omega_{max}$. In the experiment considered, the distributions in angle of particles which have left the crystal and the structure of crystal-lattice defects were not studied, and therefore we shall not discuss here the question of the radiation of de-channeled particles.

Let us turn now to comparison of theory with experiment for positrons with $\varepsilon = 10$ GeV entering the diamond crystal at angles $\theta_0 > \theta_c$. In this case almost all particles moving in the crystal are above-barrier, and therefore in the first approximation (neglecting the phenomenon of quasichanneling) we can assume that $\eta = 0$. The spectral distributions of the radiation of above-barrier positrons with various values of $\bar{\theta}(L)$ are given in Fig. 3.

We see that good agreement between theory and experiment exists for $\bar{\theta}(L) = 1.50\theta_c$. We note that the value of the mean angle of scattering of particles by the crystal corresponding to this $\bar{\theta}(L)$ is approximately twice the average value of the angular divergence of the particles in an amorphous medium, which indicates a possible increase of multiple scattering for above-barrier positrons. This conclusion is very interesting. Possible causes of an increase of the scattering are as follows: the effect of mosaic structure of the lattice on the scattering; "hovering" of positrons above the barrier, where the density of atoms is high[23]; increase of scattering as the result of multiple scattering of particles by strings of atoms.[24]

A certain discrepancy between theory and experiment in the low-frequency region may be due to the radiation of the small fraction of channeled particles which appear for $\theta_0 > \theta_c$ either as a consequence of quasichanneling or as a result of imperfections in the crystal lattice. In the approximation considered ($\eta = 0$) the radiation of this group of particles was not taken into account.

5. CONCLUSION

In conclusion we summarize the material presented.

1. On motion of positrons with energies ε of the order of several GeV under conditions in which the angle θ_0 of entry of the particles with respect to the crystallographic planes varies in the range from zero to several channeling angles θ_c there is a significant excess (by factors of ten) of the intensity of radiation of soft γ rays (see Figs. 2 and 3) in comparison with the radiation in an amorphous medium, which corresponds to the theoretical predictions of Refs. 1–7.

2. This radiation is due to channeled and above-barrier particles, and in a diamond crystal 80 μm thick the radiation of above-barrier particles is comparable with that of channeled particles even for $\theta_0 = 0$, which is due to the phenomena of de-channeling and multiple scattering in a real crystal.

In the positron energy region considered it is necessary to take into account the departure from dipole nature of the radiation for the channeled particles. Here there is a moderate depature from dipole radiation ($\gamma\theta_c \sim 1$), and the channeled particles provide the main contribution to the radiation in the frequency region

$$\omega \sim \omega_{max} = 2\gamma^{3/2}(U_0/m)^{1/2}/(1+\gamma^2\theta_c^2/2).$$

Above-barrier particles give the main contribution to the radiation for $\omega \gtrsim 2\omega_{max}$.

With increase of the entrance angle θ_0 the radiation becomes harder and less monochromatic; for $\theta_0 > \theta_c$ it is due almost completely to above-barrier particles, and here the intensity of radiation of the positrons remains of the same order as for $\theta_0 = 0$.

From comparison of theory and experiment it turned out to be possible to extract information on the dynamics of motion of particles in a real crystal. In particular, it has been shown that, even if the particles enter a diamond crystal parallel to the crystallographic planes, about half of the particles move under conditions of above-barrier motion.

We have studied the dependence of the radiation spectra of the particles on their angular distribution and have shown that this dependence is rather strong. As a result of this the study of the radiation spectra of particles entering at a small angle to the crystallographic planes can serve as an effective means of investigation of the dynamics of ultrarelativistic particles in crystals, and also of the fine details of the structure of crystals—mosaic structure, defects, and so forth.

1)Erevan Physics Institute.

2)We use the system of units with $c = \hbar = 1$.

3)The experimental data used in the present work were obtained in a joint Soviet-American experiment by the following group of authors: I. I. Miroshnichenko, J. J. Murray, R. O. Avakyan, and T. Figut.

4)Similar calculations of the radiation spectra of channeled particles were made by us for the case in which the interplanar potential is approximated by a linear function $U(x) = 2U_0|x|/d$, where $|x| \leqslant d/2$. The results of these calculations show that in the low-frequency region $\omega < \omega_{max}$ the radiation spectra of channeled positrons are practically identical when the interplanar potential is approximated by either a parabolic or a linear function.

5) A similar conclusion was reached by Bazylev *et al.*,[17] who analyzed certain data of the present experiment.

6) Similar calculations of the radiation spectra of above-barrier particles carried out by us show that a change in the shape of the distribution has an insignificant effect on the form of the calculated curves.

7) Note that at $\theta_0 - \bar{\theta}(L) < \theta_c$ quasichanneling of particles in the crystal sets in,[22] with the particles that have dropped to the top of the potential barrier as the result of multiple scattering redistributed into channeled and superbarrier particles. In the present work this effect is not taken into account, which is justified if the fraction of particles taking part in quasichanneling is small, i.e., if $\theta_0 - \bar{\theta}(L) \gtrsim \theta_c$.

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